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BASICS OF RADIOCHEMISTRY. SEMINARS

ПЕТРОВ
ВЛАДИМИР ГЕННАДЬЕВИЧ

ХИМФАК МГУ

БЛАГОДАРИМ ЗА ПОДГОТОВКУ КОНСПЕКТА
СТУДЕНТА ФИЗИЧЕСКОГО ФАКУЛЬТЕТА МГУ
БУЛГАКОВА АЛЕКСАНДРА ДМИТРИЕВИЧА

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1. Seminar 1. Types of Radioactive Decay

Decay energy

The energy of nuclear decay can be calculated from the defect of mass. Let's take the well-known mass-energy equivalence:

$$E = mc^2 \tag{1.1}$$

where the energy E is measured in Joules (J), the mass m is measured in kilograms (kg), and the speed of light c is a constant equal to 299,792,458 meter per second (m/s).

In the equation (1.2) we can see a general example of nuclear decay where the parent radionuclide ${}^{M}_{Z}X$ decays into a daughter radionuclide ${}^{M'}_{Z'}Y$ and a particle b of any type. Z and Z' are the atomic numbers (also called proton numbers) of the parent and daughter nuclide respectively. M and M' are the mass numbers of the parent and daughter nuclide respectively.

$${}^{M}_{Z}X \rightarrow^{M'}_{Z'} Y + b \tag{1.2}$$

If we take equation (3.1), the decay energy Q can be calculated as:

$$Q = \Delta mc^2 = (m_x - \sum(m_y + m_b))c^2 \tag{1.3}$$

where Q is in MeV (or in some cases in keV), Δm – difference in masses between the mass of initial (parent) radionuclide ${}^{M}_{Z}X$ and the sum of masses of produced particles ${}^{M'}_{Z'}Y$ and b. So from equation 1 we can calculate the energy value from mass, as c^2 is a constant.

As we are talking about nuclear decay, and decay is related to a single nucleus (or a single atom) it is simpler and more convenient to use masses of single atoms and energies that are released during decay of single atoms. If we would be using conventional SI units such as kilograms or Joules, the values would be very small. That is why it is more convenient to express the masses of single atoms in atomic mass units (a.m.u.). Using equation (1.3) we can calculate that the decay energy of a particle with a mass equal to 1 a.m.u. is 931501 keV = 931.5 MeV. Other useful unit that we will be using is the electron mass me = 511 keV. The masses of protons mp and neutrons m_n are very close in value (although m_n is a little bit higher than m_p) equaling approximately 1 a.m.u., which is around 1840 times more than m_e.

Types of decay

1) α-decay: ${}^{M}_{Z}X \rightarrow^{M-4}_{Z-2} Y +^{4}_{2} \alpha({}^{4}_{2}He)$.

   The α-particle is actually an atom of ${}^{4}_{2}He$ with atomic number equal 2 and mass number equal 4. The atomic number of daughter radionuclide is decreased by two and the mass number by 4, because the summary of atomic and mass numbers of decay products should be equal to the atomic and mass numbers of the parent nuclide.

2) β-decay. There are 3 types of β-decay:

- β^--decay: ${}^{M}_{Z}X \rightarrow {}^{M}_{Z+1}Y + {}^{0}_{-1}\beta(e^-) + {}^{0}_{0}\tilde{\nu}_e$

  The β^- particle is in fact an electron with Z = -1 and M = 0. However, it is better to use β^- while writing equations as we have to distinguish between β^- particles (electrons released form the nucleus) and electrons on the orbital shells of an atom. The daughter radionuclide is going to have the same mass number as the parental nuclide and an atomic number equal to Z + 1. Along with these 2 particles there is also a third particle created during decay called antineutrino $\tilde{\nu}_e$.

- β^+-decay: ${}^{M}_{Z}X \rightarrow {}^{M}_{Z-1}Y + {}^{0}_{+1}\beta(e^+) + {}^{0}_{0}\nu_e$

  The β^+ particle is a positron e^+, an antiparticle to electron. It has the same mass, the same spin as electron, but the charge of positron is +1 instead of -1 (which means Z = 1). In this case the atomic number of daughter nuclide is going to be equal to Z-1. The last particle created during decay is neutrino ν_e.

  Note to better remember which type of neutrino belongs to which type of decay: β^--decay: β^- particle is an electron, so a normal particle. To that we have an anti-particle – antineutrino β^+-decay: β^+ particle is a positron, an "antielectron". To that we have a normal particle – neutrino.

- Electron capture (E.C.): ${}^{M}_{Z}X + {}^{0}_{-1}e \rightarrow {}^{M}_{Z-1}Y + {}^{0}_{0}\nu_e$.

  During this process an electron from an electron shell of the atom is captured by the nucleus, followed by decay. The daughter nuclide is going to have the same mass number as the parent nuclide while it's atomic number is equal to Z-1. Together with the daughter nuclide a neutrino is formed.

  What is a neutrino? Neutrino is a tiny lepton particle (leptons include 3 types of particles - electrons, neutrinos and muons, and antiparticles – positrons, antineutrinos and antimuons). Atomic number of neutrinos is 0, meaning the neutrinos are neutral particles with the mass number being also close to 0. However, the exact mass of neutrino remains still unknown, we only know that it is at least 200000 smaller than the mass of electron, which makes it difficult to detect it.

3) γ-decay: ${}^{M}_{Z}X^* \rightarrow {}^{M}_{Z}X + \gamma$.

   γ-decay is in fact not a decay, because in comparison to α- and β-decay no particles are ejected from the nucleus. It is more of a transition from an excited state of the nucleus ${}^{M}_{Z}X^*$ to a ground or a lower-energy state ${}^{M}_{Z}X$. The gamma quantum γ is in fact an electromagnetic wave just like visible light, but with higher energy.

   In both α- and β-decay there is a change in atomic number Z, which means that the number of protons in the nucleus is going to change during decay and thus the chemical elements is also going to change. As in α-decay the atomic number of the resulting daughter nuclide is less than the Z of daughter nuclide by 2, we will move in the periodic table by two cells to the left (as shown in fig.1.1 left). If the process is β^--decay we go to the right by 1 cell. In case of β^+-decay and electron capture we move to the left by 1 cell. The schematic representation of the different types of decay is shown in fig.1.1 right. α-decay is symbolized by a double (or a thick) arrow to the left, β^--decay is represented by an arrow to the right, β^+-decay is represented by an arrow to the left, electron capture is symbolized by a dotted line to the left and γ-decay is represented by a line directed downwards.

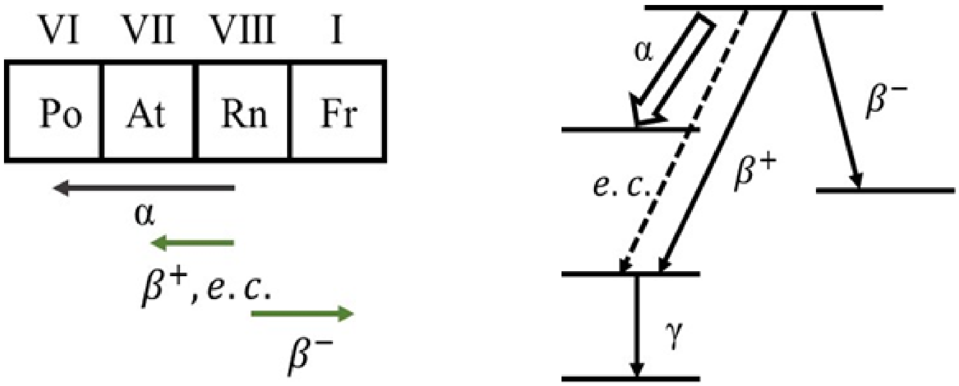


Figure 1.1. Representation of how the elements are going to chance after different types of decay (left) and schematic representation of different types of decay (right)

Task 1. Decay chain

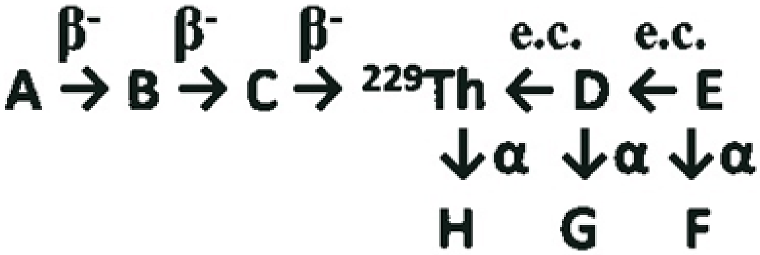


Figure 1.2. Picture for the task 1

Solution

The atomic number Z(^{229}Th) = 90, which means that we can write Thorium as ${}^{229}_{90}$Th. As we remember, during: $\alpha \rightarrow$ Z-2, $\beta^- \rightarrow$ Z+1, $\beta^+ \rightarrow$ Z-1, E.C. $\rightarrow$ Z-1 $\alpha \rightarrow$ M - 4, during any type of β-decay M does not change. So for C that underwent β^--decay, Z is going to be lower by 1: M = 229, Z = 89, which means that C is ${}^{229}_{89}$Ac.

We use the same logic for A and B:
B: M = 229, Z = 88, B is ${}^{229}_{88}$Ra
A: M = 229, Z = 87, A is ${}^{229}_{87}$Fr

D underwent electron capture and because of this, the Z of parent radionuclide D is going to be higher by 1: M = 229, Z = 91. This means that D is ${}^{229}_{91}$Pa The same logic can be used for E: M = 229, Z = 92, E is ${}^{229}_{92}$U F, G and H are obtained as the results of α-decay. In this case Z should decrease by 2 and M by four as was mentioned earlier.

F: The parent nuclide is $^{229}_{92}$U, which means that Z is going to change to 90 and M to 225. This means that F is $^{225}_{90}$Th.

G: The parent nuclide is $^{229}_{91}$Pa, so Z = 89 and M = 225 and G is $^{225}_{89}$Ac

H: The parent nuclide is $^{229}_{90}$Th, so Z = 88, M = 225 and H is $^{225}_{88}$Ra This easy task shows us how the chemical composition of the materials changes during different types of decay and that if we know the resulting radionuclide (in this case $^{229}_{90}$Th) and we know the type of decay we can easily understand what was the parental radionuclide.

Task 2. Calculation of the number of decay

How many α-decay and β^--decay accompany the transformation $^{232}_{90}\text{Th} \rightarrow {}^{208}_{82}\text{Pb}$?

Solution

Let's take a look at the wrong solution. As it is easier to count the difference between the atomic numbers ΔZ we can say that ΔZ = - 8 and thus there were only 4 α-decays. This is however incorrect.

Let's look at the difference in the mass numbers ΔM. If we went by our earlier assumption, ΔM would be equal to 16, but the actual ΔM = 232 – 208 = 24. Again, to just remind ourselves, a change in mass number can be related only to α-decay. This means that instead the 4 α-decay in our assumption there are actually 6 (24/4 = 6) α-decay is happening.

Now let's look at the number of β^--decay. We know that during $\alpha \rightarrow$ Z - 2 and during $\beta^- \rightarrow$ Z + 1. This means that if there are 6 decays, the Z would decrease by 12 (-2 · 6 = -12). However, the actual decrease of charge ΔZ = - 8. That means that this difference between the observed difference in atomic numbers ΔZ = -8 and the calculated difference in atomic numbers due to alpha decay (Z - 12) can be corrected by assuming that 4 β^--decays occur.

We can add the 4 β^--decays to the calculated Z value Z - 12 + 4 = Z - 8. As we can see we have obtained the same difference in atomic numbers as we have in the task. So to sum up we can say that even if we don't know the whole chain of radioactive decay we can assume that there should be at least 6 α-decays and 4 β^--decays. Of course as we do not have the full chain, we cannot say that there are only 6 α-decays and 4 β^--decay, there could be for example an additional β^+-decay and β^--decay as there would be no change in both Z and M.

Task 3. Calculation of maximum energy of beta-particles

Calculate the maximum energy of the β^--particles emitted during the decay of ^{15}C. The mass of the ^{15}C atom is greater than the ^{15}N mass by 0.0105207 a.m.u.

Solution

For any kind of β-decay we can estimate that the maximum energy of the β-particles $E_{\beta,max}$:

$$E_{\beta,max} = Q = \Delta mc^2 = 931.5 \cdot \Delta m \tag{1.4}$$

where the unit of $E_{\beta,max}$ is MeV and the unit of Δm is a.m.u. This equation is correct only if we don't have an additional gamma quantum. If we have additional gamma quanta that are the result if β-decay, part of the energy is going to the gamma quanta. If we know the energy of the released, we should subtract it from the decay energy Q. However, that is not the case for this task.

So, if the difference in mass Δm = 0.0105207 a.m.u., using equation (1.4) we can calculate the maximum energy of the β-particles $E_{\beta,max}$:

$$E_{\beta,max} = 931.5 \cdot 0.0105207 \text{ a.m.u.} = 9.8 \text{ MeV}$$

The value we obtained is quite high. In general of the radioactive decay is very high compared to the strength of chemical bonds $E_{chem.bonds} \sim$ 5-100 eV, in heavy elements like U and Th the energy of electron bonds on the K- shell can equal around 17-19 keV) from tens of keV to MeV. That means that the energy of nuclear interactions is 1000 stronger than the energy of chemical bonds.

Task 4. Calculation of the kinetic energy of alpha-particles

The ^{222}Rn nucleus, emitting an α-particle decays into the ground (unexcited) state of ^{218}Po. Determine the kinetic energy of the α-particle. The masses of the ^{222}Rn, ^{218}Po and ^{4}He atoms are 222.01757379, 218.00896897 and 4.002603261 a.m.u. respectively.

$$Rn \rightarrow Po + \alpha$$

Solution

Difference between α- and β-particles: In case of β-particles we can neglect the recoil energy of the nucleus as they are quite small in size meaning that they obtain almost all the energy produced during nuclear decay. This, however, does not apply to α-particles.

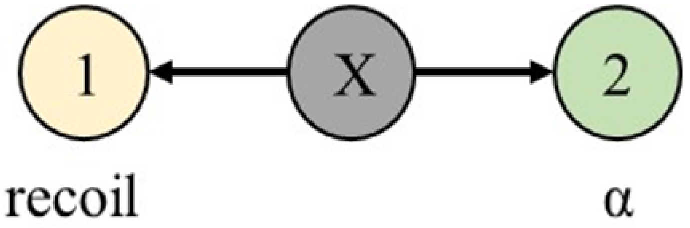


Figure 1.3. Schematic representation of the conservation of momentum during α-decay

Let's imagine that our initial nucleus of radionuclide $^{M}_{Z}$X (shown in fig.1.3) undergoes α-decay. As we know only 2 particles are produced. According to the conservation of momentum principle, in a closed system the momentum remains constant. That means that if the nucleus is not moving, the momentum $\vec{p} = 0$. This means that the sum of produced particles should also be equal to 0:

$$m_1\vec{v}_1 = -m_2\vec{v}_2 \tag{1.5}$$

where m_1 and $\vec{v}_1$ are the mass and velocity of particle 1 respectively, m_2 and $\vec{v}_2$ are the mass and velocity of particle 2 respectively. As $\vec{p}$ is a vector quantity both $\vec{v}_1$ and

$\vec{v}_2$ are also vector quantities with particle 2 going in the opposite direction of particle 1 (minus sign, see fig.1.3). The scalar equation to equation (1.5) is:

$$m_1 v_1 = m_2 v_2 \tag{1.6}$$

Now let's apply this equation to α-decay. Particle 1 is the recoil nucleus of our decay product $^{M-4}_{Z-2}\mathrm{Y}$ and particle 2 is the α-particle itself. Knowing this equation (1.6) is going to transform into:

$$m_r v_r = m_\alpha v_\alpha \tag{1.7}$$

As we know the decay energy Q of the α-decay is distributed between the recoil nucleus and the α-particle:

$$Q = E_\alpha + E_r = \frac{m_\alpha v_\alpha^2}{2} + \frac{m_r v_r^2}{2} \tag{1.8}$$

Where E_α and E_r are the kinetic energies of the recoil nucleus and the α-particle respectively.

Now after expressing v_r from equation (1.7) we get:

$$v_r = \frac{m_\alpha}{m_r} \cdot v_\alpha \tag{1.9}$$

We can take the expression obtained for v_r and put it into equation (1.8):

$$Q = \frac{m_\alpha v_\alpha^2}{2} \cdot (1 + \frac{m_\alpha}{m_r}) = E_\alpha \cdot (\frac{m_\alpha + m_r}{m_r}) \tag{1.10}$$

From this equation we can express the kinetic energy of α-particle E_α:

$$E_\alpha = Q \cdot \frac{m_r}{m_r + m_\alpha} \tag{1.11}$$

From equation (1.11) we can see that the kinetic energy of α-particle E_α will always be smaller than the decay energy Q, because the ratio $\frac{m_r}{m_r + m_\alpha} < 1$. So where does the rest of the energy go? As we mentioned earlier (equation (1.8)), the energy does not go only to the α-particle but also to the recoil nucleus. The energy of the recoil nucleus is:

$$E_r - Q \cdot \frac{m_\alpha}{m_r + m_\alpha} \tag{1.12}$$

Let's return to our task. As we have all of the masses written in the task, we can calculate the difference in mass Δm:

$$\Delta m = m_{Rn} - (m_{Po} - m_\alpha) = 222.01757379 - 218.00896897 - 4.002603261 = 0.006 \text{ a.m.u.}$$

To get the value of decay energy Q we will use equation (1.3):

$$Q = 931.5 \cdot 0.006 = 5.59 \text{ MeV}$$

Then using equation (1.11) we will calculate kinetic energy of α-particle:

$$E_\alpha = 5.49 \text{ MeV}$$

In this case we will disregard the numbers after decimal point as this will affect our result only minimally. Now to the energy of the recoil nucleus (in this case Po). As we have calculated the decay energy Q and the kinetic energy of α-particle E_α this task is rather simple. We have to calculate the difference in the two energies that is going to be equal to the energy of the recoil nucleus E_r:

$$\Delta E = Q - E_\alpha = 0.1 \text{ MeV} = E_{Po}$$

It is important to remember that energy of recoil nucleus E_r is very high in the case of α-decay. In this task it is 0.1 MeV = 100 keV. As we previously mentioned the energy of chemical bonds equals only a few eV. Thus, the energy of recoil nucleus is approximately 1000 times higher than the energy of chemical bonds. This means that if we had a molecule of an α-emitting nuclide, the recoil nucleus would break chemical bond in the molecule.

2. Seminar 2. Solving the tasks

Task 1. Determination of maximum kinetic energy of N-13 decay

Determine the maximum kinetic energy of particles emitted during the decay: ^{13}N → ^{13}C. The difference between the masses of the parent and daughter atoms is $2.38389 \cdot 10^{-3}$ a.m.u., decay is not accompanied by the emission of γ-quanta.

Solution

There is no emission of γ-quanta so we can assume that all of the decay energy will be transferred to the maximum energy of the β-particles $E_{\beta,max}$. But the 1st question we must ask is what kind of decay is in this task. As there is no change in mass number, we can assume that it is β-decay, but we have to determine the type of β-decay. Let's look at the atomic numbers of both the parent and daughter nuclides: Z of ^{13}N is 7 $\rightarrow {}^{13}_{7}$N, Z of ^{13}C is 6 $\rightarrow {}^{13}_{6}$C.

Now as we look at the decay, there should be a positron with Z = +1, as the atomic numbers on the right and left side are supposed to be the same. This means that in this case it is the β^+-decay. The last thing missing is a neutrino. ${}^{13}_{7}\text{N} \rightarrow {}^{13}_{6}\text{C} + {}^{0}_{1}e^{+} + {}^{0}_{0}\nu_e$ There is one difference when we calculate the emitted energy of positrons e^+, because the positrons annihilate with an electron: $e^+ + e^- \rightarrow 2h\nu$. So if we calculate the decay energy Q we should subtract the energy of 2 electrons (the mass of positron is the same as electron), because we are going to lose then in the decay process.

$$Q = (\Delta m - 2m_e) \cdot 931.5 \text{ MeV} = 931.5 \cdot \Delta m - 1.022 \text{ MeV} = 1.198 \text{ MeV}$$

As we know from Seminar 1, the $m_e = 511$ KeV, so $2m_e = 1022$ KeV.

