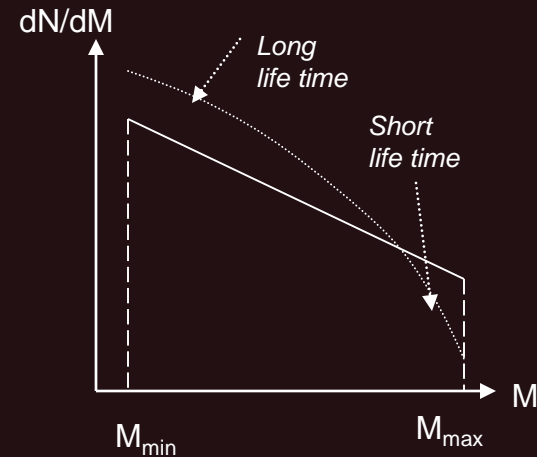


# Isolated Neutron Stars. Intro.

# Stars in the Galaxy



Salpeter (1955) mass function:  
 $dN/dM \sim M^{-2.35}$

There are many modification (Miller-Scalo, Kroupa etc.).  
At high masses the slope is usually steeper.  
Note: it is *initial* mass function, not the present day!

It is possible to estimate the number of NS and BH progenitors.  
Then using their average lifetime we can estimate the birth rate  
and total numbers (with a given age of the Galaxy and assuming constant rate)  
taking into account  $SFR \sim (3-5)$  solar mass per year.  
[see also Ch.1 in Shapiro, Teukolsky]

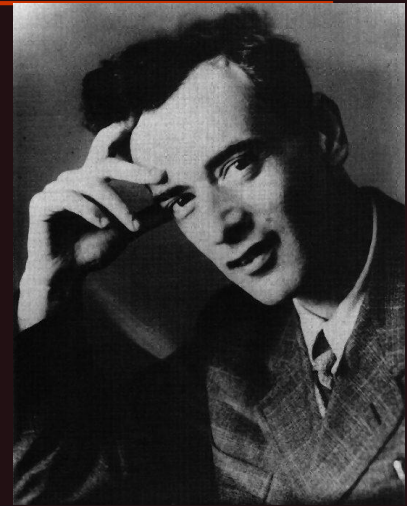


# Prediction ...

Neutron stars have been predicted in 30s:

L.D. Landau: Star-nuclei (1932) + anecdote

Baade and Zwicky:  
neutron stars and supernovae (1934)



(Landau)



(Baade)



(Zwicky)

(from lectures by D. Yakovlev)

In any case, with the discovery of X-ray sources and quasars, dozens of theoreticians focused their attention on the equilibrium properties of compact stars and on star collapse. But in spite of this mounting theoretical effort, most

<sup>2</sup>Baade and Zwicky (1934): “With all reserve we advance the view that supernovae represent the transitions from ordinary stars into *neutron stars*, which in their final stages consist of extremely closely packed neutrons.”

— According to Rosenfeld (1974), on the day that word came to Copenhagen from Cambridge telling of Chadwick’s discovery of the neutron in 1932, he, Bohr, and Landau spent the evening discussing possible implications of the discovery. It was then that Landau suggested the possibility of cold, dense stars composed principally of neutrons. Landau’s only publication on the subject was concerned with neutron cores (Landau, 1938).

<sup>3</sup>Giacconi, Gursky, Paolini, and Rossi (1962).

<sup>4</sup>Chapter 11 is devoted to this subject.

<sup>5</sup>The first QSO identified by Schmidt, 3C273, had a redshift  $\delta\lambda/\lambda = 0.158$ , which was unprecedented for a normal “star.”

<sup>6</sup>Salpeter (1965); in addition to this argument there was strong evidence that quasar redshifts were cosmological in origin.

**Shapiro, Teukolsky (1983)**

(see detailed description in the book by Haensel, Yakovlev, Potekhin and in the e-print arXiv: 1210.0682)

# Landau paper BEFORE neutron discovery

## ON THE THEORY OF STARS.

*By L. Landau.*

(Received 7 January 1932).

From the theoretical point of view the physical nature of Stellar equilibrium is considered.

The astrophysical methods usually applied in attacking the problems of stellar structure are characterised by making physical assumptions chosen only for the sake of mathematical convenience. By this is characterised, for instance, Mr. Milne's proof of the impossibility of a star consisting throughout of classical ideal gas; this proof rests on the assertion that, for arbitrary  $L$  and  $M$ , the fundamental equations of a star consisting of classical ideal gas admit, in general, no regular solution. Mr. Milne seems to have overlooked the fact, that this assertion results only from the assumption of opacity being constant throughout the star, which assumption is made only for mathematical purposes and has nothing to do with reality. Only in the case of this assumption the radius  $R$  disappears from the relation between  $L$ ,  $M$  and  $R$  necessary for regularity of the solution. Any reasonable assumptions about the opacity would lead to a relation between  $L$ ,  $M$  and  $R$ , which relation would be quite exempt from the physical criticisms put forward against Eddington's mass-luminosity-relation.

It seems reasonable to try to attack the problem of stellar structure by methods of theoretical physics, i. e. to investigate the physical nature of stellar equilibrium. For that purpose we must at first investigate the statistical equilibrium of a given mass without generation of energy, the condition for which equilibrium being the minimum of free energy  $F$  (for given temperature). The part of free energy due to gravitation is negative and inversely proportional to some

Physikalische  
Zeitschrift der Sowjetunion  
Vol. 1, No. 2, 285-188, 1932  
Written: Feb. 1931, Zurich  
Received: Jan. 7, 1932  
Published: Feb. 1932

we have no need to suppose that the radiation of stars is due to some mysterious process of mutual annihilation of protons and electrons, which was never observed and has no special reason to occur in stars. Indeed we have always protons and electrons in atomic nuclei very close together, and they do not annihilate themselves; and it would be very strange if the high temperature did help, only because it does something in chemistry (chain reactions!). Following a beautiful idea of Prof. Niels Bohr's we are able to believe that the stellar radiation is due simply to a violation of the law of energy, which law, as Bohr has first pointed out, is no longer valid in the relativistic quantum theory, when the laws of ordinary quantum mechanics break down (as it is experimentally proved by continuous-rays-spectra and also made probable by theoretical considerations).<sup>1</sup> We expect that this must occur when the density of matter becomes so great that atomic nuclei come in close contact, forming one gigantic nucleus.

On these general lines we can try to develop a theory of stellar structure. The central region of the star must consist of a core of highly condensed matter, surrounded by matter in ordinary state. If the transition between these two states were a continuous one, a mass  $M < M_0$  would never form a star, because the normal equilibrium state (i. e. without pathological regions) would be quite stable. Because, as far as we know, it is not the fact, we must conclude that the condensed and non-condensed states are separated by some unstable states in the same manner as a liquid and its vapour are, a property which could be easily explained by some kind of nuclear attraction. This would lead to the existence of a nearly discontinuous boundary between the two states.

The theory of stellar structure founded on the above considerations is yet to be constructed, and only such a theory can show how far they are true.

February 1931, Zurich.

<sup>1</sup> L. Landau und R. Peierls, ZS. f. Phys. 69, 56, 1931.

This is correct!

Disappeared in reprints,  
so we have difficulties



# Baade and Zwicky – theoretical prediction

W. Baade (Mt. Wilson Observatory)  
F. Zwicky (Caltech)

The meeting of American Physical  
Society  
(Stanford, December 15-16, 1933)  
Published in Physical Review  
(January 15, 1934)



38. Supernovae and Cosmic Rays. W. BAADE, *Mt. Wilson Observatory*, AND F. ZWICKY, *California Institute of Technology*.—Supernovae flare up in every stellar system (nebula) once in several centuries. The lifetime of a super-

nova is about twenty days and its absolute brightness a maximum may be as high as  $M_{\text{vis}} = -14^M$ . The visible radiation  $L_v$  of a supernova is about  $10^8$  times the radiation of our sun, that is,  $L_v = 3.78 \times 10^{41}$  ergs/sec. Calculations indicate that the total radiation, visible and invisible, is of the order  $L_\tau = 10^7 L_v = 3.78 \times 10^{48}$  ergs/sec. The supernova therefore emits during its life a total energy  $E_\tau \geq 10^5 L_\tau = 3.78 \times 10^{53}$  ergs. If supernovae initially are quite ordinary stars of mass  $M < 10^{34}$  g,  $E_\tau/c^2$  is of the same order as  $M$  itself. In the *supernova* process *mass in bulk is annihilated*. In addition the hypothesis suggests itself that *cosmic rays are produced by supernovae*. Assuming that in every nebula one supernova occurs every thousand years, the intensity of the cosmic rays to be observed on the earth should be of the order  $\sigma = 2 \times 10^{-3}$  erg/cm<sup>2</sup> sec. The observational values are about  $\sigma = 3 \times 10^{-3}$  erg/cm<sup>2</sup> sec. (Millikan, Regener). With all reserve we advance the view that supernovae represent the transitions from ordinary stars into *neutron stars*, which in their final stages consist of extremely closely packed neutrons.

$t_1 = 10^6$  years + 410 seconds for  $10^{11}$  volt electrons.

$t_2 =$  " + 47.6 days "  $10^9$  " "

$t_3 =$  " + 44 years "  $10^{11}$  " protons.

These time lags  $t_i - t$  would tend to smear out the change of intensity caused by the flare-up of individual supernovae. Dr. R. M. Langer in one of our seminars was the first to call attention to the straggling of simultaneously ejected particles.

#### 5. *The super-nova process*

We have tentatively suggested that the super-nova process represents the transition of an ordinary star into a neutron star. If neutrons are produced on the surface of an ordinary star they will "rain" down towards the center if we assume that the light pressure on neutrons is practically zero. This view explains the speed of the star's transformation into a neutron star. We are fully aware that our suggestion carries with it grave implications regarding the ordinary views about the constitution of stars and therefore will require further careful studies.

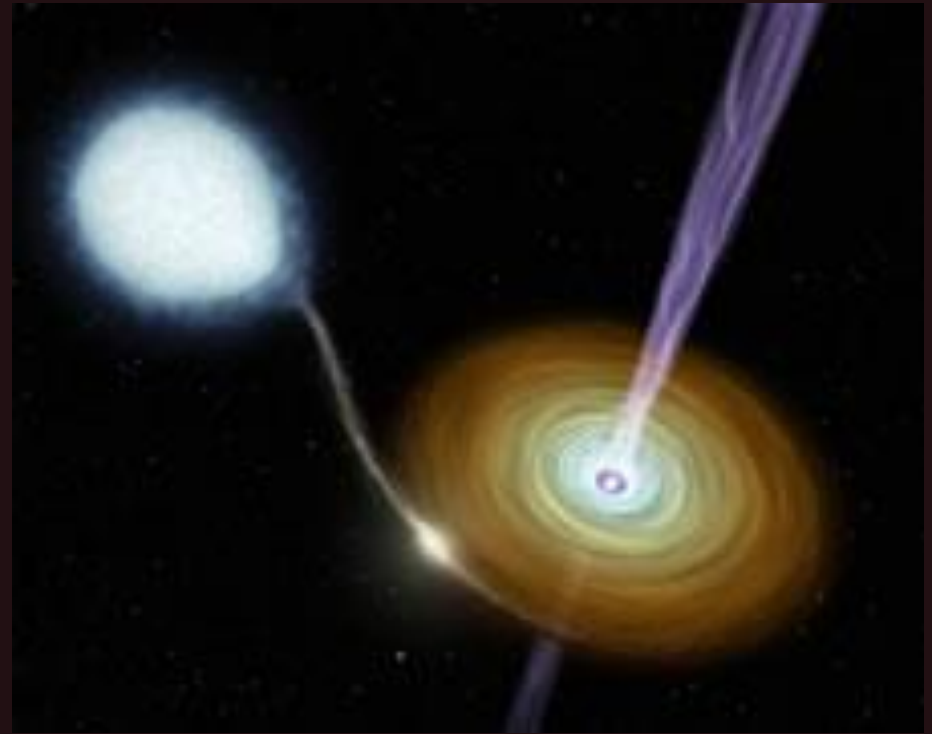
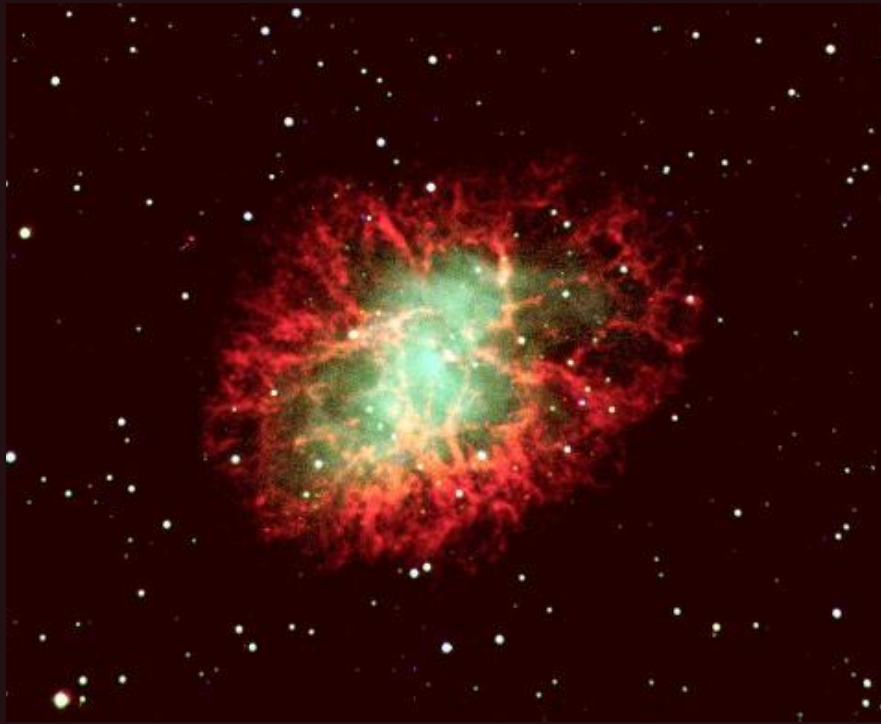
W. BAADE

F. ZWICKY

Mt. Wilson Observatory and  
California Institute of Technology, Pasadena.  
May 28, 1934.

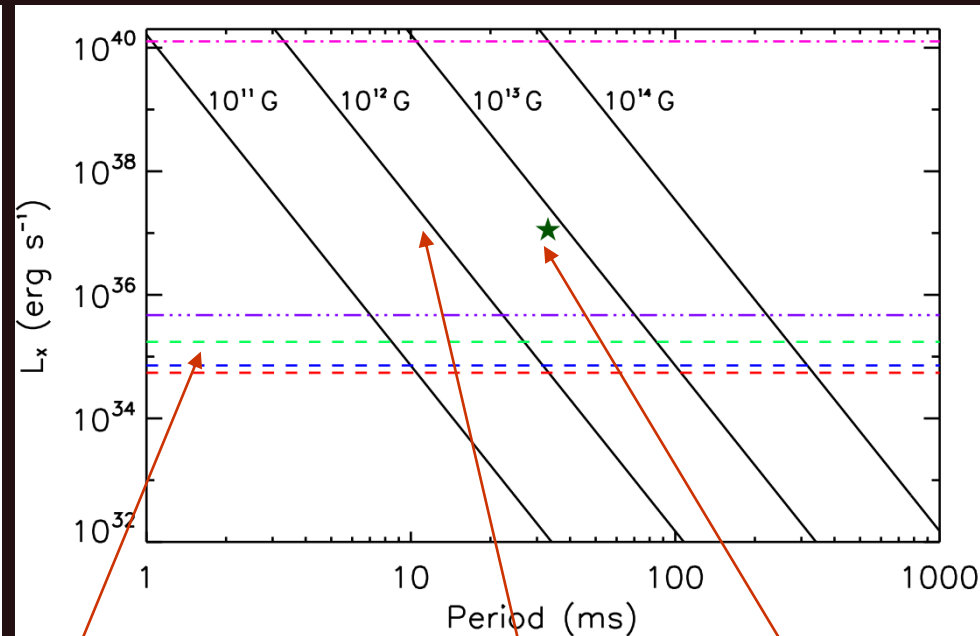
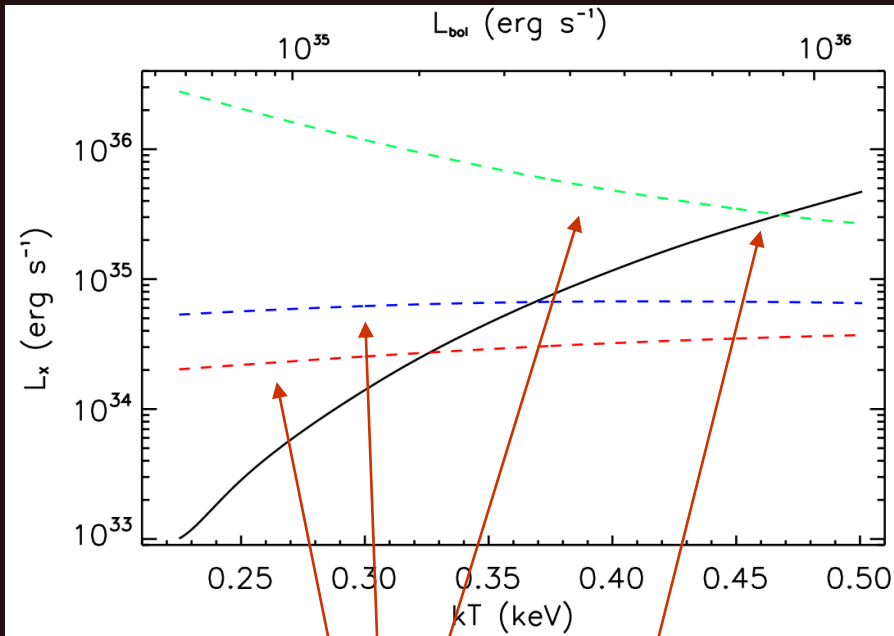
# Good old classics

For years two main types of NSs have been discussed:  
radio pulsars and accreting NSs in close binary systems



*The pulsar in the Crab nebula      A binary system*

# What formed in SN 1987A?



Limits for different  $N_H$ .

2-8 keV luminosity  
for different temperatures

Dashed and  
dot-dashed –  
different limits  
(as on the left)

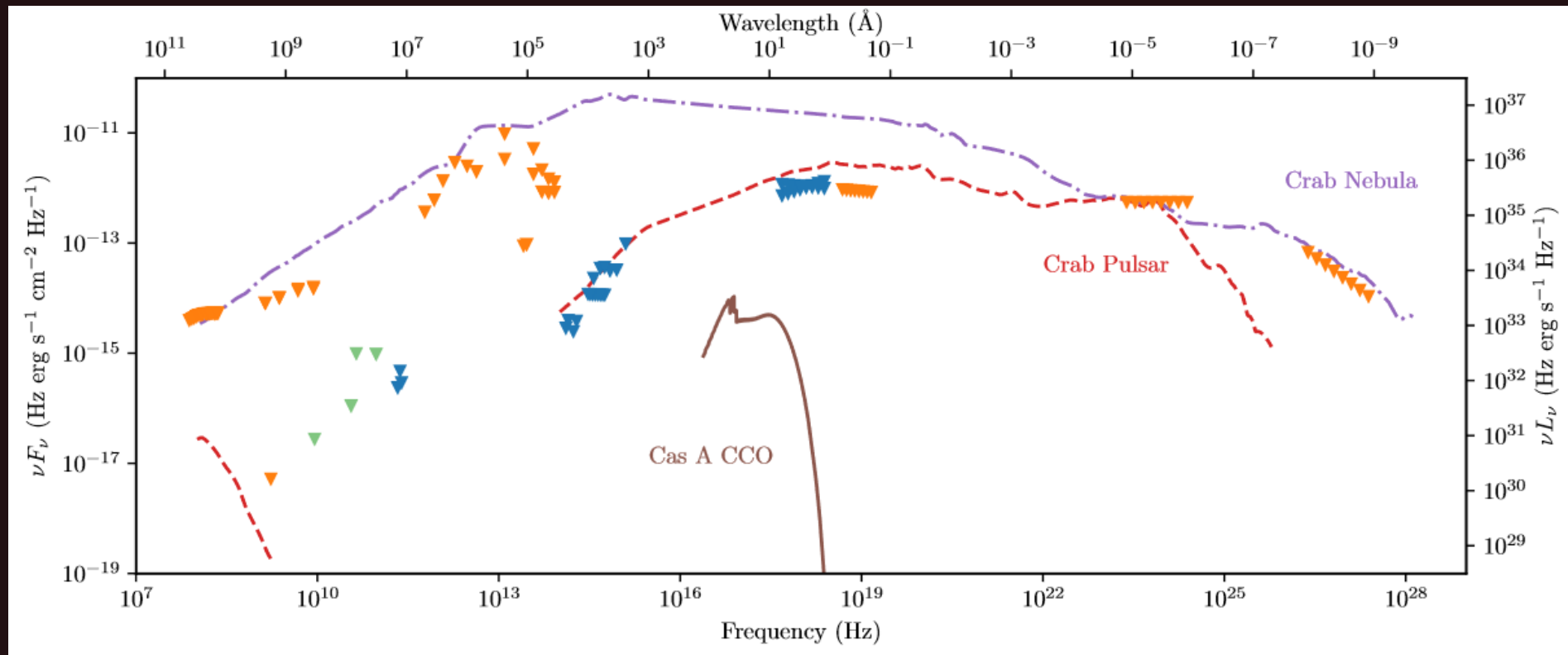
Black solid:  
2-8 keV luminosity  
for different fields.

1803.04692

See also 1805.04526



# More limits on SN1987A

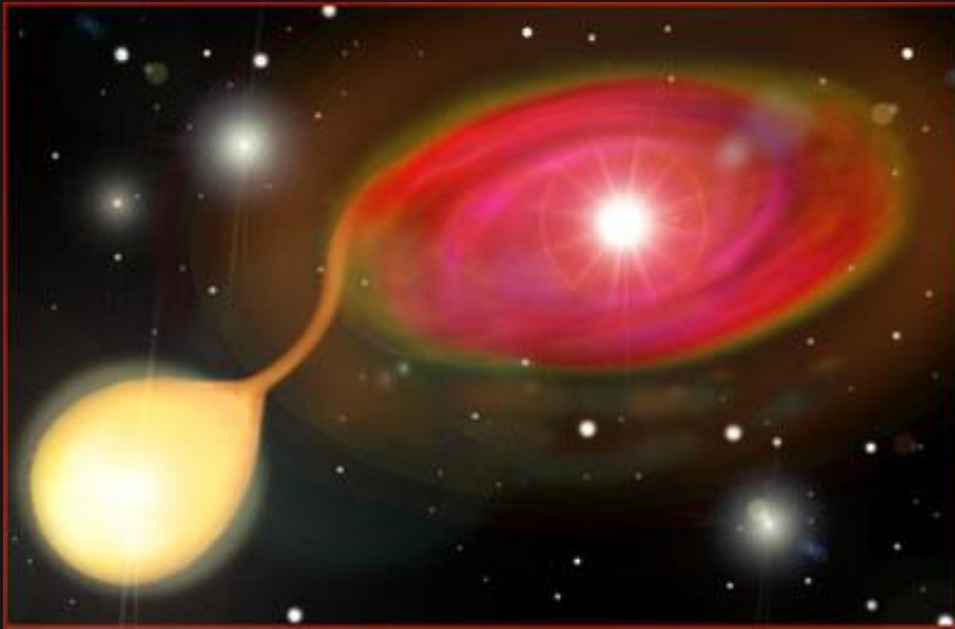


Still, it is possible that a NS is formed in SN 1987A.  
But very energetic pulsars or/and magnetars are mostly excluded  
despite strong uncertainties in absorption.  
About absorption see 1805.04528.

# The old zoo of neutron stars

In 60s the first X-ray sources have been discovered.

They were neutron stars in close binary systems, BUT ...  
.... they were «not recognized»....

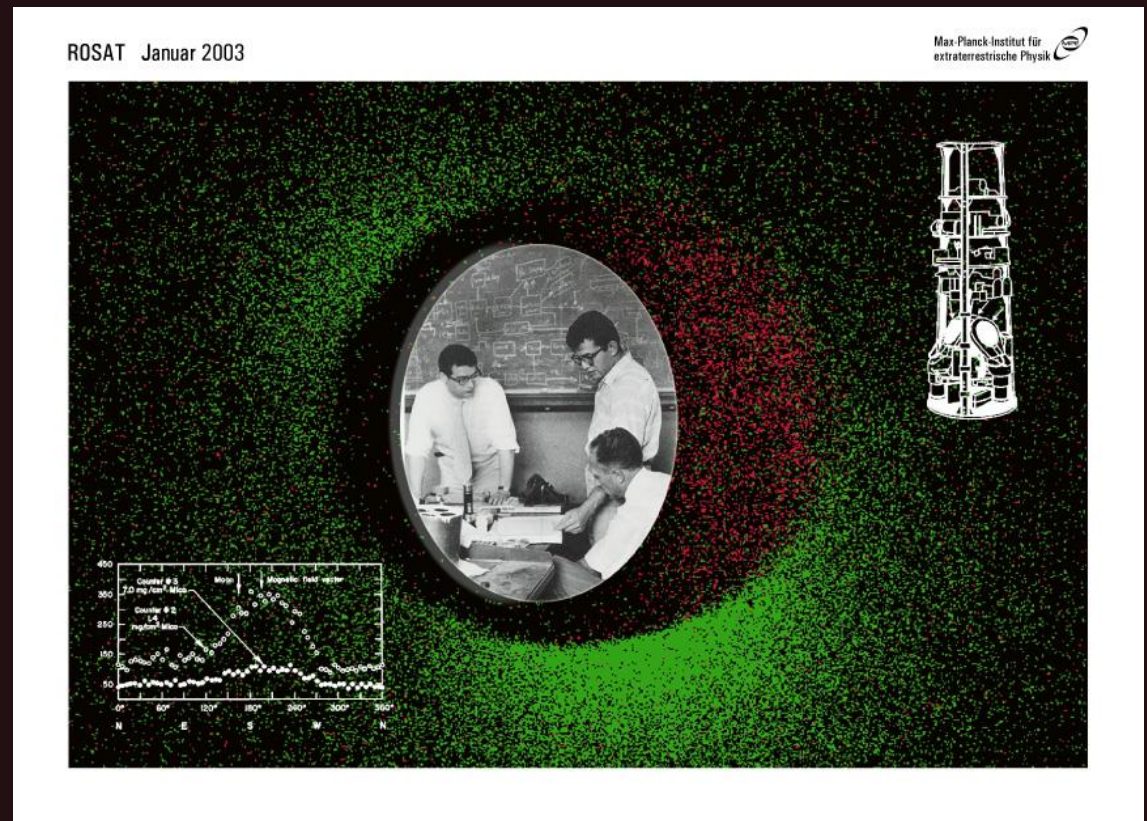


Now we know hundreds of X-ray binaries with neutron stars in the Milky Way and in other galaxies.

# Rocket experiments. Sco X-1

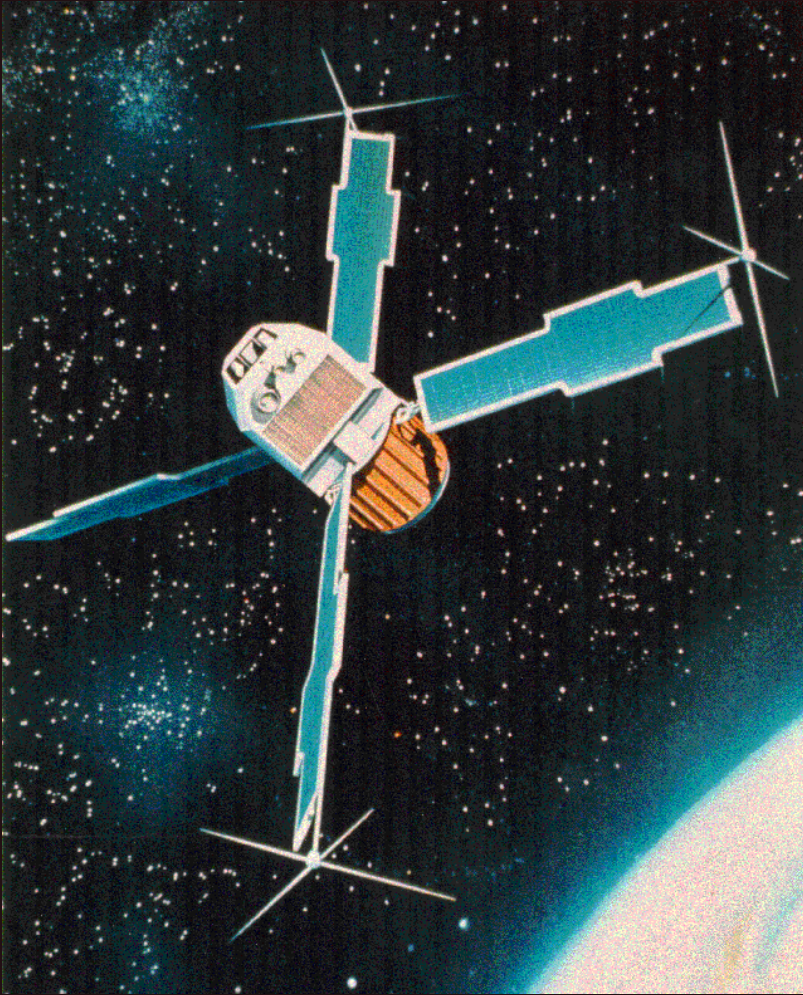
Giacconi et al. 1962

In 2002 R. Giacconi was awarded with the Nobel prize.



On the photo: Giacconi, Gursky, Hendel

# UHURU



The satellite was launched on December 12, 1970.

The program was ended in March 1973.

The other name SAS-1

2-20 keV

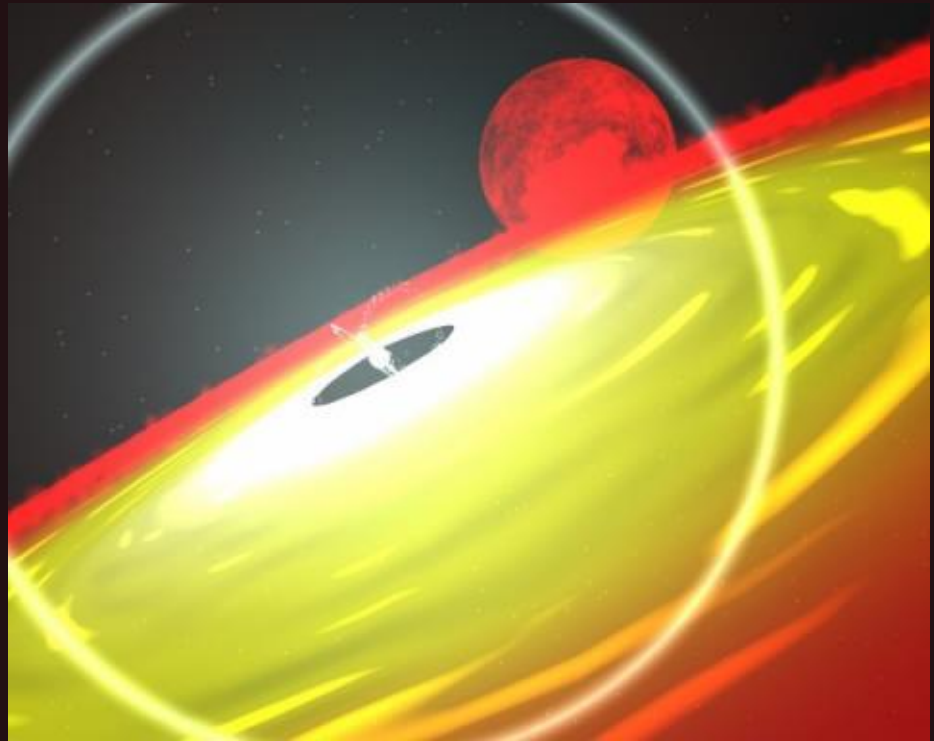
The first full sky survey.  
339 sources.