Task 2. Br-80 decay scheme. Calculation of energies

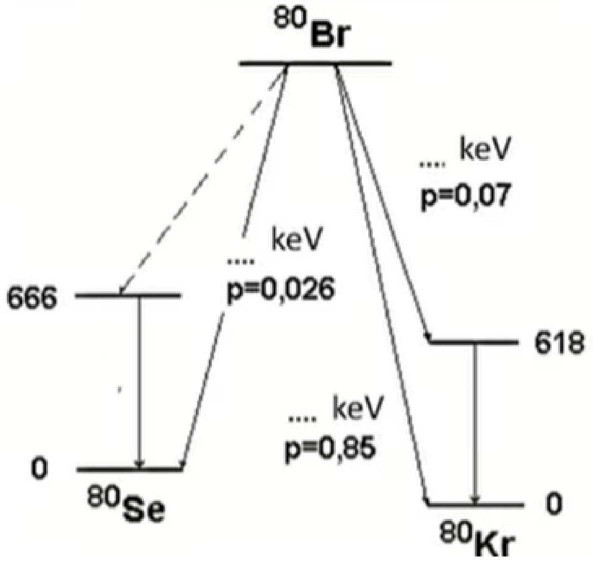


Figure 2.1. ^{80}Br decay scheme

Determine the maximum energies of β^+- and β^--particles emitted by ^{80}Br nuclei and the energy of monoenergetic neutrinos appearing during the decay of nuclei by electron capture. The masses of ^{80}Br, ^{80}Se and ^{80}Kr atoms are 79.91852827, 79.91652043 and 79.91637475 a.m.u. respectively.

Solution

We know the masses of the respective isotopes from which we can calculate the difference in masses for each decay type. Let's begin with the right part of the scheme. We know that as a result of 80Br decay 80Kr is produced. That means that it is β^--decay that occurs (the arrows are pointed to the right). To calculate the decay energy Q_{β^-} we will use equation (1.4). We should calculate the difference in masses Δm between ^{80}Br and ^{80}Kr:

$$Q_{\beta^-} = (79.91852827 - 79.91637475) \cdot 931.5 \text{ MeV} = 2.006 \text{ MeV} = 2006 \text{ KeV}$$

The value we obtained is the maximum energy of β—particles $E_{\beta^-,max1}$ in the case that ^{80}Kr decays right into the ground state of ^{80}Kr (energy going from ^{80}Br to the bottom right state in the scheme marked with 0).

Next, we can see that above ground state of ^{80}Kr there is one more excited state of ^{80}Kr with energy of 618 keV. That means that the maximum energy of β—particles for this decay $E_{\beta^-,max2}$ = 2006 keV - 618 keV=1388 keV. We can also calculate the total intensity of the β—particles. The p values on the scheme mark the probabilities of β—particles with energies of 2006 keV and 1388 keV being emitted during the decay of ^{80}Br. Or also p is the same as intensity of β—particles with the corresponding energies. The total sum of all intensities should be $\sum p_i = 1$ (100%). Among these decays 7% (p = 0.07) of atoms will decay with the emission of β—particles with the energy of 1388 keV and 85% will decay with the emission of β—particles with the energy of 2006 keV. Let's take a look on the left side of the scheme. We can see a solid line going from ^{80}Br to the ground state of ^{80}Se. This is the β^+-decay. The decay energy equals (as in equation (1.4)):

$$\begin{aligned} Q_{\beta^+} &= (79.91852827 - 79.91652043 - 2m_e) \cdot 931.5 = 1870 \text{ KeV} - 10222 \text{ KeV} = \\ &= 848 \text{ keV} = E_{\beta^+,max} \end{aligned}$$

As in task 1 when calculating the decay energy of β^+-decay we have to subtract $2m_e$.

The dashed line going from ^{80}Br to the excited state of ^{80}Se (666 KeV) corresponds to electron capture. Let's first calculate the probability (intensity) $p_{E.C.}$=1-0.07-0.85-0.026=0.054. This means that only 5.4% of all the ^{80}Br atoms will decay by electron capture decay. But what kind of particle is emitted during electron capture? Let's remember the equation for electron capture: ${}^1_1p + {}^{0}_{-1}e \rightarrow {}^1_0\text{n} + {}^0_0\nu_e$. As we can see, only neutrinos get emitted. As they have an exact value of energy, we can call them monoenergetic neutrinos and we can calculate their energy: E_{ν_e} = 1870 keV-666 keV=1204 keV.

As we can see we simply need to calculate the difference in mass Δm $\cdot$ 931.5 and subtract the energy of γ-quanta (= 666 keV). The finished scheme will look like this:

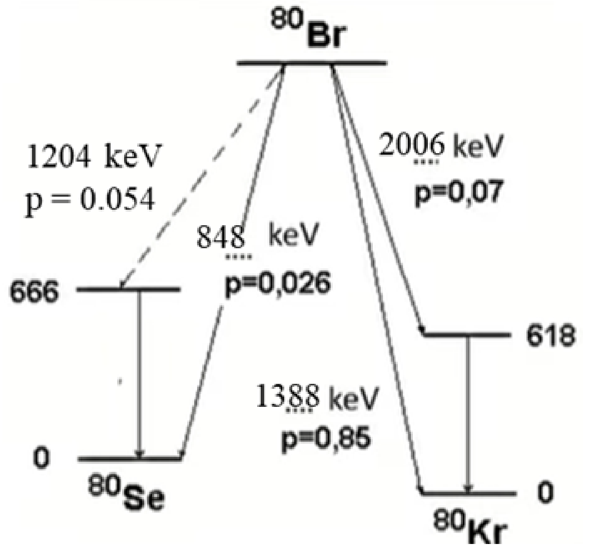


Figure 2.2. ^{80}Br decay scheme

Task 3. Building a decay scheme of Rn-219

The decay $^{219}\text{Rn} \to {}^{215}\text{Po}$ is accompanied by the emission of α-particles (energy $E_\alpha =$ 6.82, 6.56 and 6.43 MeV), as well as γ-quanta ($E_\gamma = 0.39, 0.32, 0.19, 0.13, 0.07$ MeV). Build a decay scheme for ^{219}Rn. The α-decay energy is 6.95 MeV.

Solution

Looking at the difference in masses we can clearly see that it corresponds to α-decay. There are several E_α, which means that ^{219}Rn will not decay directly into the ground state of ^{215}Po, but rather to it's excited states (and that there will be 2 or 3 of them). There are 5 different E_γ values which means that there will be 5 different transitions from an excited state to another excited state or ground state.

α-decay is represented by a double line (or a thick line) to the left (as we move to the left in the periodic table). There should be 3 α-particles emitted which means there will be 3 double lines going to 3 energy levels of ^{215}Po. We can see that the decay energy Q equals 6.95 MeV. This however is not equal to the energy of α-particles that is calculated with equation (1.11):

$$E_\alpha = Q \cdot \frac{m_{Po}}{m_{Po} + m_\alpha} = 6.95 \cdot 215/219 = 6.82 \text{ MeV}$$

This value is the same as one of the energies E_α given above. This means that the α-decay with the energy of 6.82 MeV will go directly to the ground state of ^{215}Po. From the difference of the given E_α we can calculate the energies of both excited states of ^{215}Po. Excited state 1: 6.82-6.56= 0.26 MeV, excited state 2: 6.82-6.43=0.39 MeV. As we can see, there is an energy $E_\gamma = 0.39$ MeV that corresponds to transition from energy level of 0.39 MeV to ground state (0 MeV). There is also $E_\gamma = 0.13$ MeV that corresponds to transition from energy level of 0.39 MeV to energy level of 0.26 MeV. Now only 3 E_γ values remain ($E_\gamma = 0.32, 0.19, 0.07$ MeV). This means there should be one more energy level. We can see that the sum of 0.32 and 0.07 MeV is 0.39 MeV, and the sum of 0.19 and 0.07 MeV is 0.26 MeV. This means that there is an energy level of 0.07 MeV in between ground state and energy level of 0.26 MeV. The final decay scheme looks as following:

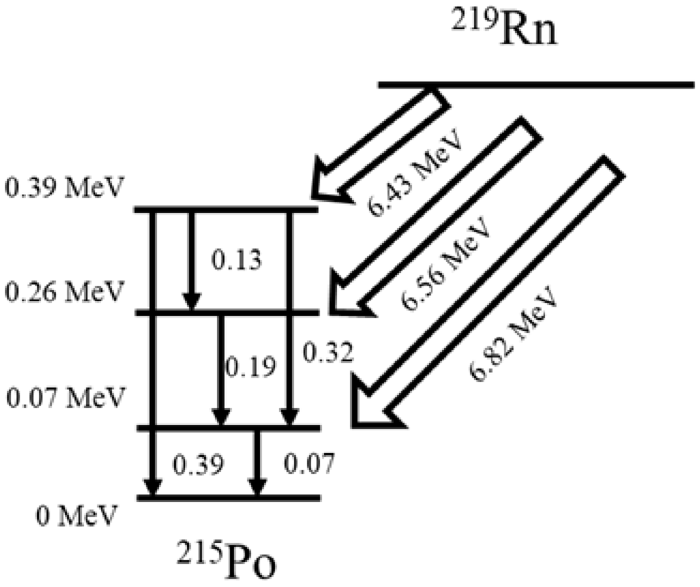


Figure 2.3. ^{219}Rn decay scheme

Task 4. Determination of absolute activity

Determine the absolute activity of 1 kg of potassium chloride (the proportion of ^{40}K in the natural mixture of isotopes is 0.000118). $T_{1/2}(^{40}K) = 1.3 \cdot 109$ years.

Solution

The expression of the main law of radioactivity is:

$$A = \lambda N \tag{2.1}$$

where A is activity, $\lambda = ln2/T_{1/2}$ and is a constant, $T_{1/2}$ is the half-life of the radionuclide and N is the number of atoms. We will begin by calculation of all the atoms of ^{40}K N. First we need to calculate the amount of substance (chemical amount):

$$n(KCl) = \frac{m(KCl)}{M(KCl)}$$

(where m is the mass, M is the molar mass). Then we will calculate the chemical amount of ^{40}K n(^{40}K). For this we need the chemical amount of potassium, but as there is only one potassium in the molecule $n(KCl) = n(K)$.

$$n(^{40}K) = n(KCl) * \omega(^{40}K)$$

where $\omega(^{40}\text{K})$ is the mass fraction of 40K in natural mixture. Then finally the number of atoms of ^{40}K:

$$N = n(^{40}K) \cdot N_A$$

N_A is the Avogadro constant, N_A = 6.022·1023 mol^{-1}.

Now let's put it all together and calculate activity:

$$A = \lambda N = \frac{\ln(2)}{T_{1/2}} \cdot \frac{m(KCl) \cdot \omega(^{40}K) \cdot N_A}{M(KCl)}$$

$m(KCl)$ should be converted into grams (g), $T_{1/2}$ should be converted into seconds (s): multiplied by 365 days, 24 hours, 60 minutes and 60 seconds.

$$A = 16118 \text{ Bq}$$

From this example we can see that even such a commonly sold substance as potassium chloride that most of the scientists (chemists) have in the lab is naturally radioactive and 1 kg generates 16118 decays per second.

Task 5. Determination of specific activity

Determine the specific activity (Bq/g) of Sm_2O_3 (the natural mixture of isotopes contains 15.07% ^{147}Sm, $T_{1/2} = 1.3 \cdot 10^{11}$ years).

Solution

Specific activity is the activity per quantity of a radionuclide (i.e. mass, volume or surface of the sample). In this task we are looking for the absolute activity divided by mass (Bq/g). To calculate the absolute activity, we will use equation (2.1). The specific activity can be calculated as:

$$a = \frac{A}{m} \tag{2.2}$$

where m is the mass of the substance (in this case Sm_2O_3). The absolute activity is calculated similarly as in the previous task. Chemical amount of Sm2O3 is calculated by the same principle: n(Sm_2O_3)= m(Sm_2O_3)/M(Sm_2O_3). However, there is a difference when calculating the chemical amount of samarium n(Sm). As there are not 1 but 2 atoms of samarium in Sm_2O_3, the chemical amount is going to be: n(Sm)=2·n(Sm_2O_3). The chemical amount of the natural isotope 147Sm is calculated as:

$$n(^{147}Sm) = n(Sm) \cdot \omega(^{147}Sm)$$

Now let's put everything together into equation (2.2):

$$a = \frac{A}{m(Sm_2O_3)} = \frac{\lambda N}{m(Sm_2O_3)} = \frac{\ln(2)}{T_{1/2}} \cdot \frac{m(Sm_2O_3) \cdot 2 \cdot \omega(^{147}Sm) \cdot N_A}{M(Sm_2O_3) \cdot m(Sm_2O_3)} = \frac{2 \cdot \omega(^{147}Sm) \cdot N_A}{M(Sm_2O_3)}$$

We remember that we must express half-life $T_{1/2}$ in seconds, the mass fraction ω should not be in %, (if it is, we divide it by 100). Now, let's put all the values into our expression:

$$a = 88.2 \, \frac{\text{Bq}}{g}$$

In task like these it is always important to remember about the stoichiometry of the compounds. If we did not remember that there are 2 atoms of Sm in Sm_2O_3, our result would be 44.1 Bq/g instead of 88.2 Bq/g.

Task 6. Calculation of impurity activity

The sample contains 50 MBq of 35S and an impurity of ^{32}P (4 MBq). What is the minimum time the sample should be stored so that the activity of the impurity does not exceed 1% of the total activity of the drug?

Solution

To solve this task, we will have to use the integral form of the decay law:

$$A = A_0 \cdot e^{-\lambda t} \tag{2.3}$$

where A is the activity, A_0 is the initial activity, λ is the decay constant and t is time. We see from this equation that we have to know the half-lives of ^{35}S and ^{32}P ($T_{1/2}(^{35}\mathrm{S})$ = 87 days, $T_{1/2}(^{32}\mathrm{P})$ = 14.4 days).

Let's solve the task. We can write the activities for ^{35}S and ^{32}P as:

$$\begin{cases} A_S = A_{0,S} \cdot e^{-\lambda_S t} \\ A_P = A_{0,P} \cdot e^{-\lambda_P t} \end{cases} \tag{2.4}$$

The activity of the impurity ^{32}P should be less than 1%. That means that the minimum time will be when the activity of ^{32}P A_P will be 1% of the total activity: $A_P = 0.01(A_S + A_P) \Rightarrow 99A_P = A_S$.

Let's add in the expressions for A_S and A_P:

$$99A_{0,P} \cdot e^{-\lambda_P t} = A_{0,S} \cdot e^{-\lambda_S t} \Rightarrow 99\frac{A_{0,P}}{A_{0,S}} = e^{(\lambda_P - \lambda_S)t}$$

Now we can take the natural logarithm of both sides of the equation:

$$\ln(99\frac{A_{0,P}}{A_{0,S}}) = (\lambda_P - \lambda_S)t$$

Now let's express the time:

$$t = \frac{\ln(99\frac{A_{0,P}}{A_{0,S}})}{\lambda_P - \lambda_S} = \frac{\ln(99\frac{A_{0,P}}{A_{0,S}})}{\frac{\ln(2)}{T_{1/2}(P)} - \frac{\ln(2)}{T_{1/2}(S)}} = 51.5 \text{ days}$$

So the minimum time the sample should be stored so that the activity of the impurity does not exceed 1% of the total activity of the drug is 51.5 days. In this task we do not have to convert half-lives from days into seconds because we are not calculating the absolute or specific activity, we are calculating time. The only important moment is that our half-lives should be in the same time units (for example all should be in days, or all should be in years, it cannot be that one half-life in in days and the second in years).

This task is quite simple, but it shows that even if you initially have a quite high content of impurity (4MBq of ^{32}P, which is around 8% of the impurity in ^{35}S), the impurity decay faster than the main component 35S and if you store your sample long enough then your component will not decay much, but your impurity will decay faster to 1% or even less.

Task 7. Determination of radioactive gas pressure

0.1g of ^{226}Ra ($T_{1/2}$ = 1620 years) is placed in a sealed vessel with a volume of 22.4 cm^3. Determine the pressure of ^{222}Rn ($T_{1/2}$ = 3.82 days) at 0 °C after 2 months.

Solution

Sometimes when we store radionuclides like ^{226}Ra their decay product is a gas, like ^{222}Rn in this case. This means that we could get a high pressure inside a vessel if it sealed. It is important to calculate this pressure in order to estimate the safety of the storage of the sealed vessel. In this task we will calculate the pressure of the ^{222}Rn gas accumulated during the ^{226}Ra storage.

To solve this task, we have to remember the ideal gas law:

$$PV = nRT \tag{2.5}$$

where P is the pressure, V is the volume, n is the chemical amount, R is the ideal gas constant (equal to 8.314 J·K^{-1}· mol^{-1}) and T is the absolute temperature (in K).

We know the volume and temperature, so to calculate the pressure of ^{222}Rn gas we have to calculate its amount. In our task, the equilibrium between ^{226}Ra and ^{222}Rn is secular, because the half-life of ^{226}Ra is much higher than the half-life of ^{222}Rn. In the case of secular equilibrium, the relationship between the activities of the daughter and parent radionuclides are as follows:

$$A_d = A_{0,P} \cdot (1 - e^{-\lambda_d t}) \tag{2.6}$$

where A_d is the activity of the daughter radionuclide, $A_{0,P}$ is the initial activity of the parent radionuclide, λ_d is the decay constant of the daughter radionuclide and t is the time of storage. If $t > 10T_{1/2}$ (daughter) then the activities of the daughter and parent radionuclides are equal $A_d = A_P$. The time of storage in our task is 10 times higher than the half-life $T_{1/2}$ ^{222}Rn which means that the activity of ^{226}Ra is equal to the activity of ^{222}Rn. The activity of ^{226}Ra can be calculated using equation (2.1):

$$A_{Ra} = \lambda_{Ra} N_{Ra} = \lambda_{Ra} \cdot \frac{m(Ra)}{A_r(Ra) \cdot N_A}$$

where A_r(Ra) is the atomic mass of ^{226}Ra.

We can write a similar equation for the activity of ^{222}Rn:

$$A_{Ra} = \lambda_{Rn} N_{Rn} = \lambda_{Rn} \cdot \frac{m(Rn)}{A_r(Rn)} \cdot N_A = \lambda_{Rn} \cdot n(Rn) \cdot N_A$$

Now using the fact that $A_{Ra} = A_{Rn}$ we can write:

$$\lambda_{Ra} \cdot \frac{m(Ra)}{A_r(Ra)} \cdot N_A = \lambda_{Rn} \cdot n(Rn) \cdot N_A$$

We can exclude the Avogadro constant N_A and express the chemical amount of radium n(Rn):

$$\lambda_{Ra} \cdot \frac{m(Ra)}{A_r(Ra)} = \lambda_{Rn} \cdot n(Rn), \quad n(Rn) = \frac{m(Ra)}{A_r(Ra)} \cdot \frac{\lambda_{Ra}}{\lambda_{Rn}}$$

As we know $\lambda = \ln 2/T_{1/2}$. As λ is at the top and the bottom of fraction, we can exclude $\ln 2$, leaving only the half-life $T_{1/2}$, inversely proportional to decay constant λ ($\lambda = 1/T_{1/2}$). Let's add this into our equation:

$$n(Rn) = \frac{m(Ra)}{A_r(Ra)} \cdot \frac{T_{1/2}(Rn)}{T_{1/2}(Ra)} = 2.86 \cdot 10^{-9} \text{ mol}$$

It is important that the half-lives are in the same units (in this case days). Now that we have the chemical amount of ^{222}Rn n(Rn), let's calculate the pressure using equation (2.5):

$$P = \frac{nRT}{V} = 0.29 \text{ Pa}$$

It is important that every variable is expressed in correct units: n in mol, T in K, and V in m^3 and P in Pa.