# Accretion in close binaries

Accretion is the most powerful source of energy realized in Nature, which can give a huge energy output.

When matter fall down onto the surface of a neutron star up to 10% of  $mc^2$  can be released.



# Accretion disc



The theory of accretion discs was developed in 1972-73 by N.I. Shakura and R.A. Sunyaev.

Accretion is important not only in close binaries, but also in active galactic nuclei and many other types of astrophysical sources.

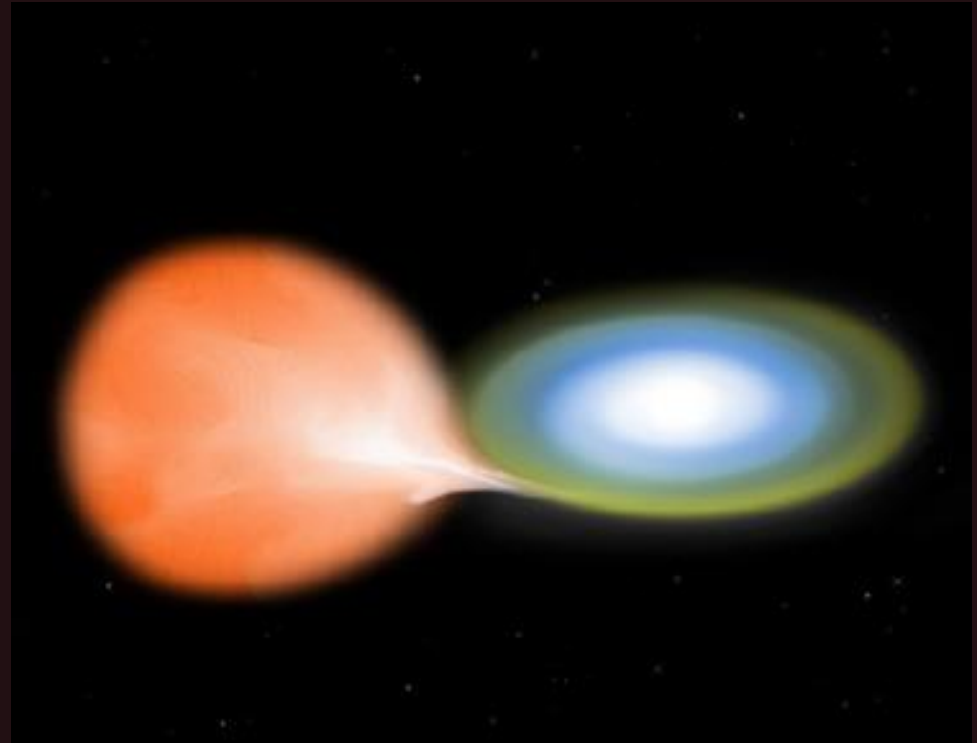
# Close binary systems

About 1/2 of massive stars  
Are members of close binary  
systems.

Now we know many dozens  
of close binary systems with  
neutron stars.

$$L = \dot{M} \eta c^2$$

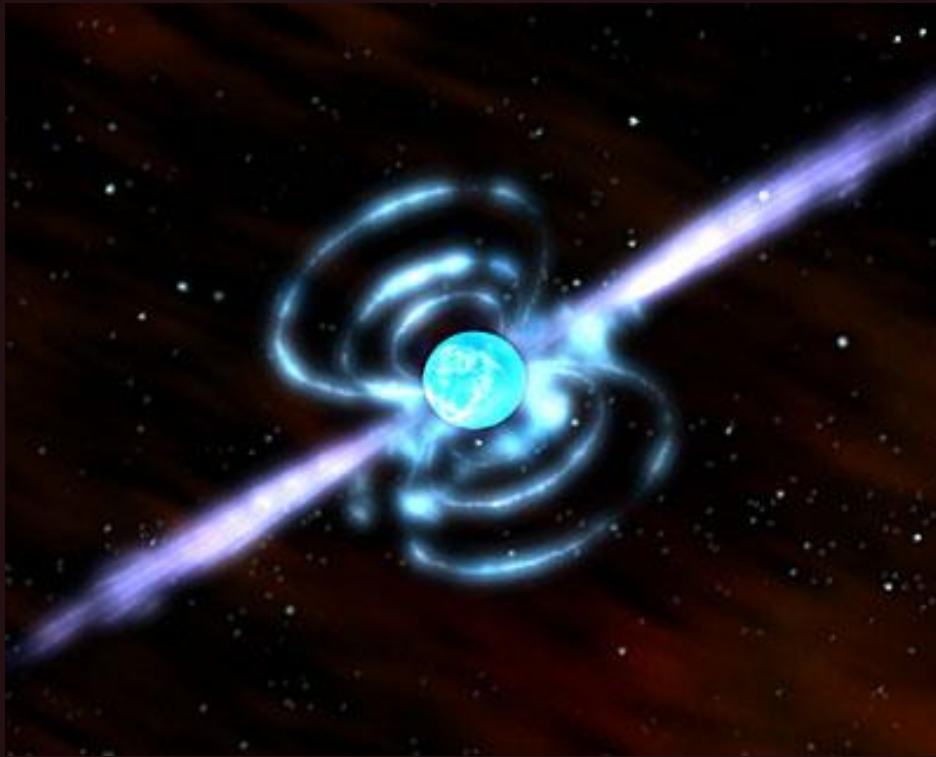
The accretion rate can be up to  $10^{20}$  g/s;  
Accretion efficiency – up to 10%;  
Luminosity – thousands of hundreds of the solar.



# Discovery !!!!

1967: Jocelyn Bell. Radio pulsars.

Serendipitous discovery.

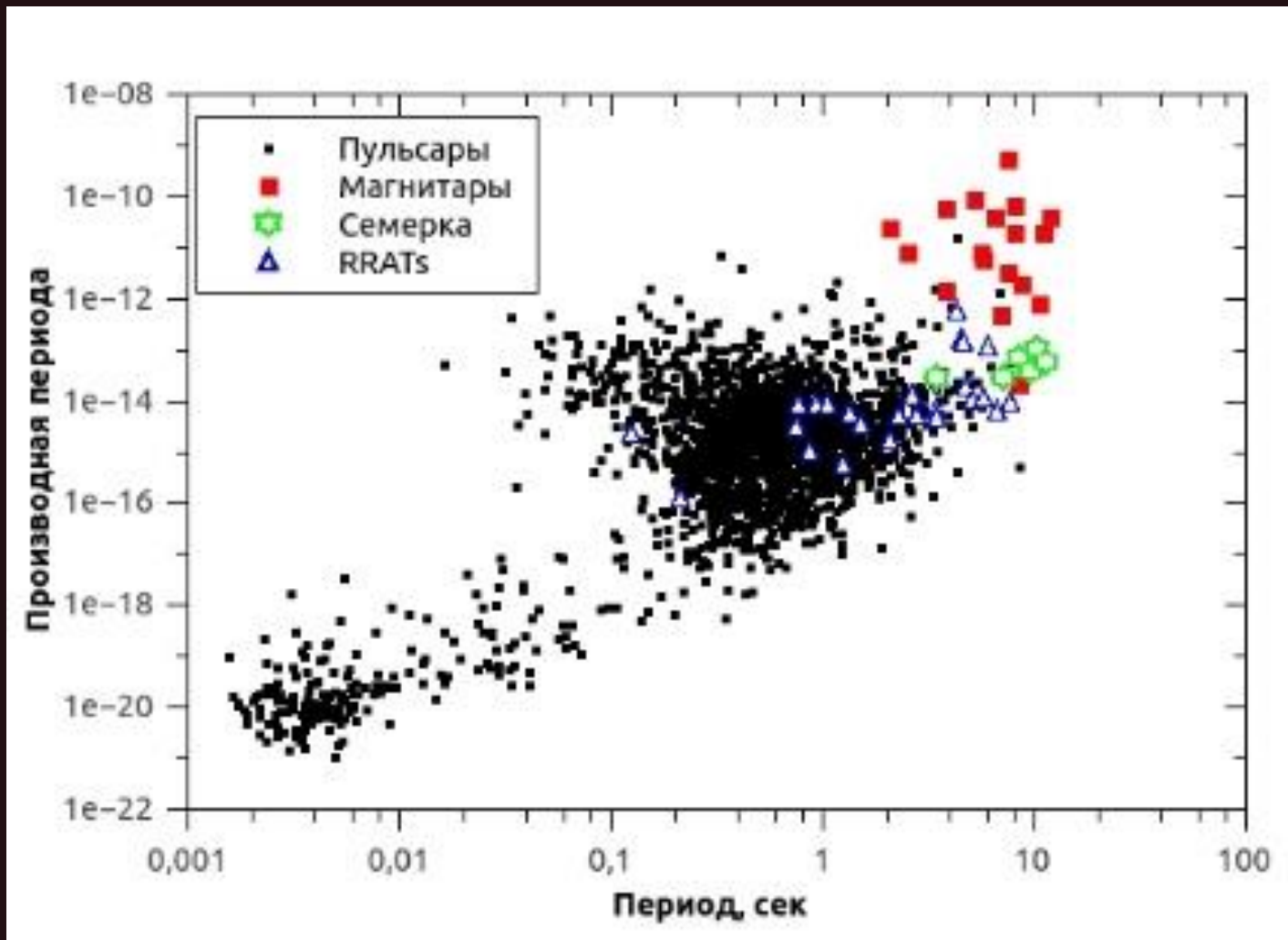




# The pulsar in the Crab nebula



# Pulsar spin-down: P-Pdot diagram

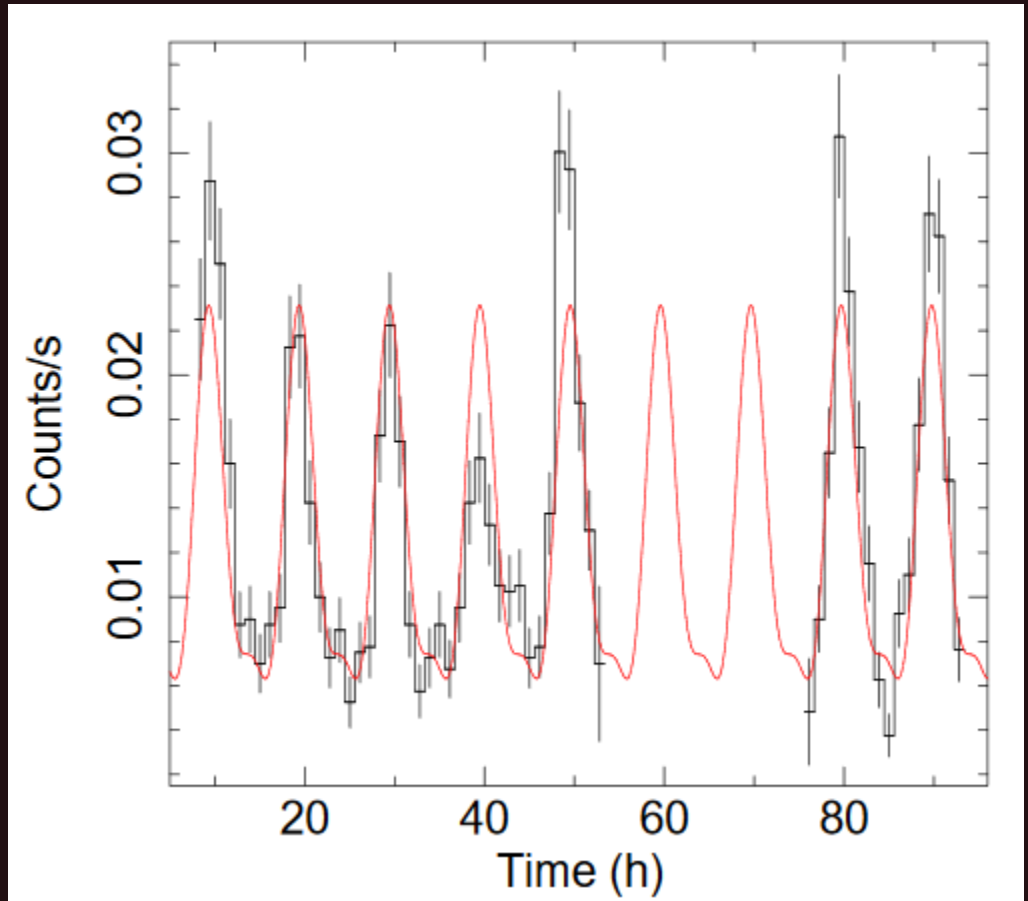


$$L_m = \frac{2}{3} \frac{\mu^2 \omega^4}{c^3} \sin^2 \beta = \kappa_t \frac{\mu^2}{R_l^3} \omega,$$

# Slowly rotating NSs – in binaries

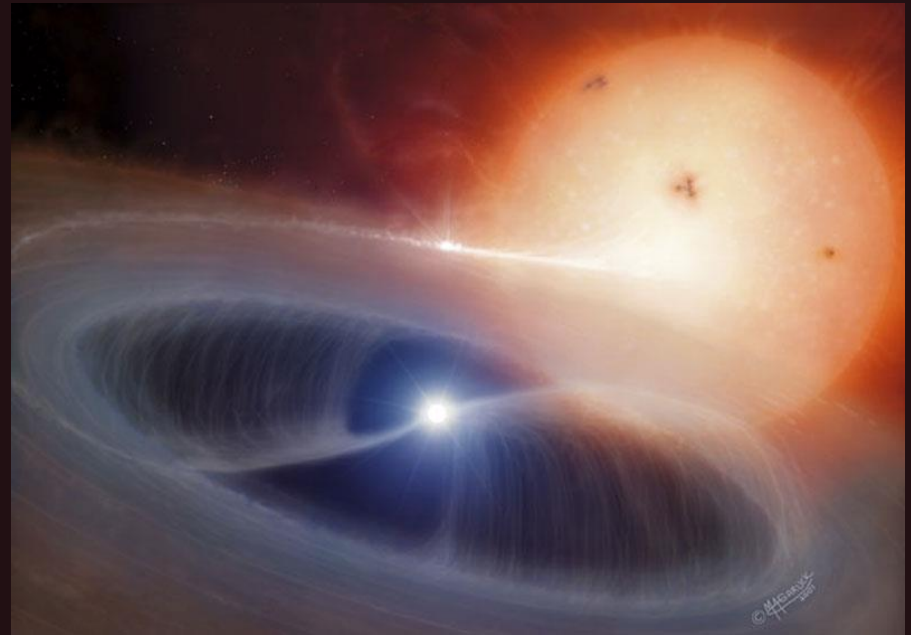
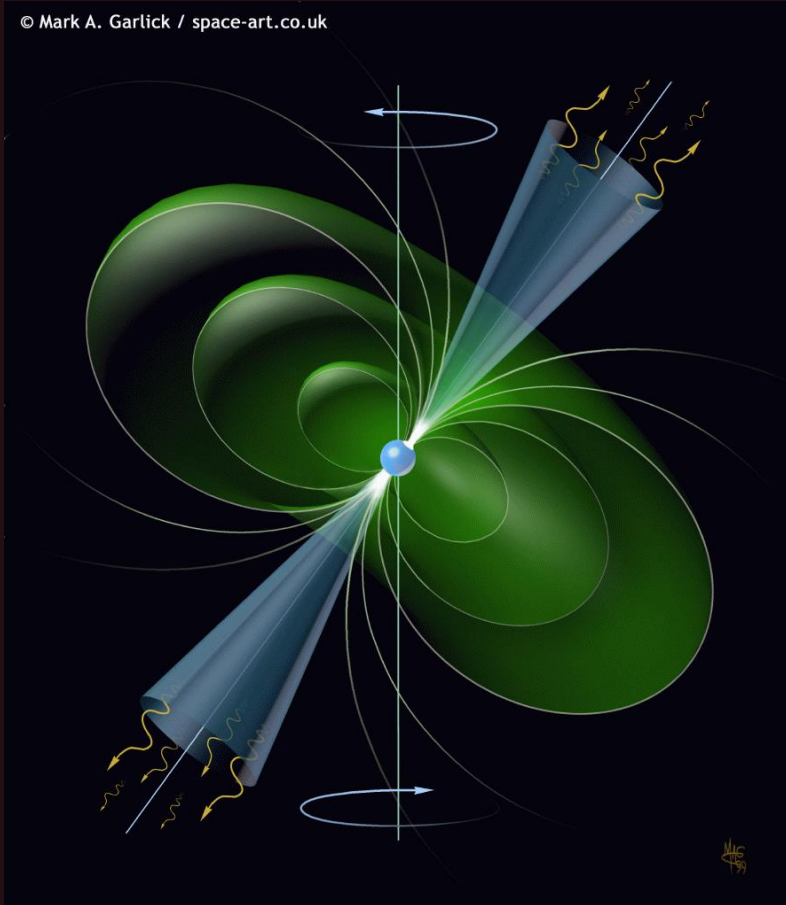
AX J1910.7+0917

$P > 10$  hours! ( $36200 \pm 100$  sec)



# The old Zoo: young pulsars & old accretors

© Mark A. Garlick / space-art.co.uk



# The new zoo of young neutron stars

During last  $\sim 25$  years  
it became clear that neutron stars  
can be born very different.  
In particular, absolutely  
non-similar to the Crab pulsar.

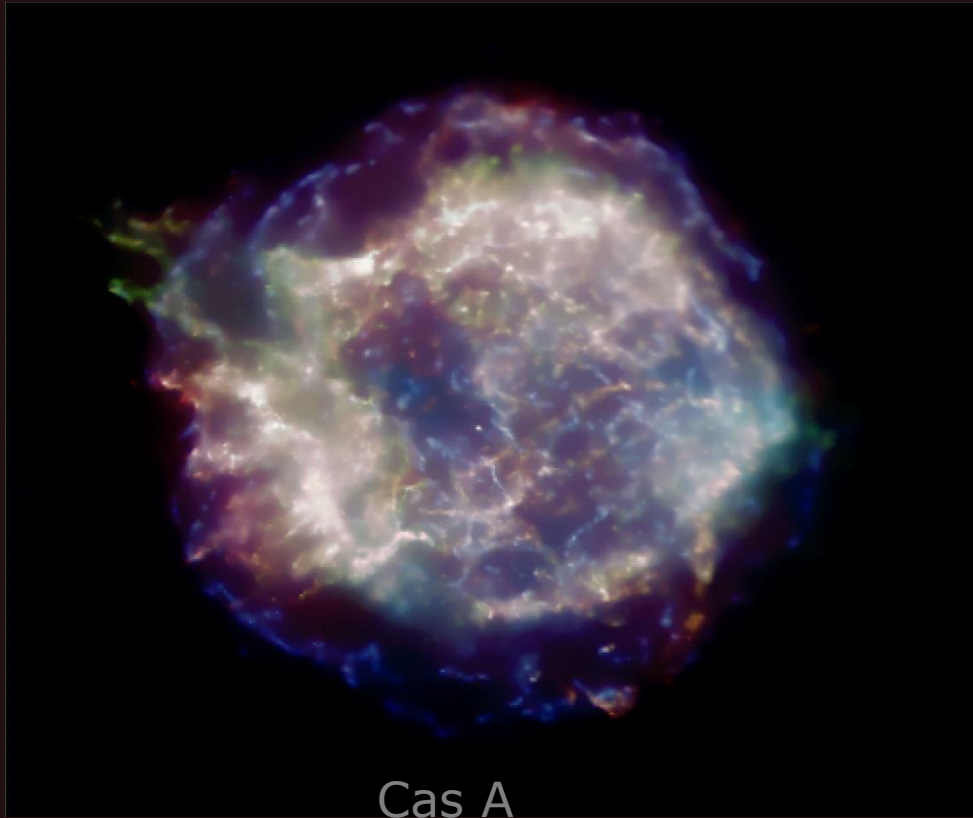
- o High-B PSRs
- o Compact central X-ray sources  
in supernova remnants.
- o Anomalous X-ray pulsars
- o Soft gamma repeaters
- o The Magnificent Seven
- o Transient radio sources (RRATs)



Old and new zoos: Harding [arXiv:1302.0869](https://arxiv.org/abs/1302.0869)

See a more recent review in [1712.06040](https://arxiv.org/abs/1712.06040)

# Compact central X-ray sources in supernova remnants



Cas A

Rapid cooling  
(Heinke et al. 1007.4719)



RCW 103

6.7 hour period  
(de Luca et al. 2006)



# CCOs in SNRs

		Age	Distance
J232327.9+584843	Cas A	0.32	3.3–3.7
J085201.4–461753	G266.1–1.2	1–3	1–2
J082157.5–430017	Pup A	1–3	1.6–3.3
J121000.8–522628	G296.5+10.0	3–20	1.3–3.9
J185238.6+004020	Kes 79	~9	~10
J171328.4–394955	G347.3–0.5	~10	~6

[Pavlov, Sanwal, Teter: astro-ph/0311526,  
de Luca: [arxiv:0712.2209](#)]

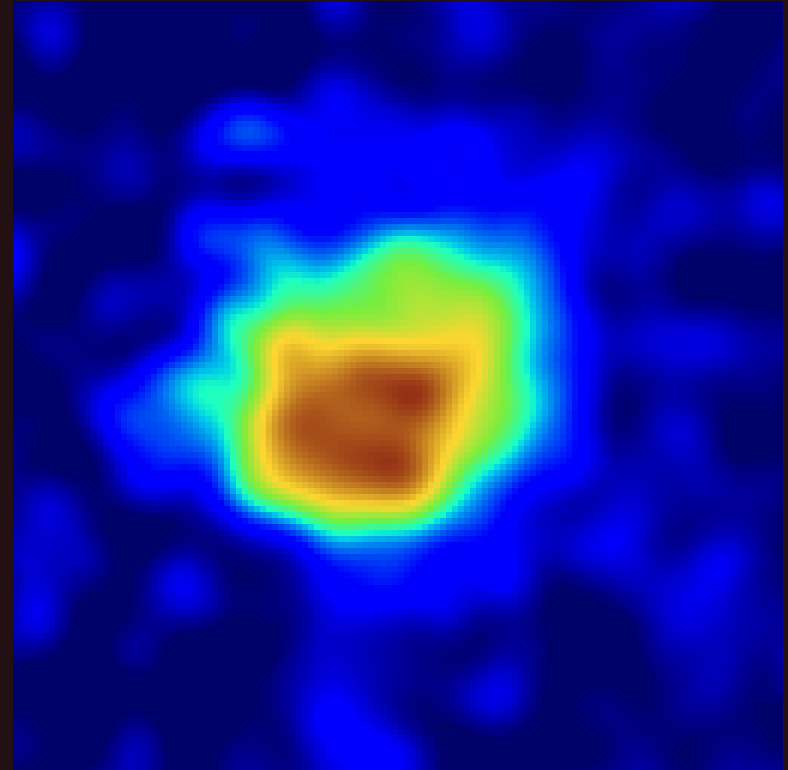
For three sources there are strong indications for  
large ( $>\sim 100$  msec) initial spin periods and  
low magnetic fields:

1E 1207.4-5209 in PKS 1209-51/52

PSR J1852+0040 in Kesteven 79

PSR J0821-4300 in Puppis A

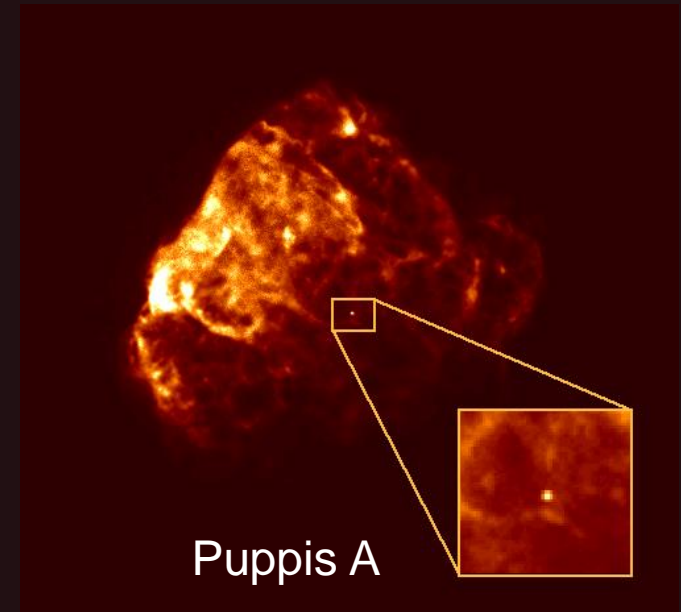
[see Halpern et al. [arxiv:0705.0978](#) and 1301.2717]



# CCOs

High proper motion of CCO in Pup A.  
Velocity 672 +/- 115 km/s

1204.3510



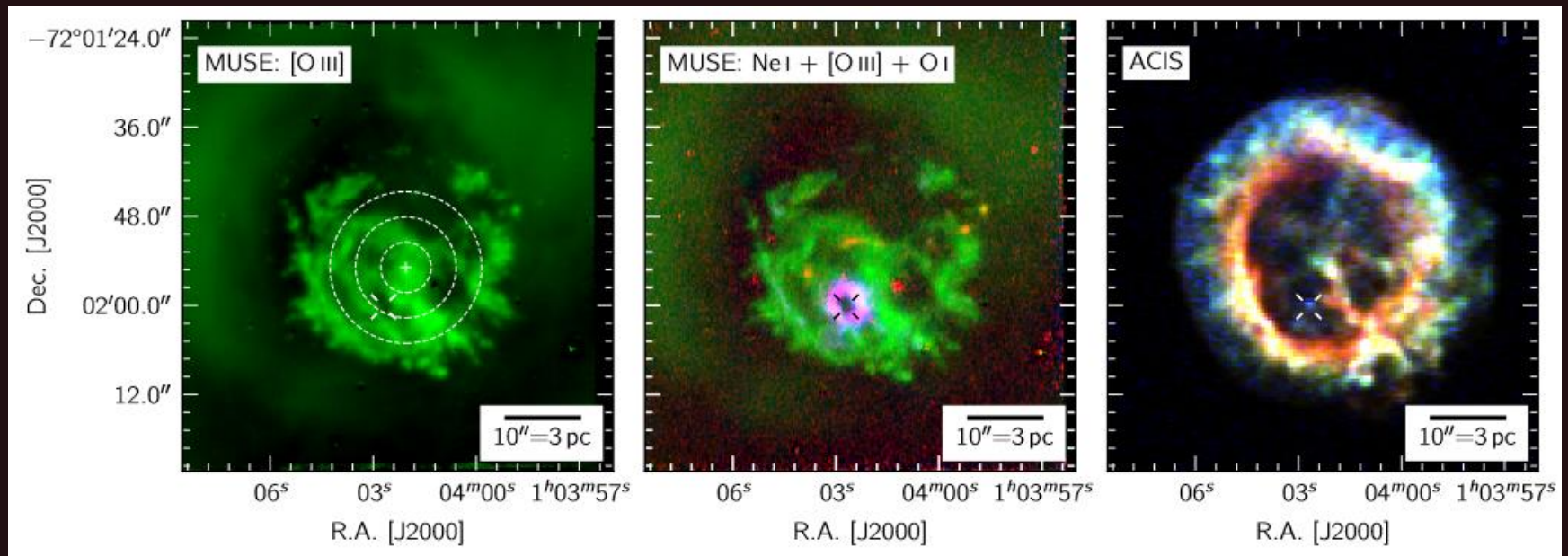
CCO	SNR	Age (kyr)	$d$ (kpc)	$P$ (s)	$f_p^a$ (%)	$B_s$ ( $10^{11}$ G)	$L_{x,bol}$ ( $\text{erg s}^{-1}$ )	References
RX J0822.0–4300	Puppis A	3.7	2.2	0.112	11	$< 9.8$	$6.5 \times 10^{33}$	1,2
CXOU J085201.4–461753	G266.1–1.2	1	1	...	$< 7$	...	$2.5 \times 10^{32}$	3,4,5,6,7
1E 1207.4–5209	PKS 1209–51/52	7	2.2	0.424	9	$< 3.3$	$2.5 \times 10^{33}$	8,9,10,11,12
CXOU J160103.1–513353	G330.2+1.0	$\gtrsim 3$	5	...	$< 40$	...	$1.5 \times 10^{33}$	13,14
1WGA J1713.4–3949	G347.3–0.5	1.6	1.3	...	$< 7$	...	$\sim 1 \times 10^{33}$	7,15,16
CXOU J185238.6+004020	Kes 79	7	7	0.105	64	0.31	$5.3 \times 10^{33}$	17,18,19,20
CXOU J232327.9+584842	Cas A	0.33	3.4	...	$< 12$	...	$4.7 \times 10^{33}$	20,21,22,23,24
XMMU J172054.5–372652	G350.1–0.3	0.9	4.5	...	...	...	$3.4 \times 10^{33}$	25
XMMU J173203.3–344518	G353.6–0.7	$\sim 27$	3.2	...	...	...	$1.0 \times 10^{34}$	26,27,28
CXOU J181852.0–150213	G15.9+0.2	1–3	(8.5)	...	...	...	$\sim 1 \times 10^{33}$	29

0911.0093

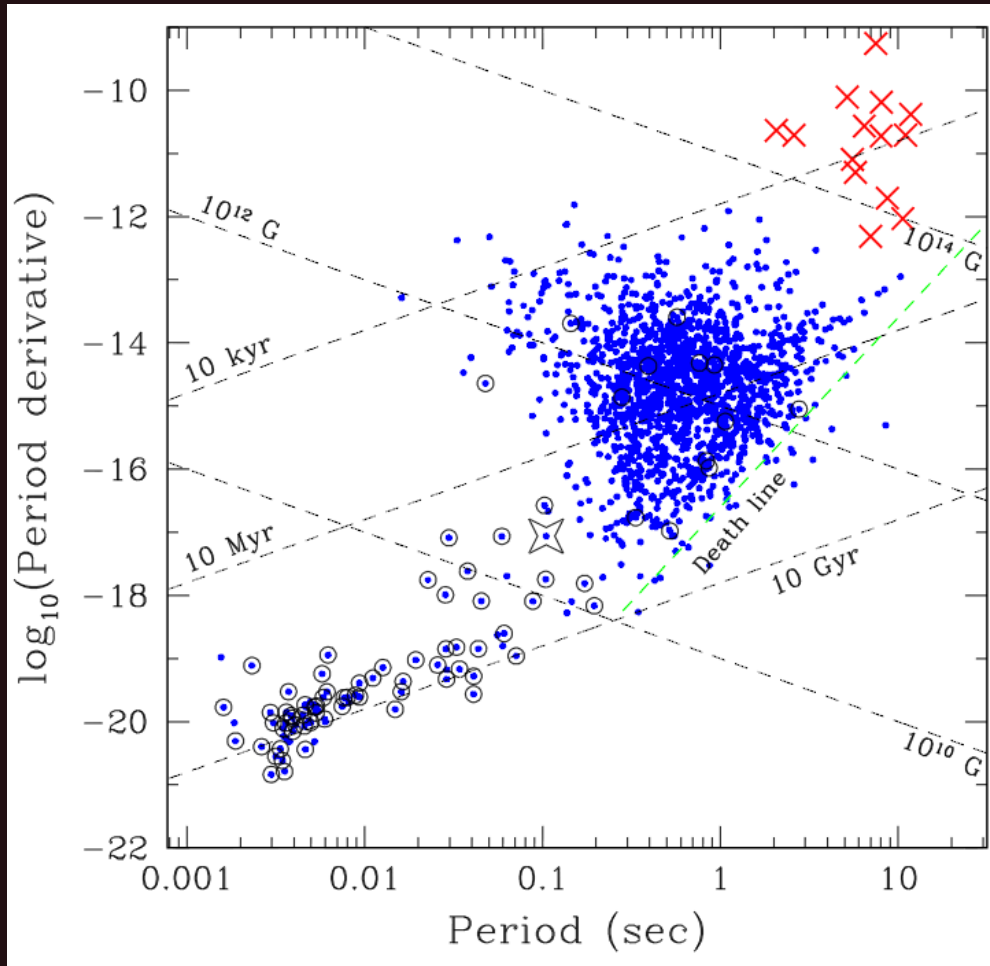


# The first CCO in SMC 1E 0102.2-7219

The first CCO identified outside the Galaxy.  
 $L \sim 10^{33}$  erg/s.



# Anti-magnetars



Star marks the CCO from 0911.0093

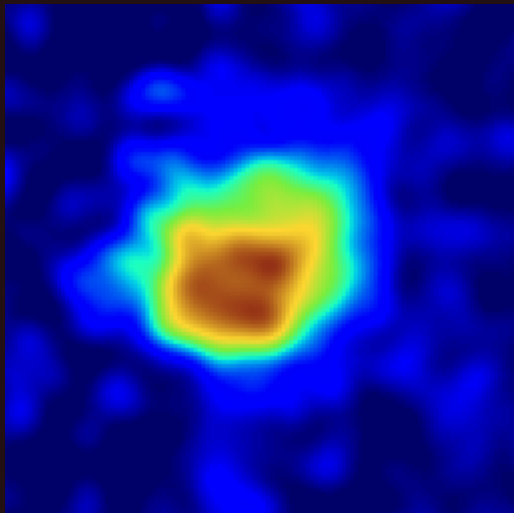
New results 1301.2717  
Spins and derivative are measured for  
PSR J0821-4300 and  
PSR J1210-5226

0911.0093

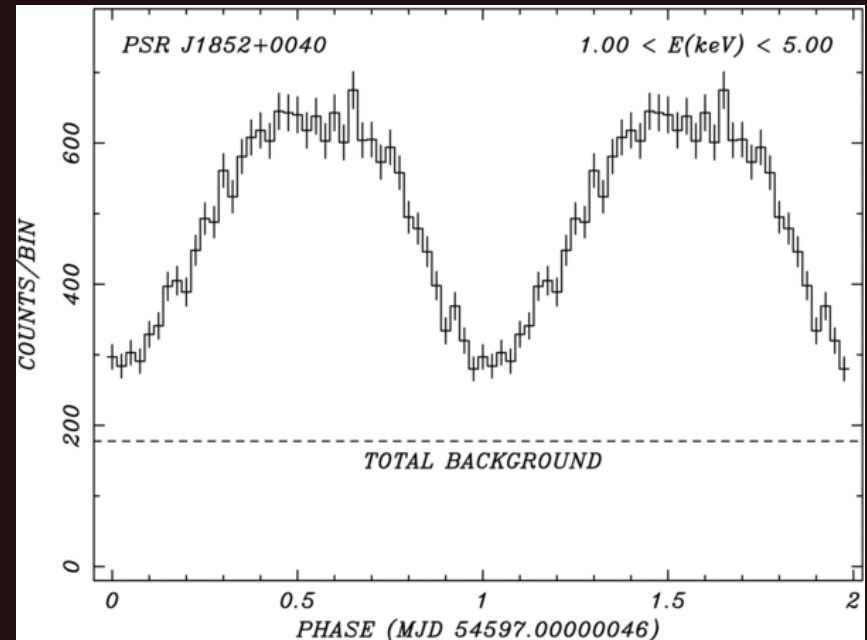
# “Hidden” magnetars

Kes 79. PSR J1852+0040.  $P \sim 0.1$  s

Shabaltas & Lai (2012) show that large pulse fraction of the NS in Kes 79 can be explained if its magnetic field in the crust is very strong:  $\text{few} \times 10^{14}$  G.



Kes 79

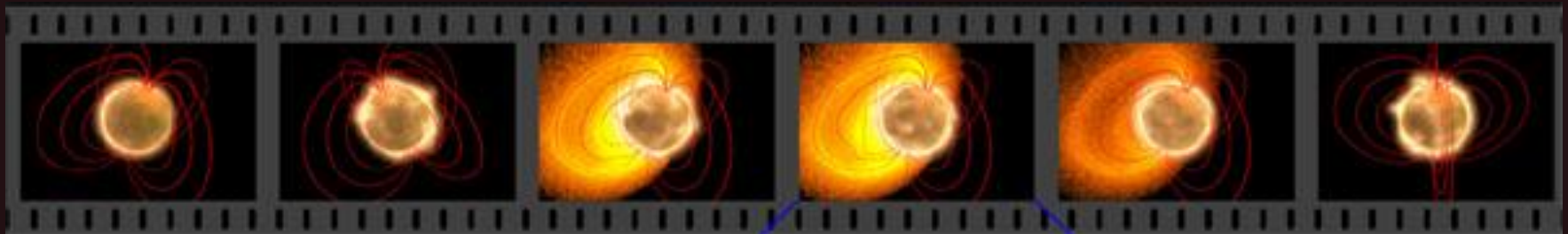


- If submergence of the field happens rapidly, so the present day period represents the initial one
- Then, the field of PSR 1852 was not enhanced via a dynamo mechanism
- Detection of millisecond “hidden” magnetars will be a strong argument in favour of dynamo.

# Magnetars

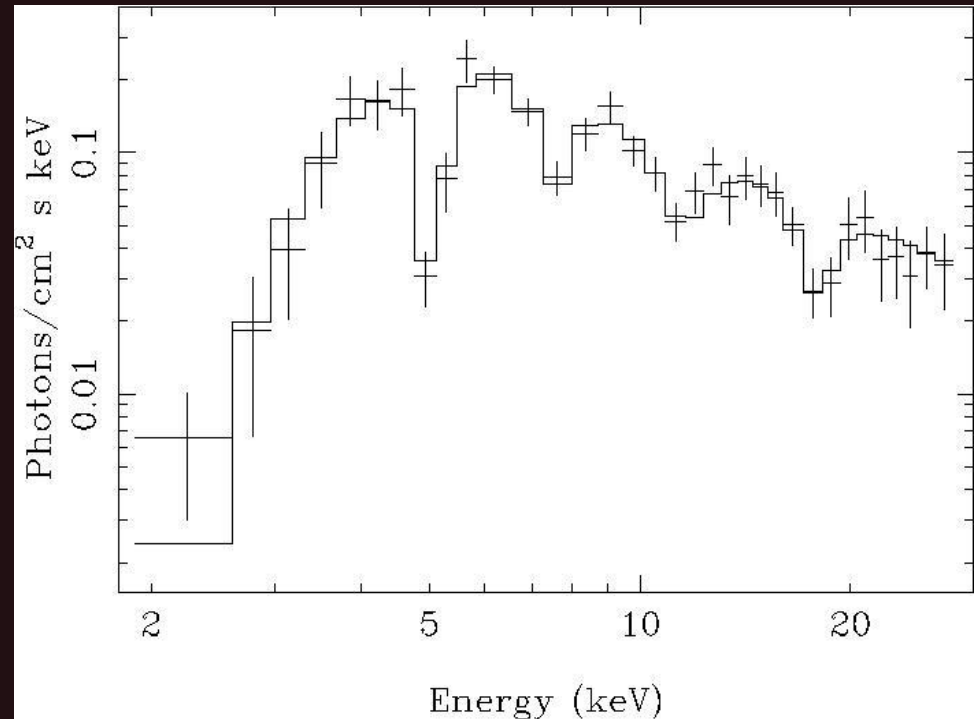
- $dE/dt > dE_{\text{rot}}/dt$
- By definition: The energy of the magnetic field is released

Magnetic fields  $10^{14}$ – $10^{15}$  G



# Magnetic field estimates

- Spin down
- Long spin periods
- Energy to support bursts
- Field to confine a fireball (tails)
- Duration of spikes (alfven waves)
- Direct measurements of magnetic field (cyclotron lines)

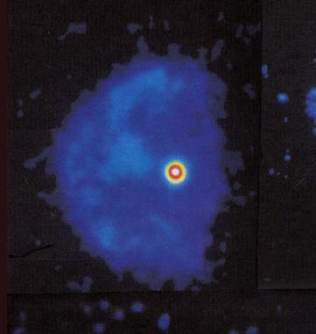


Ibrahim et al. 2002

# Some of known magnetars

## SGRs

- 0526-66
- 1627-41
- 1806-20
- 1900+14
- 0501+4516
- 0418+5729
- 1833-0832
- 1822-1606
- 1834-0846
- 1801-23 (?)
- 2013+34 (?)

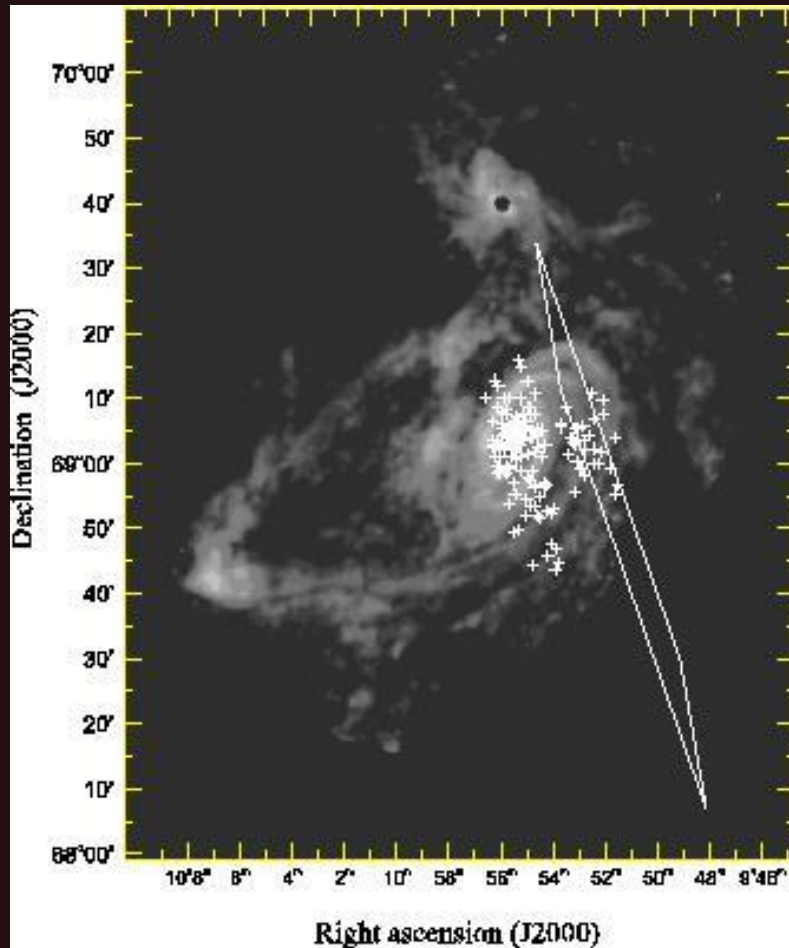


(CTB 109)

## AXPs

- CXO 010043.1-72
- 4U 0142+61
- 1E 1048.1-5937
- CXO J1647-45
- 1 RXS J170849-40
- XTE J1810-197
- 1E 1841-045
- AX J1845-0258
- 1E 2259+586
- 1E 1547.0-5408
- PSR J1622-4950
- CXO J171405-381031

# Extragalactic SGRs



It was suggested long ago (Mazets et al. 1982) that present-day detectors could already detect giant flares from extragalactic magnetars.

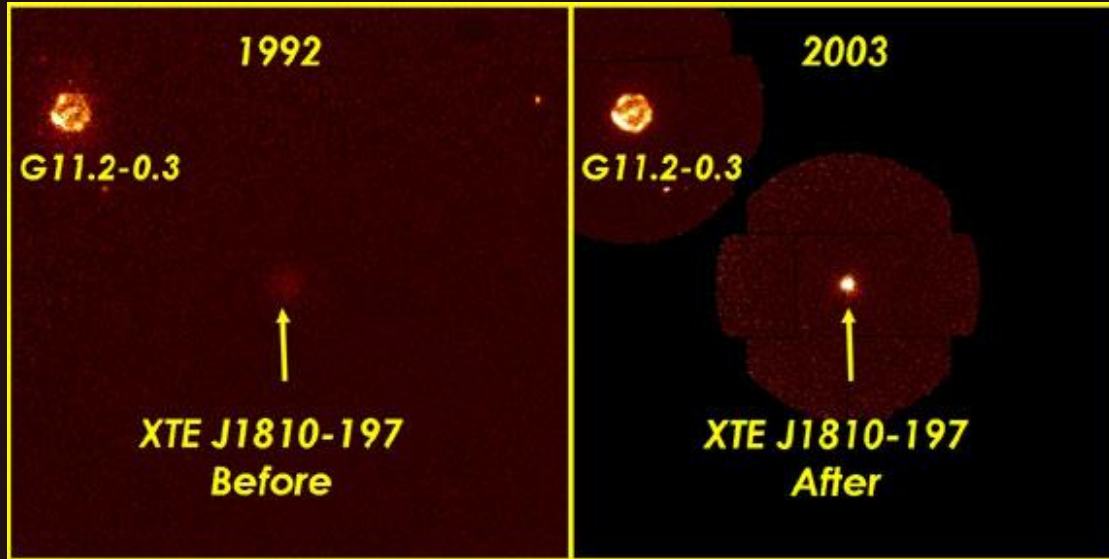
However, all searches in, for example, BATSE database did not provide good candidates (Lazzati et al. 2006, Popov & Stern 2006, etc.).

Finally, recently several good candidates have been proposed by different groups (Mazets et al., Frederiks et al., Golenetskii et al., Ofek et al, Crider ...., see [arxiv:0712.1502](https://arxiv.org/abs/0712.1502) and references therein, for example).

Burst from M31



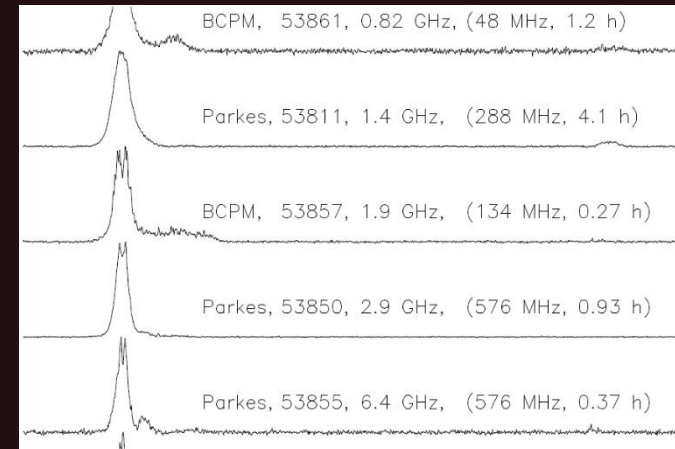
# Transient radio emission from AXP



← ROSAT and XMM images  
an X-ray outburst  
happened in 2003.

AXP has spin period 5.54 s

Radio emission was detected from XTE J1810-197  
during its active state.  
Clear pulsations have been detected.  
Large radio luminosity.  
Strong polarization.  
Precise  $\dot{P}$  measurement.  
Important to constrain models, for better distance  
and coordinates determinations, etc.





# Another AXP detected in radio

**1E 1547.0-5408**

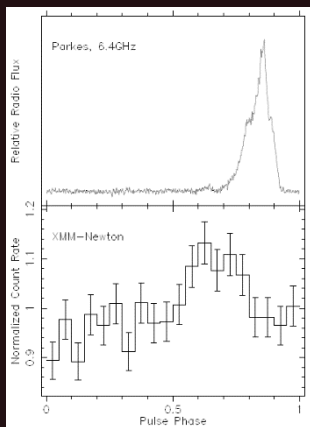
**P= 2 sec**

**SNR G327.24-0.13**

Pdot changed significantly on the scale of just  
~few months

Rotation and magnetic axis seem to be aligned

Also this AXP demonstrated weak  
SGR-like bursts (Rea et al. 2008, GCN 8313)

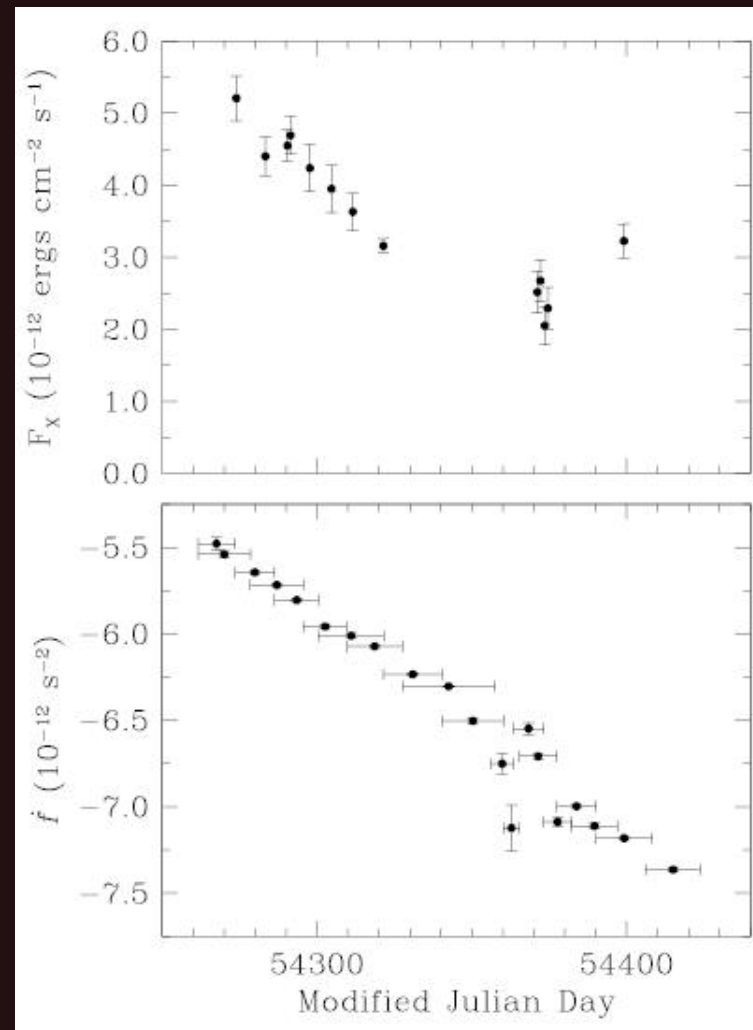


Radio

[simultaneous]



X-rays



# Transient radiopulsar

PSR J1846-0258

P=0.326 sec

B=5  $10^{13}$  G

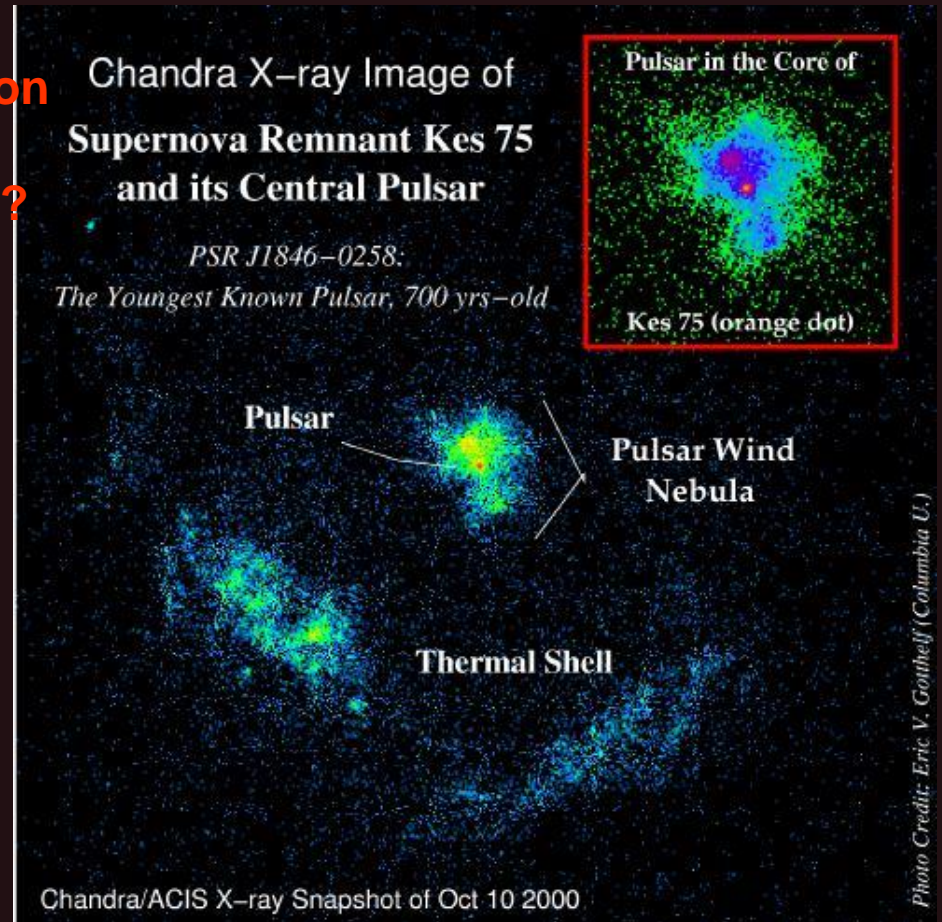
However,  
no radio emission  
detected.  
Due to beaming?

Among all rotation powered  
PSRs it has the largest  $\dot{E}$ .  
Smallest spindown age (884 yrs).

The pulsar increased  
its luminosity in X-rays.  
Increase of pulsed X-ray flux.  
Magnetar-like X-ray bursts (RXTE).  
Timing noise.

See additional info about this pulsar  
at the web-site

[http://hera.ph1.uni-koeln.de/~heintzma/SNR/SNR1\\_IV.htm](http://hera.ph1.uni-koeln.de/~heintzma/SNR/SNR1_IV.htm)



# Bursts from the transient PSR

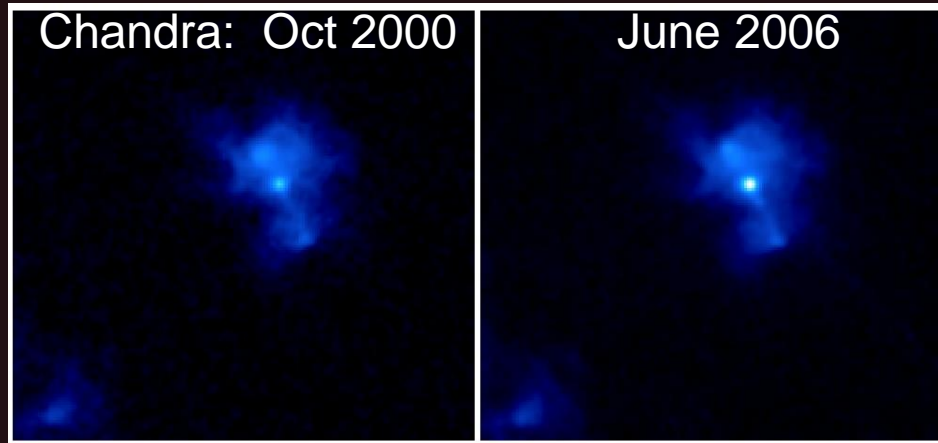
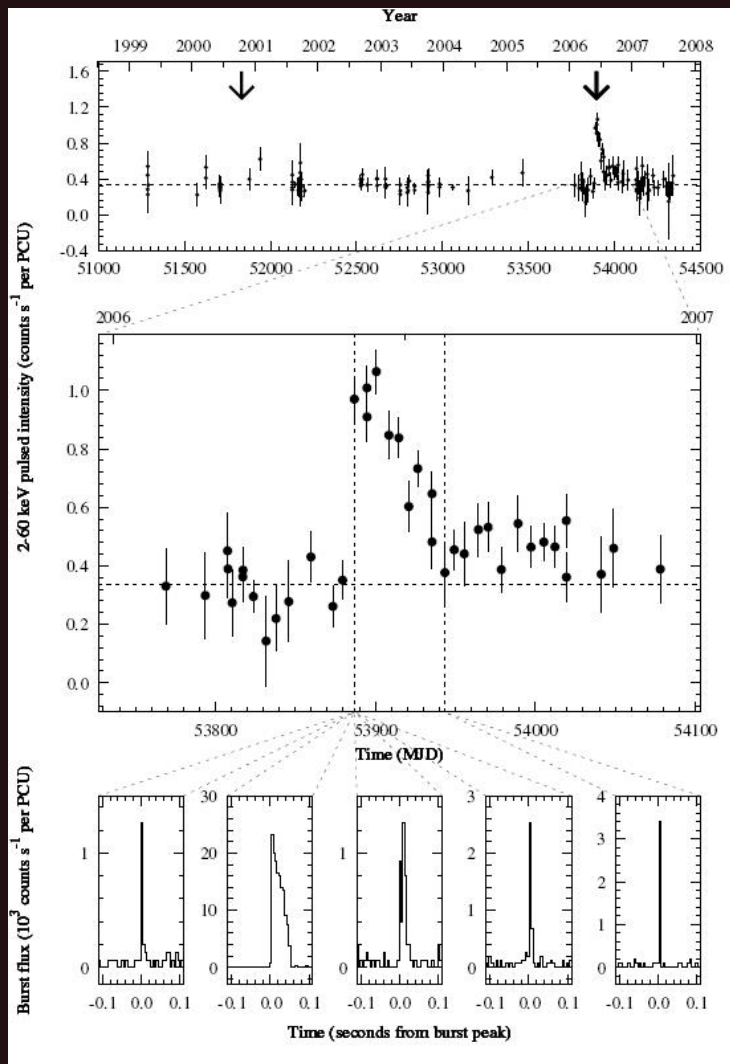
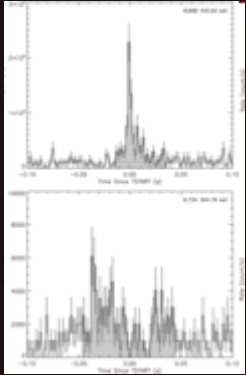


Table 1 PSR J1846–0258 Burst Temporal and Spectral Properties					
	Burst 1	Burst 2	Burst 3	Burst 4	Burst 5
<b>Temporal properties</b>					
Burst day (MJD)	53886	53886	53886	53886	53943
Burst start time (fraction of day)	0.92113966(5)	0.93247134(1)	0.93908845(2)	0.94248467(5)	0.45543551(1)
Rise time, $t_r$ (ms)	$4.2^{+3.5}_{-2.0}$	$1.1^{+0.9}_{-0.5}$	$1.90^{+1.7}_{-0.9}$	$4.1^{+3.1}_{-1.9}$	$0.9^{+2.2}_{-0.7}$
$T_{90}$ (ms)	$71.8^{+38.0}_{-5.5}$	$42.9^{+0.3}_{-0.2}$	$137.0^{+11.4}_{-36.2}$	$33.4^{+29.1}_{-23.1}$	$65.3^{+0.7}_{-0.5}$
Phase (cycles)	-0.49(1)	-0.04(1)	-0.20(1)	-0.05(1)	-0.08(1)
<b>Fluences and fluxes</b>					
$T_{90}$ Fluence (counts/PCU)	$8.9 \pm 0.7$	$712.8 \pm 2.5$	$18.3 \pm 0.7$	$18.4 \pm 0.7$	$18.4 \pm 1.1$
$T_{90}$ Fluence ( $10^{-10}$ erg/cm <sup>2</sup> )	$4.1 \pm 2.4$	$289.9 \pm 13.1$	$6.6 \pm 2.5$	$5.8 \pm 1.7$	$5.3 \pm 2.0$
Flux for 64 ms ( $10^{-10}$ erg/s/cm <sup>2</sup> )	$57 \pm 36$	$4533 \pm 227$	$99 \pm 41$	$97 \pm 31$	$79 \pm 32$
Flux for $t_r$ ( $10^{-10}$ erg/s/cm <sup>2</sup> )	$678 \pm 427$	$5783 \pm 885$	$810 \pm 385$	$828 \pm 284$	$2698 \pm 1193$
<b>Spectral properties</b>					
Power-law index	$0.89 \pm 0.58$	$1.05 \pm 0.04$	$1.14 \pm 0.34$	$1.36 \pm 0.25$	$1.41 \pm 0.31$
$\chi^2/\text{DoF}$ (DoF)	0.42 (1)	1.16 (55)	0.97 (3)	0.35 (2)	1.18 (2)

# Weak dipole field magnetar

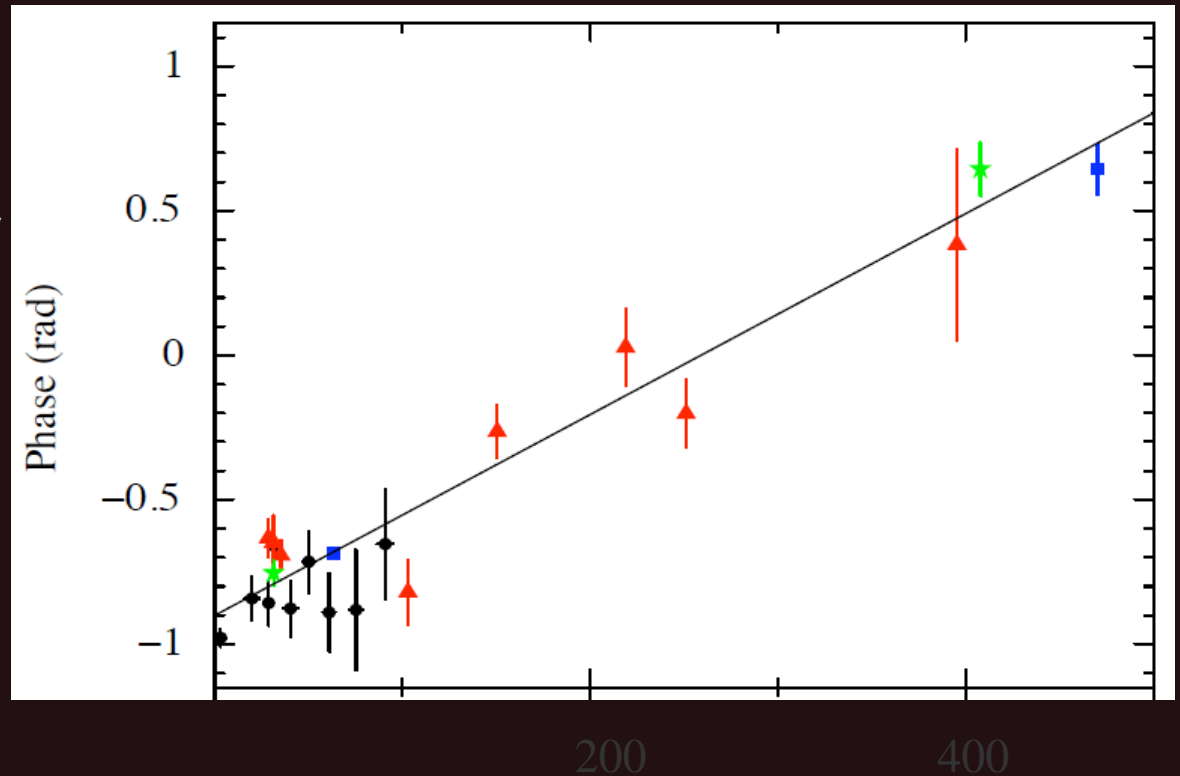


Spin period of a neutron star grows.  
The rate of deceleration is related to the dipole magnetic field.  
Measuring the spin-down rate we measure the field.