The calculated pressure value is quite low (to compare the atmospheric pressure = 101325 Pa). However, during ^{226}Ra storage $^4_2$He is also accumulated. (^{226}Ra decay: $^{226}\text{Ra} \rightarrow {}^{222}\text{Rn} + \alpha$ ($^4_2$He)). The helium particles are stable, they don't decay so during ^{226}Ra storage the amount of $^4_2$He keeps increasing and can at a certain point reach a significantly big value.

Task 8. Secular equilibrium

There is a decay chain: $^{210}\text{Pb} \rightarrow {}^{210}\text{Bi} \rightarrow {}^{210}\text{Po} \rightarrow {}^{206}\text{Pb}_{stab}$. At the initial moment, the activity of the purified ^{210}Pb preparation ($T_{1/2}$ = 22.2 years) was 1 MBq. Evaluate the activity of the sample after: a) 5 days, b) 276 days, c) 22.2 years.

$T_{1/2}(^{210}\text{Bi})$ = 5 days, $T_{1/2}(^{210}\text{Po})$ = 138 days.

Solution

In the first two decays, the mass number does not change, and we are "moving to the left" in the periodic table, which means that they are β^--decays. In the last decay the mass number changes by four, indicating that it is α-decay. Let's create a table (fig.2.4) to solve this task in a comprehensible manner.

In the first line of the table the initial activity of all radionuclides – in this case only of ^{210}Pb equal to 1 MBq. (sum of activities = 1 MBq). Now let's look at the half-lives of all the radionuclides. We can see that the half-life of parental nuclide ^{210}Pb is much higher than the half-life of it's daughter radionuclides which means that there is secular equilibrium. Thus, we can use equation (2.6) in this case. We have to remember that the decay constant λ_d is always related to the daughter radionuclide (not parental). We can also rewrite equation (2.6) as:

$$A_d = A_{0,P} \cdot (1 - 2^{-\frac{t}{T_{1/2}(d)}}) \tag{2.7}$$

a) t = 5 days. For ^{210}Pb 5 days is a very short time for it to decay. We can assume, that it's activity will change insignificantly and will be equal to 1MBq (we can neglect the small amount that decays). To calculate the activity of ^{210}Bi we can use equation (2.7). The time t = 5 days = $T_{1/2}$(Bi), so the fraction $-t/(T_{1/2}(d) = -1$. The activity of ^{210}Bi is equal to:

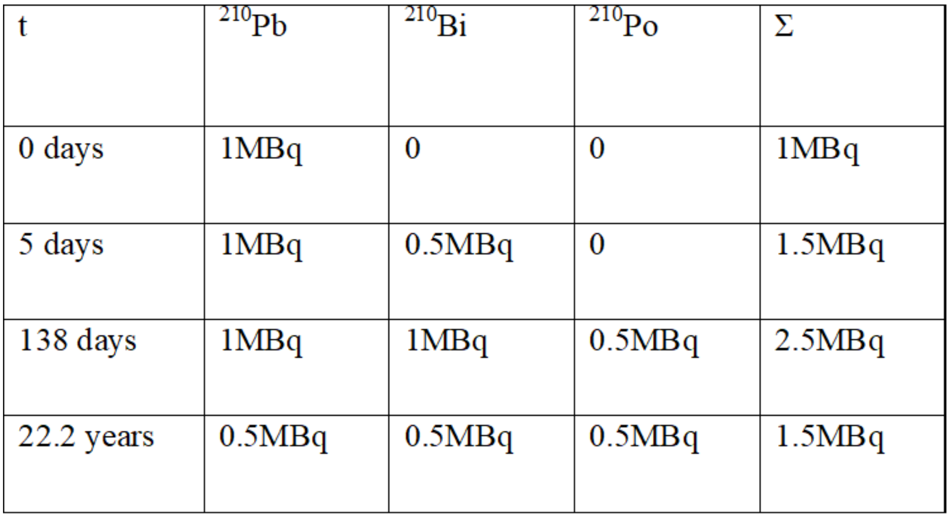
$$A_Bi = A_{0,Pb} \cdot (1 - 1/2) = A_{0,Pb} \cdot 0.5 = 0.5 \text{ MBq}$$

For ^{210}Po 5 days is much shorter than it's half-life so there won't be any polonium accumulation after five days (or will be close to 0). Thus, the total activity of the sample equals 1.5MBq. The total activity is increasing with time due to the accumulation of the daughter radionuclide.

b) t = 138 days (note: should be 276 days, it's a mistake by the lector). Again, the time is too short for ^{210}Pb to decay, so it's activity will be 1MBq. $10 \cdot T_{1/2}$(Bi) = 50 days $< t$ which means that the secular equilibrium for ^{210}Bi was already reached. The activity of the daughter radionuclide equals the activity of the parent radionuclide ($A_{Bi} = A_{Pb}$ = 1MBq). For ^{210}Po t = 138 days = $T_{1/2}$(Po) so using the same principle as in part a), it's activity will be 0.5MBq. Thus, the total activity will be 2.5 MBq. If the t = 276 days, for ^{210}Pb and ^{210}Bi nothing would change. For ^{210}Po t = 276 days = $2 \cdot T_{1/2}$(Po) and thus it's activity would be 0.75MBq.

c) t = 22.2 years. The half-life of ^{210}Pb $T_{1/2}$(Pb) = 22.2 years = t, the activity will be equal to 0.5MBq. For ^{210}Bi and ^{210}Po $t > 10 \cdot T_{1/2}$, the secular equilibrium has was reached and their activities will be 0.5MBq. The total activity of the sample will be 1.5MBq.

This is an example of how the total activity of the sample is increasing due to the accumulation of the activity from the daughter radionuclides. When the secular equilibrium is reached the activity parental and the daughter radionuclide will decrease with time according to the law or radioactive decay.



t	^{210}Pb	^{210}Bi	^{210}Po	Σ
0 days	1MBq	0	0	1MBq
5 days	1MBq	0.5MBq	0	1.5MBq
138 days	1MBq	1MBq	0.5MBq	2.5MBq
22.2 years	0.5MBq	0.5MBq	0.5MBq	1.5MBq

Figure 2.4. The activities of the radionuclides from the decay chain

3. Seminar 3. Attenuation of fluxes

Task 1. Calculation of the fraction of particles retained by the shield

Parallel fluxes of monoenergetlc electrons, β^--particles and α-particles pass through aluminum shield with the thickness of 0.25 of the maximum path of β^--particles. The energy of electrons, α-particles, and the maximum energy of the β^--spectrum are equal to 0.7 MeV. How many (fraction) of electrons, β^-- and α-particles are retained by the shield?

Solution

First of all, we have to calculate how many particles will go the shield and to do that we need the maximum range of penetration for given energy of the particles: $R_{max} = 250$ mg/cm^2. The linear depth of penetration l_{max} is therefore given by:

$$l_{max} = \frac{R_{max}}{\rho} = 0.0926 \text{ cm} \approx 1 \text{ mm} \tag{3.1}$$

where $\rho = 2.7$ g/cm^3 is the density of the aluminum.

The thickness of the shield l_s will be:

$$l_s = 0.25 \cdot l_{max} = 0.023 \text{ cm} \tag{3.2}$$

For monoenergetic electrons the maximum range will be ≈ 1 mm. The dependence of α-particles flux F_α on the thickness of the shield d is shown in fig.3.1

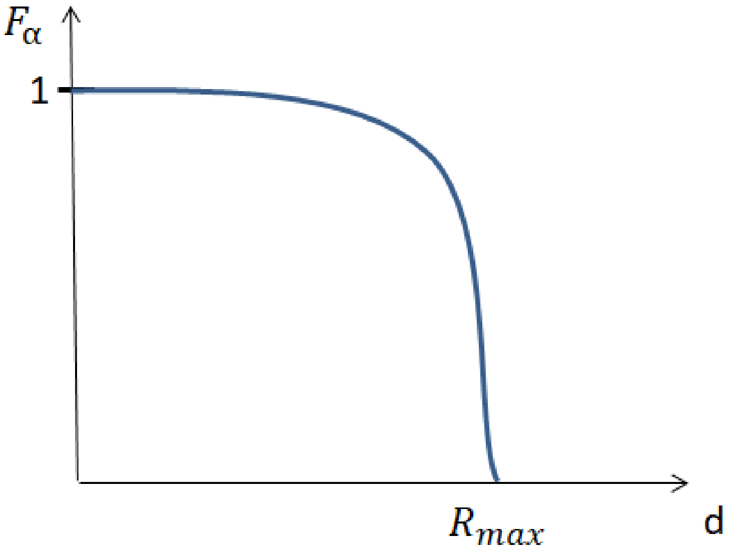


Figure 3.1. F_α vs d

It means that if a shield has a thickness bigger than the R_{max} no α-particle will penetrate it, if the thickness is smaller than R_{max} all of them will go through the shield. The maximum range of α-particles penetration is 10-100 μm. the thickness of our screen is 0.23 mm = 230 μm $\Rightarrow$ 0 % of α-particles will go through the screen.

The flux of monoenergetic electrons decreases linearly with the thickness of the shield d (fig.3.2) therefore 25% of the electrons will be absorbed by the screen and 75% will go through.

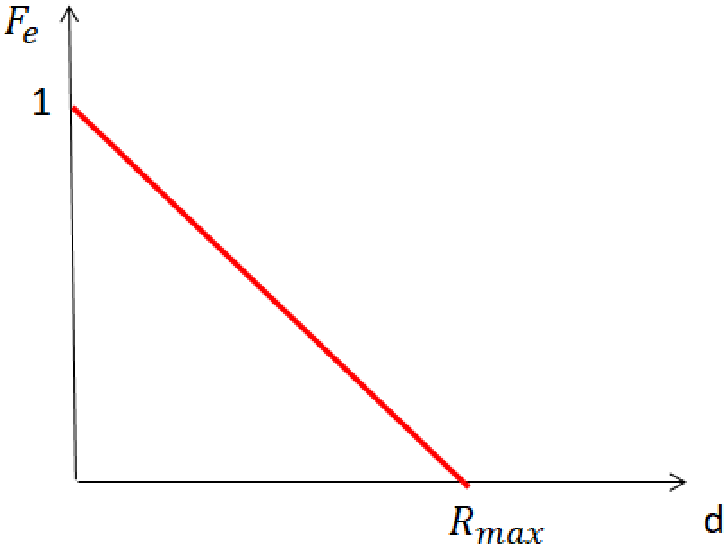


Figure 3.2. F_e vs d

Such dependence for β^--particles is less trivial (3.3). If the thickness of the screen is less than 0.3·R_{max} the attenuation coefficient k is given by:

$$k = e^{-\mu d} = e^{-23 \cdot 0.25 \cdot 0.25} = 0.24 \tag{3.3}$$

where $\mu = 23$ cm^2/g is the mass-attenuation coefficient.

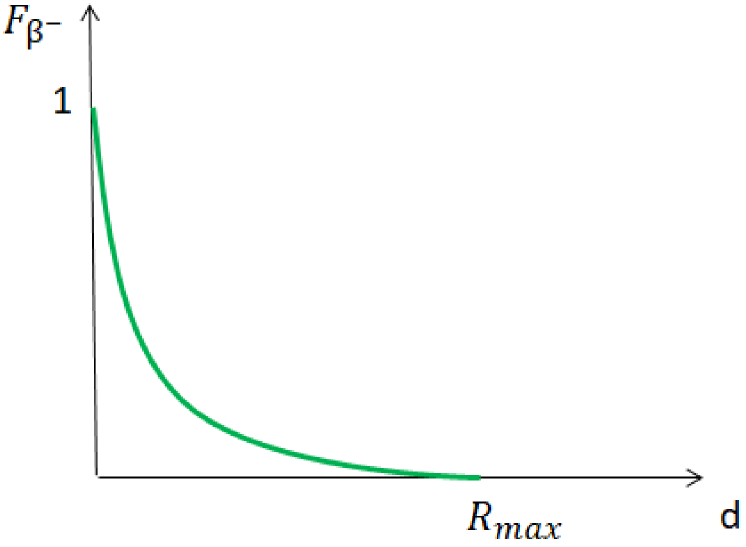


Figure 3.3. F_{β^-} vs d

As a result of our calculations, only 24% of the β^- particles will go through the screen and 76% will be stopped.

Task 2. Calculation of the fraction of particles retained by the screens

Parallel fluxes of monoenergetic electrons, α-particles and β-particles pass through screens, the thickness of which is equal to 0.6 of the path of each type of radiation. What fraction of electrons, α-particles and β-particles are retained by screens?

Solution

From the previous task we can say that in this case 100% of α-particles will go through and 0% will be retained, as the maximum range is bigger than the thickness of the screen.

As for the monoenergetic electrons, 40% of them will go through, 60% will be retained. We can use the following formula for the attenuation coefficient for monoenergetic electrons:

$$k_e = 1 - \frac{d}{R_{max}} \tag{3.4}$$

We can only use the expression (3.3) for β-particles if $d < 0.3 \cdot R_{max}$ which is not our case. Instead, we are going to use experimentally determined dependence of the attenuation coefficient k on the ratio $\frac{d}{R_{max}}$ shown in fig.3.4.

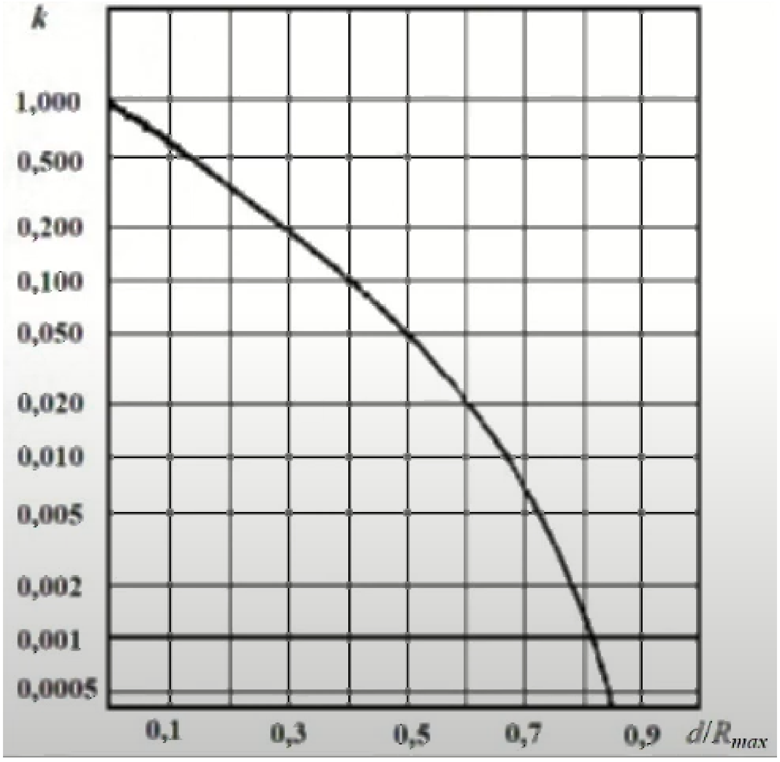


Figure 3.4. k vs d/R_{max} experimental data

We discover, that the attenuation coefficient is $k = 0.02 \Rightarrow 98\%$ of β-particles will be retained. Therefore only a small fraction of the β-spectrum (fig.3.5) can pass through the shield.

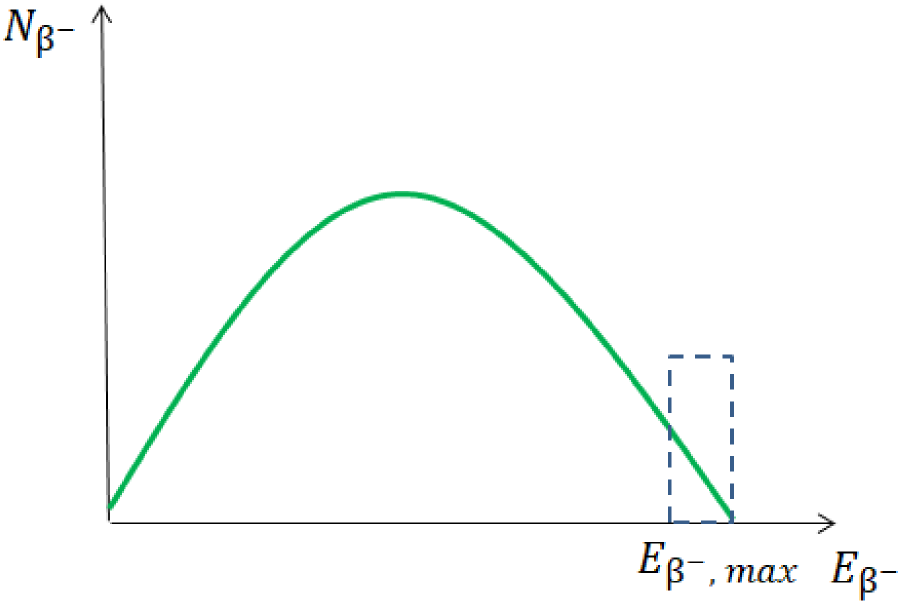


Figure 3.5. β-spectrum

Task 3. Calculation of the fraction of particles retained by the screens

Determine the maximum range (in cm) of β-particles emitted by ^{32}P in air at normal conditions.

Solution

First of all, we should know the energy of the β-particles, emitted by phosphorus. From databases: $E_{\beta,max} = 1710$ KeV. Again, consulting with databases we find out the maximum range: $R_{max} = 779$ mg/cm^2. To transfer it into cm we use the expression (3.1). The density of air can be obtained as following:

$$\rho = \frac{M_{air}}{V_m} = 1.29 \text{ mg/cm}^3 \tag{3.5}$$

where $M_{air} = 29$ g/mol is the average molar mass of air, V_m=22.4 dm^3/mol is the molar volume of ideal gas.

As a result, the answer: $l_{max} = \frac{R_{max}}{\rho} = 604$ cm.

Task 4. Attenuation of beta-particles flux

Calculate the minimum thickness of a plexglass screen ($\rho = 1.07$ g/cm^3) that completely stops the ^{89}Sr β-particles.

Solution

Let's look up the maximum energy of the β-particles and the maximum range of penetration: $E_{\beta,max} = 1492$ KeV, $R_{max} = 666$ mg/cm^2. The maximum range in cm: $l_{max} = \frac{R_{max}}{\rho} = 0.62$ cm $= 62$ mm. It means, that the thickness of 6.2 mm will be enough to stop the β-particles. This range in air would have been $l_{max}(air) = 516$ cm. So using a thin shield made of plexglass completely protects a human from β-particles emitted by ^{89}Sr - it is very efficient.

Task 5. Attenuation of beta-particles and gamma quanta fluxes

Determine the attenuation of the fluxes of β-particles and γ-quanta emitted by ^{40}K by aluminum plates with a thickness of 270 and 810 mg/cm^2 ($\rho_{Al} = 2.7$ g/cm^3)

Solution

We look up the maximum energies of β-particles and γ-quanta: $E_{\beta,max} = 1312$ KeV, $E_\gamma = 1460$ KeV, then we get the mass attenuation coefficient and maximum range for β-particles: $\mu_\beta = 8.83$ cm^2/g, $R_{max} = 572$ mg/cm^2 and the liner attenuation coefficient for γ-quanta: μ_γ =0.13 cm^{-1}.

Then we find the maximum range of penetration for γ-quanta:

$$\begin{cases} l_1 = \frac{270}{2700} = 0.1 \text{ cm} \\ l_2 = \frac{810}{2700} = 0.3 \text{ cm} \end{cases} \tag{3.6}$$

For γ-quanta we can always use the expression (3.3) for attenuation coefficient k:

$$\begin{cases} k_1 = e^{-0.13\cdot 0.1} = 0.987 \;\Rightarrow 98.7\% \text{ of } \gamma\text{-quanta will go through} \\ k_2 = 0.962 \;\Rightarrow 96.2\% \text{ of } \gamma\text{-quanta will go through} \end{cases} \tag{3.7}$$

For β-particles we will have to use the experimental data in fig.3.4 and find out: $k_1 \approx 0.05 \Rightarrow 95\%$ of the β-particles will be stopped with the 1st screen. In the second case $k_2 = 0$ as the thickness of the shield is greater than the maximum range of the β-particles.