The source is a soft gamma-ray  
repeater: SGR 0418+5729  
 $P=9.1$  s

The straight line in the plot  
corresponds to a constant  
spin periods: i.e. no spin-down  
 $B < 7.5 \cdot 10^{12}$  G (arXiv:1010.2781)

Old magnetar ? (1107.5488)



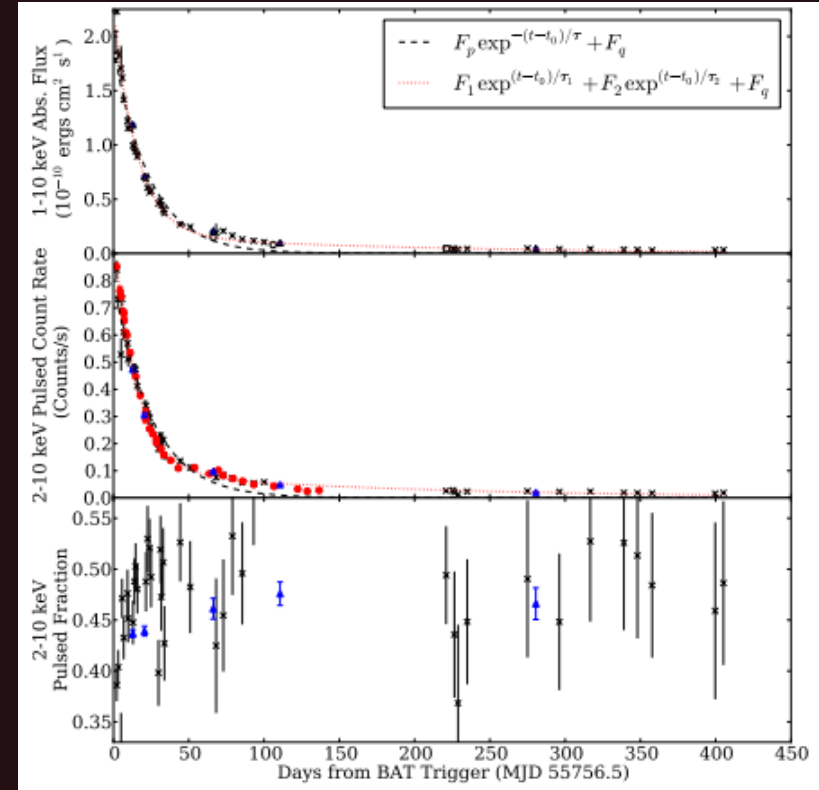
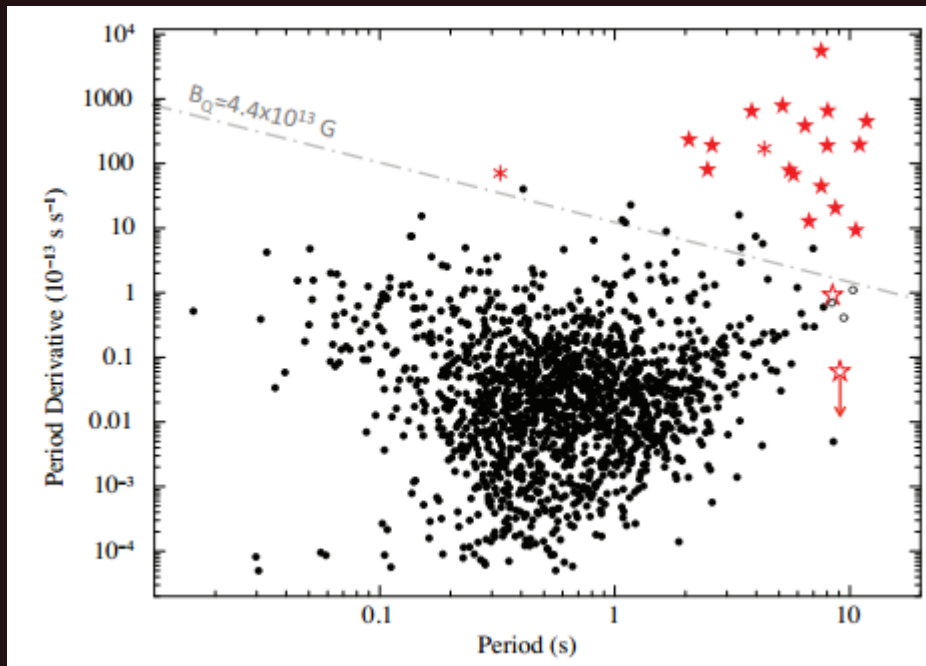
Spectral data suggests high field on the surface: 1103.3024

# Another low field magnetar

Swift J1822.3-1606 (SGR 1822-1606)

$P=8.44$  s

$B=3-5 \times 10^{13}$  G



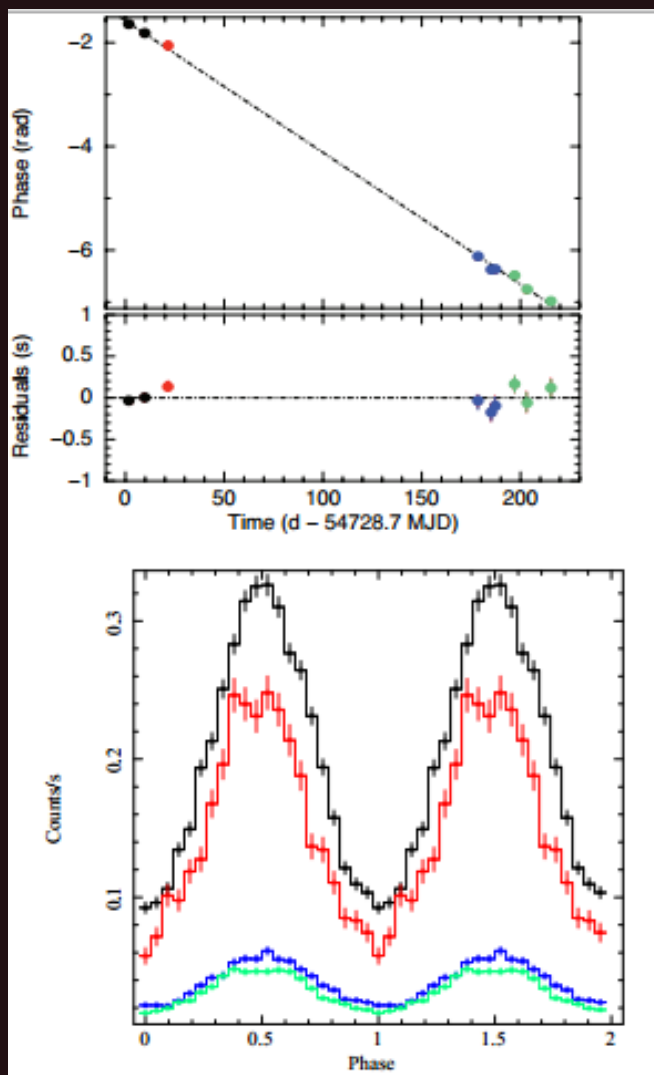
1204.1034

1203.6449

New data: 1211.7347



# One more low-field magnetar

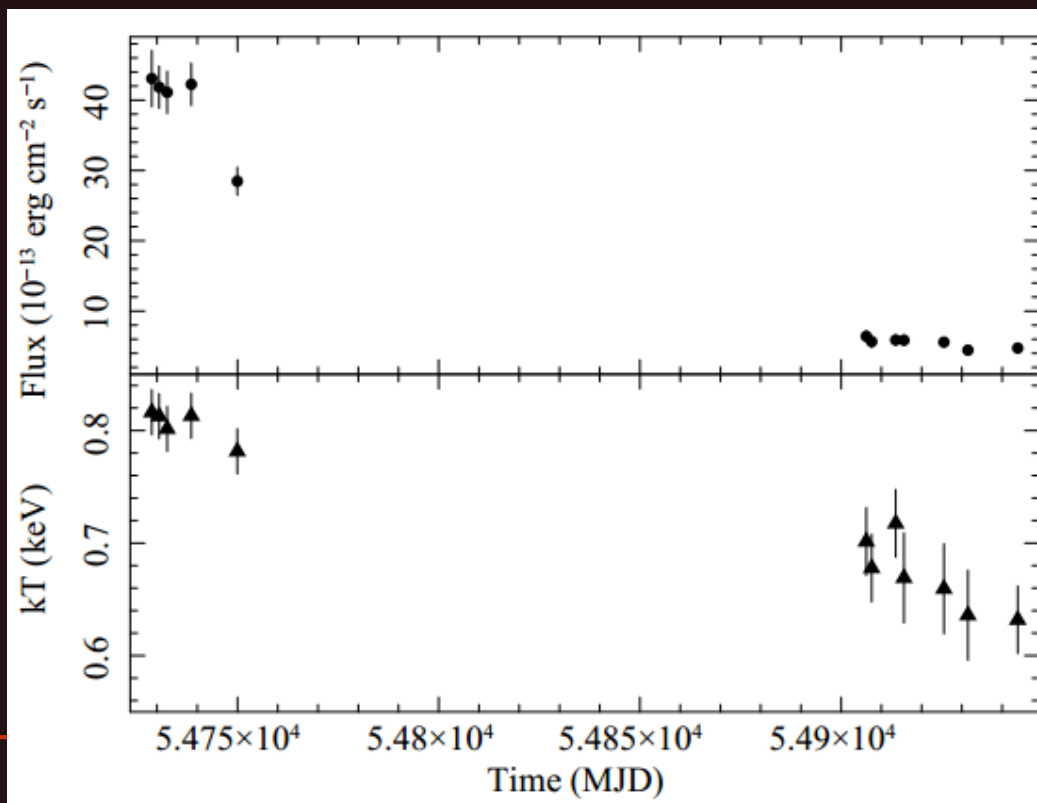


3XMM J185246.6+003317

$P=11.5$  s No spin-down detected after 7 months

$B < 4 \times 10^{13}$  G

Transient magnetar





# Quiescent magnetar J1622–4950

Normally, magnetars are detected via their strong activity: gamma-ray bursts or enhanced X-ray luminosity.

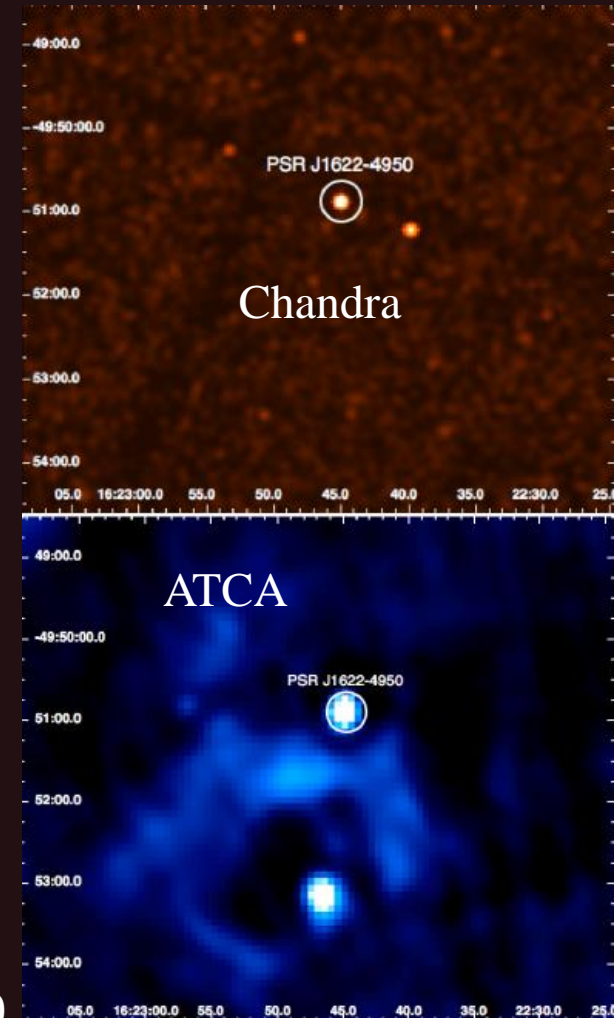
This one was detected in radio observations  
The field is estimated to be  $B \sim 3 \times 10^{14}$  G

It seems to be the first magnetar to be  
Detected in a quiescent state.

PSR J1622–4950 was detected in a radio survey  
As a pulsar with  $P=4.3$  s.

Noisy behavior in radio

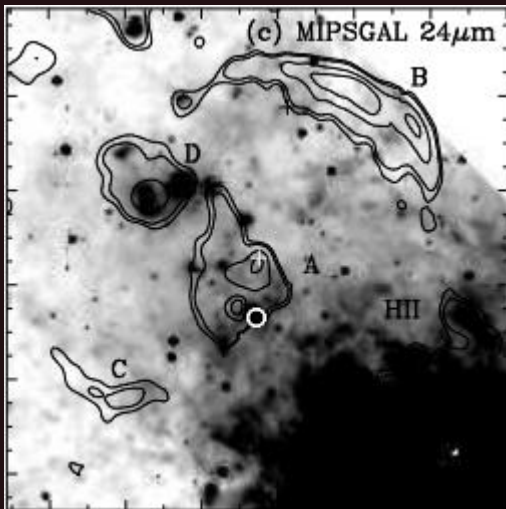
See reviews on high-B PSRs in 1010.4592, 1805.01680



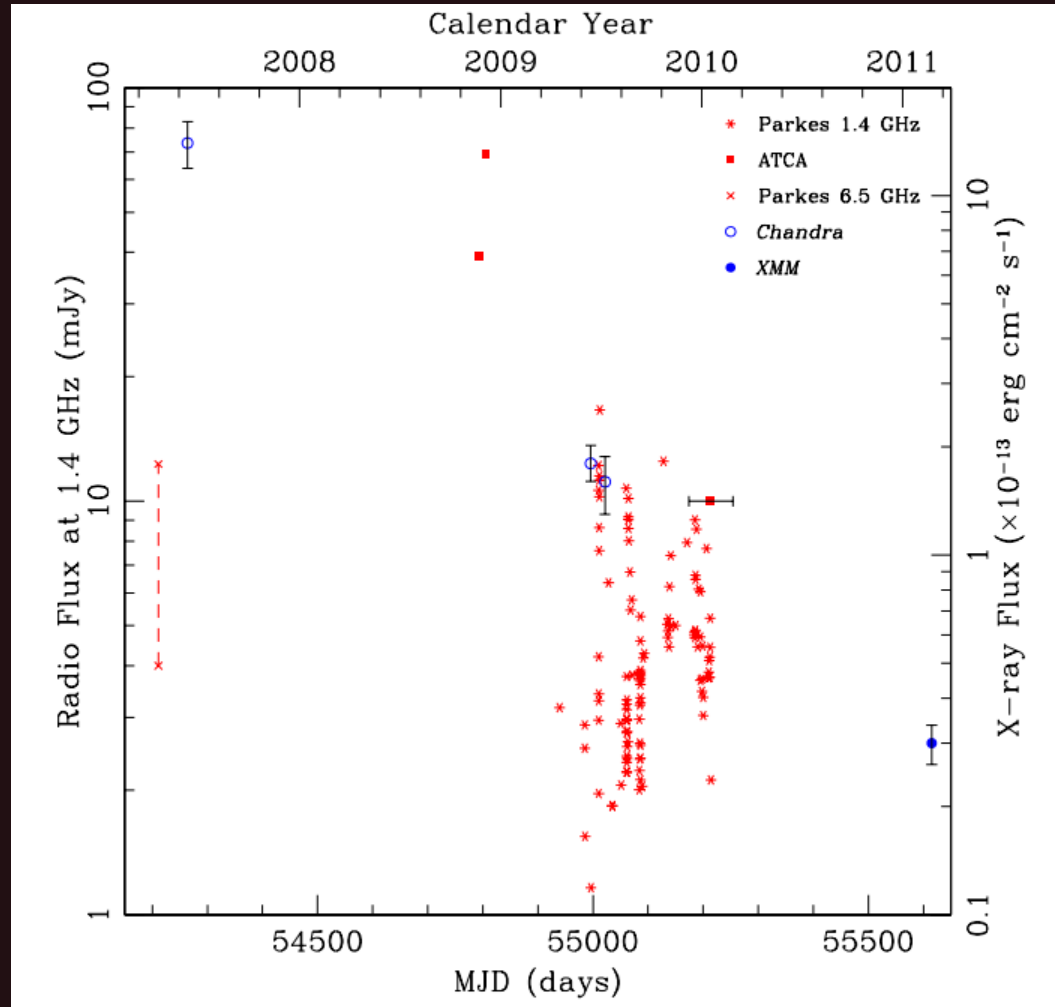
# Is J1622–4950 a transient magnetar?

PSR J1622–4950

X-ray flux is decaying  
for several years.  
Probably, the source  
was active years before.



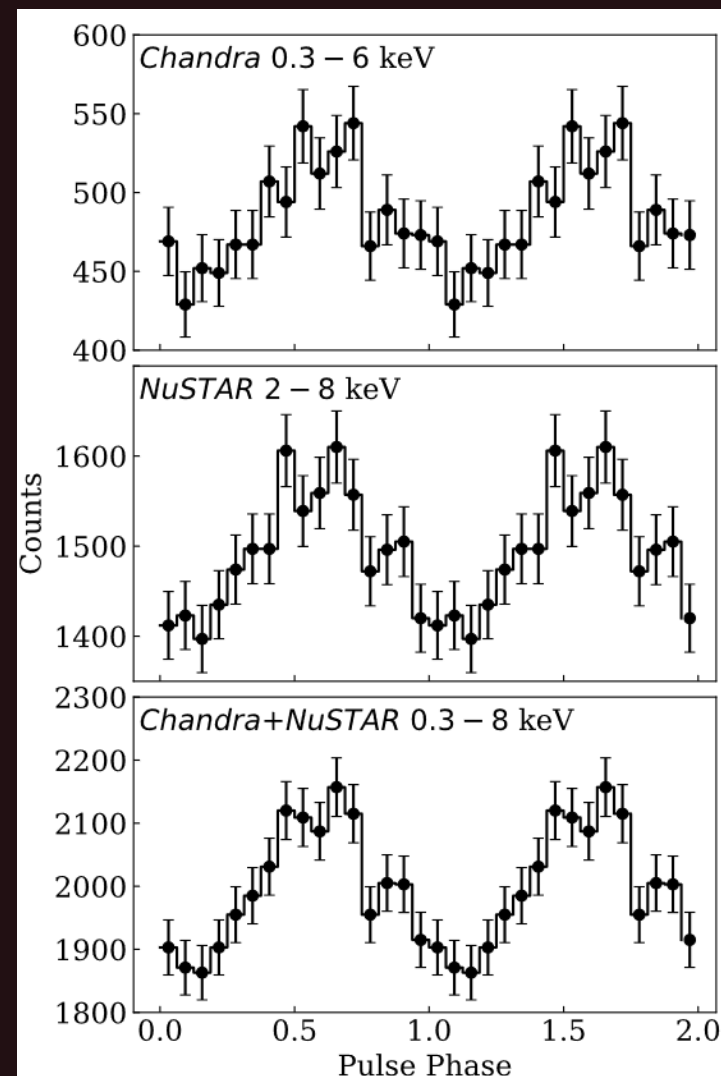
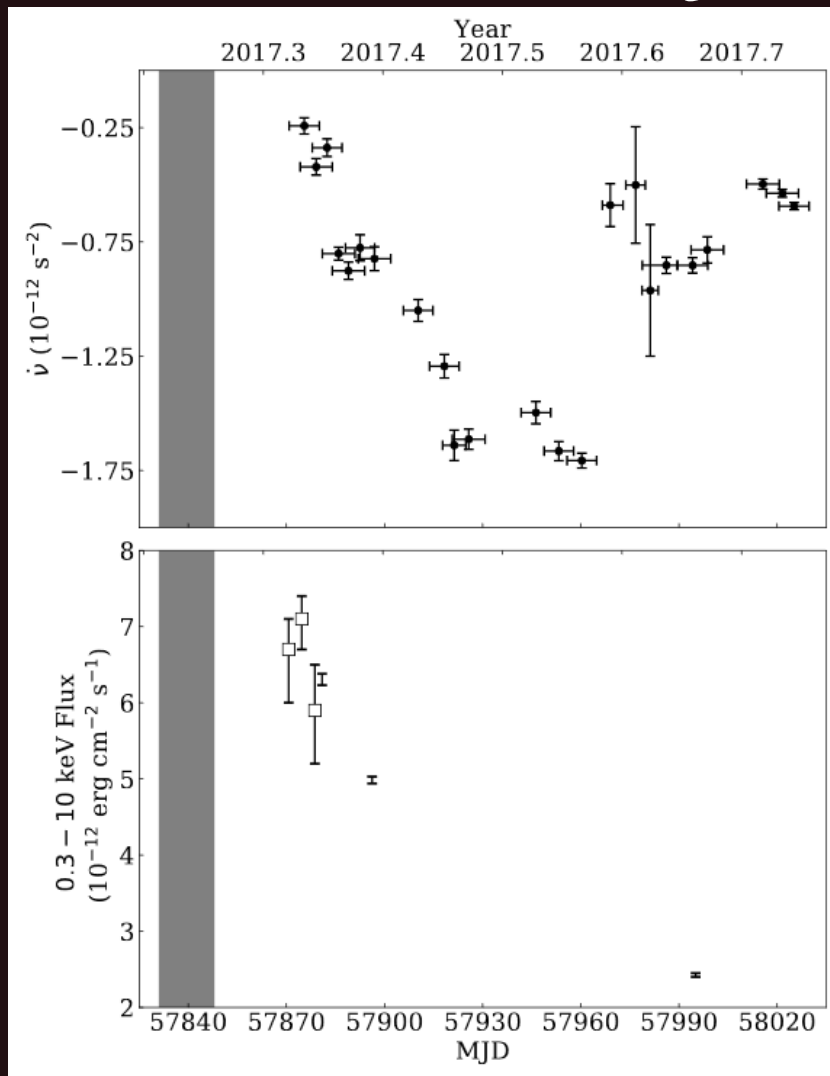
G333.9+0.0 SNR ?



1203.2719

See also 1204.2045

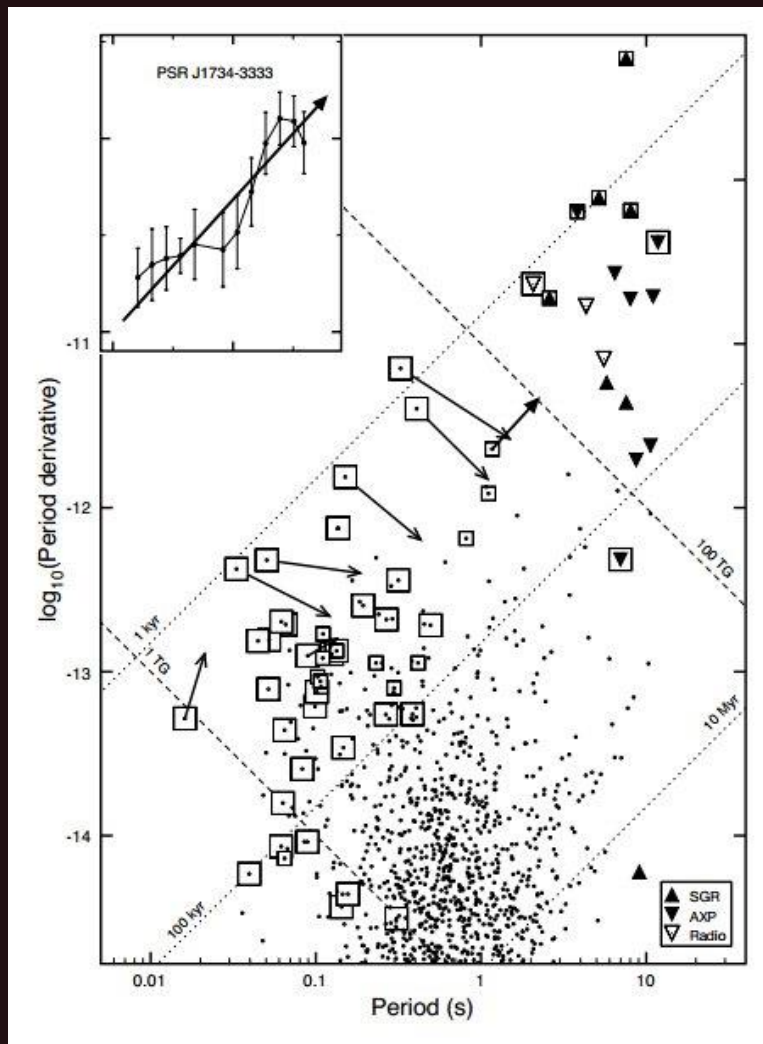
# Yes! Revival of J1622–4950



1804.01933

Among few (~5) magnetars with detected radio emission.

# A pulsar with growing field?

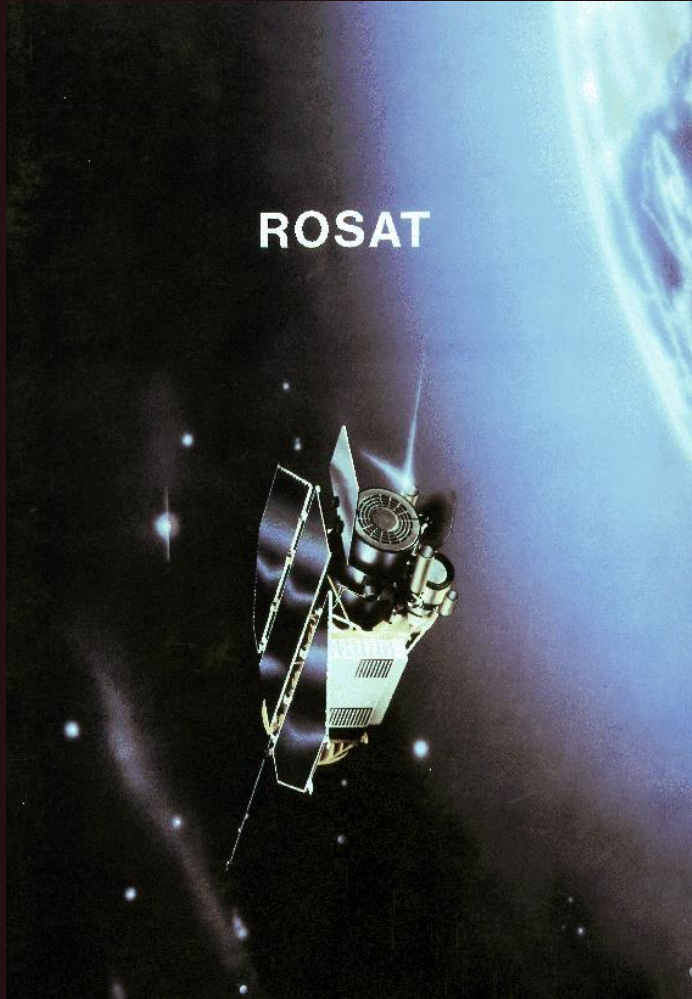


PSR J1734-3333

$n=0.9\pm0.2$

Will it become a magnetar?

# ROSAT

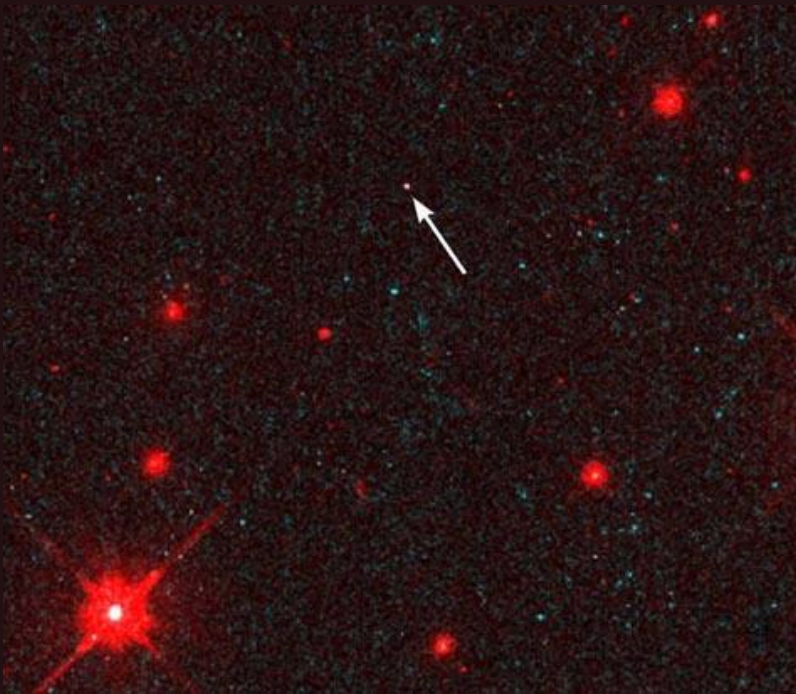


## ROentgen SATellite

German satellite  
(with participation of US and UK).

Launched 01 June 1990.  
The program was successfully ended  
on 12 Feb 1999.

# Close-by radioquiet NSs



RX J1856.5-3754

- Discovery: Walter et al. (1996)
- Proper motion and distance: Kaplan et al.
- No pulsations
- Thermal spectrum
- Later on: six brothers



# Magnificent Seven

Name	Period, s
RX 1856	7.05
RX 0720	8.39
RBS 1223	10.31
RBS 1556	3.39?
RX 0806	11.37
RX 0420	3.45
RBS 1774	9.44



**Radioquiet**  
**Close-by**  
**Thermal emission**  
**Absorption features**  
**Long periods**

For RBS 1556 (RX J1605) the priod is uncertain: 1901.08533

# Spin properties and other parameters

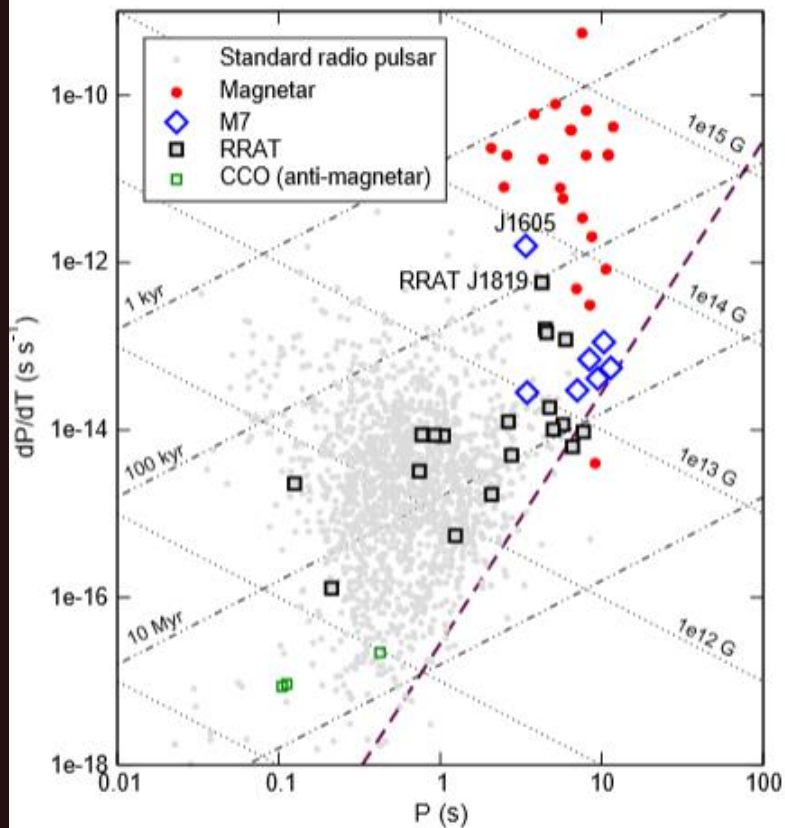
RX J	Spin <sup>*</sup>			Spectrum <sup>†</sup>					Astrometry <sup>**</sup>		References
	P	$\dot{P}$	PF	$N_{H,20}$	kT	PN	$E_{abs}$	m <sub>B</sub>	$\mu$	d	
	(s)	(10 <sup>-14</sup> )	(%)	(cm <sup>-2</sup> )	(eV)	(s <sup>-1</sup> )	(keV)	(mag)	(mas yr <sup>-1</sup> )	(pc)	
1856.5–3754	7.06	...	1	0.8	62	8.3	...	25.2	333	160	<a href="#">14</a> , <a href="#">15</a> , <a href="#">18–20</a>
0720.4–3125 <sup>‡</sup>	8.39	7	11	1.0	87	7.6	0.3	26.6	97	360	<a href="#">21–26</a>
1605.3+3249	...	...	< 3	0.8	93	5.6	0.5(0.6,0.8)	27.2	155	390	<a href="#">27–31</a>
1308.6+2127	10.31	11	18	1.8	102	2.5	0.2(0.4)	28.4 <sup>§</sup>	200 <sup>¶</sup>	...	<a href="#">32–36</a>
2143.0+0654	9.44	...	4	3.6	102	2.0	0.7	> 26 <sup>  </sup>	...	430	<a href="#">37–39</a>
0806.4–4123	11.37	...	6	1.1	92	1.8	0.3(0.6)	> 24	...	250	<a href="#">29</a> , <a href="#">40</a>
0420.0–5022	3.45	...	17	2.1	45	0.2	0.3	26.6	...	345	<a href="#">29</a> , <a href="#">40</a>

Kaplan arXiv: 0801.1143

Updates:

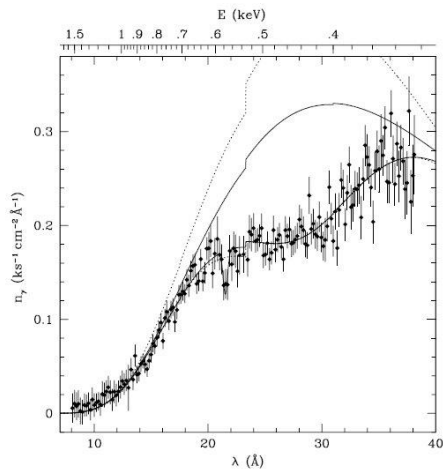
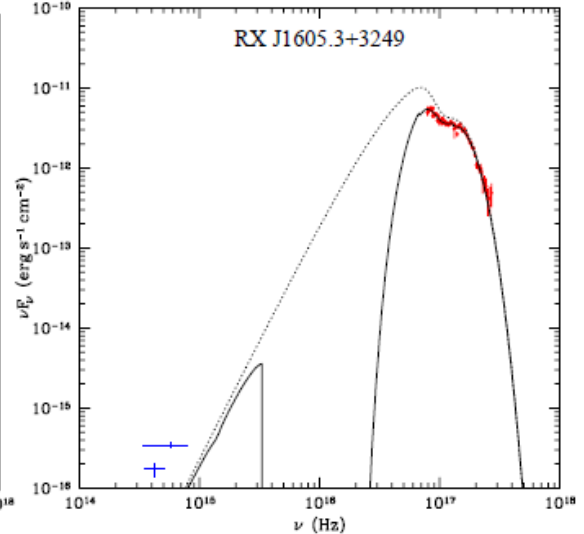
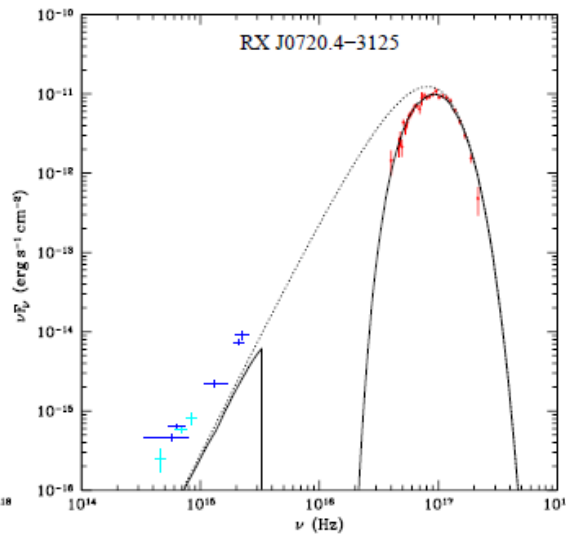
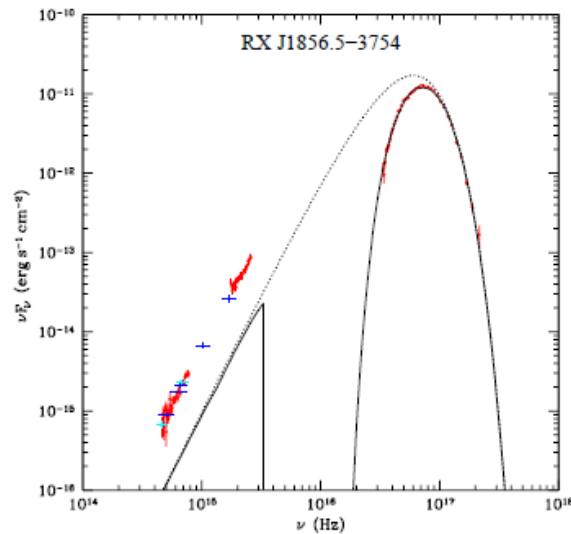
- 1856.  $\dot{v} = -6 \cdot 10^{-16}$  ( $|\dot{v}| < 1.3 \cdot 10^{-14}$ ) van Kerkwijk & Kaplan arXiv: 0712.3212
- 2143.  $\dot{v} = -4.6 \cdot 10^{-16}$  Kaplan & van Kerkwijk arXiv: 0901.4133
- 0806.  $|\dot{v}| < 4.3 \cdot 10^{-16}$  Kaplan and van Kerkwijk arXiv: 0909.5218

Object	$kT_\infty$ (eV)	$P$ (s)	$p_f$ (%)	$\log(\dot{P})$ ( $\text{s s}^{-1}$ )	$\log(\dot{E})$ ( $\text{erg s}^{-1}$ )	$\log(\tau_{\text{ch}})$ (yr)	$\log(t_{\text{kin}})$ (yr)	$\log(B_{\text{dip}})$ ( $10^{13}$ G)	$\log(B_{\text{cyc}})$ ( $10^{13}$ G)	Reference
RX J1856.5-3754	61	7.06	1	-13.527	30.580	6.58	5.62	13.17	—	[1]
RX J0720.4-3125	84 – 93	8.39	11	-13.156	30.726	6.28	5.93	13.39	13.75	[2]
RX J1605.3+3249	100	3.39	4	-11.796	33.267	4.53	5.65	13.87	13.92	[3]
RX J1308.6+2127	100	10.31	18	-12.951	30.663	6.16	5.95	13.54	13.60	[4]
RX J2143.0+0654	104	9.43	4	-13.398	30.332	6.57	—	13.29	14.15	[5]
RX J0806.4-4123	95	11.37	6	-13.260	30.227	6.51	—	13.40	13.96	[6]
RX J0420.0-5022	48	3.45	17	-13.553	31.487	6.29	—	13.00	—	[7]



1401.7147

# Spectral properties



Spectra are blackbody plus one or several wide absorption features.

The origin of features is not understood, yet.

New data: Kaplan et al. 1105.4178

# The isolated neutron star candidate 2XMM J104608.7-594306

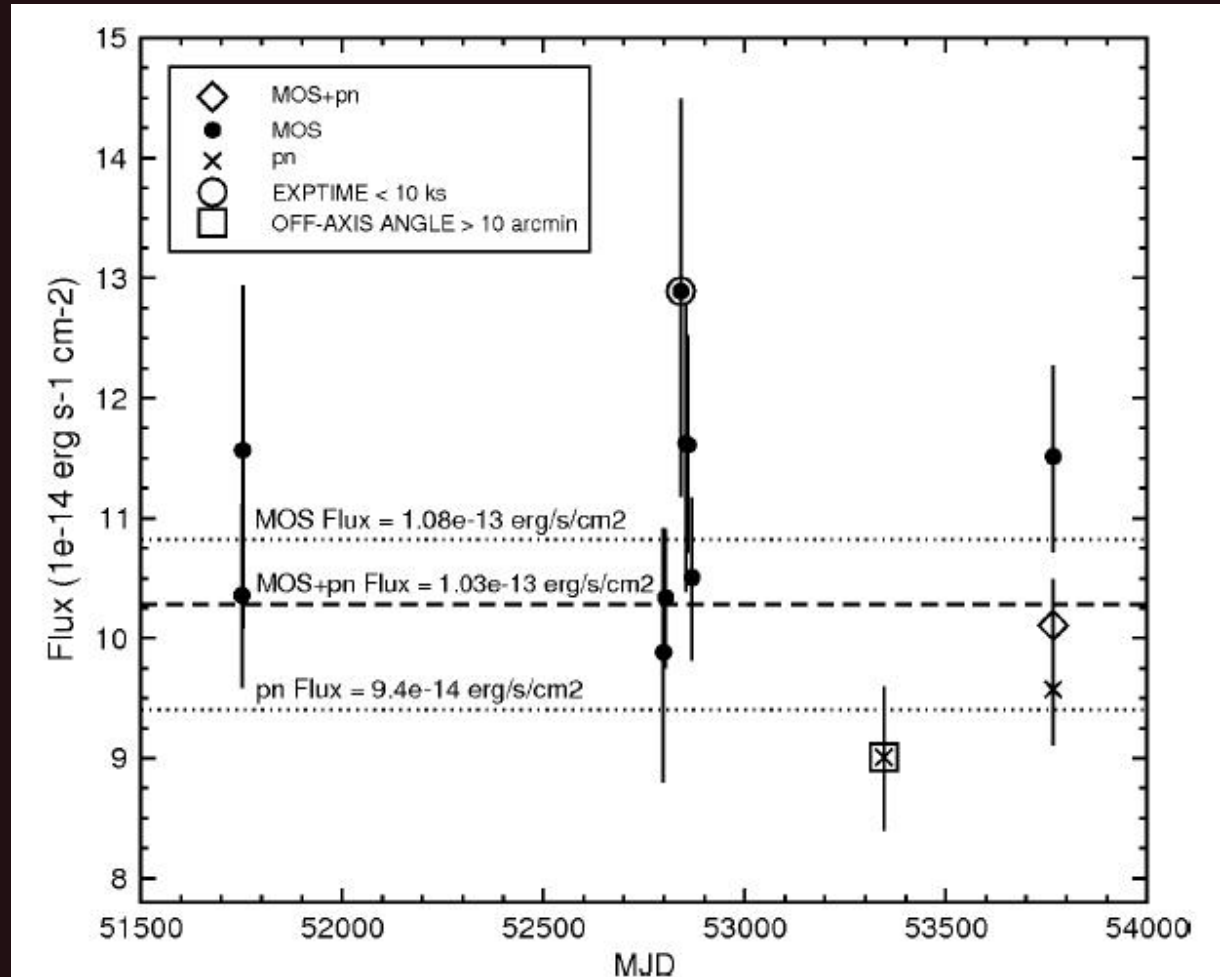
A new INS candidate.

$B > 26$ ,  $V > 25.5$ ,  $R > 25$   
(at  $2.5\sigma$  confidence level)

$\log(F_X/F_V) > 3.1$   
 $kT = 118 \pm 15$  eV

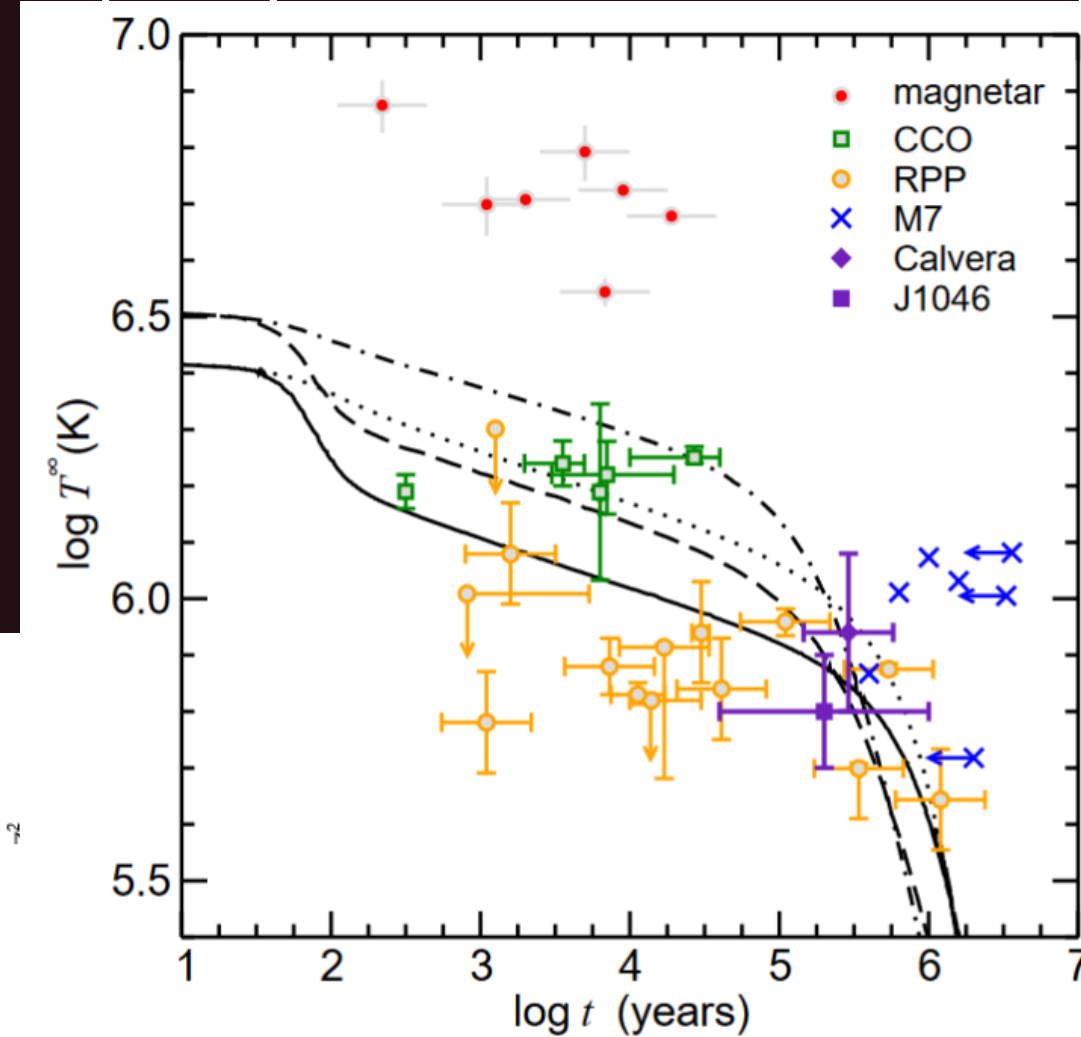
unabsorbed X-ray flux:  
 $F_X \sim 1.3 \cdot 10^{-12}$  erg s $^{-1}$  cm $^{-2}$   
in the 0.1–12 keV band.

At 2.3 kpc (Eta Carina)  
the luminosity is  
 $L_X \sim 8.2 \cdot 10^{32}$  erg s $^{-1}$   
 $R_\infty \sim 5.7$  km

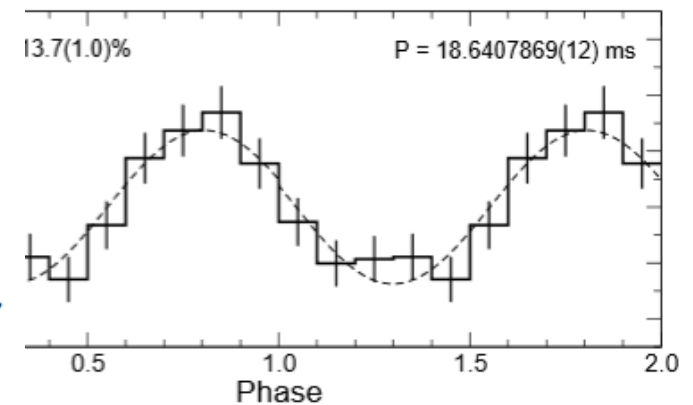


**M7-like? Yes!** Pires & Motch arXiv: [0710.5192](#) and Pires et al. arXiv: 0812.4151  
**But P=19 msec**

# Spin period of 2XMM J1046-5943



Calvera-like?



$$T_{\text{eff}} = (6 - 10) \times 10^5 \text{ K, and } L_X = (1.1 - 7.4) \times 10^{32} \text{ erg s}^{-1}$$

1508.05246



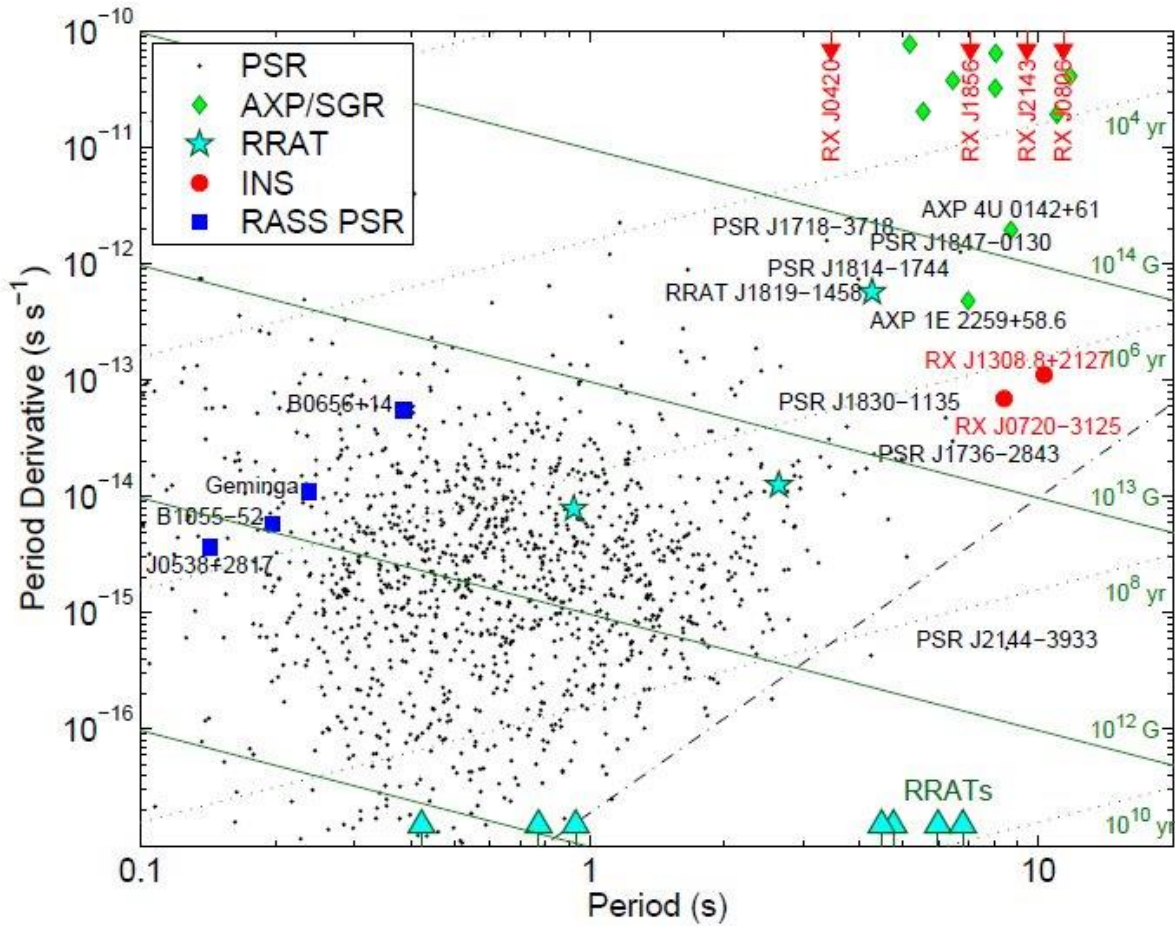
# Radio observations

Up to now the M7 are not detected for sure at radio wavelengths, however, there was a paper by Malofeev et al., in which the authors claim that they had detect two of the M7 at very low wavelength ( $< \sim 100$  MHz).

At the moment the most strict limits are given by Kondratiev et al. Non-detection is still consistent with narrow beams.

XDINS	Pulsed emission			Bursty emission		
	$S_{\text{lim}}$ ( $\mu\text{Jy}$ )	$L_{1400}^{\text{p,max}}$ (mJy kpc <sup>2</sup> )	$L_{820}^{\text{p,max}}$ (mJy kpc <sup>2</sup> )	rate upper limit (hr <sup>-1</sup> )	$S_{\text{lim}}^{\text{sp}}$ (mJy)	$L_{1400}^{\text{b,max}}$ (mJy kpc <sup>2</sup> )
RX J0720.4–3125	8	$4 \times 10^{-4}$	$10^{-3}$	0.25	21	1
RX J0806.4–4123	10	$4 \times 10^{-3}$	$10^{-2}$	0.32	18	6.9
RX J1308.6+2127	10	$4 \times 10^{-3}$	$10^{-2}$	0.24	17	6.5
RX J1605.3+3249	8	$3 \times 10^{-3}$	$8 \times 10^{-3}$	0.25	22	8.4
RX J1856.5–3754	14	$1.4 \times 10^{-4}$	$3.6 \times 10^{-4}$	0.32	24	0.2
RX J2143.0+0654	13	$5 \times 10^{-3}$	$1.3 \times 10^{-2}$	0.36	20	7.6

# M7 among other NSs



Evolutionary links of M7 with other NSs are not clear, yet.

M7-like NSs can be numerous.

They can be descendants  
of magnetars.

Can be related to RRATs.

Or, can be a different population.

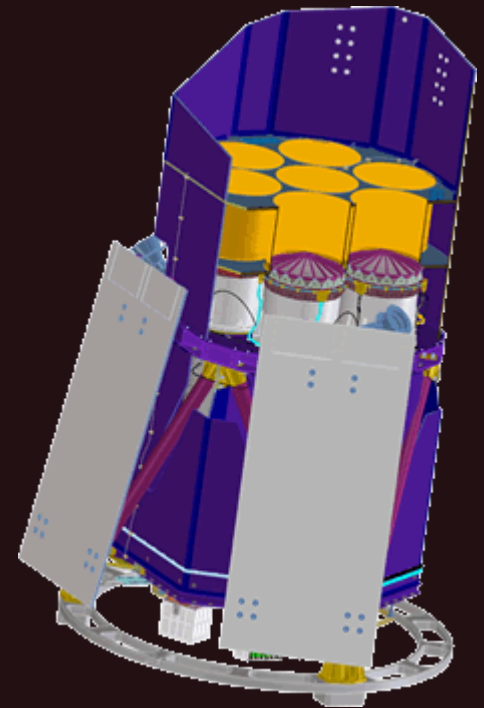
# How to find new candidates?

## 1. Digging the data

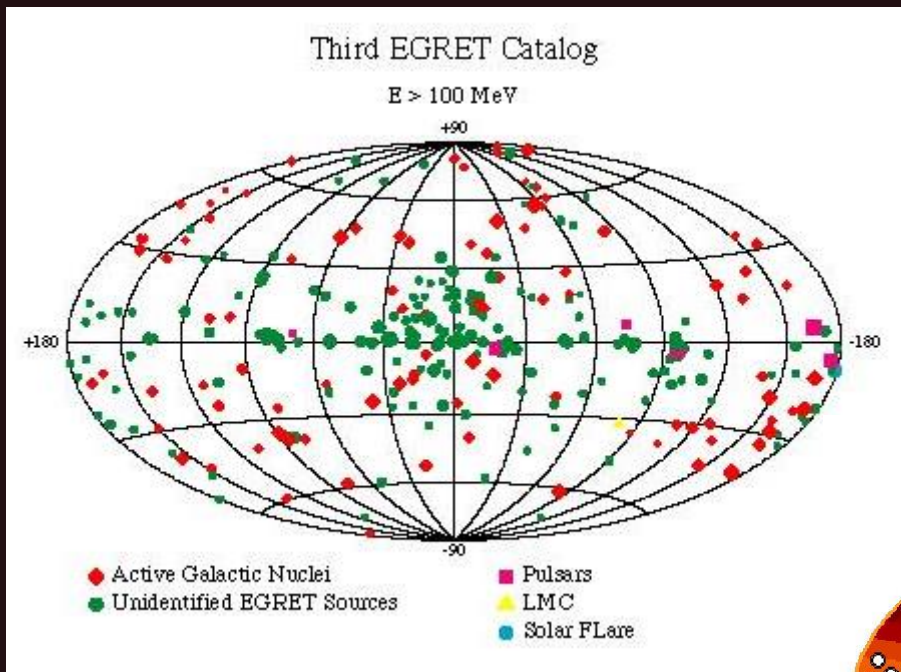
Many attempts failed. One of the latest used SDSS optical data together with ROSAT X-ray. Candidates have been observed by Chandra. Nothing was found (Agueros et al. arXiv: 1103.2132).

## 2. eROSITA is coming!

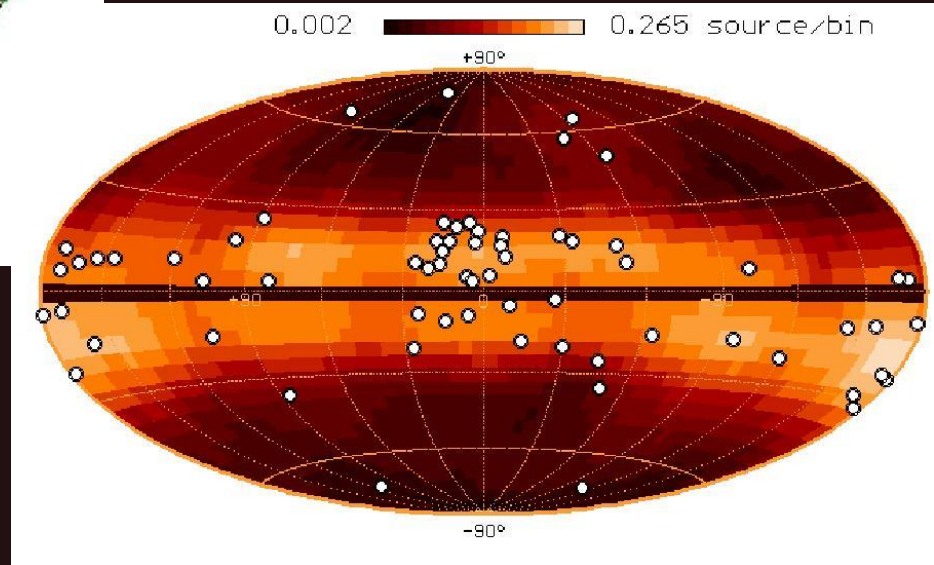
In 2019 spectrum-RG with eROSITA will be launched. It is expected that with this telescope tens of new M7-like NSs can be found (Boldin et al., Pires et al.)



# Pulsars invisible in radio?



EGRET data  
Many unidentified sources

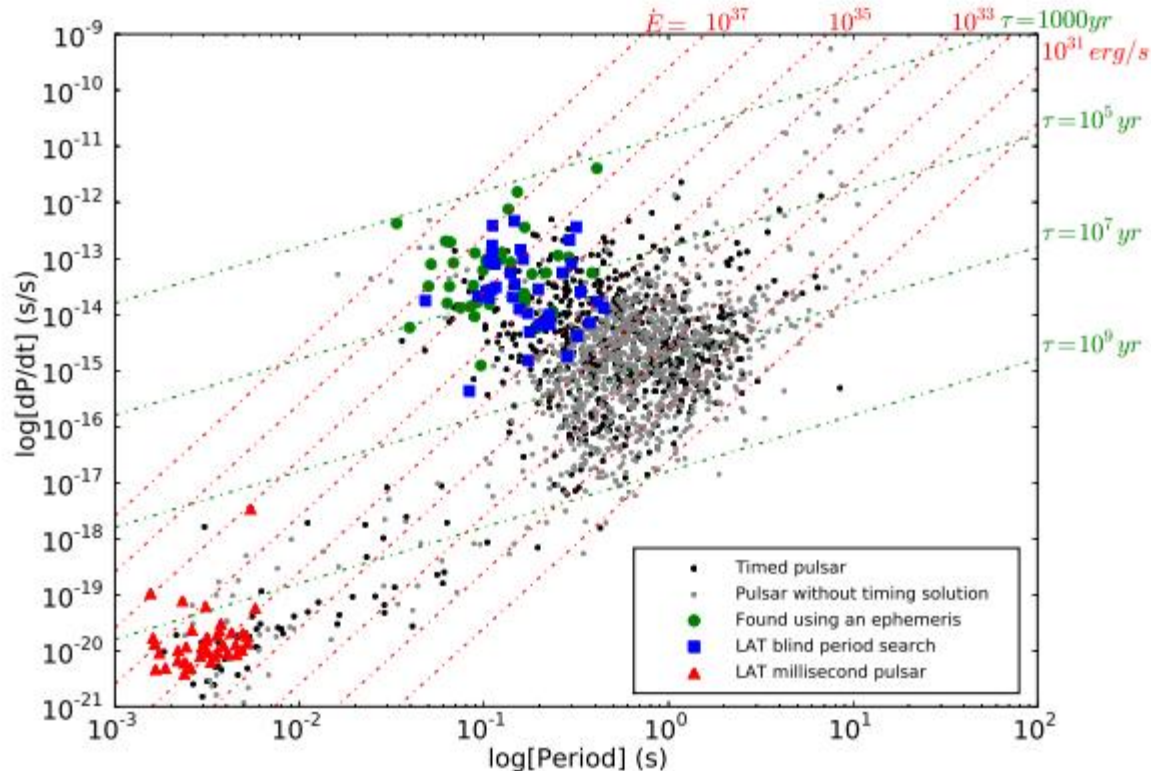


(Nolan et al. astro-ph/9607079)

(Grenier astro-ph/0011298)



# Fermi pulsars



In the 2<sup>nd</sup> catalogue there are 117 pulsars.

1/3 mPSR

The rest are young:  
1/3 radio-loud  
1/3 radio-quiet

1211.3726

Full 2<sup>nd</sup> catalogue is presented in 1305.4385

In the 3<sup>rd</sup> catalogue there are 167 pulsars

[https://fermi.gsfc.nasa.gov/ssc/data/access/lat/4yr\\_catalog/3FGL-table/](https://fermi.gsfc.nasa.gov/ssc/data/access/lat/4yr_catalog/3FGL-table/)

# Discovery of radio transients



McLaughlin et al. (2006) discovered a new type of sources— RRATs (Rotating Radio Transients).

For most of the sources periods about few seconds were discovered. The result was obtained during the Parkes survey of the Galactic plane.

Burst duration 2-30 ms, interval 4 min-3 hr

Periods in the range 0.4-7 s

>100 sources known.

Thermal X-rays were observed from one of the RRATs (Reynolds et al. 2006). This one seems to me the youngest.