Task 6. Attenuation of beta-particles flux (+secular equilibrium)

A sample containing an equilibrium mixture of ^{90}Sr and ^{90}Y was coated with a layer of aluminum sufficient for complete absorption of ^{90}Sr radiation. What fraction of ^{90}Y β-particles will be retained in this layer?

Solution

In this task ^{90}Sr and ^{90}Y are in secular equilibrium. ^{90}Sr transforms into ^{90}Y, whose β-particles have much more energy and we have to take them into consideration. For ^{90}Sr: $E_{\beta,max} = 546$ KeV, $R_{max} = 196$ mg/cm^2, $\mu = 35$ cm^2/g. For ^{90}Y: $E_{\beta,max} = 2274$ KeV, $R_{max} = 1098$ mg/cm^2, $\mu = 4.45$ cm^2/g.

We notice that $\frac{196}{1098} < 0.3 \cdot 1098 \;\Rightarrow$ we use (3.3).

$$k_Y = e^{-4.45\cdot 196\cdot 10^{-3}} = 0.418 \;\Rightarrow\; 42\% \text{ of the particles will go through} \tag{3.8}$$

The linear thickness of the screen will be: $l_{max} = \frac{196}{2700} = 0.0726 \text{ cm} \approx 0.7$ mm.

4. Seminar 4. Detection of ionizing radiation

Equations and parameters

The count rate, that we measure is given by:

$$J_c = p \cdot \varphi \cdot A + B \tag{4.1}$$

where p is the yield of the certain decay type, φ is detector efficiency, A is the activity of the studied radionuclide, B is the background count rate.

We also have to correct for the dead time of the detector τ:

$$J = \frac{J_c}{1 - \tau \cdot J_c} \tag{4.2}$$

J_c is the measured count rate.

The efficiency of the detector φ depends on several parameters:

$$\varphi = \varepsilon \cdot \eta \cdot S \cdot k \cdot q \tag{4.3}$$

where ε is detector efficiency - the probability that a particle that deposited some of its energy into the atoms of the detecting medium will be detected, η is geometry factor - the relative position of the specimen and the counter, k is the attenuation - absorption and scattering of particles by matter between the source and the detector, S is self-attenuation - absorption and scattering of particles by the material of the source itself, q is the back-scattering coefficient - accounts for the scattering of particles from the substrate material.

The radiation from the point source spreads spherically in all directions. Let's say we know the distance from the source to the detector - r, the area of the detector is S_{det} (see fig.4.1), then we can calculate the geometry factor as η:

$$\eta = \frac{S_{det}}{4\pi r^2} \tag{4.4}$$

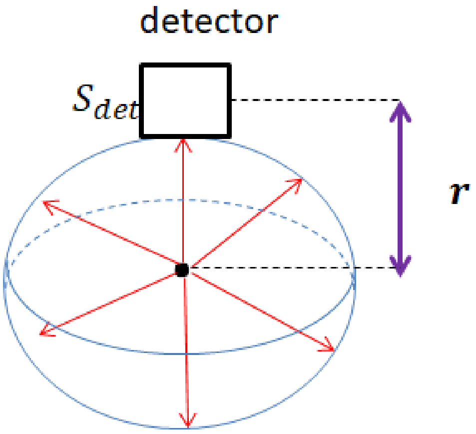


Figure 4.1. The source and the detector

When we use liquid-scintillator detectors we put radioactive source inside the detector (fig.4.2) and in this case the geometry factor is equal to 1.

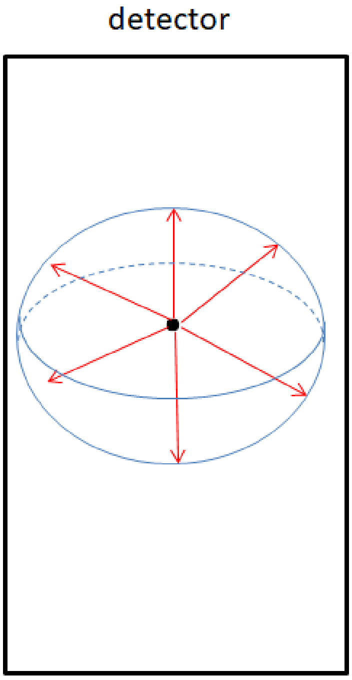


Figure 4.2. The source inside the detector

When the source is placed right next to the detector and contacts its surface half of radiation goes into the detector, the other half misses it. In this case $\eta = 0.5$.

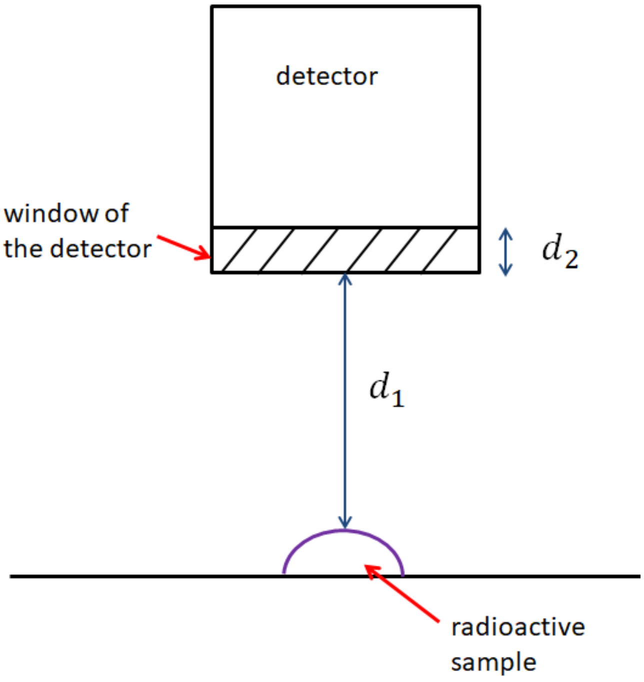


Figure 4.3. The source beside the detector

The attenuation coefficient is given by:

$$k = e^{-\mu d} \tag{4.5}$$

where $d = d_1 + d_2$ - the sum of distances (see fig.4.3).

Also, if we have a sample with thickness d_s some of the released radiation will be absorbed inside the sample and will not go to the detector (fig.4.4).

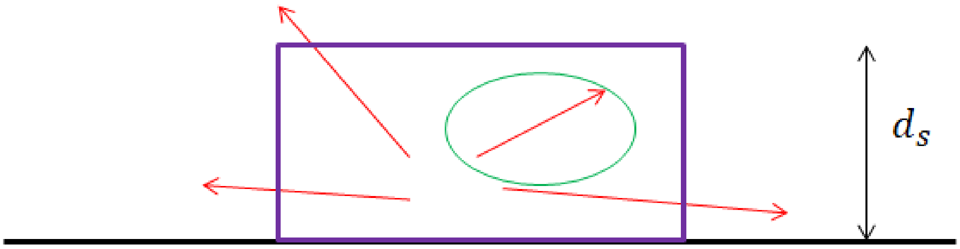


Figure 4.4. Absorption of radiation inside the sample

Self-attenuation coefficient S accounts for that and given by the following expression:

$$S = \frac{1 - e^{-\mu d_s}}{\mu d_s} \tag{4.6}$$

Task 1. Calculation of registration coefficient and determination of absolute activity

The measured count rate of the preparation containing ^{204}Tl is 80 cps. Calculate the registration coefficient and determine the activity of the radionuclide if:

1) resolution time $\tau = 3 \cdot 10^{-4}$s,
2) the background counting rate is 30 cpm,
3) the distance to the counter is 2 cm,
4) the counter window thickness is 4 mg/cm^2,
5) the thickness of the sample is 40 mg/cm^2,
6) the geometric factor is 0.06

The rest of the correction factors are taken equal to 1. R_{max}287.8 mg/cm^2, mass attenuation coefficient: $\mu = 20.28$ cm^2/g.

Solution

First thing, let's re-calculate mass attenuation coefficient into the linear one:

$$\mu_l = \mu \cdot \rho \tag{4.7}$$

The thickness of the material d:

$$d = \frac{l}{\rho} \tag{4.8}$$

where l is the linear size of the material.

The registration coefficient:

$$\varphi = \varepsilon \eta k S q \tag{4.9}$$

We already know that $\eta = 0.06$, $\varepsilon = 1$, $q = 1$.

Let's calculate the self-attenuation coefficient:

$$S = \frac{1 - e^{-\mu \cdot d_s}}{\mu d_s} = \frac{1 - e^{-20.28 \cdot 40 \cdot 10^{-3}}}{20.28 \cdot 40 \cdot 10^{-3}} = 0.685 \tag{4.10}$$

and the attenuation coefficient:

$$k = e^{-\mu d} = e^{-\mu(d_1 + d_2)} = e^{-20.28 \cdot (2 \cdot 1.29 \cdot 10^{-3} + 4 \cdot 10^{-3})} = 0.875 \tag{4.11}$$

As a result:

$$\varphi = 0.036 \tag{4.12}$$

The absolute activity:

$$A = \frac{1}{\underbrace{p}_{=1} \cdot \varphi} \left(\frac{J_c}{1 - \tau \cdot J_c} - B \right) = \frac{1}{0.036} \left(\frac{80}{1 - 3 \cdot 10^{-4} \cdot 80} - \frac{30}{60} \right) = 2263 \text{ Bq} \tag{4.13}$$

This task is a great example of how the measured count rate differs from the real activity of the sample.

Task 2. Estimating the sample counting rate.

The count rate of a thin sample of ^{90}Sr, purified from the daughter radionuclide, was determined using a detector with a window thickness of 3 mg/cm^2. The sample was located 2 cm from the detector window. Estimate the sample counting rate after 64 and 128 hours, if at the initial moment it was (after subtracting background) 3600 cpm. Disregard the correction for the detector resolution time.

Solution

We are going to need the following information. For ^{90}Sr: $T_{1/2}(^{90}\text{Sr}) = 28.7$ years, $R_{max} = 196$ mg/cm^2, $\mu = 35$ cm^2/g. For ^{90}Y: $T_{1/2}(^{90}\text{Y}) = 64$ h, $R_{max} = 1098$ mg/cm^2, $\mu = 4.45$ cm^2/g.

We have a mixture of 2 radionuclides, simultaneously present in our sample. As far as we can neglect the resolution time, the count rate after subtracting the background:

$$J = p \varphi A \tag{4.14}$$

where $\varphi = \varepsilon \eta k S q$

Since both radionuclides are located in the same sample and the efficiency of the detector does not depend on the energy of the emitted β-particles we can say that ε, η, q, p are the same for both radionuclides and all equal to 1.

Initially there was no yttrium in the system, therefore the given count rate is the count rate of strontium only:

$$J_{Sr} = k_{Sr} \cdot S_{Sr} \cdot A_{Sr} \Rightarrow A_{Sr} = \frac{J_{Sr}}{k_{Sr} \cdot S_{Sr}} = A_{Sr} = \frac{3600 \text{ cpm}}{k_{Sr} \cdot S_{Sr}} \tag{4.15}$$

And for yttrium:

$$J_Y = k_Y \cdot S_Y \cdot A_Y \tag{4.16}$$

What about the activity of yttrium? Comparing the half-lives of strontium and yttrium we see that the first is much longer ⇒ the case of secular equilibrium. Therefore:

$$A_Y = A_{Sr} \cdot (1 - 2^{-t/T_{1/2}(Y)}) \tag{4.17}$$

- After 64 hours:

$$A_{Y,1} = 0.5 \cdot A_{Sr} \tag{4.18}$$

- After 128 hours:

$$A_{Y,2} = A_{Sr} \cdot (1 - 2^{-2}) = 0.75 \cdot A_{Sr} \tag{4.19}$$

The activity of the whole sample:

$$A_1 = A_{Sr} + A_Y \tag{4.20}$$

The count rate of the whole sample:

$$J_1 = J_{Sr} + J_Y \tag{4.21}$$

where $J_Y = k_Y S_Y \cdot \frac{J_{Sr}}{k_{Sr} S_{Sr}} \cdot 0.5 A_{Sr}$ or $J_Y = k_Y S_Y \cdot \frac{J_{Sr}}{k_{Sr} S_{Sr}} \cdot 0.75 A_{Sr}$
The attenuation coefficient:

$$k_{Sr} = e^{-\mu_{Sr} d} = e^{-35 \cdot (3 + 2 \cdot 1.29) \cdot 10^{-3}} = 0.8226 \tag{4.22}$$

$$k_Y = e^{-\mu_Y d} = 0.975 \tag{4.23}$$

The self-attenuation coefficient equals to 1 since we have a thin sample.

In the case of 64 hour:

$$J_Y = 2133 \text{ cpm} \tag{4.24}$$

And the total count rate:

$$J_1 = 3600 + 2133 = 5733 \text{ cpm} \tag{4.25}$$

In the case of 128 hours:

$$J_Y = 3200 \text{ cpm} \tag{4.26}$$

And the total count rate:

$$J_1 = 3600 + 3200 = 6800 \text{ cpm} \tag{4.27}$$

Notice, that the mass-attenuation coefficients for strontium and yttrium are very different - it is because the maximum energy of beta-particles for strontium: $E_{\beta,max} =$ 546 KeV and for yttrium: $E_{\beta,max} =$ 2274 KeV. The higher the energy of particles the easier they penetrate the matter.

5. Seminar 5. Use of radioanalitical tracers

Task 1.

In 50 mL of diluted hydrochloric acid $8 \cdot 10^{-7}$ mol of ^{95}Zr-labeled zirconium hydroxide with an activity of 12.5 GBq was dissolved. It is assumed that the absorbed ions can cover the glass surface with a monolayer of 25 cm^2 and the area occupied by the ion is $2.5 \cdot 10^{-15}$ cm^2. How can the specific activity of the solution change due to molecular absorption.

Solution

The half-life time of the ^{95}Zr is $T_{1/2} = 64$ days. Let's calculate how many molecules of zirconium will be adsorbed. The total surface area is $S = 25$ cm^2 and the surface area occupied by 1 ion is $S_1 = 2.5 \cdot 10^{-15}$ cm^2. So the number of the adsorbed ions:

$$N_{ads} = \frac{S}{S_1} = 10^{16} \tag{5.1}$$

The amount of ions in the solution:

$$N_{sol} = \nu N_A = 48 \cdot 10^{16} \tag{5.2}$$

where ν is the amount of matter in mol, N_A is the Avogadro constant.

The final amount of ions:

$$N_f = N_{tot} - N_{abs} = 4.7 \cdot 10^{17} \tag{5.3}$$

The activity that is absorbed:

$$A_{ads} = \frac{12.5}{48}\ \text{GBq} = 0.26\ \text{GBq} \tag{5.4}$$

The activity remaining in the solution:

$$A_{sol} = \frac{47}{48} \cdot 12.5\ \text{GBq} = 12.24\ \text{GBq} \tag{5.5}$$

The specific activity:

$$a = \frac{12.24}{0.05}\ \frac{\text{GBq}}{\text{L}} = 244.8\ \frac{\text{GBq}}{\text{L}} \tag{5.6}$$

Now let's assume that there is no stable zirconium - only radioactive ^{95}Zr is present. Then the total amount of ^{95}Zr in the system:

$$N = \frac{A}{\lambda} = \frac{A}{\ln(2)} T_{1/2} = \frac{12.5 \cdot 10^9}{\ln(2)} \cdot 64 \cdot 24 \cdot 3600 = 10^{17}\ \text{ions} \tag{5.7}$$

Using (5.1) we can say that the activity in the solution:

$$A_{sol} = 12.5 \cdot 0.9 = 11.25\ \text{GBq} \tag{5.8}$$

And the specific activity in this case:

$$a = 225\ \frac{\text{GBq}}{\text{L}} \tag{5.9}$$

Task 2.

The researcher has a setup that reliably records $J = 10$ cpm with a registration coefficient $\varphi = 0.1$. What is the detection limit of element M by the emission of its ^{100}M isotope (content in the natural mixture is 10%, decay constant is $\lambda = 10^{-18}$ min^{-1})?

Solution

Let's determine the minimum mass m_{min} that we can measure with this detector. The minimum activity that can be measured:

$$A_{min} = \frac{J}{\varphi} = \frac{10\ \text{cpm}}{0.1} = 100\ \text{dpm} \tag{5.10}$$

where dpm stands for "disintegrations per minute".

From this we can calculate the number of atoms:

$$N_{min} = \frac{A_{min}}{\lambda} = \frac{100\ \text{dpm}}{10^{-18}\ \text{min}^{-1}} = 10^{20}\ \text{atoms} \tag{5.11}$$

Taking into consideration that the content of ^{100}M isotope is 10%:

$$N_{tot,min} = 10^{21}\ \text{atoms} \tag{5.12}$$

So the minimum mass:

$$m_{min} = \frac{N}{N_A} \cdot 100 = 0.167\ \text{g} \tag{5.13}$$

Task 3.

The researcher has a setup that reliably records $J = 54$ cpm with a registration coefficient $\varphi = 0.05$. What should be the specific activity (Bq/g) of ^{22}Na:NaCl in order to detect $m_{min} = 2 \cdot 10^{-7}$ g of NaCl using this setup. $T_{1/2} = 2.6$ years.

Solution

This is an example of a task for determining the amount of tracer we should put in the system to detect very low concentrations of matter.

The activity that we measure:

$$A = \frac{J}{\varphi} = 1080\ \text{dpm} \tag{5.14}$$

The specific activity:

$$a = \frac{A}{m_{min}} = 9 \cdot 10^{7}\ \text{Bq/g} \tag{5.15}$$

The amount of the radionuclide:

$$N = \frac{A}{\lambda} = \frac{A}{\ln(2)} \cdot T_{1/2} = 1.06 \cdot 10^{16}\ \text{atoms} \tag{5.16}$$

And thus the mass of this isotope:

$$m({}^{22}\text{Na}) = \frac{N}{N_A} \cdot 22 = 3.9 \cdot 10^{-7}\ \text{g} \tag{5.17}$$

So, even if we add such large activity to our system there will be only a small addition of the radioactive tracer and it will not have an effect on the mass balance of the whole system.

Task 4.

To determine the blood volume, the animal was injected intravenously with V =0.5 ml of tritium-labeled water with a specific activity of $a = 2$ kBq/mL, and then V_2 =0.2 mL of blood was taken. The counting rate of this sample, measured using an LS spectrometer with a detection coefficient of φ =0.6, turned out to be J =100 cpm. Calculate the blood volume of the animal.

Solution

The initial activity of the tritium-labeled water:

$$A_1 = a \cdot V = 2000 \text{ Bq/mL} \cdot 0.5 \text{ mL} = 1000 \text{ Bq} \tag{5.18}$$

The activity of the blood sample:

$$A_2 = \frac{J}{\varphi} = 2.78 \text{ Bq} \tag{5.19}$$

Obviously:

$$\frac{A_1}{A_2} = \frac{V_0}{V_2} \Rightarrow V_0 = 72 \text{ ml} \tag{5.20}$$

where V_0 is the volume of all blood.

This task introduces a very powerful method that allows us to indirectly measure not only small volumes of liquid but also huge vessels.

Task 5.

To obtain $Sr_3(PO_4)_2$, a 0.005 mol/ L $SrCl_2$ solution containing ^{89}Sr was used. The volume recorded activity of a 50-times diluted $^{89}SrCl_2$ solution was 17050 cpm/mL. The $Sr_3(PO_4)_2$ precipitate was obtained by adding a slight excess of sodium orthophosphate to 100 mL of $^{89}SrCl_2$ solution. A suspension of a part of the precipitate was stirred for a long time at 298 K in 50 mL of 0.001 mol/L sodium perchlorate solution. The volumetric recorded activity of the saturated solution was 52 cpm/mL.

Determine the solubility (mol/L) of $Sr_3(PO_4)_2$ in 0.001 mol/L $NaClO_4$ solution. Estimate the solubility product of this salt.