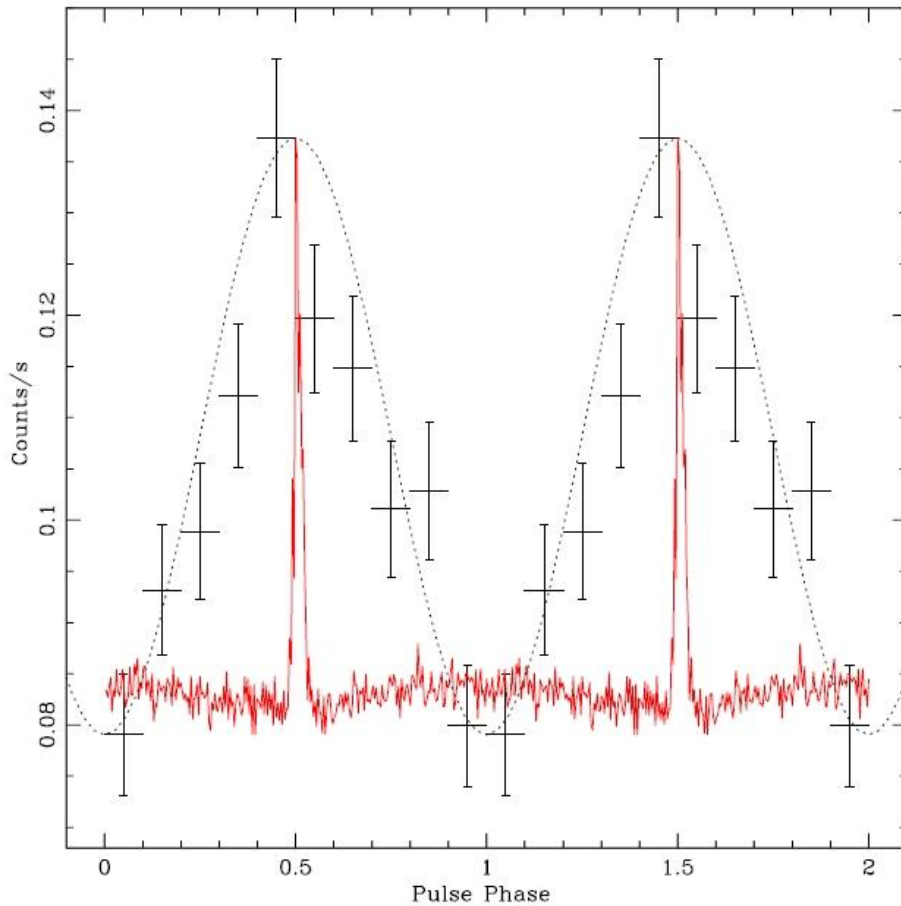
Review: 1109.6896

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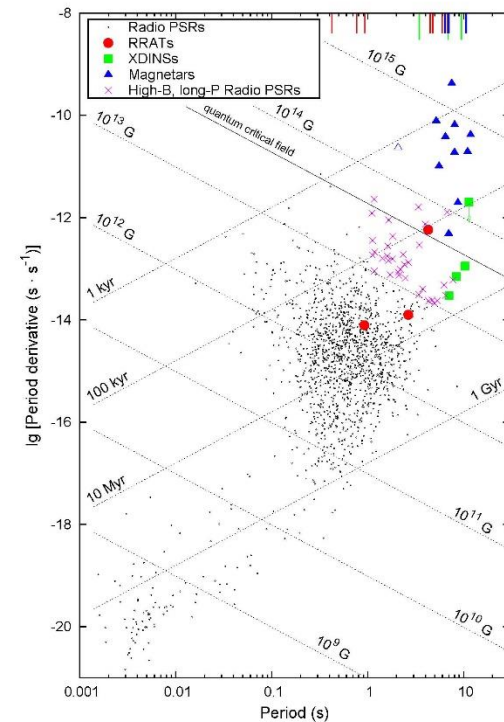
Catalogue: <http://www.as.wvu.edu/~pulsar/rratalog/>



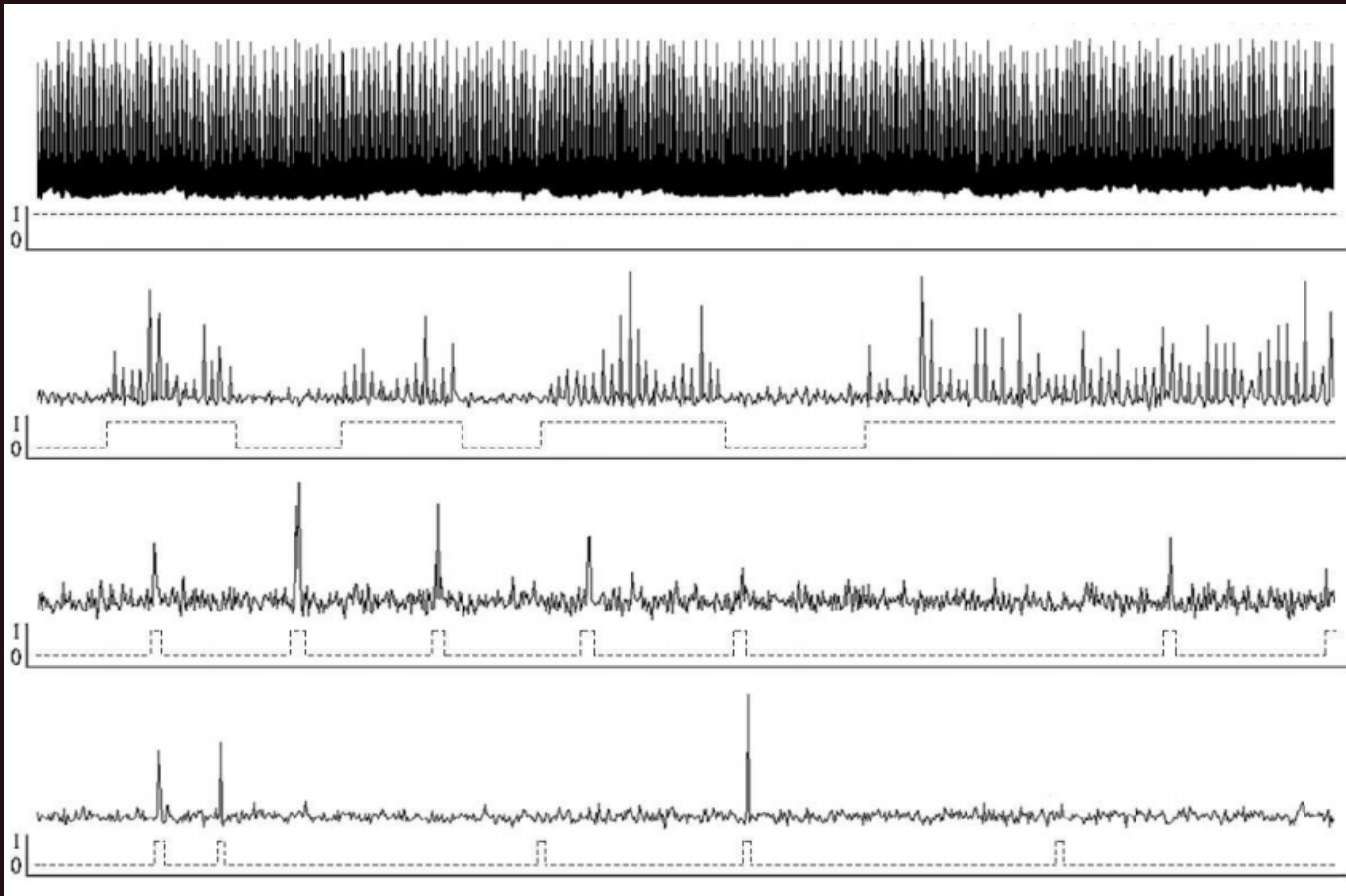
# RRATs. X-ray + radio data



X-ray pulses overlapped on  
radio data of RRAT J1819-1458.



# RRAT – are pulsars?



Vela

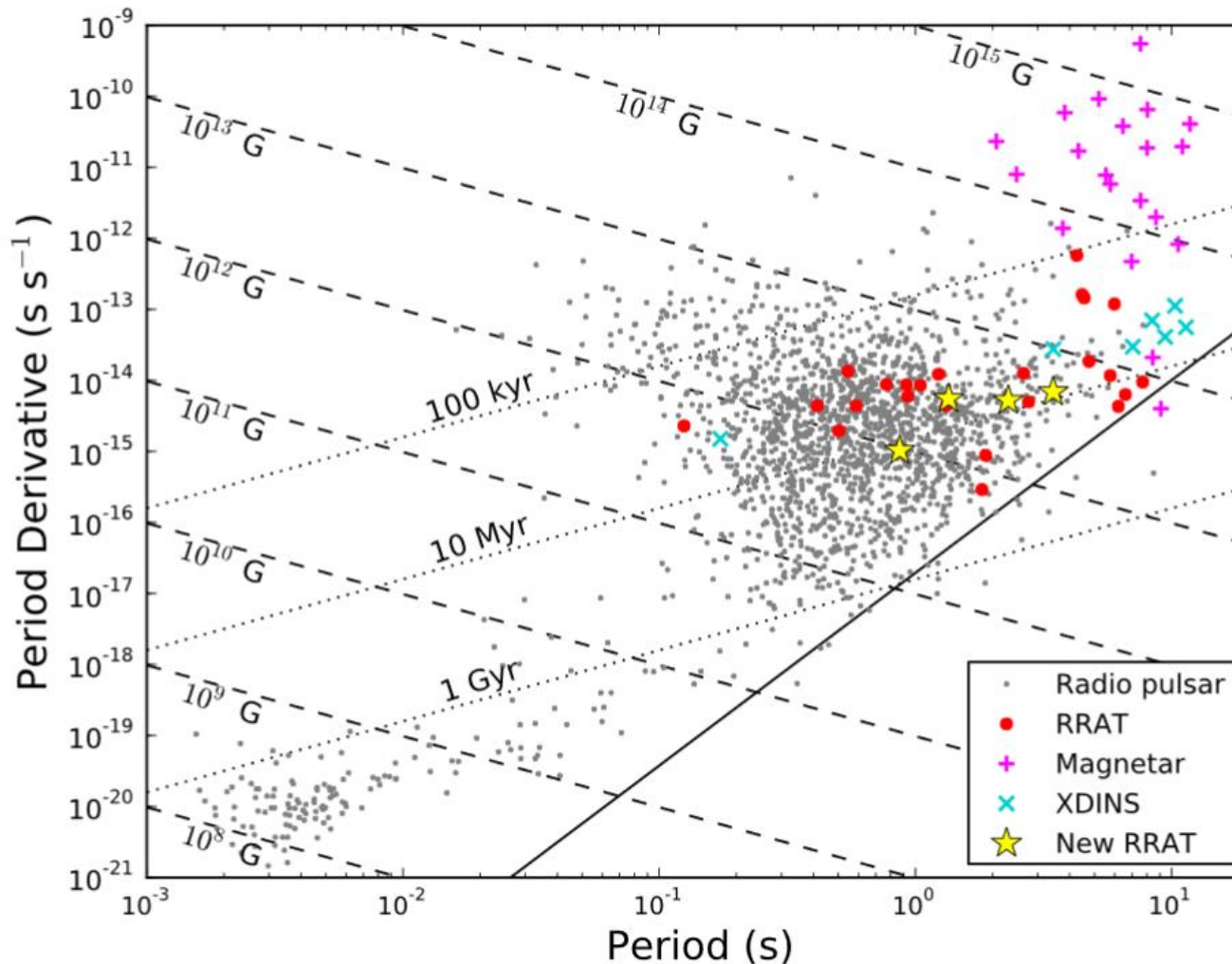
PSR  
J1646-6831

J1647-36

J1226-32

It looks like RRATs bursts are just some kind of magnetospheric activity.  
Some PSRs have similar bursts.  
It is not easy to plot a boarder line between RRATs and PSRs.

# RRATs properties



RRATs with  $P\text{-}\dot{P}$  seem to be similar to PSRs

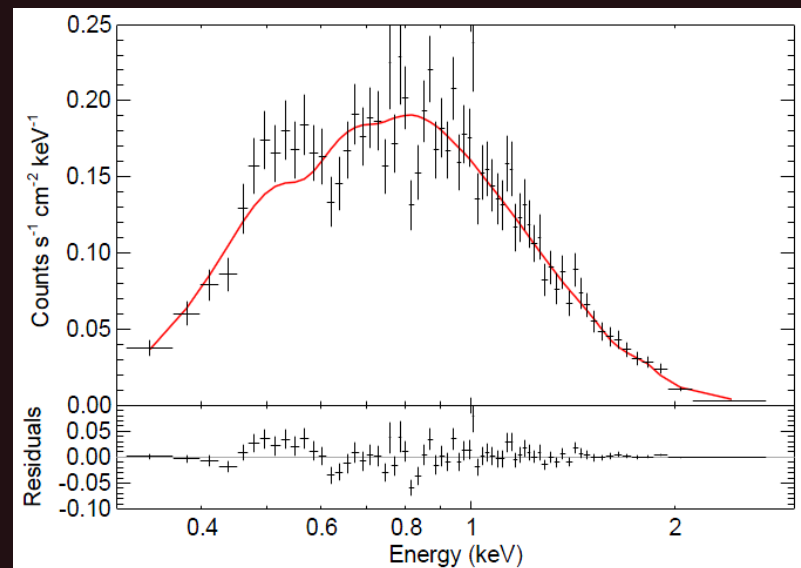
About low-frequency detection see 1807.07565.

# Calvera et al.

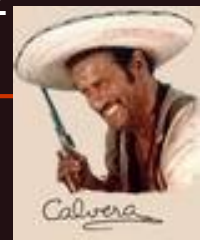


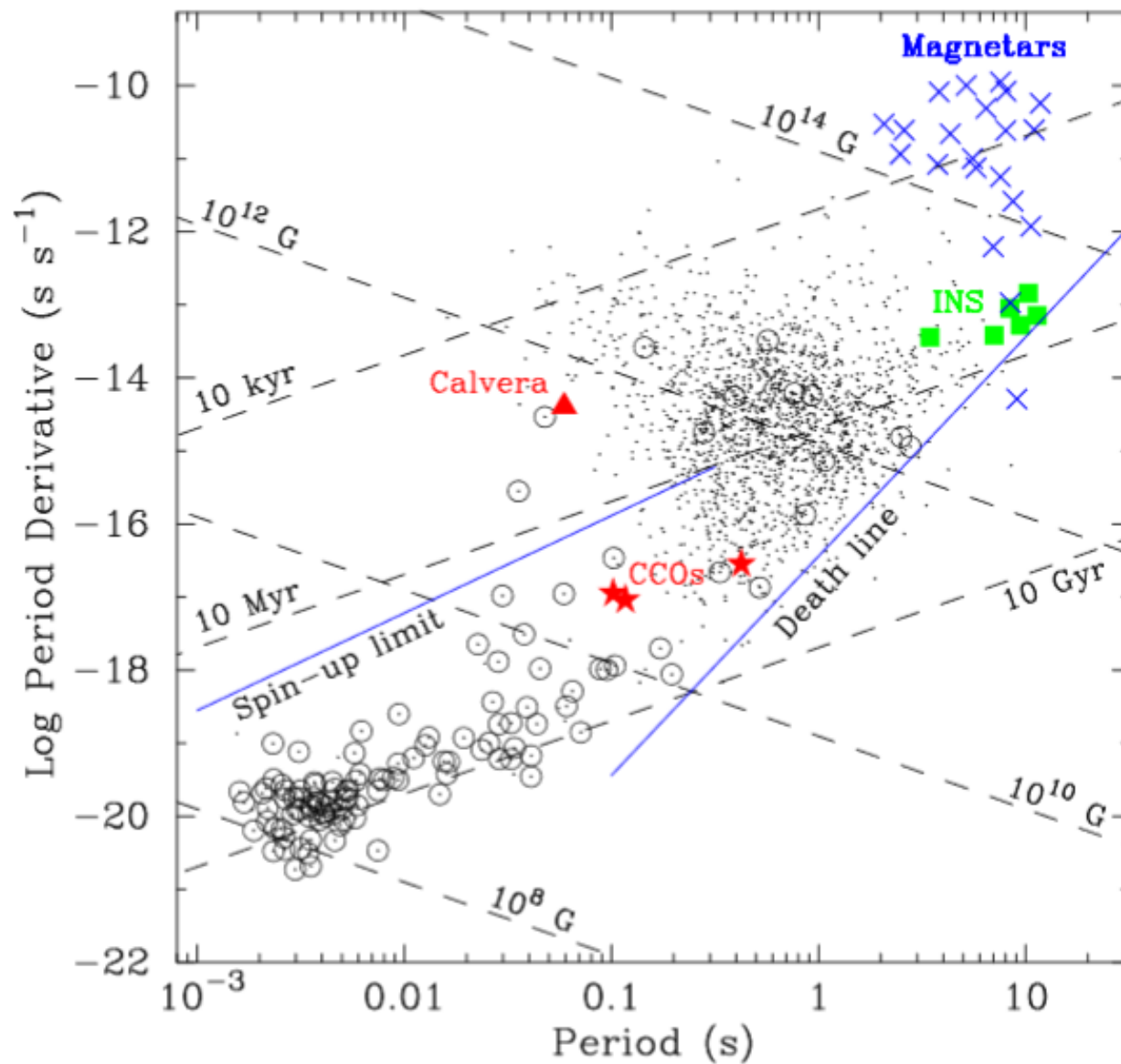
In 2008 Rutledge et al. reported the discovery of an enigmatic NS candidate dubbed *Calvera*. It is high above the galactic plane.

Characteristic	Value
Right Ascension (J2000)	14 <sup>h</sup> 12 <sup>m</sup> 55 <sup>s</sup> .759
Declination (J2000)	+79° 22' 03".41
Uncertainty Ellipse	0.31" (R.A.) × 0.25" (Dec.)
Absorbed Blackbody <sup>a</sup>	
$N_{\text{H}}$	0 (limit)
$kT_{\text{eff}}$	229 eV
$(R_{\text{km}}/D_{10 \text{ kpc}})^2$	26.6
Observed X-ray Flux (0.3–9.5 keV)	$7.1 \times 10^{-13} \text{ erg cm}^{-2} \text{ s}^{-1}$
$\chi^2_{\nu} (\nu)$	2.04 (67 dof)
NS Hydrogen Atmosphere (NSA) <sup>b</sup>	
$N_{\text{H}}$	$3.1^{+0.9}_{-0.9} \times 10^{20} \text{ cm}^{-2}$
$kT_{\text{eff}}$	$109^{+1}_{-1} \text{ eV}$
$D_{\text{kpc}}^{-2}$	$7.71^{+0.41}_{-0.38} \times 10^{-2}$
Observed X-ray Flux (0.3–9.5 keV)	$7.62 \times 10^{-13} \text{ erg cm}^{-2} \text{ s}^{-1}$
$\chi^2_{\nu} (\nu)$	1.31 (67 dof)

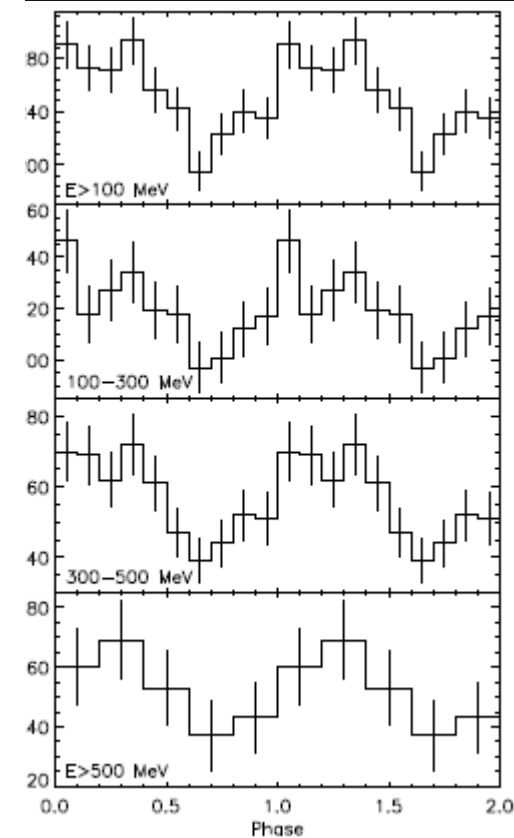


Shevchuk et al. arXiv: 0907.4352





50 eV and  $\sim 80/250 \text{ eV}$ )



New data 1310.6789, 1510.00683

# Some LIGO results

[arxiv:1701.07709](https://arxiv.org/abs/1701.07709)

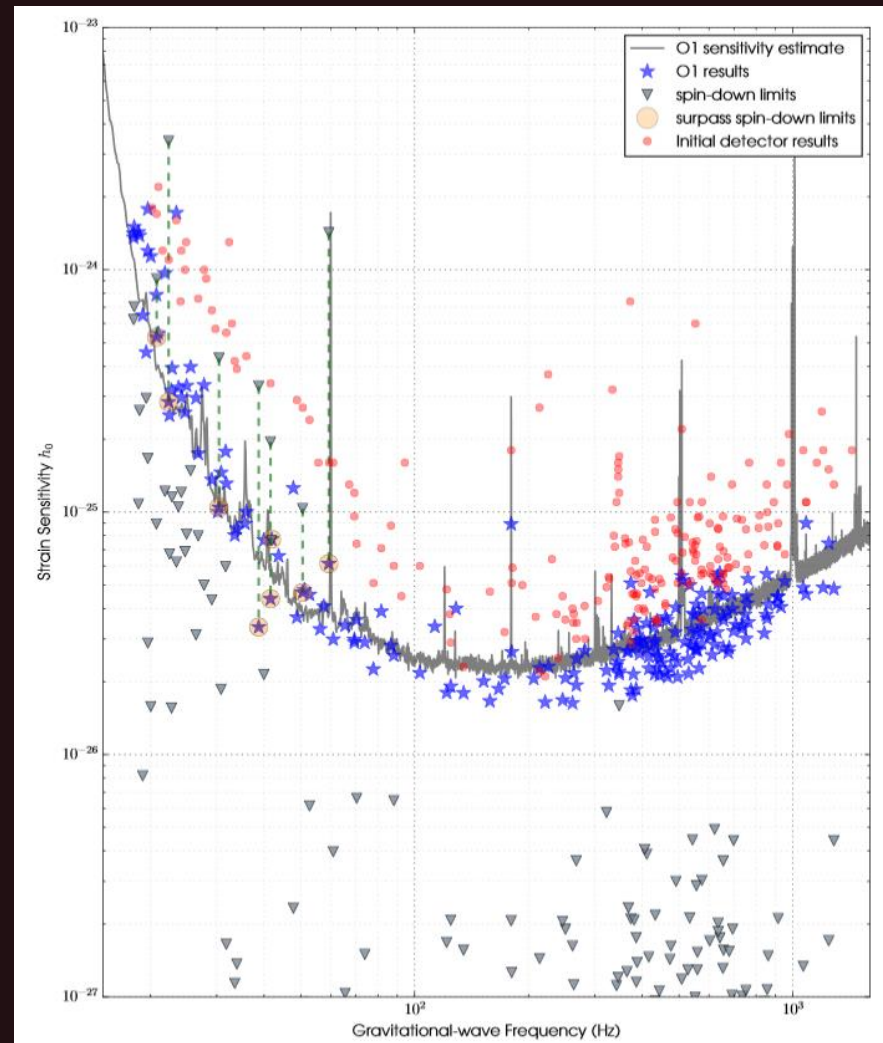
## First search for gravitational waves from known pulsars with Advanced LIGO

~200 pulsars searched.

Upper limits on GW emission from some known pulsars are becoming constraining.

For Crab  $< 0.002$  and for Vela  $< 0.01$  of the spin-down can be due to GW.

See a review on GW from NSs in 0912.0384,  
GEO600 results in 1309.4027



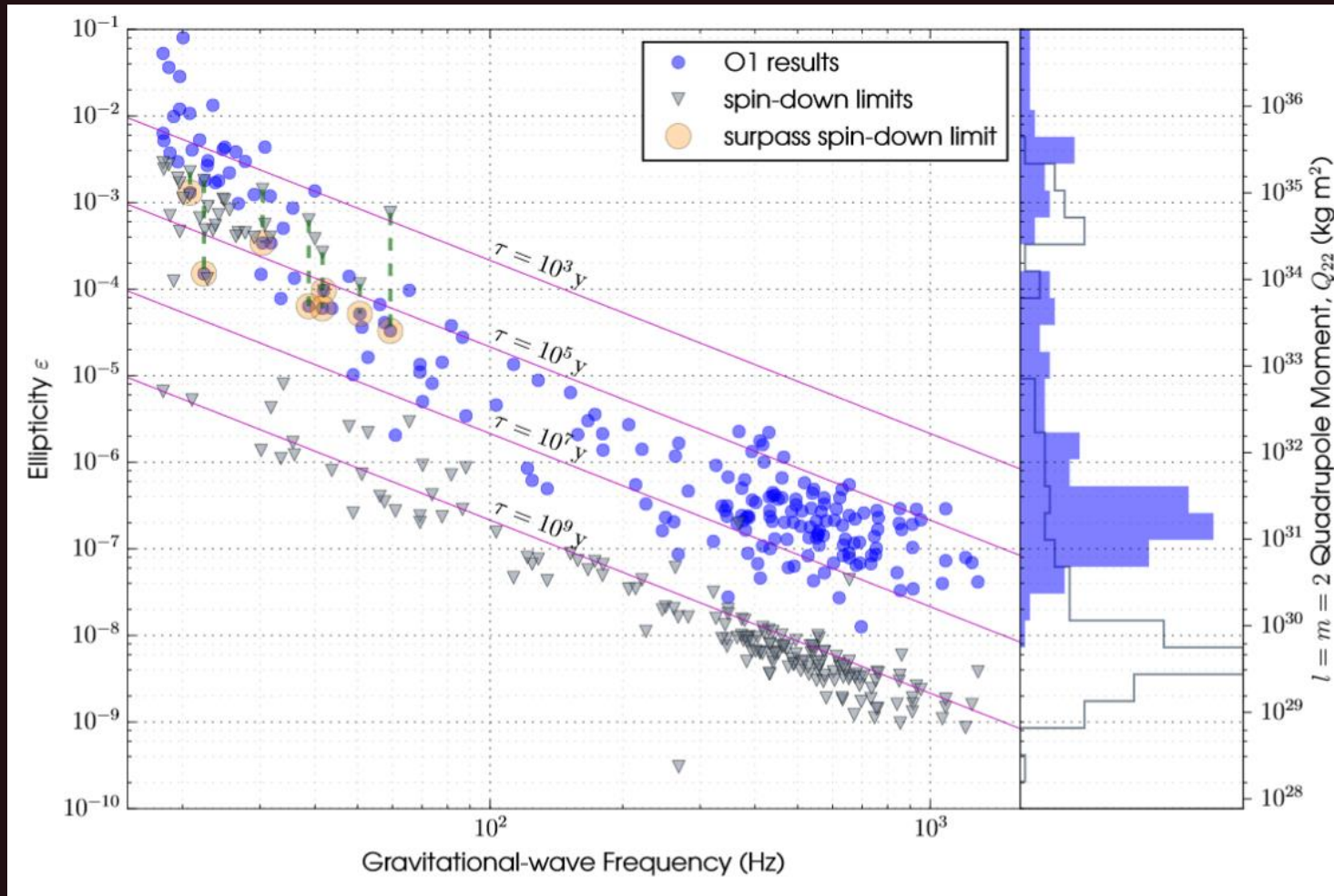
[arxiv:1701.07709](https://arxiv.org/abs/1701.07709)



# Limits on ellipticity

$$Q_{22} = h_0 \left( \frac{c^4 d}{16\pi^2 G f_{\text{rot}}^2} \right) \sqrt{\frac{15}{8\pi}}$$

$$\varepsilon = \frac{Q_{22}}{I_{zz}} \sqrt{\frac{8\pi}{15}}$$



# New search

Eleven pulsars in Advanced LIGO first observing run

1710.02327

Name	distance[kpc]	$h_{\text{sd}} \cdot 10^{-25}$	$\epsilon_{\text{sd}} \cdot 10^{-4}$	$h_{\text{ul}}$ $10^{-25}$	$\epsilon_{\text{ul}}$ $\cdot 10^{-4}$	$h_{\text{ul}}/h_{\text{sd}}$	$\dot{E}_{\text{rot}}/\dot{E}_{\text{GW}}$
J0205+6449 <sup>a</sup>	$2.0 \pm 0.3$ <sup>b</sup>	$6.9 \pm 1.1$	14	3.76	7.7	$0.54 \pm 0.09$	0.29
J0534+2200 (Crab)	$2.0 \pm 0.5$ <sup>c</sup>	$14 \pm 3.5$	7.6	1.08	0.58	$0.07 \pm 0.02$	0.005
J0835-4510 (Vela)	$0.28 \pm 0.02$ <sup>c</sup>	$34 \pm 2.4$	18	9.28	5.3	$0.27 \pm 0.02$	0.07
J1400-6326	$10 \pm 3$ <sup>d</sup>	$0.90 \pm 0.27$	2.1	1.17	2.7	$1.3 \pm 0.4$	-
J1813-1246	$> 2.5$ <sup>e</sup>	$< 1.8$	$< 2.4$	1.80	2.5	$> 1.0$	-
J1813-1749	$4.8 \pm 0.3$ <sup>f</sup>	$3.0 \pm 0.2$	7.0	1.9	4.8	$0.64 \pm 0.04$	0.41
J1833-1034	$4.8 \pm 0.4$ <sup>g</sup>	$3.1 \pm 0.3$	13	3.08	13	$0.99 \pm 0.09$	-
J1952+3252	$3.0 \pm 0.5$ <sup>h</sup>	$1.0 \pm 0.2$	1.1	1.31	1.4	$1.31 \pm 0.22$	-
J2022+3842	$10 \pm 2$ <sup>i</sup>	$1.0 \pm 0.3$	6.0	1.90	11	$1.77 \pm 0.35$	-
J2043+2740	$1.5 \pm 0.6$ <sup>j</sup>	$6.9 \pm 2.8$	23	14.4	47	$2.07 \pm 0.83$	-
J2229+6114	$3.0 \pm 2$ <sup>c</sup>	$3.4 \pm 2.2$	6.2	1.78	3.4	$0.54 \pm 0.35$	0.30

More results on 16 NSs: 1812.11656

# Search for periodic signals by LIGO

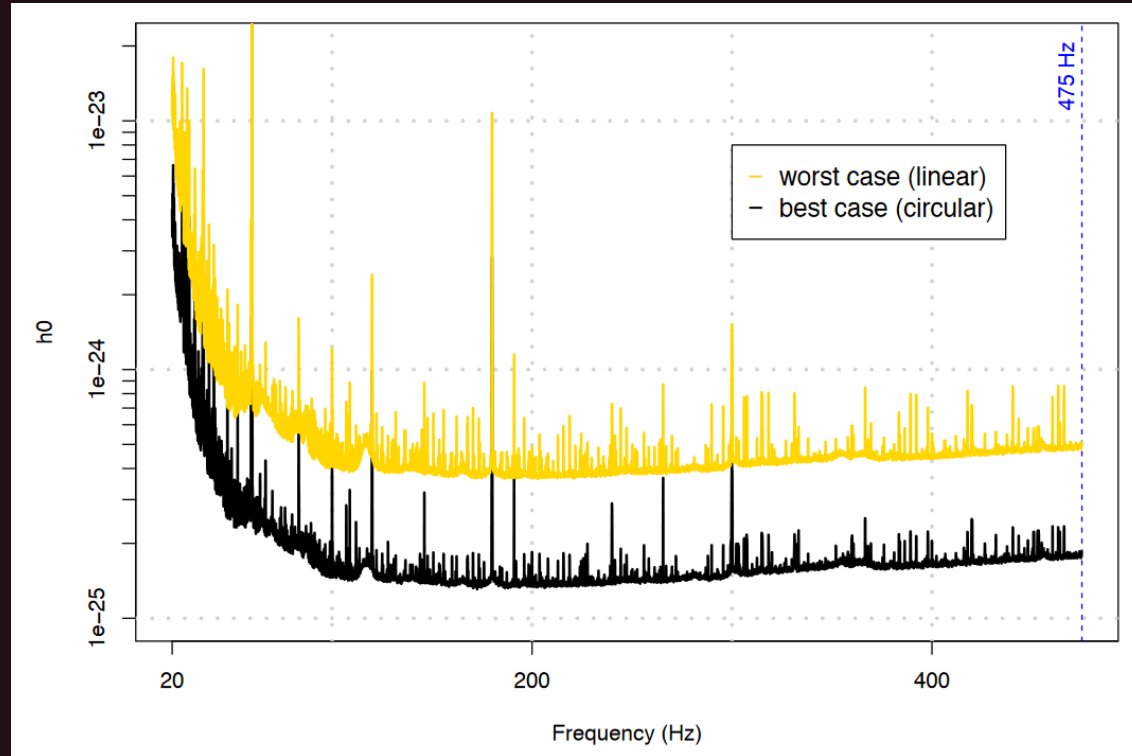
20-475 Hz

No signals

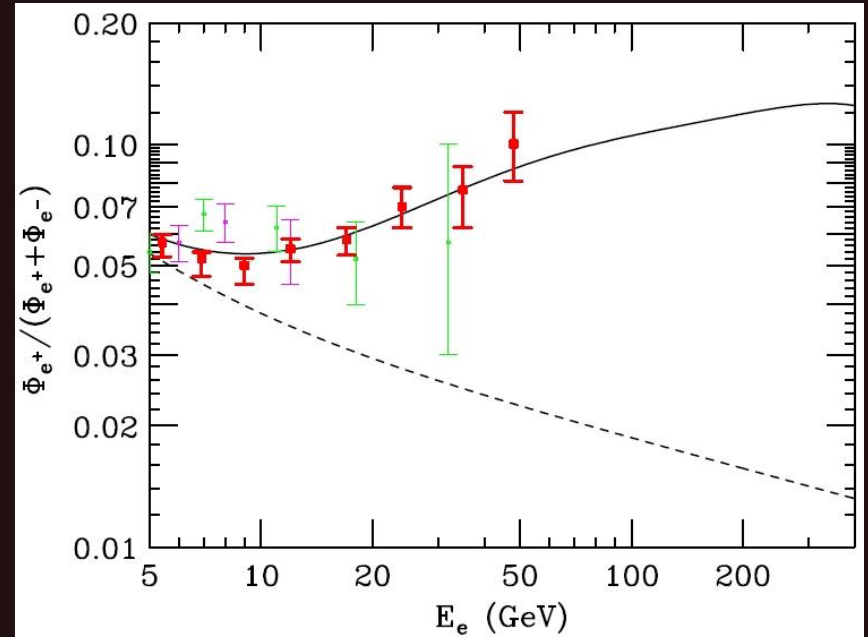
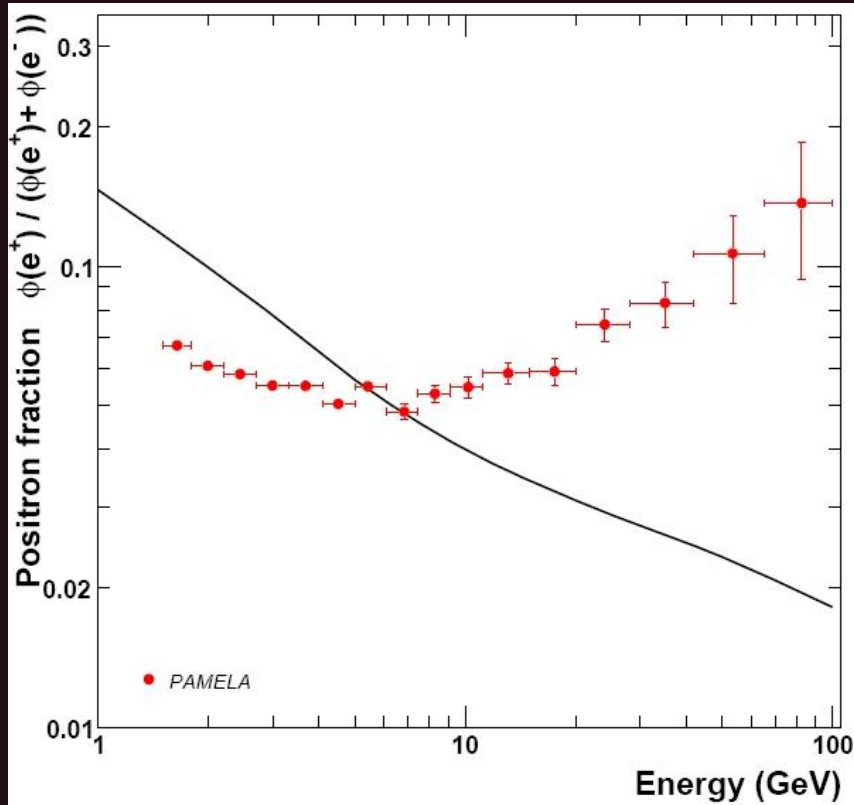
Could detect ellipticity  $8 \times 10^{-7}$  from 1 kpc at the upper limit.

And  $10^{-5}$  from 1 kpc at 200 Hz.

See a review on GW emission  
from isolated NSs in 1709.07049



# Pulsars, positrons, PAMELA

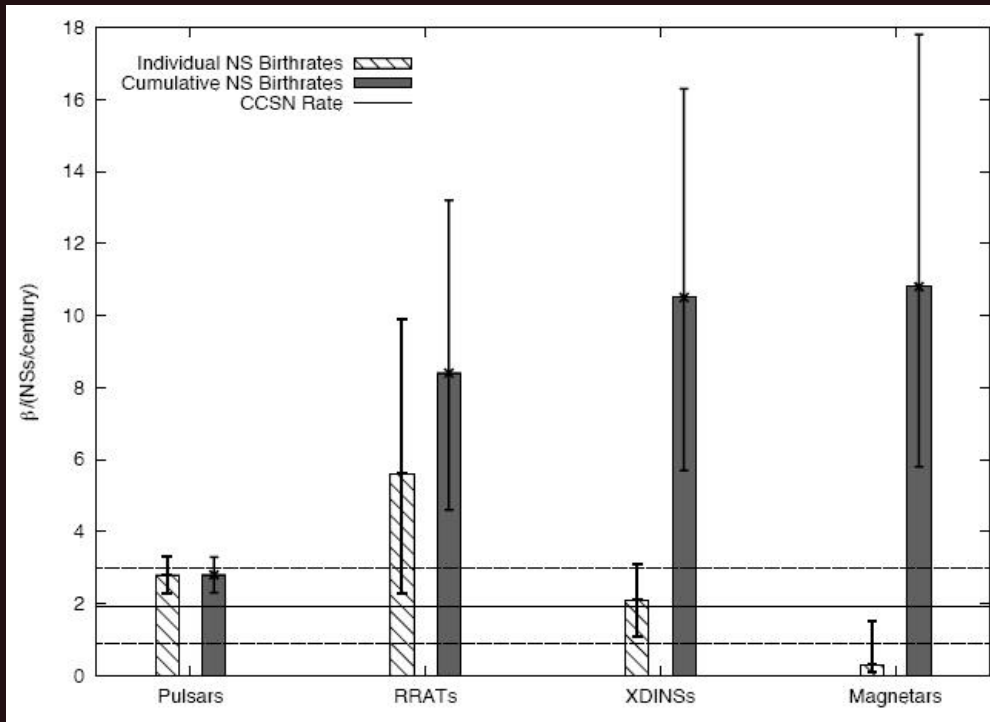
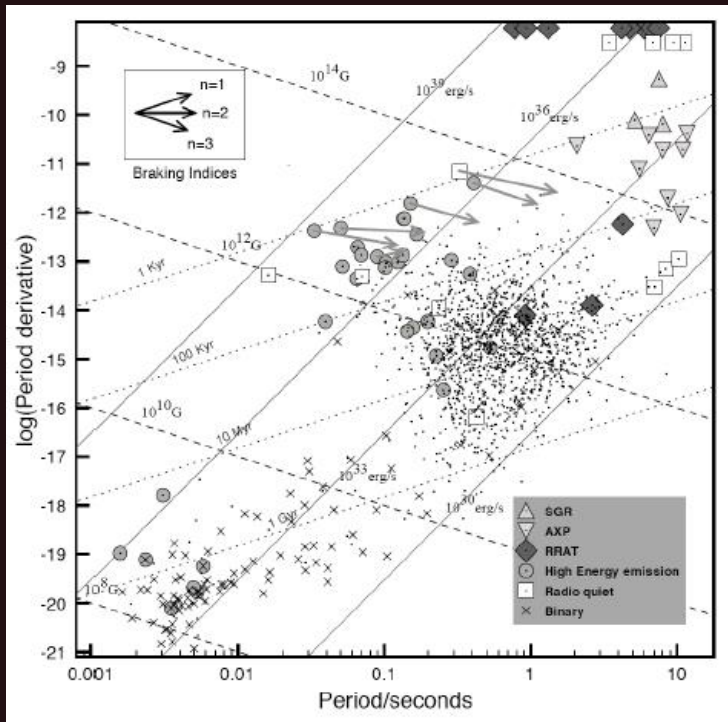


Geminga, PSR B0656+14, and all PSRs

[O. Adriani et al.] [arXiv:0810.4995](https://arxiv.org/abs/0810.4995)

[Dan Hooper et al. 2008  
[arXiv: 0810.1527](https://arxiv.org/abs/0810.1527)]

# NS birth rate



# Too many NSs???

$\beta_{\text{PSR}}, n_e$	PSRs	RRATs	XDINSs	Magnetars	Total	CCSN rate
FK06, NE2001	$2.8 \pm 0.5$	$5.6^{+4.3}_{-3.3}$	$2.1 \pm 1.0$	$0.3^{+1.2}_{-0.2}$	$10.8^{+7.0}_{-5.0}$	$1.9 \pm 1.1$
L+06, NE2001	$1.4 \pm 0.2$	$2.8^{+1.6}_{-1.6}$	$2.1 \pm 1.0$	$0.3^{+1.2}_{-0.2}$	$6.6^{+4.0}_{-3.0}$	$1.9 \pm 1.1$
L+06, TC93	$1.1 \pm 0.2$	$2.2^{+1.7}_{-1.3}$	$2.1 \pm 1.0$	$0.3^{+1.2}_{-0.2}$	$5.7^{+4.1}_{-2.7}$	$1.9 \pm 1.1$
V+04, NE2001	$1.6 \pm 0.3$	$3.2^{+2.5}_{-1.9}$	$2.1 \pm 1.0$	$0.3^{+1.2}_{-0.2}$	$7.2^{+5.0}_{-3.4}$	$1.9 \pm 1.1$
V+04, TC93	$1.1 \pm 0.2$	$2.2^{+1.7}_{-1.3}$	$2.1 \pm 1.0$	$0.3^{+1.2}_{-0.2}$	$5.7^{+4.1}_{-2.7}$	$1.9 \pm 1.1$

It seems, that the total birth rate is larger than the rate of CCSN.  
 $e^-$  - capture SN cannot save the situation, as they are  $< \sim 20\%$ .

Note, that the authors do not include CCOs.

So, some estimates are wrong, or some sources evolve into others.

See also [astro-ph/0603258](#).

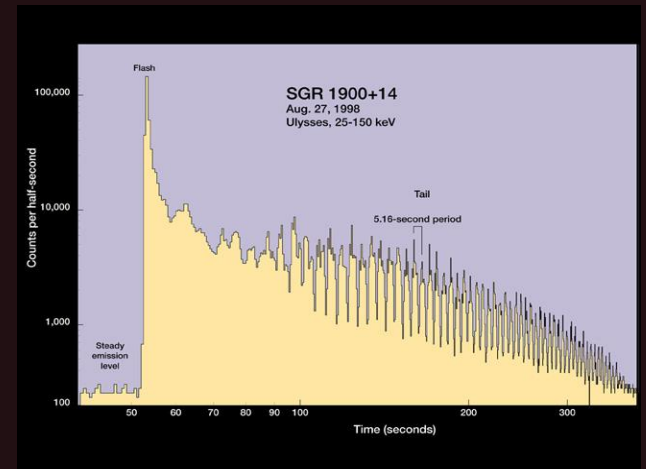
GRAND UNIFICATION: 1005.0876

Keane, Kramer 2008, arXiv: 0810.1512



# Conclusion

- There are several types of sources: CCOs, M7, SGRs, AXPs, RRATs ...
- Magnetars
- Significant fraction of all newborn NSs are not similar to the Crab pulsar
- Unsolved problems:
  1. Are there links?
  2. Reasons for diversity



# Some reviews on isolated neutron stars

- NS basics: [physics/0503245](#)  
[astro-ph/0405262](#)  
[1507.06186](#)
- Thermal emission [1507.02924](#)
- Magnetars: [arXiv: 1101.4472](#)
- Magnetar bursts: [astro-ph/0311526](#)
- CCOs: [arxiv:0712.2209](#)
- Quark stars: [arxiv:0809.4228](#)
- The Magnificent Seven: [astro-ph/0609066](#)  
[arxiv:0801.1143](#)
- RRATs: [arXiv:1008.3693](#)
- Cooling of NSs: [astro-ph/0402143](#)
- NS structure [arXiv:0705.2708](#)
- EoS [arXiv: 1001.3294](#)  
[1512.07820](#)
- NS atmospheres [1403.0074](#)
- NS magnetic fields [arxiv:0711.3650](#)
- Different types [arXiv:1005.0876](#)  
[arXiv:1302.0869](#)  
[arXiv: 1712.06040](#)
- Radio pulsars [1602.07738](#)
- Internal structure and astrophysics [1603.02698](#)
- SN and compact remnants [1806.07267](#)

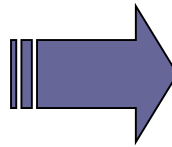
Lectures can be found  
at my homepage:

[http://xray.sai.msu.ru/~polar/  
html/presentations.html](http://xray.sai.msu.ru/~polar/html/presentations.html)

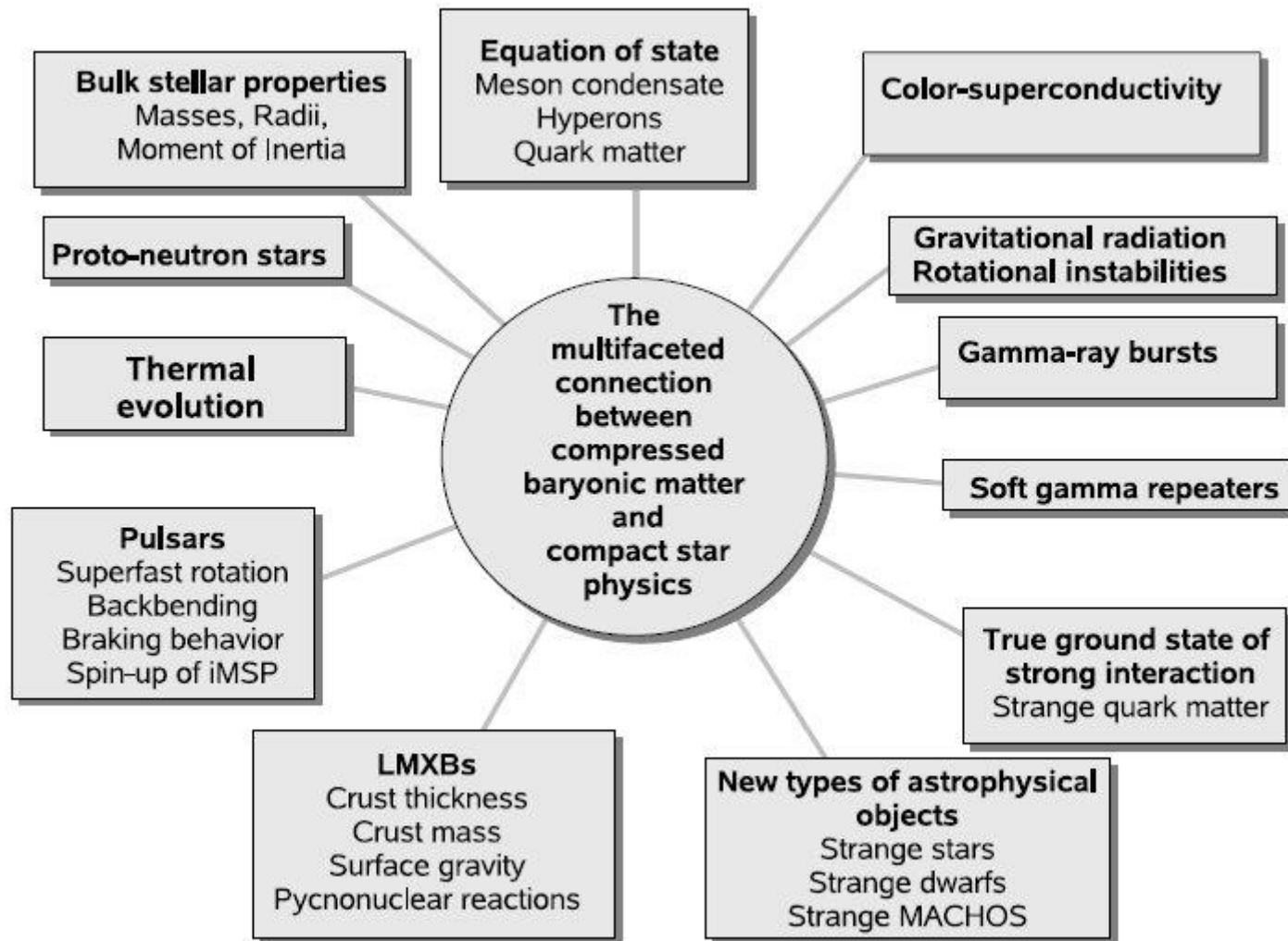
Read the OVERVIEW in the book by Haensel, Yakovlev, Potekhin

# Internal structure of Neutron Stars

# Artistic view



# Astronomy meets QCD





# Hydrostatic equilibrium for a star

$$\left\{ \begin{array}{ll} (1) & \frac{dP}{dr} = -\frac{Gm\rho}{r^2} \quad m = m(r) \\ (2) & \frac{dm}{dr} = 4\pi\rho r^2 \\ (3) & \cancel{\frac{dS}{dt} = Q} \\ (4) & P = P(\rho) \end{array} \right.$$

For NSs we can take  $T=0$   
and neglect the third equation

For a NS effects of GR are also important.

$$r_g = \frac{2GM}{c^2} \approx 2.95 \frac{M}{M_{\text{SUN}}} \text{ km}$$

$$M/R \sim 0.15 (M/M_{\odot})(R/10 \text{ km})^{-1}$$

$$J/M \sim 0.25 (1 \text{ ms}/P) (M/M_{\odot})(R/10 \text{ km})^2$$



# Lane-Emden equation. Polytropes.

$$P = K\rho^\gamma, \quad K, \gamma = \text{const}, \quad \gamma = 1 + \frac{1}{n}$$

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} = g\rho, \quad g = -\frac{Gm}{r^2} = -\frac{d\varphi}{dr}$$

$$\frac{dP}{dr} = -\rho \frac{d\varphi}{dr}, \quad \Delta\varphi = 4\pi G\rho$$

$$\rho = \rho_c \Theta^n, \quad \Theta = 1 \text{ при } r = 0$$

$$P = K\rho_c^{1+1/n} \Theta^{1+n}, \quad \frac{dP}{dr} = (n+1)K\rho_c^{1+1/n} \Theta^n \frac{d\Theta}{dr}$$

$$\frac{d\varphi}{dr} = -(n+1)K\rho_c^{1/n} \frac{d\Theta}{dr}$$

$$\Delta\Theta = -\frac{4\pi G\rho_c^{1-1/n}}{(n+1)K} \Theta^n$$

$$\xi = r/a, \quad a^2 = (n+1)K\rho_c^{1/n-1}/(4\pi G)$$

$$\frac{1}{\xi^2} \frac{d}{d\xi} \xi^2 \frac{d}{d\xi} \Theta = -\Theta^n$$

$$\Theta = \Theta(\xi)$$

$$0 \leq \xi \leq \xi_1$$

$$\Theta(0) = 1, \quad \Theta'(0) = 0$$

$$\Theta(\xi_1) = 0$$

# Properties of polytropic stars

## Analytic solutions:

$$\begin{array}{lll}
 n=0 & \Theta = 1 - \frac{\xi^2}{6} & \xi_1 = \sqrt{6} \\
 n=1 & \Theta = \frac{\sin \xi}{\xi} & \xi_1 = \pi \\
 n=5 & \Theta = \frac{1}{\sqrt{1 + \xi^2/3}} & \xi_1 = \infty
 \end{array}$$

$$M = 4\pi \int_0^R dr r^2 \rho = 4\pi \rho_c a^3 \xi_1^2 |\Theta'(\xi_1)|$$

$$\frac{\rho_c}{\rho} = \frac{4\pi R^3 \rho_c}{3M} = \frac{\xi_1}{3|\Theta'(\xi_1)|}$$

↓  $\gamma=5/3$

↓  $\gamma=4/3$

$n$	0	1	1.5	2	3
$\xi_1$	2.449	3.142	3.654	4.353	6.897
$ \Theta'_1 $	0.7789	0.3183	0.2033	0.1272	0.04243
$\rho_c / \bar{\rho}$	1	3.290	5.991	11.41	54.04

$$M \sim \rho_c^{(3-n)/(2n)}$$

$$R \sim \rho_c^{(1-n)/(2n)}$$

$$M \sim R^{(3-n)/(1-n)}$$

$$n=0 \quad M \sim R^3$$

$$n=1 \quad M \sim \rho_c \quad R = \text{const}$$

$$n=1.5 \quad M \sim \sqrt{\rho_c} \sim R^{-3}$$

$$n=3 \quad M = \text{const} \quad R \sim \rho_c^{-1/3}$$

# Useful equations

## White dwarfs

### 1. Non-relativistic electrons

$$\gamma=5/3, K=(3^{2/3} \pi^{4/3} /5) (\hbar^2/m_e m_u^{5/3} \mu_e^{5/3});$$

$\mu_e$ -mean molecular weight per one electron

$$K=1.0036 \cdot 10^{13} \mu_e^{-5/3} \text{ (CGS)}$$

### 2. Relativistic electrons

$$\gamma=4/3, K=(3^{1/3} \pi^{2/3} /4) (\hbar c/m_u^{4/3} \mu_e^{4/3});$$
$$K=1.2435 \cdot 10^{15} \mu_e^{-4/3} \text{ (CGS)}$$

## Neutron stars

### 1. Non-relativistic neutrons

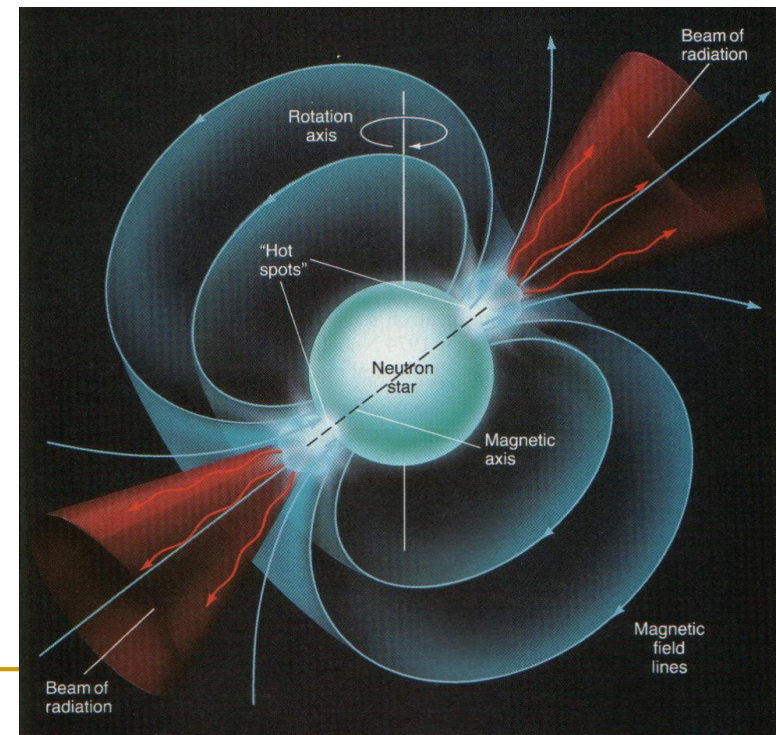
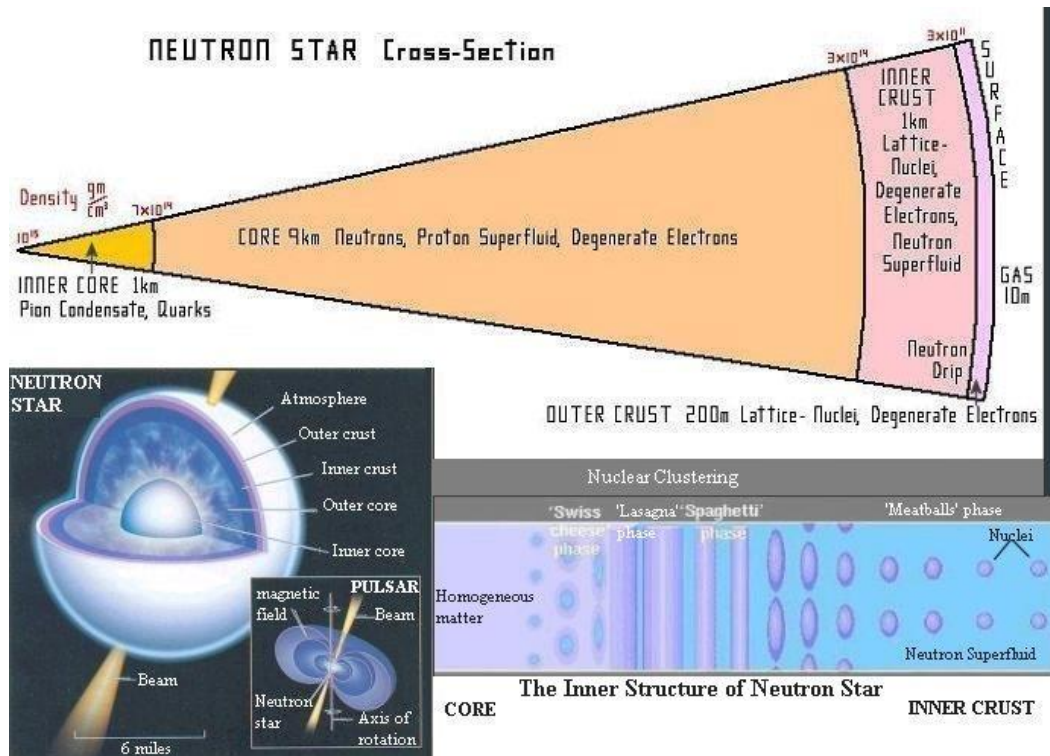
$$\gamma=5/3, K=(3^{2/3} \pi^{4/3} /5) (\hbar^2/m_n^{8/3});$$
$$K=5.3802 \cdot 10^9 \text{ (CGS)}$$

### 2. Relativistic neutrons

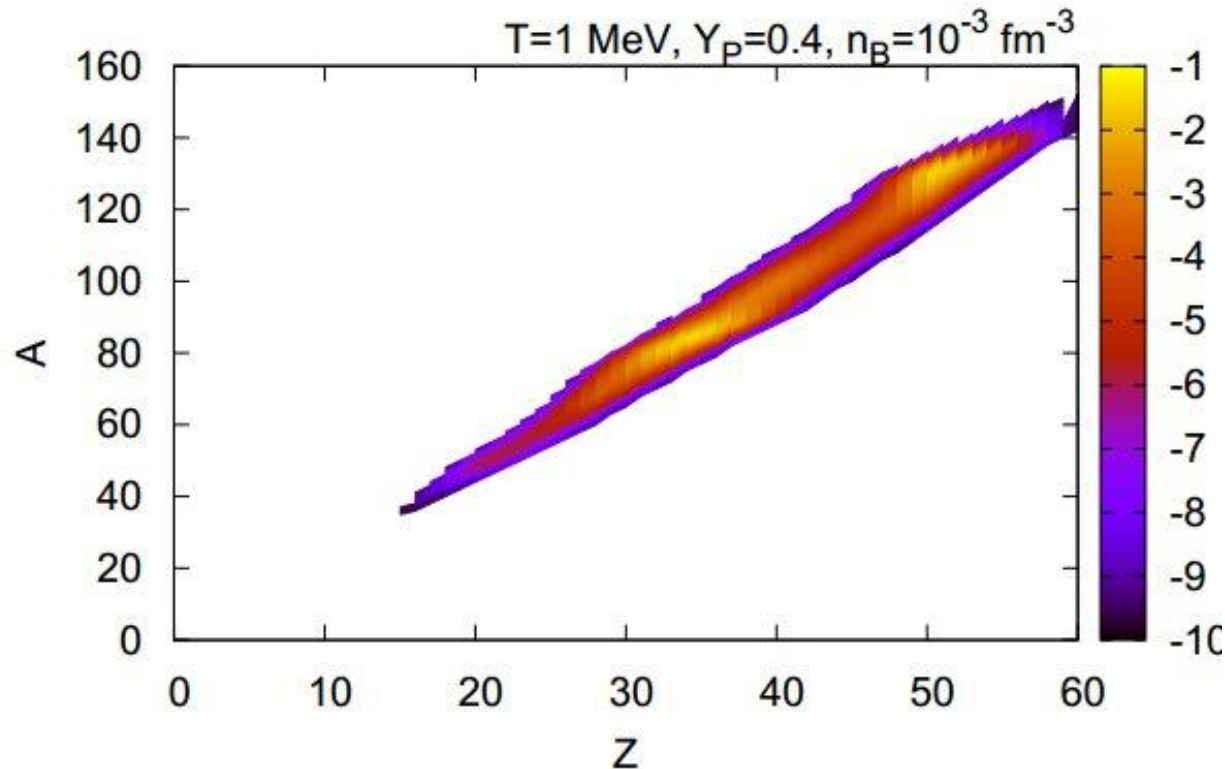
$$\gamma=4/3, K=(3^{1/3} \pi^{2/3} /4) (\hbar c/m_n^{4/3});$$
$$K=1.2293 \cdot 10^{15} \text{ (CGS)}$$

# Neutron stars

## Superdense matter and superstrong magnetic fields



# Proto-neutron stars



Mass fraction of nuclei in the nuclear chart for matter at  $T = 1 \text{ MeV}$ ,  $n_B = 10^{-3} \text{ fm}^{-3}$ , and  $Y_p = 0.4$ . Different colors indicate mass fraction in  $\text{Log}_{10}$  scale.

1202.5791

---

NS EoS are also important for SN explosion calculation, see 1207.2184

# EoS for core-collapse, proto-NS and NS-NS mergers

	Core-collapse supernovae	Proto-neutron stars	Mergers of compact binary stars
$n/n_s$	$10^{-8} - 10$	$10^{-8} - 10$	$10^{-8} - 10$
$T(\text{MeV})$	0 - 30	0 - 50	0 - 100
$Y_e$	0.35 - 0.45	0.01 - 0.3	0.01 - 0.6
$S(k_B)$	0.5 - 10	0 - 10	0 - 100

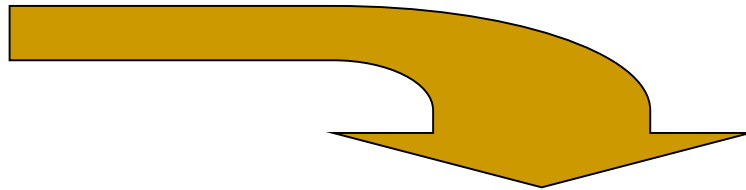
Wide ranges of parameters



# Astrophysical point of view

**Astrophysical appearance of NSs  
is mainly determined by:**

- Spin
- Magnetic field
- Temperature
- Velocity
- Environment






The first four are related to the NS structure!

# Equator and radius

$$ds^2 = c^2 dt^2 e^{2\Phi} - e^{2\lambda} dr^2 - r^2 [d\theta^2 + \sin^2\theta d\varphi^2]$$

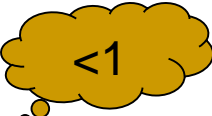
In flat space  $\Phi(r)$  and  $\lambda(r)$  are equal to zero.