The coefficients of thermodynamic activity of the cation and anion, respectively, $\gamma_{Sr} = 0.87$ and γ_P 0.73.

Solution

This task demonstrates how we can use radionuclide tracers in thermodynamical studies. The process in the solution:

$$\mathrm{Sr_3(PO_4)_2} \leftrightarrows \mathrm{3Sr^{2+} + 2PO_4^{3-}} \tag{5.21}$$

The solubility product:

$$K_{SP} = [\mathrm{Sr^{2+}}]^3[\mathrm{PO_4^{3-}}]^2 \cdot \gamma_{Sr}^3 \cdot \gamma_P^2 \tag{5.22}$$

Strontium and phosphate ions can appear from the $Sr_3(PO_4)_2$ only. So, using (5.21), if we take the concentration of the $Sr_3(PO_4)_2$ as x, then the concentration of Sr^{2+} will be $3x$, PO_4^{3-} - $2x$.

We get an equation:

$$K_{SP} = (3x)^3(2x)^2 \cdot \gamma_{Sr} \cdot \gamma_P = 108x^5\gamma_{Sr}^3 \cdot \gamma_P^2 \tag{5.23}$$

We know that for 0.005 mol/L of $SrCl_2$ the activity of the 50-times diluted solution is 17050 cpm/mL. The activity of the non-diluted solution then is $a_V = 17050 \cdot 50 \cdot 1000 = 8525 \cdot 10^8$ cpm/L. The specific activity of strontium:

$$a_{Sr} = \frac{a_V}{0.005} = 1705 \cdot 10^1 1 \text{ cpm/mol} \tag{5.24}$$

We know activity of the saturated solution: $a_2 = 52$ cpm/mL, so we find the concentration of strontium in this solution:

$$C_2 = \frac{a_2}{a_{Sr}} = 3.05 \cdot 10^{-7} \text{mol/L} \tag{5.25}$$

Concentration of $Sr_3(PO_4)_2$:

$$C(Sr_3(PO_4)_2) = C_2/3 \approx 10^{-7} \text{ mol/L} \tag{5.26}$$

Going back to the expression (5.22):

$$K_{SP} = 3.8 \cdot 10^{-34} \tag{5.27}$$

6. Seminar 6. Dosimetry. Part 1

This seminar is dedicated to calculation of different doses of radiation.

Task 1.

Determine the flux density of particles in air at a distance of r = 50 cm from point isotropic source ^{204}Tl with an activity A = 2.4 MBq. What is the particle and energy fluence in t =1 hour? Air density ρ = 0.00129 g/cm^3, $E_{\beta,max}$ = 763 KeV, R_{max} = 288 mg/cm^2, μ = 20.5 cm^2/g.

Solution

We should remember, that radiation from a point source goes in all directions.

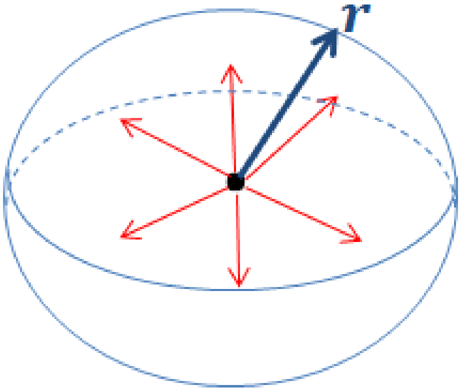


Figure 6.1. Radiation from a point source

The flux density is the number of particles N, going through an area S over a period of time t:

$$F = \frac{N}{S \cdot t} \tag{6.1}$$

where in our task $S = S_{sphere} = 4\pi r^2$.

Using the definition of activity:

$$A = \frac{N_\beta}{t} \tag{6.2}$$

However, since the source is located not in the vacuum but in the air a fraction of the emitted β-particles will be absorbed by the air before they cover the distance r. Thus we have to calculate the attenuation coefficient for β-particles. First thing we re-calculate the linear maximum range:

$$l_{max} = \frac{R_{max}}{\rho} = 223 \text{ cm} \tag{6.3}$$

The attenuation coefficient:

$$k = e^{-\mu r \rho} = 0.267 \tag{6.4}$$

So only 26.7 % of the β-particles will go over the distance r. The number of particles going through the area S:

$$N = A \cdot k \cdot t \tag{6.5}$$

As a result, the flux density:

$$F = \frac{A \cdot k}{4\pi r} = 20.4 \text{ cm}^{-2}\text{s}^{-1} \tag{6.6}$$

The particle fluence is the total number of particles N going through an area S over a certain time:

$$\Phi = \frac{N}{S} = F \cdot t = 73440 \text{ cm}^{-2} \tag{6.7}$$

The energy fluence Φ_E is easily found multiplying the particle fluence by the average energy of β-particles: $\overline{E}_\beta = 0.4 \cdot E_{\beta,max}$.

$$\Phi_E = \Phi \cdot 0.4 \cdot E_{\beta,max} = 2.24 \cdot 10^7 \text{ KeV cm}^{-2} \tag{6.8}$$

Task 2.

Compare the energy flux densities of β- and γ-radiation in air at a distance $r = 50$ cm from the source. Linear attenuation coefficients in air (μ', cm^{-1}) for γ-quanta with energies of 0.19 and 1.0 MeV are equal to $0.161 \cdot 10^{-3}$ and $0.0822 * 10^{-3}$, respectively.
$\mu_1 = 10.1$ cm^2/g (1.2 MeV)
$\mu_2 = 4.6$ cm^2/g (2.2 MeV)

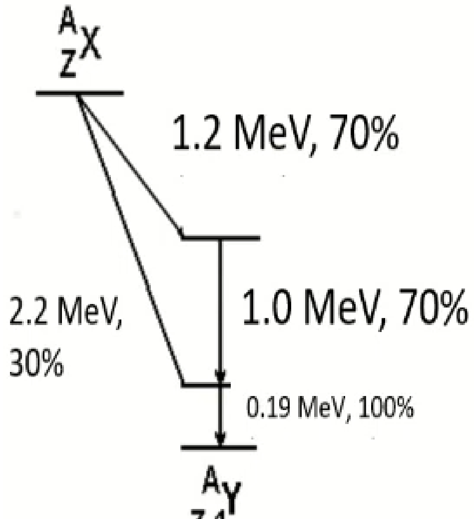


Figure 6.2. The scheme for the task 2

Solution
The energy flux:

$$F_E = \frac{N \cdot E}{S \cdot t} \tag{6.9}$$

$$E = \begin{cases} 0.4 \cdot E_{\beta,max}, \text{ for } \beta\text{-particles} \\ E_\gamma, \text{ for } \gamma\text{-quanta} \end{cases} \tag{6.10}$$

Since there are 2 beta-decays and 2 gamma-decays:

$$F_{tot} = \frac{\sum\limits_{i=1}^{2} N_i E_i}{S \cdot t} = \frac{A \sum\limits_{i=1}^{2} p_i E_i k_i}{S} \tag{6.11}$$

where p_i is the yield, A is the activity, k_i is the attenuation coefficient.

The attenuation coefficient for β-particles with the energy 1.2 MeV:

$$k_1 = e^{-\mu_1 \cdot r \cdot \rho} = 0.52 \tag{6.12}$$

The attenuation coefficient for β-particles with the energy 2.2 MeV:

$$k_1 = e^{-\mu_2 \cdot r \cdot \rho} = 0.74 \tag{6.13}$$

Now let's calculate the energy flux density for β-particles:

$$F_\beta = \frac{A \cdot (p_1 \cdot 0.4 \cdot E_{\beta,max,1} \cdot k_1 + p_2 \cdot 0.4 \cdot E_{\beta,max,2} \cdot k_2)}{S} = \tag{6.14}$$

$$= \frac{A \cdot (0.7 \cdot 0.4 \cdot 1.2 \cdot 0.52 + 0.3 \cdot 0.4 \cdot 2.2 \cdot 0.74)}{S} = 0.37 \cdot \frac{A}{S} \tag{6.15}$$

As for gamma-quanta we see, that the linear attenuation coefficient is very small, hence we can neglect the absorption and take the attenuation coefficient for gamma-quanta $k_\gamma = 1$.

The energy flux density for γ-particles:

$$F_\gamma = \frac{A \cdot (p_1 \cdot E_{\gamma 1} \cdot k_\gamma + p_2 \cdot E_{\gamma 2} \cdot k_\gamma)}{S} = \tag{6.16}$$

$$= \frac{A \cdot (0.7 \cdot 1.0 \cdot 1 + 1 \cdot 0.19 \cdot 1)}{S} = 0.89 \cdot \frac{A}{S} \tag{6.17}$$

So the ratio:

$$\frac{F_\beta}{F_\gamma} = \frac{0.37}{0.89} = 0.416 \tag{6.18}$$

Task 3.

Calculate the ratio of the absorbed dose rates in the air created by the β- and γ-radiation from a point isotropic ^{60}Co source at a distance $r = 10$ cm. $E_{\beta,max} = 318$ KeV, $\mu = 91.9$ cm^2/g, the KERMA-constant: $K_\gamma = 84.6$ aGy·m^2/(s·Bq), for β-particles of energy 0.3 MeV: $R_{max,1} = 76$ mg/cm^2, for β-particles of energy 0.4 MeV: $R_{max,2} = 115$ mg/cm^2.

Solution

The absorbed dose rate for γ-quanta :

$$P_\gamma = K_\gamma \cdot \frac{A}{r^2} = \frac{84.6 \cdot 10^{-18} \cdot A}{(0.1)^2} = 84.6 \cdot 10^{-16} \text{Gy/s} \cdot A \tag{6.19}$$

Assuming that the R_{max} depends linearly on the E_β (fig. 6.3) we can calculate the R_{max} for 318 KeV (0.318 MeV):

$$R_{max}(0.318\ \text{MeV}) = 83\ \text{mg/cm}^2 \tag{6.20}$$

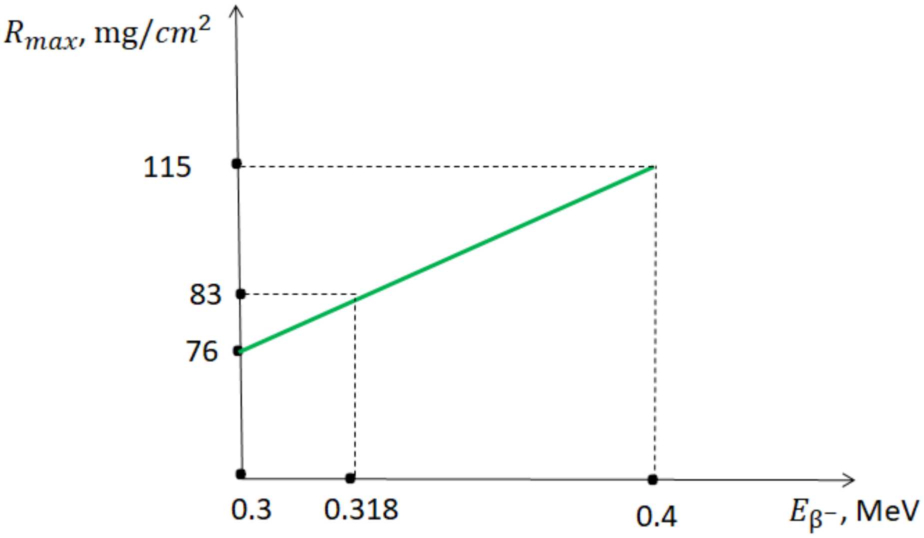


Figure 6.3. Linear dependence of R_{max} on E_β

Knowing the density of the air ρ we can calculate the linear l_{max}:

$$l_{max} = R_{max}/\rho = 64.4\ \text{cm} \tag{6.21}$$

So at the distances greater than 65 cm the dose is created by γ-quanta only.

In this task $r < 0.3 \cdot l_{max} \Rightarrow$ we can use the following expression for the attenuation coefficient:

$$k = e^{-\mu d} = e^{-\mu r \rho} = 0.306 \tag{6.22}$$

The absorbed dose rate for β-particles:

$$P_\beta = \frac{A}{4\pi r^2} \cdot \frac{0.4 E_{\beta,max} \cdot k}{R_{max}} \cdot 1.6 \cdot 10^{-16} = \frac{A \cdot 0.4 \cdot 318 \cdot 0.306 \cdot 1000}{4\pi \cdot 10^2 \cdot 0.083} \cdot 1.6 \cdot 10^{-16} =$$
$$= 5.97 \cdot 10^{-14} \text{Gy/s} \cdot A$$

So the ratio:

$$\frac{P_\beta}{P_\gamma} = 7 \tag{6.23}$$

Task 4.

Is it necessary to install a protective screen if at the workplace the equivalent dose rate from sources of ionizing radiation is 2.3 nSv/s? The radiation dose is distributed evenly throughout the year. The work is carried out 1700 hours a year.

Solution

First thing, let's convert nSv/s into μSv/h:

$$P = 2.3 \text{ nSv/s} = 8.28\ \mu\text{Sv/h} \tag{6.24}$$

The dose that the personnel will receive over 1700 hours of work per year:

$$D = 8.28 \cdot 1700 \approx 14 \text{ mSv/year} \tag{6.25}$$

And since the maximum allowed dose is equal to 20 mSv/year we see that it is still safe and **no shielding is needed**.

Task 5.

In the laboratory, work is carried out every day for $t = 2$ hours with a point source of ^{137}Cs(^{137m}Ba) with an activity $A = 394$ MBq at a distance $r = 30$ cm. The sources of ionizing radiation are not operated at other times.

Determine: **a)** the daily dose that the experimenter can receive in the absence of protection; **b)** the minimum thickness of the lead shield to ensure a safe working environment.

The KERMA-constant $K_\gamma = 21.4$ aGy·m^2/(s·Bq)

Solution

The dose rate:

$$P = \frac{A \cdot K_\gamma}{r^2} \tag{6.26}$$

The absorbed dose:

$$D = P \cdot t = \frac{A \cdot K_\gamma \cdot t}{r^2} = 674.53\ \mu\text{Gy} \tag{6.27}$$

Then we have to re-calculate the absorbed dose into the equivalent dose:

$$H = w_R \cdot D \cdot 1.09 = 735\ \mu\text{Sv} \tag{6.28}$$

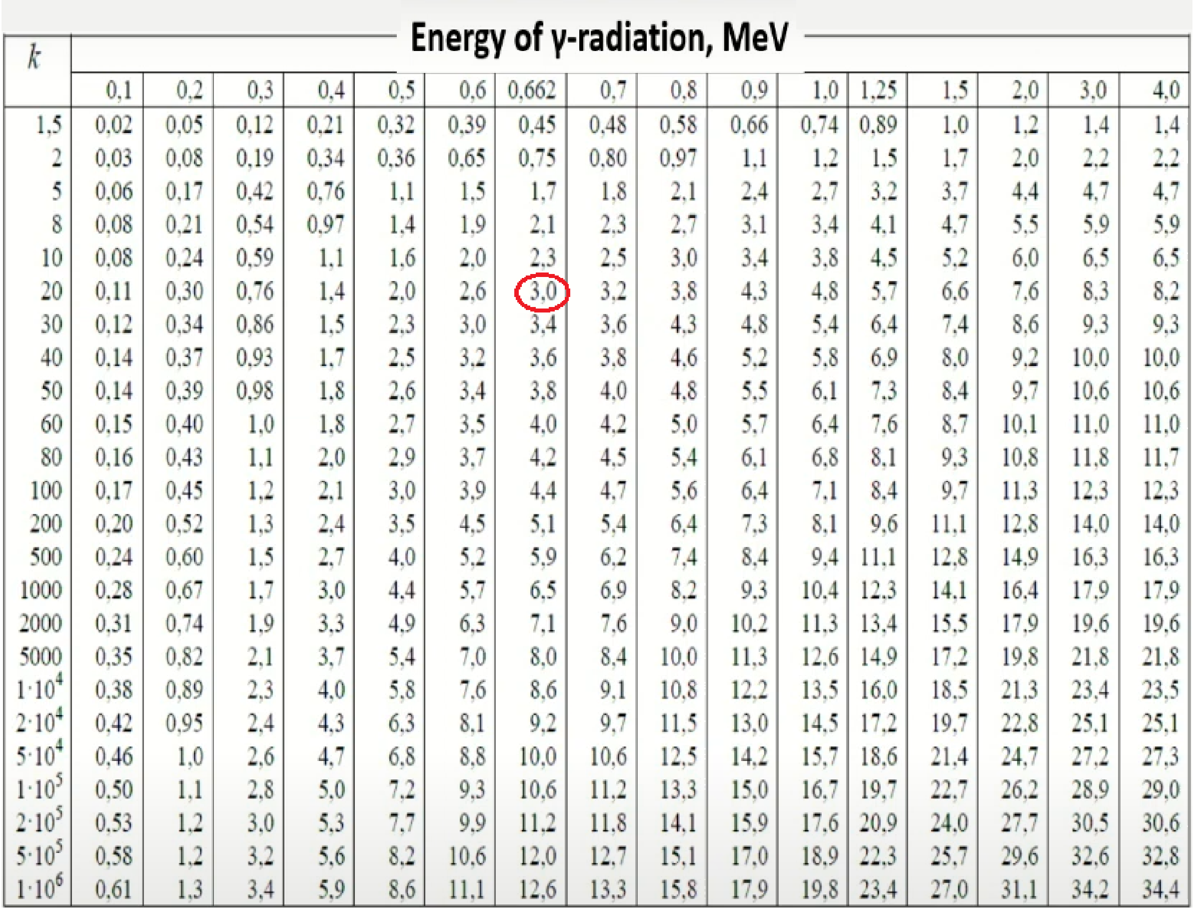
where $w_R = 1$ is the weighing factor for γ and β particles, 1.09 is the coefficient for re-calculation of the absorbed dose into the equivalent dose.

The allowed dose in our case is 72 μSv/day $\Rightarrow$ we have to use some shielding to mitigate the hazardous effects of radiation.

To calculate the thickness of the lead shielding let's use the table in fig.6.4. It shows the thickness of lead (cm) for a given attenuation factor and for a given energy of gamma-quanta.

So, the researcher receives 735 μSv, the allowed dose is 72 μSv. To ensure that the working conditions are safe we take $k = 2 \cdot \frac{735}{72} \approx 20$. The energy of γ-quanta emitted by ^{137}Cs(^{137m}Ba) source is 662 KeV. Consulting with the table gives us the answer:

$$l_{lead} = 3 \text{ cm} \tag{6.29}$$



k	Energy of γ-radiation, MeV															
	0,1	0,2	0,3	0,4	0,5	0,6	0,662	0,7	0,8	0,9	1,0	1,25	1,5	2,0	3,0	4,0
1,5	0,02	0,05	0,12	0,21	0,32	0,39	0,45	0,48	0,58	0,66	0,74	0,89	1,0	1,2	1,4	1,4
2	0,03	0,08	0,19	0,34	0,36	0,65	0,75	0,80	0,97	1,1	1,2	1,5	1,7	2,0	2,2	2,2
5	0,06	0,17	0,42	0,76	1,1	1,5	1,7	1,8	2,1	2,4	2,7	3,2	3,7	4,4	4,7	4,7
8	0,08	0,21	0,54	0,97	1,4	1,9	2,1	2,3	2,7	3,1	3,4	4,1	4,7	5,5	5,9	5,9
10	0,08	0,24	0,59	1,1	1,6	2,0	2,3	2,5	3,0	3,4	3,8	4,5	5,2	6,0	6,5	6,5
20	0,11	0,30	0,76	1,4	2,0	2,6	3,0	3,2	3,8	4,3	4,8	5,7	6,6	7,6	8,3	8,2
30	0,12	0,34	0,86	1,5	2,3	3,0	3,4	3,6	4,3	4,8	5,4	6,4	7,4	8,6	9,3	9,3
40	0,14	0,37	0,93	1,7	2,5	3,2	3,6	3,8	4,6	5,2	5,8	6,9	8,0	9,2	10,0	10,0
50	0,14	0,39	0,98	1,8	2,6	3,4	3,8	4,0	4,8	5,5	6,1	7,3	8,4	9,7	10,6	10,6
60	0,15	0,40	1,0	1,8	2,7	3,5	4,0	4,2	5,0	5,7	6,4	7,6	8,7	10,1	11,0	11,0
80	0,16	0,43	1,1	2,0	2,9	3,7	4,2	4,5	5,4	6,1	6,8	8,1	9,3	10,8	11,8	11,7
100	0,17	0,45	1,2	2,1	3,0	3,9	4,4	4,7	5,6	6,4	7,1	8,4	9,7	11,3	12,3	12,3
200	0,20	0,52	1,3	2,4	3,5	4,5	5,1	5,4	6,4	7,3	8,1	9,6	11,1	12,8	14,0	14,0
500	0,24	0,60	1,5	2,7	4,0	5,2	5,9	6,2	7,4	8,4	9,4	11,1	12,8	14,9	16,3	16,3
1000	0,28	0,67	1,7	3,0	4,4	5,7	6,5	6,9	8,2	9,3	10,4	12,3	14,1	16,4	17,9	17,9
2000	0,31	0,74	1,9	3,3	4,9	6,3	7,1	7,6	9,0	10,2	11,3	13,4	15,5	17,9	19,6	19,6
5000	0,35	0,82	2,1	3,7	5,4	7,0	8,0	8,4	10,0	11,3	12,6	14,9	17,2	19,8	21,8	21,8
$1\cdot10^4$	0,38	0,89	2,3	4,0	5,8	7,6	8,6	9,1	10,8	12,2	13,5	16,0	18,5	21,3	23,4	23,5
$2\cdot10^4$	0,42	0,95	2,4	4,3	6,3	8,1	9,2	9,7	11,5	13,0	14,5	17,2	19,7	22,8	25,1	25,1
$5\cdot10^4$	0,46	1,0	2,6	4,7	6,8	8,8	10,0	10,6	12,5	14,2	15,7	18,6	21,4	24,7	27,2	27,3
$1\cdot10^5$	0,50	1,1	2,8	5,0	7,2	9,3	10,6	11,2	13,3	15,0	16,7	19,7	22,7	26,2	28,9	29,0
$2\cdot10^5$	0,53	1,2	3,0	5,3	7,7	9,9	11,2	11,8	14,1	15,9	17,6	20,9	24,0	27,7	30,5	30,6
$5\cdot10^5$	0,58	1,2	3,2	5,6	8,2	10,6	12,0	12,7	15,1	17,0	18,9	22,3	25,7	29,6	32,6	32,8
$1\cdot10^6$	0,61	1,3	3,4	5,9	8,6	11,1	12,6	13,3	15,8	17,9	19,8	23,4	27,0	31,1	34,2	34,4

Figure 6.4. The thickness of lead (cm) for a given attenuation factor and for a given energy of gamma-quanta

7. Seminar 7. Dosimetry. Part 2

In task 5 from the previous seminar we had γ-quanta emitted with one energy only (662 KeV). Let's solve the task in which there will be γ-quanta with several different energies.

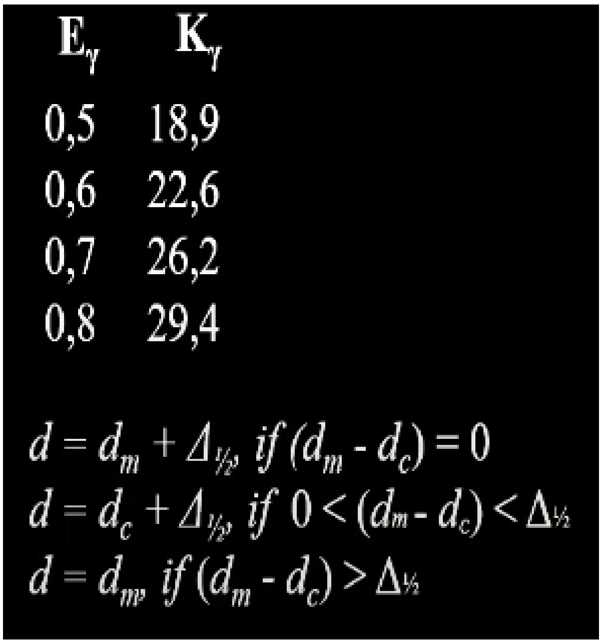
Task 1.

Calculate the minimum thickness of the lead shield to ensure safe working conditions with $A = 4$ GBq ^{134}Cs source at a distance $r = 50$ cm from the source.

From the database:

$$\begin{cases} E_\gamma = 567 \text{ KeV}, \ p_\gamma = 0.23 \\ E_\gamma = 604 \text{ KeV}, \ p_\gamma = 0.97 \\ E_\gamma = 797 \text{ KeV}, \ p_\gamma = 0.93 \end{cases} \tag{7.1}$$

$K_\gamma = 57$ aGy·m^2/(s^2·Bq)



E_γ	K_γ
0,5	18,9
0,6	22,6
0,7	26,2
0,8	29,4

$d = d_m + \Delta_{1/2},\ if\ (d_m - d_c) = 0$

$d = d_c + \Delta_{1/2},\ if\ 0 < (d_m - d_c) < \Delta_{1/2}$

$d = d_m,\ if\ (d_m - d_c) > \Delta_{1/2}$

Figure 7.1. The data for the task 1

Solution

Let's calculate the K_γ coefficient for the given in this task energies using the table from fig.7.1 and proportions:

$$\begin{cases} K_\gamma(567 \text{ KeV}) = 21.38 \\ K_\gamma(604 \text{ KeV}) = 22.74 \\ K_\gamma(797 \text{ KeV}) = 29.3 \end{cases} \tag{7.2}$$

Partial value of a gamma constant:

$$K^* = p_i \cdot K_\gamma \tag{7.3}$$

where p_i is the intensity of a γ-line

Using fig.7.1

$$\begin{cases} K^*(567 \text{ KeV}) = 4.9 \\ K^*(604 \text{ KeV}) = 22.1 \\ K^*(797 \text{ KeV}) = 27.2 \end{cases} \tag{7.4}$$

By how many times we have to reduce the dose created by our source of ionizing radiation at a distance r?

The dose rate of the equivalent dose:

$$P(H) = 1.09 \cdot \frac{K_\gamma \cdot A}{r^2} = 3.6 \cdot 10^{-3} \text{ Sv/h} \tag{7.5}$$

The limit of the dose rate:

$$P(H_{lim}) = 6 \ \mu\text{Sv/h} = 6 \cdot 10^{-6} \text{ Sv/h} \tag{7.6}$$

So our coefficient will be:

$$k = \frac{3.6 \cdot 10^{-3}}{6 \cdot 10^{-6}} = 600 \tag{7.7}$$

Now let's calculate the attenuation coefficient for each energy of γ-quanta:

$$k_i = k \cdot (\frac{K_i^*}{K_\gamma}) = 600 \cdot \frac{K_i^*}{57} \tag{7.8}$$

$$\begin{cases} k_i = 51.5 \\ k_i = 232.6 \\ k_i = 286.3 \end{cases} \tag{7.9}$$

And the thickness of lead shielding in cm for each k_i – $d(k_i)$ (we use doubled k-value to ensure safety). We consult with the table in fig.6.4.

$$\begin{cases} d(51.5) \approx 4 \text{ cm} \\ d(232.6) = 4.4 \text{ cm} \\ d(286.3) = 7.5 \text{ cm} \end{cases} \tag{7.10}$$

The next step - we neglect the photons with energy 567 KeV and double the attenuation coefficients (7.9) for the two remaining energies:

$$\begin{cases} 2k_i \approx 465 \\ 2k_i \approx 570 \end{cases} \tag{7.11}$$

Then calculate (using the table) new d' thicknesses:

$$\begin{cases} d'(465) \approx 5.6 \text{ cm} \\ d'(570) \approx 8.3 \text{ cm} \end{cases} \tag{7.12}$$

Now let's calculate the $\Delta_{1/2}$-value, which is the thickness of shielding that will reduce the equivalent dose 2 times.

$$\Delta_{1/2} = d_i' - d_i \tag{7.13}$$

$$\begin{cases} \Delta_{1/2}(604 \text{ KeV}) = 1.2 \text{ cm} \\ \Delta_{1/2}(797 \text{ KeV}) = 0.8 \text{ cm} \end{cases} \tag{7.14}$$

We take the greater value: $\Delta_{1/2}(604 \text{ KeV}) = 1.2$ cm.

Then we consider the difference of d between the two γ-lines:

$$\Delta d = 7.5 - 4.4 = 3.1 \text{ cm} \tag{7.15}$$

and again have a look at fig.7.1 (the bottom). We get that $\Delta d > \Delta_{1/2} \Rightarrow$ the thickness of lead shielding should be equal to the thickness for the main γ-line (797 KeV): $d_{result} = 7.5$ cm.

So in this task the shielding minimizes the most energetic γ-line.

Task 2.

Estimate the equivalent dose when working with the ^{24}Na sample with the activity at the time of preparation $a = 1$ MBq, at a distance $r = 30$ cm for $t = 6$ hours. Consider the sample as a point isotropic source.

$T_{1/2} = 15$ hours, $E_{\beta,max} = 1390$ KeV, $\mu = 8.1$ cm^2/g, $R_{max} = 0.613$ g/cm^2,
$K_\gamma = 112$ aGy·m^2/(s·Bq).

Solution ^{24}Na is not only γ-emitter but also high-energy β-emitter. So we have to calculate dose rates for both γ and β particles: P_γ, P_β and combine the results into the total dose rate P_0.

$$P_\beta = 1.6 \cdot 10^{-10} \cdot \frac{A}{4\pi r^2} \cdot \frac{0.4 \cdot E_{\beta,max} \cdot e^{-\mu\rho r}}{R_{max}} = 9.6 \cdot 10^{-9} \text{ Sv/s} \tag{7.16}$$

where ρ is the density of air.

$$P_\gamma = 1.09 \cdot \frac{A \cdot K_\gamma}{r^2} = 1.36 \cdot 10^{-9} \text{ Sv/s} \tag{7.17}$$

The total dose rate:

$$P_0 = P_\gamma + P_\beta = 10.96 \cdot 10^{-9} \text{ Sv/s} \tag{7.18}$$

^{24}Na is decaying, thus its activity decreases $\Rightarrow$ dose rate decreases with time. Let's calculate the dose integrating the dose rate as a function of time:

$$D = \int\limits_{t=0 \text{ h}}^{t=6 \text{ h}} P_0 \cdot e^{-\lambda \cdot t} dt = -\frac{1}{\lambda} P_0 \cdot e^{-\lambda \cdot t} \Big|_0^6 = 200 \ \mu\text{Sv} \tag{7.19}$$

Whereas the allowed equivalent dose is 72 μSv $\Rightarrow$ the working is not safe. The main contribution to the total dose rate comes from β-particles.

An aluminum shield of what thickness should we use to for protection?

The linear maximum range for β-particles in aluminum for this task:

$$l_{max} = \frac{R_{max}}{\rho_{Al}} = 0.23 \text{ cm} \tag{7.20}$$

Thereby a ~3 mm thick shield would suffice to protect workers.
If we decide to use water as shielding material we would need $\sim$ 6.1 mm shield.

Task 3.

Determine the absorbed dose rate in elementary volume from the α-radiation of a point isotropic source of radionuclide *Y (A =10 KBq, E_α = 6 MeV, the intensity p = 100%) located in biological tissue. the density of the tissue is $\rho = 1$ g/cm^3, the range of α-particles is $R_{max} = 0.0056$ g/cm^2.

Solution

The absorbed dose by definition is the ratio of the absorbed energy E and the mass of the matter m in which this absorption occurs. The energy in our case is the number of α-particles N_α multiplied by the energy of one α-particle E_α. The mass is the density ρ times its volume V.

$$D = \frac{E}{m} = \frac{N_\alpha \cdot E_\alpha}{\rho V} \tag{7.21}$$

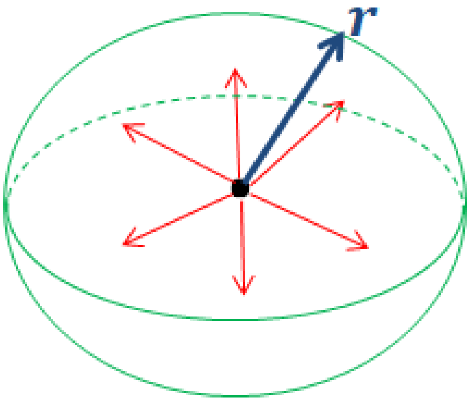


Figure 7.2. Point isotropic source inside the body

Since the source is point and isotropic (fig.7.2), the volume of mater in which absorption occurs is equal to the volume of a sphere with the radius $r = \frac{R_{max}}{\rho} = 0.0056$ cm - the linear range of α-particles penetration:

$$V = \frac{4}{3}\pi r^3 \tag{7.22}$$

The number of α-particles is the activity multiplied by time:

$$N_\alpha = A \cdot t \tag{7.23}$$

As a result:

$$D = \frac{A \cdot t \cdot E_\alpha}{\frac{4}{3}\pi r^3 \cdot \rho} \tag{7.24}$$

The dose rate:

$$P_D = \frac{D}{t} = \frac{A \cdot E_\alpha}{\frac{4}{3}\pi r^3 \cdot \rho} = 13 \text{ Gy/s} \tag{7.25}$$

That's an extremely large amount of radiation! We can re-calculate it into the rate of the equivalent dose:

$$P(H) = w_R \cdot P_D = 260 \text{ Sv/s} \tag{7.26}$$

where the weighing factor $w_R = 20$ for α-radiation.

As we can see, a mere 1 second would be enough to cause devastating damage to internal organs close to the source of radiation $\Rightarrow$ α-emitting sources are very dangerous if they are inside a human body - they kill all cells close to them. But we can also use it for treatment of cancer by destroying tumors.

Task 4.

For radionuclide diagnostics in a PET center, a patient weighing 95 kg was injected with 2-[^{18}F] fluoro-2-deoxy-D- glucose ([^{18}F]FDG) with a radioactivity $A = 0.24$ GBq. Determine the equivalent dose if the biological half- life of FDG $T_{1/2}^{bio} = 90$ minutes, the half-life of ^{18}F $T_{1/2}^{r} = 110$ minutes. β^+ particles are emitted with a probability 97%, $E_{max} = 0.635$ MeV, and electron capture (EC) occurs with a probability of 3%. We assume a uniform distribution of [^{18}F]FDG throughout the body, all e^+ are absorbed in the body, and 50% of the photons leave the body. Neglect the energy release during EC.

Solution

The dose rate at the initial moment P_0 is the sum of the contributions from β-particles and γ-quanta :

$$P_0 = P_\beta + P_\gamma \tag{7.27}$$

$$P_\beta = \frac{A \cdot p_\beta \cdot 0.4 \cdot E_{\beta,max}}{m} \tag{7.28}$$

Since the γ-quanta are emitted in the process of annihilation: $e^+ + e^- \rightarrow 2\gamma$ the intensity of γ-quanta is 2 times the intensity of β^+, but only 50% of the γ-quanta leave the body $\Rightarrow$: $p_\gamma = 2 \cdot p_\beta \cdot 0.5 = p_\beta$

$$P_\gamma = \frac{A \cdot p_\beta \cdot E_\gamma}{m} \tag{7.29}$$

Thus

$$P_0 = \frac{A \cdot p_\beta}{m} \cdot (0.4 E_{\beta,max} + E_\gamma) = 3 \cdot 10^{-7} \text{ Sv/s} \tag{7.30}$$

The absorbed dose:

$$D = \int_0^\infty P_0 \cdot e^{-\lambda t} dt = \frac{P_0}{\lambda} = \frac{P_0}{\ln(2)} \frac{T_{1/2}^{r} \cdot T_{1/2}^{bio}}{T_{1/2}^{r} + T_{1/2}^{bio}} = 1.3 \text{ mSv} \tag{7.31}$$

where $\lambda = \lambda_{eff} = \lambda_r + \lambda_{bio} = \frac{\ln(2)}{T^r_{1/2}} + \frac{\ln(2)}{T^{bio}_{1/2}}$, which takes into account the decay due to radioactivity and due to biological release of the compound.

All in all, we can see, that the absorbed dose is quite safe for the patient.

8. Seminar 8. Radionuclide tracers

The following task features neutron-activation analysis.

Task 1.

Determine the mass of As in a sample irradiated to saturation in a flux of $2 \cdot 10^{12}$ neutron/(cm$^2\cdot$ s). Arsenic is a mononuclide element, the activity of ^{76}As in the sample after irradiation is $A = 5$ Bq, cross section $\sigma = 4.5$ barn.

Solution

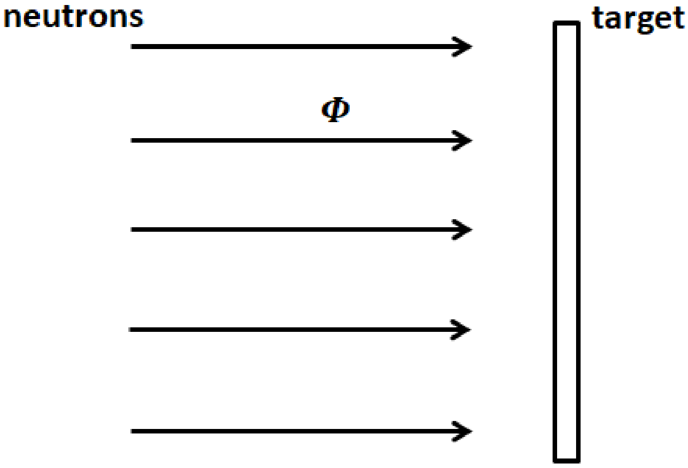


Figure 8.1. Irradiating a target with neutrons

The basic idea of neutron activation analysis: suppose we have a target that is irradiated with neutrons with the flux Φ (fig.8.1). Due to neutron-induced nuclear reactions such as $^{75}\text{As}(n, \gamma)^{76}\text{As}$, we produce another radionuclide in the sample that emits γ-rays.

Let's calculate the activity of the accumulated radionuclide.

$$A = \Phi \cdot \sigma \cdot N \cdot (1 - e^{-n\sigma l}) \cdot (1 - e^{\lambda t}) \tag{8.1}$$

where σ is the cross section of the neutron induced reaction, N is the number of irradiated atoms in the sample, n is the concentration of atoms in the sample, l is the thickness of the target, the second brackets show secular equilibrium.

If we have a very thin target we can neglect the first brackets and get:

$$A = \Phi \cdot \sigma \cdot N \cdot (1 - e^{\lambda t}) \tag{8.2}$$

In the case of secular equilibrium the activity of the daughter radionuclide A_2 is accumulated like this:

$$A_2 = A_{1,0} \cdot (1 - e^{\lambda_2 t}) \tag{8.3}$$

where $A_{1,0}$ is the initial activity of the mother radionuclide, λ_2 is the decay constant for the daughter radionuclide.

The conditions for the secular equilibrium:

$$T_{1/2}^{mother} \gg T_{1/2}^{daughter} \tag{8.4}$$

We assume that ^{75}As has a very long half-life, then comparing (8.2) and (8.3) we get that:

$$\Phi \cdot \sigma \cdot N = A_{1,0} \tag{8.5}$$

It will be reached over a very long time (fig.8.2):

$$A = \Phi \cdot \sigma \cdot N \cdot (1 - e^{\lambda t})\Big|_{t \to \infty} = \Phi\sigma N \tag{8.6}$$

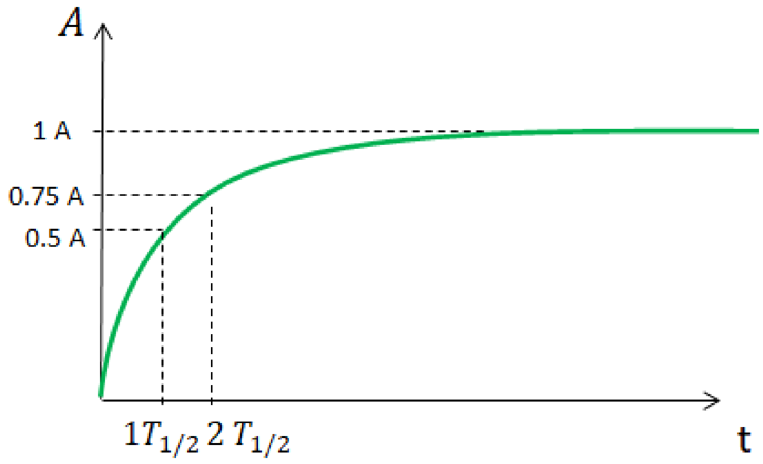


Figure 8.2. The activity over time

And since we know that a sample was irradiated to saturation ⇒ we reach that maximum activity and equilibrium. But when we stop irradiation at some point the radionuclide will decay ⇒ the activity, according to the law of radioactive decay will decrease exponentially with time (fig.8.3):

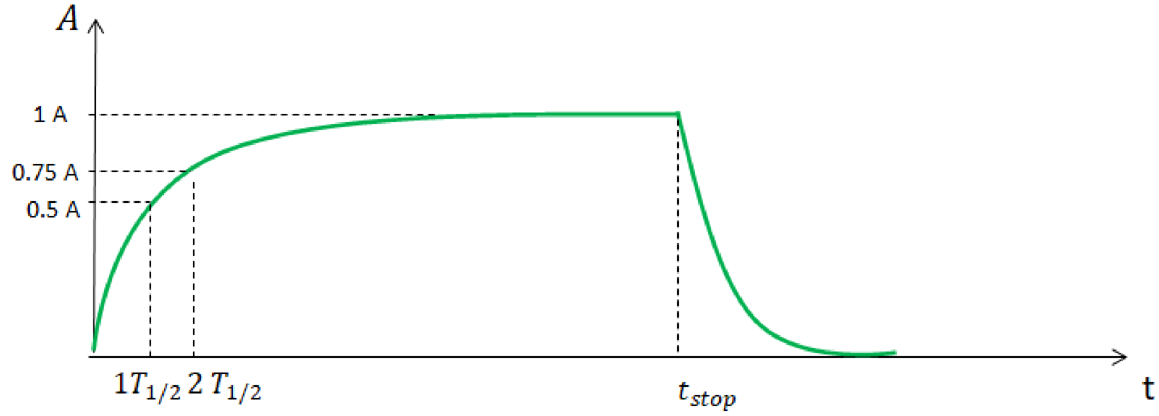


Figure 8.3. The activity decreases when we stop irradiating

The number of atoms in the target:

$$N = \frac{A}{\Phi\sigma} = 5.56 \cdot 10^{11} \text{ atoms} \tag{8.7}$$

The mass of the target therefore is:

$$m = \frac{N}{N_A} \cdot \mu_{As} = 6.9 \cdot 10^{-11} \text{ g} \tag{8.8}$$

where μ_{As} is the molar mass of As.

All in all, the neutron-activation analysis method allows us to determine extremely small masses an is a very powerful tool to perform the chemical analysis when we have subtle impurities in the sample. Moreover, the neutron-activation analysis is a non-destructive method - shortly after the experiment we will have the same sample as before.

Task 2.

Calculate the minimum irradiation time required to obtain a Sc_2O_3 sample with a specific activity $a = 400$ MBq/g. A neutron flux $\Phi = 10^{11}$ $cm^{-2} \cdot s^{-1}$. Scandium is a mononuclide element, the ^{45}Sc thermal neutron absorption cross section $\sigma = 27.2 \cdot 10^{-24}$ cm^2 and the ^{46}Sc half-life $T_{1/2} = 83.8$ days.

Solution

We use the expression (8.2) from the previous task assuming that we have a very thin target.

$$A = \Phi \cdot \sigma \cdot N \cdot (1 - e^{\lambda t}) \tag{8.9}$$

where N is the number of atoms in the sample.

We can express $N' = n \cdot N_A$, where $n = \frac{m}{\mu_r}$, μ_r is the molar mass of Sc_2O_3 $\Rightarrow N'$ is the amount of Sc_2O_3, not Sc $\Rightarrow N = 2N'$. Using this and $\frac{A}{m} = a$ (the definition of specific activity) and $\lambda = \frac{\ln(2)}{T_{1/2}}$ we get:

$$(1 - e^{\lambda t}) = \frac{A}{\Phi \cdot \sigma \cdot N} = \frac{a\mu_r}{2\Phi\sigma N_A} \tag{8.10}$$

Therefore:

$$e^{-\lambda t} = 1 - \frac{a}{2\Phi\sigma N_A} \Rightarrow t = -\frac{\ln(1 - \frac{a}{2\Phi\sigma N_A})}{\ln(2)} \cdot T_{1/2} \approx 2 \text{ days} \tag{8.11}$$

The neutron activation is a way to produce radionuclides for medical, research or other purposes.

Task 3.

The sample of molybdenum-99 is purified from decay products. How long does it take for the ^{99m}Tc to reach its maximum activity? The half-lives: $T^{(1)}_{1/2} = 2.8$ days for ^{90}Mo, $T^{(2)}_{1/2} = 6.6$ h for ^{99m}Tc, $T^{(3)}_{1/2} = 2.1 \cdot 10^5$ years for ^{99}Tc.

Solution

In this task, very much like in the 1st one we have accumulation of the daughter radionuclide and then its decay.

In the case of transition equilibrium the activity of the daughter radionuclide:

$$A_2 = \frac{\lambda_2 \cdot A_{1,0}}{\lambda_2 - \lambda_1} \cdot (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \tag{8.12}$$

where index 1 is related to parental (mother) radionuclide, 2 - to the daughter radionuclide.

Let's take $C = \frac{\lambda_2 \cdot A_{1,0}}{\lambda_2 - \lambda_1}$. To find the moment of time when the activity reaches its maximum we should differentiate (8.12) and take it equal to 0:

$$\frac{dA}{dt} = C \cdot (-\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t}) = 0 \tag{8.13}$$

Therefore:

$$\lambda_1 e^{\lambda_1 t} = \lambda_2 e^{-\lambda_2 t} \Rightarrow e^{(\lambda_2 - \lambda_1)t} = \frac{\lambda_2}{\lambda_1} \Rightarrow t_{max} = \frac{\ln(\frac{\lambda_2}{\lambda_1})}{\lambda_2 - \lambda_1} \tag{8.14}$$

Or in terms of half-lives:

$$t_{max} = \frac{\ln(\frac{T_{1/2}^{(1)}}{T_{1/2}^{(2)}})}{\ln(2)} \cdot \frac{T_{1/2}^{(1)} \cdot T_{1/2}^{(2)}}{T_{1/2}^{(1)} - T_{1/2}^{(2)}} = 24.5 \text{ hours} \tag{8.15}$$

After 24.5 hours we can elute the accumulated radionuclide, so the activity will be 0, then it starts to accumulate again and we can repeat the elution. That is the scheme of using ^{99}Mo for production of ^{99}Tc in a radionuclide generator (fig.8.4).

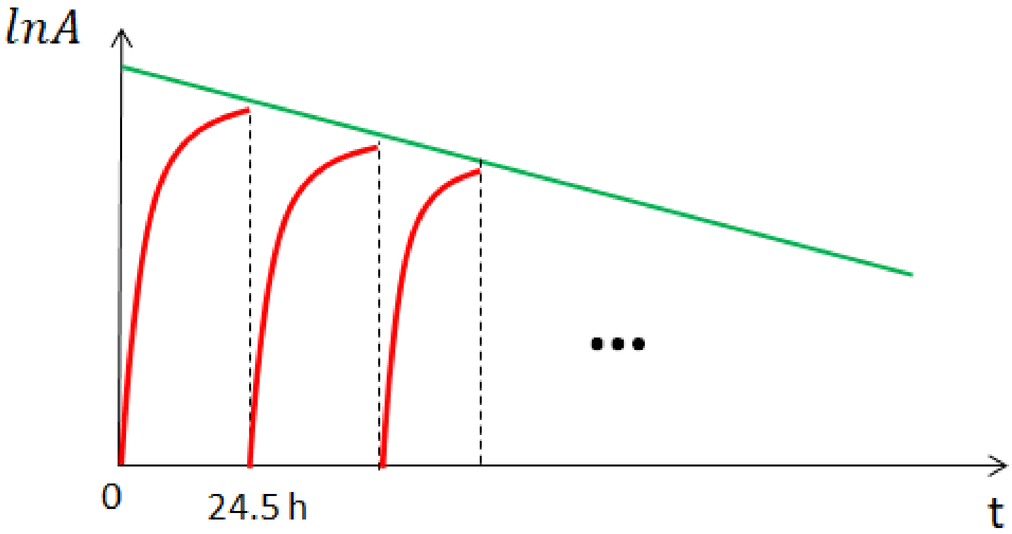


Figure 8.4. The elution of the daughter radionuclide

Task 4.

Into V_0 =50 ml of a $C_0 = 10^{-7}$ mol/L $Sr(NO_3)_2$ solution was added $V_1 = 1$ mL of $^{89}Sr(NO_3)_2$ solution (without carrier) with a specific activity $a = 47.7$ MBq/mL. Determine the activity of the solution after monomolecular adsorption on glass with area $S = 10$ cm^2. The area occupied by the hydrated ion assumed to be $S_1 = 5 \cdot 10^{-19}$ m^2.

Solution

First of all, we estimate the number of atoms that will be adsorbed by the glass:

$$N_{ads} = \frac{S}{S_1} = 2 \cdot 10^{15} \text{ atoms} \tag{8.16}$$

The amount of the adsorbed atoms:

$$n_{ads} = \frac{N_{ads}}{N_A} = 3.3 \cdot 10^{-9} \text{ mol} \tag{8.17}$$

The amount of strontium in our initial solution:

$$n_0 = C_0 \cdot V_0 = 5 \cdot 10^{-9} \text{ mol} \tag{8.18}$$

The initial activity of $^{89}Sr(NO_3)_2$:

$$A_0 = a \cdot V_1 = 47.7 \text{ MBq} \tag{8.19}$$

The activity after the adsorption will be proportional to the amount of atoms remaining in the solution:

$$A = \frac{(n_0 - n_{ads})}{n_0} \cdot A_0 = 16.2 \text{ MBq} \tag{8.20}$$

We see that the activity decreases due to the adsorption in the walls of our vessel.

The half-life of ^{89}Sr: $T_{1/2} = 50.5$ days. Let's assume that there is no stable strontium and calculate the number of ^{89}Sr atoms:

$$A = \lambda N \Rightarrow N = \frac{A}{\ln(2)} \cdot T_{1/2} = 3 \cdot 10^{14} \text{ atoms} \tag{8.21}$$

It means that all of them would be adsorbed in the glass if there were no stable strontium.

Task 5.

Determine maximum activity of ^{95}Zr ($\lambda = 1.25 \cdot 10^{-7}$ s^{-1}) can be adsorbed by silica gel with a specific surface area $S = 80$ m^2/g if the monomolecular layer of adsorbed ions occupies 40% of the surface, and the area of an ion containing one zirconium atom on the surface of the adsorbent is approximately $S_1 = 5 \cdot 10^{-15}$ cm^2.

Solution

The number of atoms that can be adsorbed per 1 gram of the silica gel:

$$N = \frac{S \cdot 0.4}{S_1} = 6.4 \cdot 10^{19} \text{ atoms/g} \tag{8.22}$$

The specific activity:

$$a = \lambda N = 8 \cdot 10^{12}\ \text{Bq/g} \tag{8.23}$$

That's quite a high value. One of the ways to deliver radio-pharmaceuticals to internal organs is the use of labeled nanoparticles. We can calculate the specific activity of nanoparticles and determine the dose that will be accumulated.

Task 6.

Estimate what dose a person receives annually from internal exposure to ^{40}K. The potassium content in the body is $\omega_K = 0.25$ wt.%, the ^{40}K content in the natural mixture of isotopes is $\omega_{^{40}K}$ =0.0118 wt.%, the half-life of ^{40}K is $1.3 \cdot 10^9$ years.

1) β-decay: $p = 89\%$, maximum energy of β-particles $E_{\beta,max} = 1312$ keV

2) electron capture: $p = 11\%$, energy of of accompanying y-quanta $E_\gamma = 1460$ KeV

wt.% stands for weight percent.

Solution

To estimate the dose accumulated in the body we have to add the energies released in the processes of β- and γ-decays. But since the probability of the electron capture in which γ-quanta are emitted is just 11 % and γ-quanta have high penetration ability $\Rightarrow$ they hardly deposit any energy inside the body, we can neglect the contribution of the γ-quanta.

$$D = D_\beta + \underbrace{D_\gamma}_{\to 0} = D_\beta \tag{8.24}$$

$$D_\beta = \frac{0.4 \cdot E_{\beta,max} A \cdot p \cdot t}{m(body)} \tag{8.25}$$

where p is the yield of β-particles, $t = 1$ year, $m(body)$ is a mass of a human body.

$$A = \lambda N = \frac{\ln(2)}{T_{1/2}} \cdot n(^{40}\text{K}) \cdot N_A = \frac{\ln(2) N_A}{T_{1/2}} \cdot \frac{m(^{40}\text{K})}{\mu_r} = \frac{\ln(2) N_A}{T_{1/2}} \cdot \frac{m(^{40}\text{K})}{\mu_r} \tag{8.26}$$

where μ_r is the molar mass of ^{40}K

$$m(^{40}\text{K}) = m(\text{K}) \cdot \omega_{^{40}K} = m(body) \cdot \omega_k \cdot \omega_{^{40}K} \tag{8.27}$$

As a result:

$$D_\beta = 1.77 \cdot 10^{-4}\ \text{Gy} \tag{8.28}$$

The equivalent dose will be:

$$H = 0.177\ \text{mSv/year} \tag{8.29}$$

9. Seminar 9. Isotopic exchange

Suppose we have a acid molecules with stable and radioactive iodine. They can exchange it like this:

$$HI + HI^* \leftrightarrows HI^* + HI \tag{9.1}$$

or another example:

$$HI + NaI^* \leftrightarrows HI^* + NaI \tag{9.2}$$

Such processes are crucial for radio-pharmaceuticals: the radioactive compound in medicine can be exchanged for stable compounds in the body and thereby dilute the specific activity.

The equations (9.1) and (9.2) can be generalized:

$$AX + BX^* \leftrightarrows AX^* + BX \tag{9.3}$$

The thermodynamic constant for this reaction:

$$k = \frac{a_{AX^*} \cdot a_{BX}}{a_{AX} \cdot a_{BX^*}} \tag{9.4}$$

where a_i are activities of corresponding compounds.

At thermodynamic equilibrium the specific activity will distribute between all compounds homogeneously $\Rightarrow$ the specific activity of each compound will be the same.

Suppose we know the total amount of activity A_{tot}, and total number of X: $N_X = N_{AX} + N_{BX^*} = N_{AX^*} + N_{BX}$, then the specific activity of X:

$$a_X = \frac{A_{tot}}{N_X} \tag{9.5}$$

But a system needs some time to reach the state of thermodynamic equilibrium and before that time the distribution of specific activity is not homogeneous. In this case we use the **degree of isotopic exchange** F:

$$F = \frac{a_t - a_0}{a_\infty - a_0} \tag{9.6}$$

where a_t is the activity of the compound at the time t, a_0 - its initial activity, a_∞ - its equilibrium activity.

Task 1.

What is the minimum volume V of $Ca^{131}I_2$ solution (without a carrier, specific activity $a = 200$ Bq/ml) in order to obtain $n_{C_2H_5I}$ =0.1 mol C_2H_5I labeled with ^{131}I with a specific activity $a_{C_2H_5I}$ =5000 Bq/mol using isotope exchange?

Solution

The specific activity of iodine:

$$a_I = \frac{A_{tot}(I)}{n(C_2H_5I) + 2n(CaI_2)} \tag{9.7}$$

Due to the short half-life of ^{131}I:

$$n(CaI_2) \ll n(C_2H_5I) \tag{9.8}$$

So we can neglect the second term in the denominator of (9.7):

$$a_I = \frac{A_{tot}(I)}{n(C_2H_5I)} \tag{9.9}$$

It is said that there is no carrier (no CaI with the stable iodine) ⇒ all the activity will go to the main component - C_2H_5I.

The total activity that we add to the system:

$$A_{tot} = a_{C_2H_5I} \cdot n_{C_2H_5I} = 500 \text{ Bq} \tag{9.10}$$

So the volume:

$$V = \frac{A_{tot}}{a} = 2.5 \text{ ml} \tag{9.11}$$

Task 2.

In a separating funnel, V =50 ml of C_1 = 0.03 mol/L solution of 1,3-diiodopropane in benzene and V = 50 ml of C =0.04 mol/L Na^{131}I solution in water with a volumetric activity a =500 Bq/ml are stirred. After 2 hours, the specific activity of $C_3H_6I_2$ became equal to a_1 = 150 Bq/ml. Determine the degree of isotope exchange.

Solution

In this task there is a carrier of stable iodine in our system. We need to find the degree of isotopic exchange:

$$F = \frac{a_t - a_0}{a_\infty - a_0} \tag{9.12}$$

But $a_0 = 0$, therefore:

$$F = \frac{a_t}{a_\infty} \tag{9.13}$$

Let's calculate the equilibrium activity of diiodopropane a_∞. The specific activity of iodine at equilibrium:

$$a_{\infty I} = \frac{A_{tot}}{n(NaI) + 2n(C_3H_6I_2)} = \frac{a \cdot V}{C \cdot V + 2 \cdot C_1 \cdot V} = 5 \cdot 10^6 \text{ Bq/mol} \tag{9.14}$$

The specific activity of diiodopropane after reaching the equilibrium:

$$a(C_3H_6I_2)_\infty = a_{\infty I} \cdot 2n(C_3H_6I_2) = a_{\infty I} \cdot 2C_1V = 1.5 \cdot 10^4 \text{ Bq} \tag{9.15}$$

Activity at after 2 hours:

$$a_t = a_1 \cdot V = 7500 \text{ Bq} \tag{9.16}$$

So the answer:

$$F = \frac{a_t}{(C_3H_6I_2)_\infty} = 0.5 \tag{9.17}$$

Task 3.

Stirring of an aqueous solution of $Sr^{82}Br_2$ (without a carrier) and a 0.1 mol/L solution of C_2H_5Br in benzene for $t = 9$ h resulted in the decrease of activity of the aqueous phase from a_0 =3000 to $a_t = 500$ Bq. Find the degree of isotope exchange. The half-life of ^{82}Br is $T_{1/2} = 36$ hours.

Solution

As far as there is no carrier, amount of $Sr^{82}Br_2$ is small ⇒ at equilibrium the activity should be equal to almost 0 Bq ⇒ in the expression for the degree of isotopic exchange (9.6) $a_\infty = 0$. We would think that the expression looks like this

$$F_? = \frac{a_0 - a_t}{a_0} = 0.83 \tag{9.18}$$

But we need to re-calculate a_0 considering the decay of ^{82}Br:

$$a_0' = a_0 \cdot e^{-\lambda t} = a_0 \cdot e^{-\frac{\ln(2)}{T_{1/2}} \cdot t} = 2523 \text{ Bq} \tag{9.19}$$

So the correct expression:

$$F = \frac{a_0' - a_t}{a_0'} = 0.8 \tag{9.20}$$

The results of (9.18) and (9.20) are fairly close, but if the time of stirring had been longer, we would have got absolutely different answers.

Task 4.

The reactor of a volume of $V = 50$ ml with barium carbonate (m =296 mg) labeled with ^{14}C was filled with carbon dioxide (T =24 °C, p =99 kPa), hermetically closed, and kept for several hours at 100 °C. Determine the degree of isotope exchange if it is established that the specific activity of $Ba^{14}C0_3$ has decreased by 1.25 times during heating.

Solution

Knowing the mass of barium carbonate allows us to find its amount in the system:

$$n(BaCO_3) = \frac{m}{\mu_r} = 1.5 \cdot 10^{-3} \text{ mol} \tag{9.21}$$

μ_r is the molar mass of $Ba^{14}C0_3$.

We know temperature and pressure in the reactor ⇒ we can find the amount of CO_2 gas using the famous Clapeyron-Mendeleev equation:

$$pV = nRT \Rightarrow n(CO_2) = \frac{pV}{RT} = 2 \cdot 10^{-3} \text{ mol} \tag{9.22}$$

The activity at the moment t:

$$a_t = \frac{a_0}{1.25} = \frac{4}{5} a_0 \tag{9.23}$$

The activity of $Ba^{14}C0_3$ at equilibrium:

$$a_\infty = a_0 \cdot \frac{n(Ba^{14}C0_3)}{n(Ba^{14}C0_3) + n(CO_2)} = \frac{3}{7}a_0 \tag{9.24}$$

As a result:

$$F = \frac{\frac{4}{5}a_0 - a_0}{\frac{3}{7}a_0 - a_0} = 0.35 \tag{9.25}$$

Task 5.

In a separating funnel, V =50 ml of $C_1 = 0.03$ mol/L solution of 1,3-diiodopropane in benzene and $V = 50$ ml of C =0.04 mol/L $Na^{131}I$ solution in water with a volumetric activity a =500 Bq/ml are stirred. After 2 hours, the specific activity of $C_3H_6I_2$ became equal to $a_1 = 150$ Bq/ml. Determine the degree of isotope exchange **after 8 hours**.

Solution

Many reactions of isotope exchange depend on 1 component: usually the amount of radionuclide is very small ⇒ we can neglect the presence of stable atoms. The equation for the rate of chemical reaction v:

$$v = kC_a^\alpha C_b^\beta \tag{9.26}$$

C_a, C_b are concentrations, α, $\beta \in \mathcal{N}$ - order of the reaction, k is constant.

So if $C_a \ll C_b \Rightarrow$ reactions will not cause significant changes in $C_b \Rightarrow C_b \approx const$. Assume we have the 1st order reaction:

$$v = k'C_a^\alpha = k'C_a^1 \tag{9.27}$$

By definition the rate of a chemical reaction v:

$$v = -\frac{dC_a}{dt} = k'C_a \tag{9.28}$$

The solution of such differential equation is given by the exponent, therefore we can write the degree of isotope exchange as:

$$F = 1 - e^{-kt} \tag{9.29}$$

This expression is very similar to the accumulation of activity of daughter radionuclide in the case of secular equilibrium:

$$A_2 = A_{1,0} \cdot (1 - e^{-\lambda t}) \tag{9.30}$$

But here λ is the decay constant, k in the previous equation is the isotope exchange constant.

The answer to the Task 2 is $F = 0.5$, the the isotope exchange constant k:

$$k = \frac{\ln(2)}{t_{0.5}} \tag{9.31}$$

where $t_{0.5}$ is the time required for F to become equal to 0.5 (in the Task 2 $t_{0.5} = 2$ hours).

Therefore, the answer to this task:

$$F = 1 - 2^{-\frac{t}{t_{0.5}}} = 1 - 2^{-\frac{8}{2}} = \frac{15}{16} \approx 0.94 \tag{9.32}$$

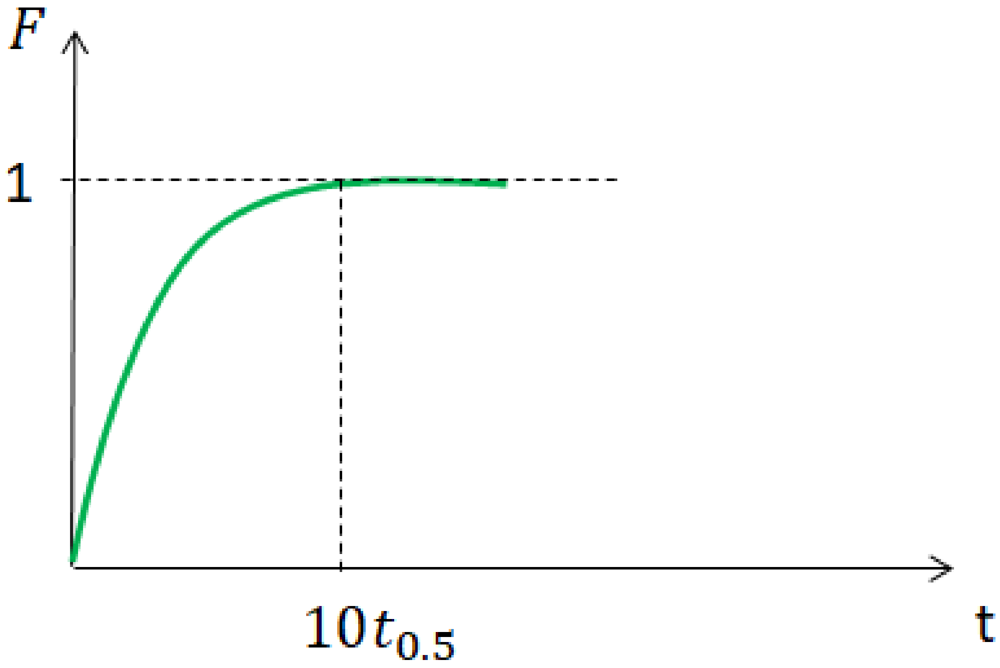


Figure 9.1. We reach the equilibrium for isotope exchange after ~ 20 hours

10. Seminar 10. Solving the tasks: the overview.

This seminar concludes the course and is dedicated to solving tasks from different topics. We kick things off with the law of radioactive decay.

Task 1.

Determine the specific activity of rubidium fluoride. The natural mixture of rubidium isotopes contains 72. 2% of stable ^{85}Rb and 27. 8% of radioactive ^{87}Rb.

The half-life of ^{87}Rb is $4.88 \cdot 10^{10}$ years.

Solution

The specific activity:

$$a = \frac{A}{m} \tag{10.1}$$

The activity depends on the amount of rubidium fluoride atoms (RbF):

$$A = \lambda N \tag{10.2}$$

The number of Rb atoms:

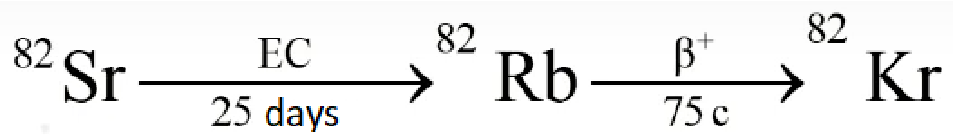
$$N = n \cdot N_A = \frac{m}{\mu_r} \cdot N_A \tag{10.3}$$

where μ_r is the molar mass of RbF

So we put (10.3) → (10.2) → (10.1) and get:

$$a = \frac{\lambda N_A}{\mu_r} = \frac{\ln(2) \cdot N_A}{T_{1/2} \cdot \mu_r} = 2607 \text{ Bq/g} \tag{10.4}$$

Task 2.



$$^{82}\text{Sr} \xrightarrow[\text{25 days}]{\text{EC}} {}^{82}\text{Rb} \xrightarrow[\text{75 c}]{\beta^+} {}^{82}\text{Kr}$$

Figure 10.1. Decay scheme of ^{82}Sr

The ^{82}Sr sample with a radioactivity $A_{1,0} = 50$ MBq was purified of decay products. Determine:

a) what is the minimum time when it will accumulate $A' = 35$ MBq of ^{82}Rb;

b) radioactivity of ^{82}Rb after 50 days;

c) the ratio of the numbers of atoms N(^{82}Sr), N(^{82}Rb) in the preparation after 65 days;

d) after what time the total activity of the sample will be 25 MBq?

Solution

It is the case of secular equilibrium: the half-life of the mother radionuclide is much longer than the daughter's one. Let's calculate the accumulation of the activity A_2 daughter radionuclide:

$$A_2 = A_{1,0} \cdot (1 - e^{-\lambda_2 t}) \tag{10.5}$$

When the equilibrium is reached the activities of the two radionuclides become equal:

$$A_2 = A_1 \tag{10.6}$$

a) We find the time using (10.5):

$$e^{-\lambda_2 t} = 1 - \frac{A'}{A_{1,0}} \Rightarrow t = -\frac{\ln(1 - \frac{A'}{A_{1,0}})}{\ln(2)} \cdot T_{1/2}^{Rb} = 130.3 \text{ s} \tag{10.7}$$

Such Sr-Rb chain is used in generators for production of ^{82}Rb which plays important role in radiomedicine.

b) After 50 days the equilibrium is already reached - since 50 days is much more than $10 \cdot T_{1/2}^{Rb} \Rightarrow$ we can use (10.6): the activities of Sr and Rb are equal. Since 50 days $= 2 \cdot T_{1/2}^{Sr}$ the activity of ^{82}Sr ($A_{1,t}$) will decrease 4 times according to the law of radioactive decay:

$$A_{1,t} = \frac{1}{4} A_{1,0} = 12.5 \text{ MBq} = A(^{82}Rb) \tag{10.8}$$

c) Again, after 65 days there still will be equilibrium:

$$A_2 = A_1 \Leftrightarrow \lambda_2 N_2 \cdot \lambda_1 N_1 \tag{10.9}$$

Therefore the ratio:

$$\frac{N_1}{N_2} = \frac{N(^{82}Sr)}{N(^{82}Rb)} = \frac{T_{1/2}^{Sr}}{T_{1/2}^{Rb}} = 28800 \tag{10.10}$$

d) The total activity is the sum of Sr and Rb activities. Soon enough their activities become equal (equilibrium), therefore we need to determine time after which each activity is equal to 12.5 MBq (so that they sum up to 25 MBq). From the point b) we know that it happens after 50 days (see expression (10.8)).

Task 3.

Two thin scandium samples weighing m =150 mg were irradiated with a neutron flux (density $\Phi = 2 \cdot 10^{10}$ cm^{-2} s^{-1}) for t_1 =30 days and t_2 =100 days. What will be the activity of each sample immediately after the end of the irradiation? Scandium is a mononuclide element, the cross section for thermal neutron absorption by ^{45}Sc nuclei is $\sigma = 27.2 \cdot 10^{-24}$ cm2, the half-life of ^{46}Sc is $T_{1/2} = 83.8$ days.

Solution

The activity of a radionuclide after being irradiated by neutrons:

$$A = \Phi \sigma N (1 - e^{-\lambda t}) \tag{10.11}$$

N is the number of atoms converted into radionuclides.

The activity of the first sample will be:

$$A_1 = \Phi \sigma \frac{m N_A}{\mu_r} \cdot (1 - e^{\frac{\ln(2)}{T_{1/2}} \cdot t_1}) = 2.4 \cdot 10^8 \text{ Bq} \tag{10.12}$$

where μ_r is the molar mass of Sc.

And similar for the second target:

$$A_2 = \Phi \sigma \frac{m N_A}{\mu_r} \cdot (1 - e^{\frac{\ln(2)}{T_{1/2}} \cdot t_2}) = 6.1 \cdot 10^8 \text{ Bq} \tag{10.13}$$

Task 4.

The maximum range of β-radiation of the radionuclide *X in water is R_{max} =800 mg/cm^2. What layer of water (in cm) will attenuate the parallel flow of such particles by 10%?

Solution We use the fact that the attenuation coefficient should be equal to 100% - 10% = 90% = 0.9:

$$0.9 = k = e^{-\mu d} \Rightarrow d = -\frac{\ln(0.9)}{\mu} = 0.018 \text{ g/cm}^2 \tag{10.14}$$

where $\mu = 5.8$ cm^2/g is the mass attenuation coefficient.

The needed linear thickness:

$$l = \frac{d}{\rho} = 0.018 \text{ cm} \tag{10.15}$$

where $\rho = 1$ g/cm^3 is the density of water.

Task 5.

Parallel flows of α—particles, monoenergetic electrons and β- particles (each $\Phi = 3000$ s^{-1}) fall on an aluminum screen $l = 0.3$ mm in thickness. The density of aluminum is $\rho = 2.7$ g/cm^3. The energy of particles of each type (in the case of β-particles the maximum energy of the spectrum) is $E = 2.0$ Mev. Calculate the particle flux behind the screen. The maximum range of electrons in aluminum is equal to $d_{max} = 950$ mg/cm^2.

Solution

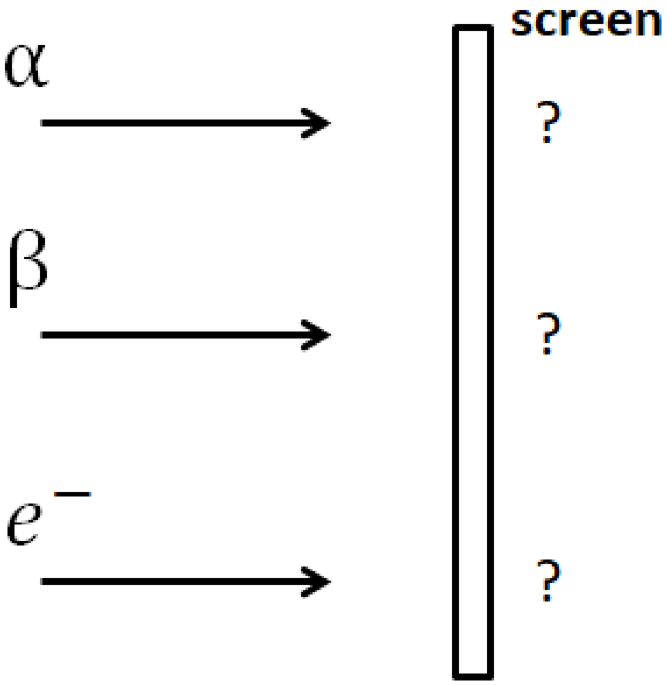


Figure 10.2. A picture to the Task 5.

Since α-particles have penetration range of tens maybe a hundred μm we can say for sure that none of them will penetrate the given screen $\Rightarrow$ the flux of α-particles $F_\alpha = 0$.

For monoenergetic electrons the attenuation coefficient is given by:

$$k_e = 1 - \frac{d}{d_{max}} = 1 - \frac{l \cdot \rho}{d_{max}} = 0.915 \tag{10.16}$$

So the flux of monoenergetic electrons:

$$F_e = \Phi \cdot k_e \cdot = 2745 \text{ s}^{-1} \tag{10.17}$$

For β-particles we use the following expression for the attenuation coefficient:

$$k_\beta = e^{-\mu d} = e^{-\mu l \rho} = 0.667 \tag{10.18}$$

where $\mu = 5$ cm^2/g is the mass attenuation coefficient.

So the flux of β-particles:

$$F_\beta = \Phi \cdot k_\beta = 2001 \text{ s}^{-1} \tag{10.19}$$

Task 6.

An employee of category A carries out daily work throughout the year with a closed source of ^{60}Co radiation with a radioactivity A =175 MBq. Calculate:

A) the distance r_1 from the source at which you can work t_1 6 hours a day;

B) operating time t_2 at a distance of r_2 =0.5 m;

C) the thickness of the lead screen for daily work for $t_1 = 6$ hours at a distance r_2 50 cm.

The KERMA constant $K_\gamma = 84.6$ aGy m^{2}Bq^{-1}s^{-1}

Solution

The absorbed dose depends on the distance r and time of exposure t like this:

$$D = K_\gamma \frac{A}{r^2} \cdot t \tag{10.20}$$

The dose rate:

$$P = \frac{D}{t} = K_\gamma \frac{A}{r^2} \tag{10.21}$$

If we want to re-calculate it into the equivalent dose rate, we have to multiply it by 1.09:

$$P_H = 1.09 \cdot K_\gamma \frac{A}{r^2} = \frac{5.76 \cdot 10^{-5}}{r^2}\ \text{h}^{-1} \tag{10.22}$$

The allowed P_H should not exceed $12 \cdot 10^{-6}$ h^{-1}, therefore:

$$r_1 = \sqrt{\frac{5.76 \cdot 10^{-5}}{12 \cdot 10^{-6}}} = 2.2\ \text{m} \tag{10.23}$$

We know the time t_1 of safe operation at distance r_1, therefore the time t_2 of safe operation at distance r_2:

$$\frac{t_1}{t_2} = \frac{r_1^2}{r_2^2} \Rightarrow t_2 = t_1 \cdot \frac{r_2^2}{r_1^2} = 18.6\ \text{min} \tag{10.24}$$

So we would need to work really quickly. We can also use a lead screen for protection.

We should decrease flux by $k = 2 \cdot (\frac{r_1}{r_2})^2 \approx 40$ times to ensure safe operation. using the table in fig.6.4. The maximum energy of γ-rays for ^{60}Co is equal to 1333 KeV. So the thickness of the lead screen will be: $d \approx 7.3$ cm.

Task 7.

For radionuclide diagnostics in a PET center, a patient weighing $m = 95$ kg was injected with 2-[^{18}F]-fluoro-2-deoxy-D- glucose ([^{18}F]-FDG) with a radioactivity A = 0.24 GBq. Determine the equivalent dose to the patient if the biological half-life of FDG is $T_{1/2}^{bio} = 90$ minutes, the half- life of ^{18}F is $T_{1/2}^{r} = 110$ minutes. In decay, β^+ particles are emitted with a probability of p =97%, $E_{\beta,max} = 0.635$ Mev, and electron capture (EC) occurs with a probability of 3%. We assume a uniform distribution of [^{18}F] FDG throughout the body, all e^+ are absorbed in the body, and p' =50% of the photons leave the body. Neglect the energy release during EC.

The energy of γ-quanta born in the e^+e^- annihilation process is $E_\gamma = 0.511$ MeV.

Solution

The equivalent dose H is related to the absorbed dose D through the weighing factor w_R:

$$H = D \cdot w_R \tag{10.25}$$

in case of β and γ radiation $w_R = 1$.
The absorbed dose is the energy absorbed per kg:

$$D = \frac{E}{m} = \frac{E_\beta + E_\gamma}{m} \tag{10.26}$$

The energy of β-particles per second is:

$$\frac{dE_\beta}{ddt} = A \cdot p \cdot 0.4 \cdot E_{\beta,max} \tag{10.27}$$

For γ-quanta that are born in pairs (hence the factor 2) in e^+e^- annihilation:

$$\frac{dE_\gamma}{ddt} = A \cdot p \cdot 2 \cdot p' \cdot E_\gamma \tag{10.28}$$

The dose rate:

$$P = \frac{dE_\beta}{ddt} + \frac{dE_\gamma}{ddt} = \frac{A \cdot p \cdot (0.4 \cdot E_{\beta,max} + E_\gamma)}{m} \tag{10.29}$$

The total adsorbed dose will be:

$$D = \int\limits_0^\infty \frac{A(t) \cdot p \cdot (0.4 \cdot E_{\beta,max} + E_\gamma)}{m} dt \tag{10.30}$$

where $A(t) = A_0 \cdot e^{-\lambda_{eff} t}$, $\lambda_{eff} = \lambda_r + \lambda_{bio} = \ln(2) \cdot \frac{T^r_{1/2} + T^{bio}_{1/2}}{T^r_{1/2} \cdot T^{bio}_{1/2}}$ is the effective decay constant that takes into account radioactive and biological disintegration.

$$\begin{aligned} D &= \frac{A_0 \cdot p \cdot (0.4 \cdot E_{\beta,max} + E_\gamma)}{m} \int\limits_0^\infty e^{-\lambda_{eff} t} = \\ &= \frac{A_0 \cdot p \cdot (0.4 \cdot E_{\beta,max} + E_\gamma)}{m} \cdot \frac{1}{\lambda_{eff}} \underbrace{e^{-\lambda_{eff} t}\Big|_\infty^0}_{=1} = \\ &= \frac{A_0 \cdot p \cdot (0.4 \cdot E_{\beta,max} + E_\gamma)}{m\lambda_{eff}} \end{aligned}$$

As a result:

$$D = \frac{A_0 \cdot p \cdot (0.4 \cdot E_{\beta,max} + E_\gamma)}{\ln(2) m} \cdot \frac{T^r_{1/2} \cdot T^{bio}_{1/2}}{T^r_{1/2} + T^{bio}_{1/2}} = 1.3 \text{ mSv} \tag{10.31}$$

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