•  $t = \text{const}, r = \text{const}, \theta = \pi/2, 0 < \varphi < 2\pi$    $l = 2\pi r$

•  $t = \text{const}, \theta = \text{const}, \varphi = \text{const}, 0 < r < r_0$    $dl = e^\lambda dr$    $l = \int_0^{r_0} e^\lambda dr \neq r_0$

# Gravitational redshift

$$d\tau = dt e^{\Phi},$$



$$\nu_r = \frac{dN}{d\tau} = e^{-\Phi} \frac{dN}{dt} \longrightarrow \text{Frequency emitted at } r$$

$$r \rightarrow \infty \quad \Phi \rightarrow 0 \quad \nu_{\infty} = \frac{dN}{dt} \longrightarrow \text{Frequency detected by an observer at infinity}$$

$$\nu_{\infty} = \nu_r e^{\Phi} \Rightarrow \Phi(r) \longrightarrow \text{This function determines gravitational redshift}$$

$$e^{2\lambda} \equiv \frac{1}{1 - \frac{2Gm}{c^2 r}}$$


It is useful to use  $m(r)$  – gravitational mass inside  $r$  – instead of  $\lambda(r)$

# Outside of the star

$$r > R \Rightarrow m(r) = M = \text{const}$$

$$e^{2\Phi} = 1 - \frac{2GM}{c^2 r} = 1 - \frac{r_g}{r}, \quad r_g = \frac{2GM}{c^2}$$

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

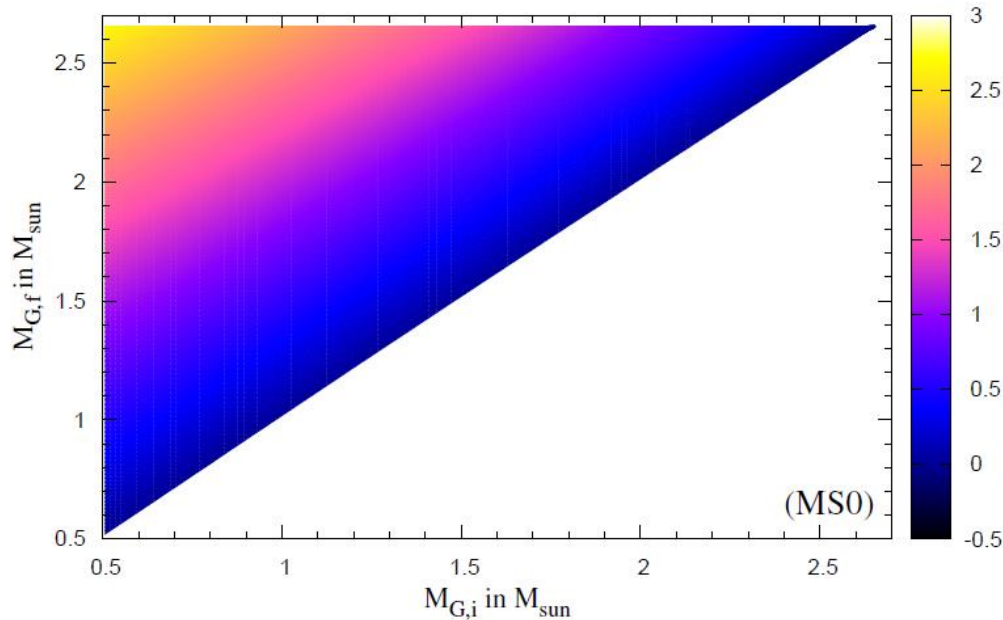
$$v_\infty = v_r \sqrt{1 - \frac{r_g}{r}}$$


redshift

Bounding energy  $\longrightarrow \Delta M = M_b - M \sim 0.2 M_{\text{sun}}$

Apparent radius  $\longrightarrow R_\infty = R / \sqrt{1 - r_g / R}$

# Bounding energy



If you drop a kilo on a NS, then you increase its mass for < kilo

$M_{\text{acc}}$  is shown with color

$M_{G,i}$ ( $M_{\odot}$ )	$\Delta M_G$ ( $M_{\odot}$ )	$M_{B,i}$ ( $M_{\odot}$ )		$M_{\text{acc}} (\Delta M_B)$ ( $M_{\odot}$ )	
		APR	MS0	APR	MS0
1.4	0.57	1.554	1.525	0.768	0.712
1.5	0.47	1.681	1.647	0.641	0.591
1.6	0.37	1.811	1.767	0.511	0.470
1.7	0.27	1.943	1.892	0.379	0.345
1.8	0.17	2.080	2.018	0.242	0.219
1.9	0.07	2.221	2.146	0.101	0.091


$$M_{\text{acc}} = \Delta M_G + \Delta BE / c^2 = \Delta M_B$$

BE- binding energy

$$BE = (M_B - M_G) c^2$$

# TOV equation

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik}$$



(1)  $\frac{dP}{dr} = -\frac{G\rho m}{r^2} \left(1 + \frac{P}{\rho c^2}\right) \left(1 + \frac{4\pi r^3 P}{mc^2}\right) \left(1 - \frac{2Gm}{rc^2}\right)^{-1}$

(2)  $\frac{dm}{dr} = 4\pi r^2 \rho$

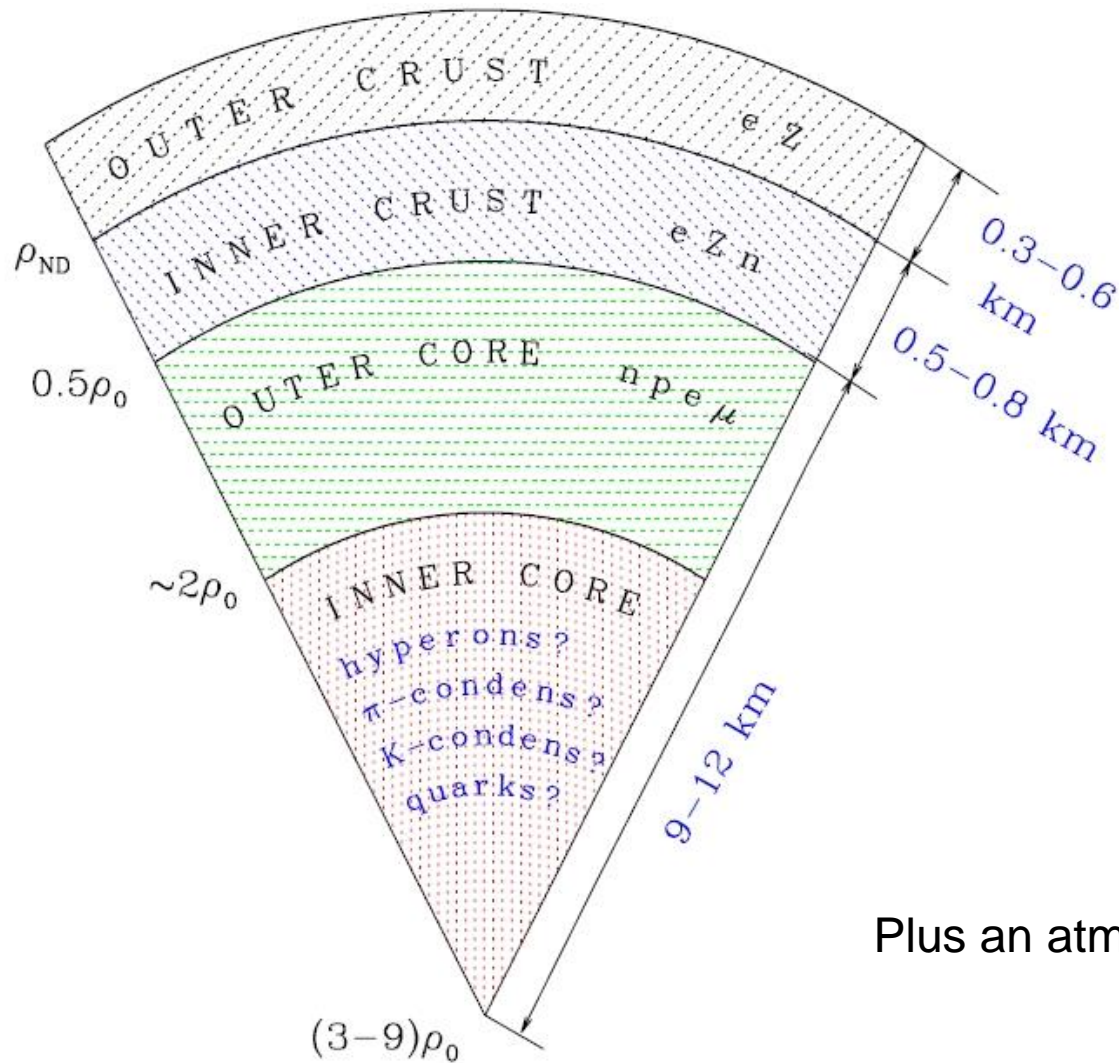
(3)  $\frac{d\Phi}{dr} = -\frac{1}{\rho c^2} \frac{dP}{dr} \left(1 + \frac{P}{\rho c^2}\right)^{-1}$

(4)  $P = P(\rho)$

**Tolman (1939)  
Oppenheimer-  
Volkoff (1939)**

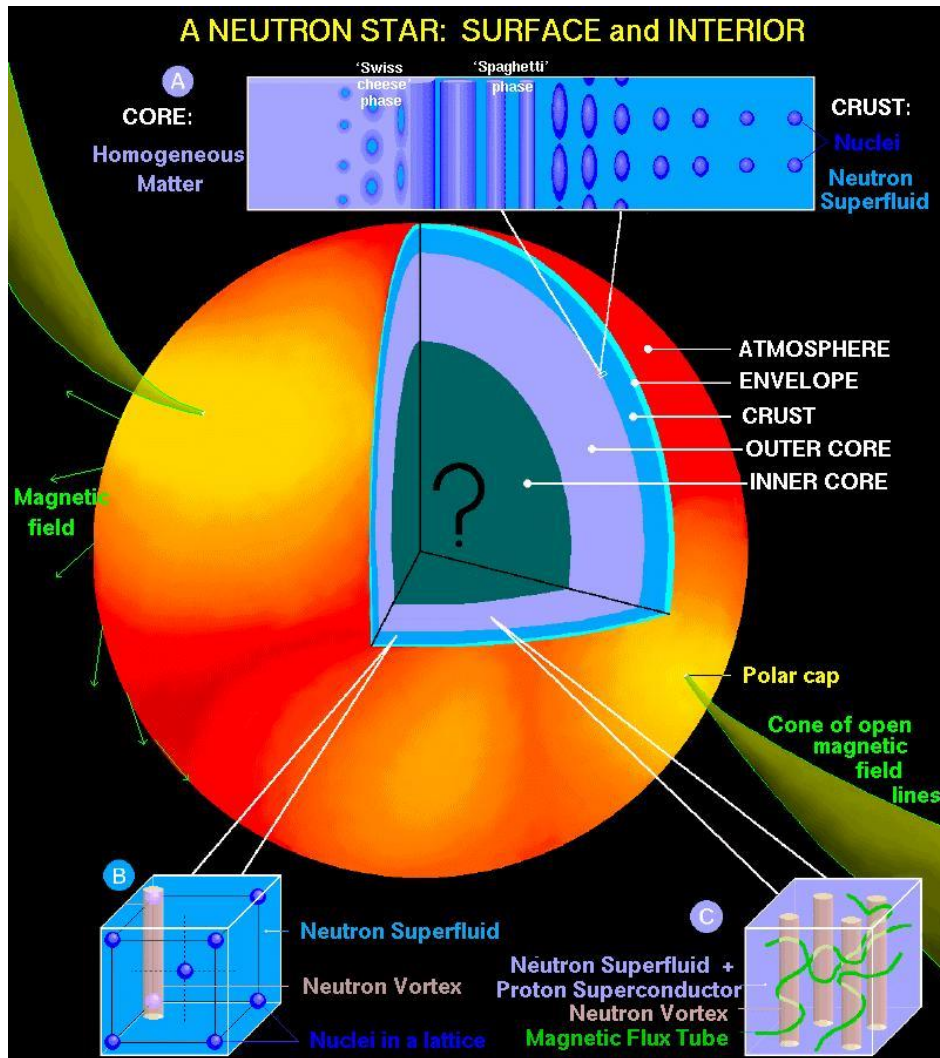


# Structure and layers



Plus an atmosphere...

# Neutron star interiors

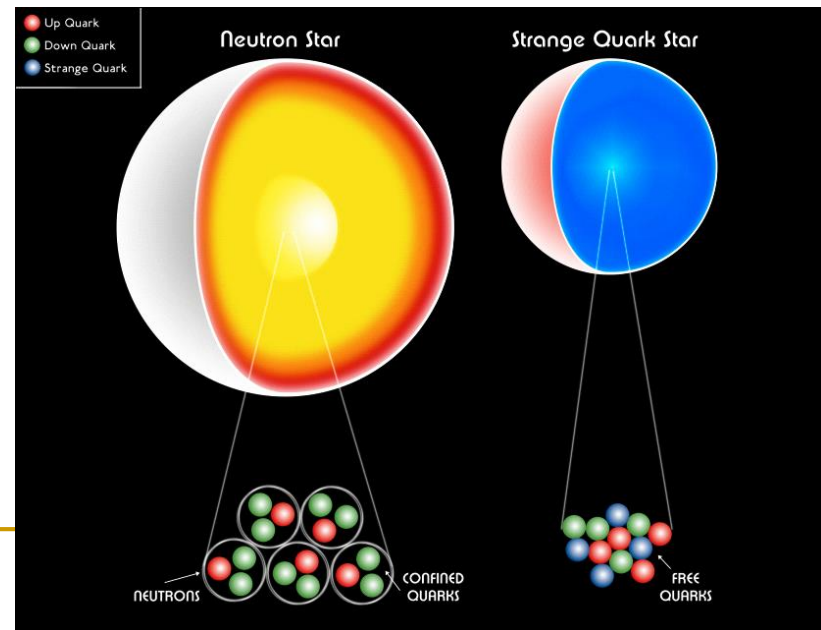


Radius: 10 km

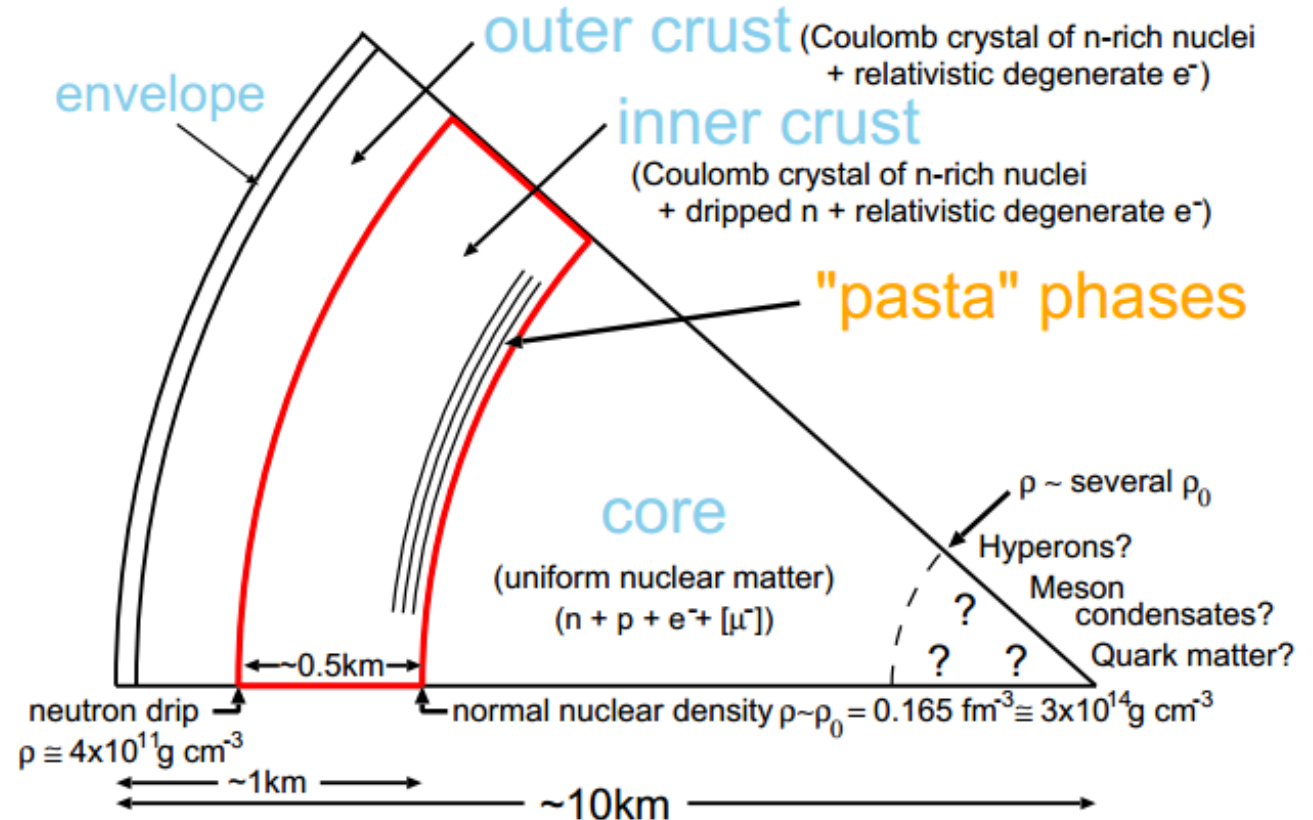
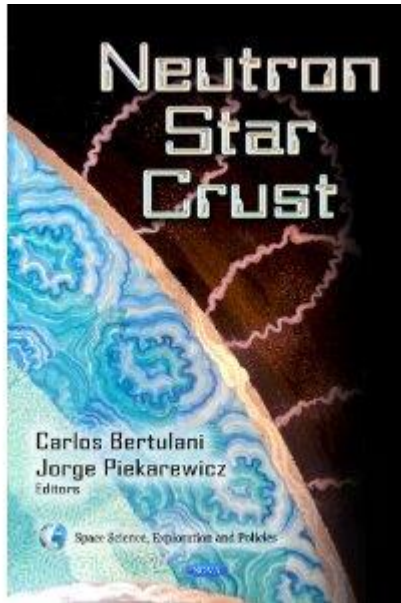
Mass: 1-2 solar

Density: above the nuclear

Strong magnetic fields



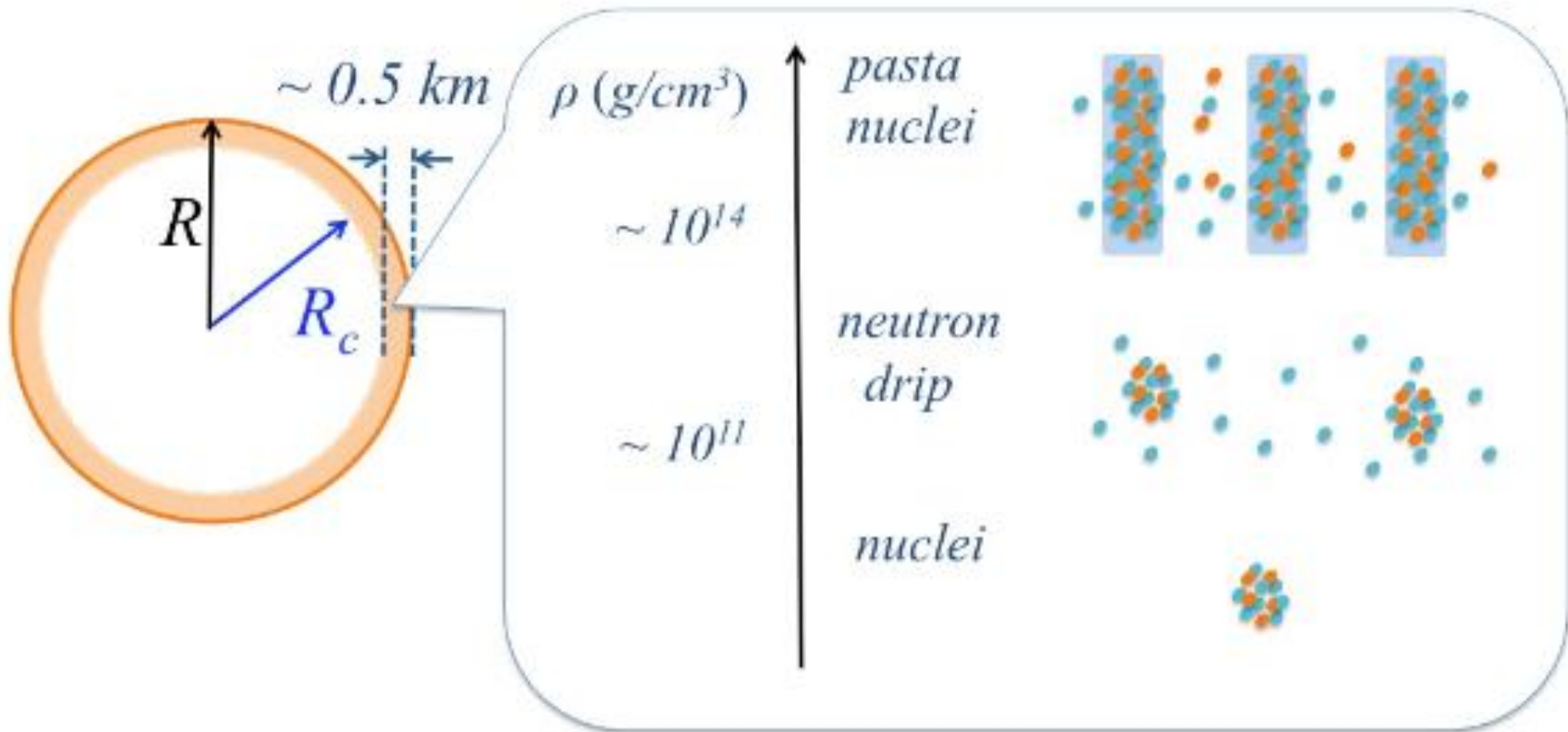
# Neutron star crust



Many contributions to the book are available in the arXiv.

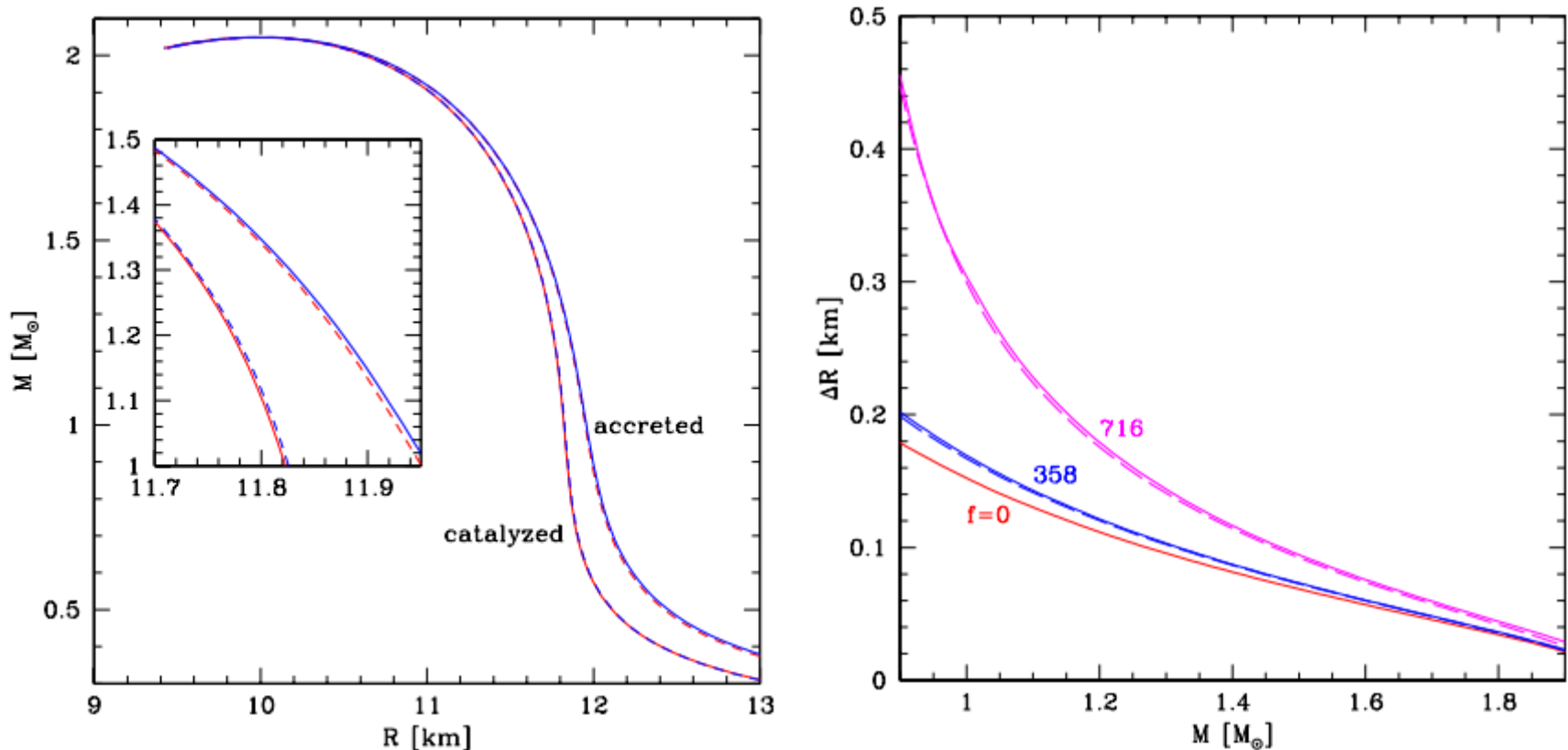
Mechanical properties of crusts are continuously discussed, see 1208.3258

# Inner crust properties



# Accreted crust

It is interesting that the crust formed by accreted matter differs from the crust formed from catalyzed matter. The former is thicker.

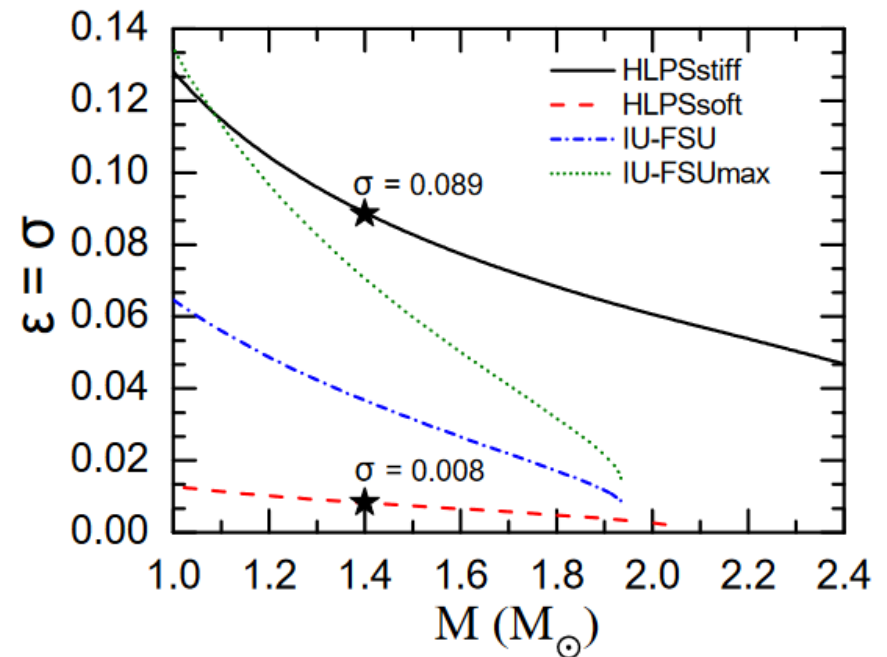




# Crust and limiting rotation

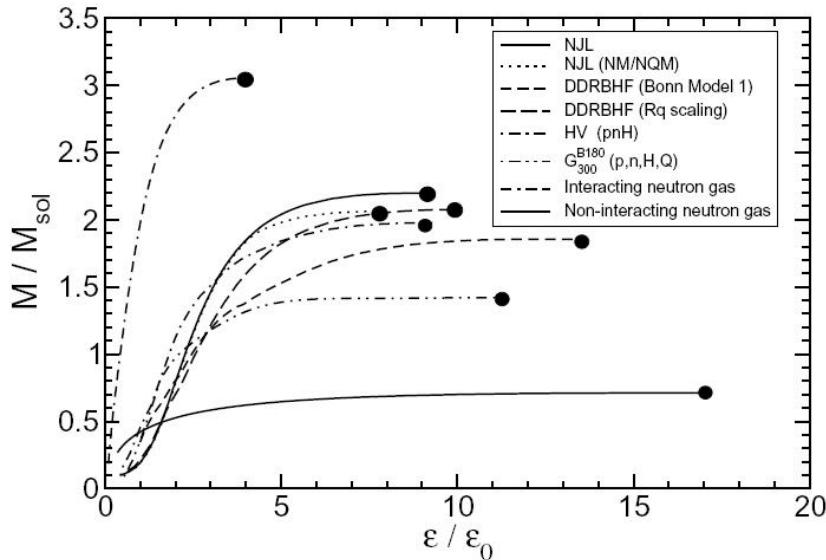
Model	$\sigma$	$f_{\text{in}}^{1.4}$ (Hz)	$f_{\text{fin}}^{1.4}$ (Hz)	$f_{\text{in}}^{1.8}$ (Hz)	$f_{\text{fin}}^{1.8}$ (Hz)
HLPSStiff	0.05	0	326	35	368
	0.10	136	479	236	569
IU-FSU	0.05	349	515	909	1022
	0.10	781	947	1875	1988
IU-FSU <sub>max</sub>	0.05	35	358	374	586
	0.10	232	555	854	1066

Failure of the crust can be the reason of the limiting frequency.  
 Spinning-up of a NS due to accretion can result in crust failure.  
 Then the shape of the star is deformed, it gains ellipticity.  
 So, GWs are emitted which slow down the compact object.





# Configurations

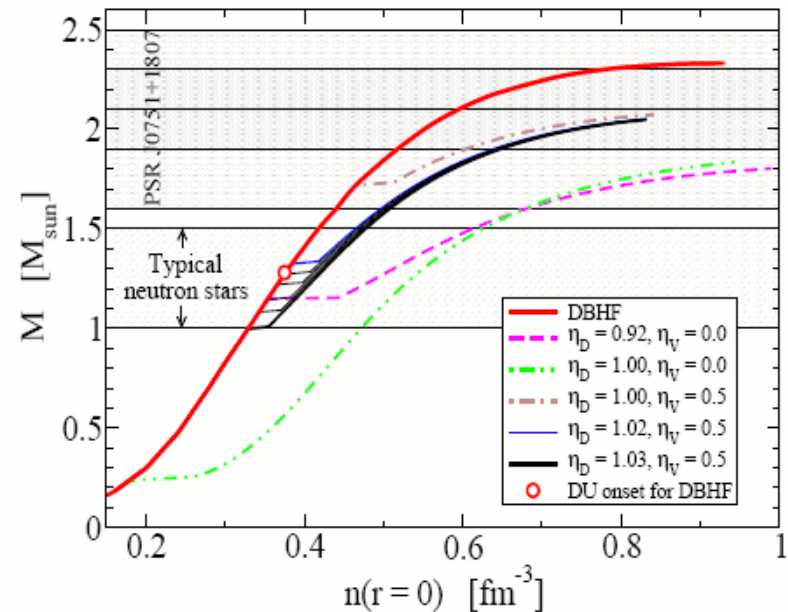


A RNS code is developed  
and made available to the public  
by Sterlgioulas and Friedman  
ApJ 444, 306 (1995)  
<http://www.gravity.phys.uwm.edu/rns/>

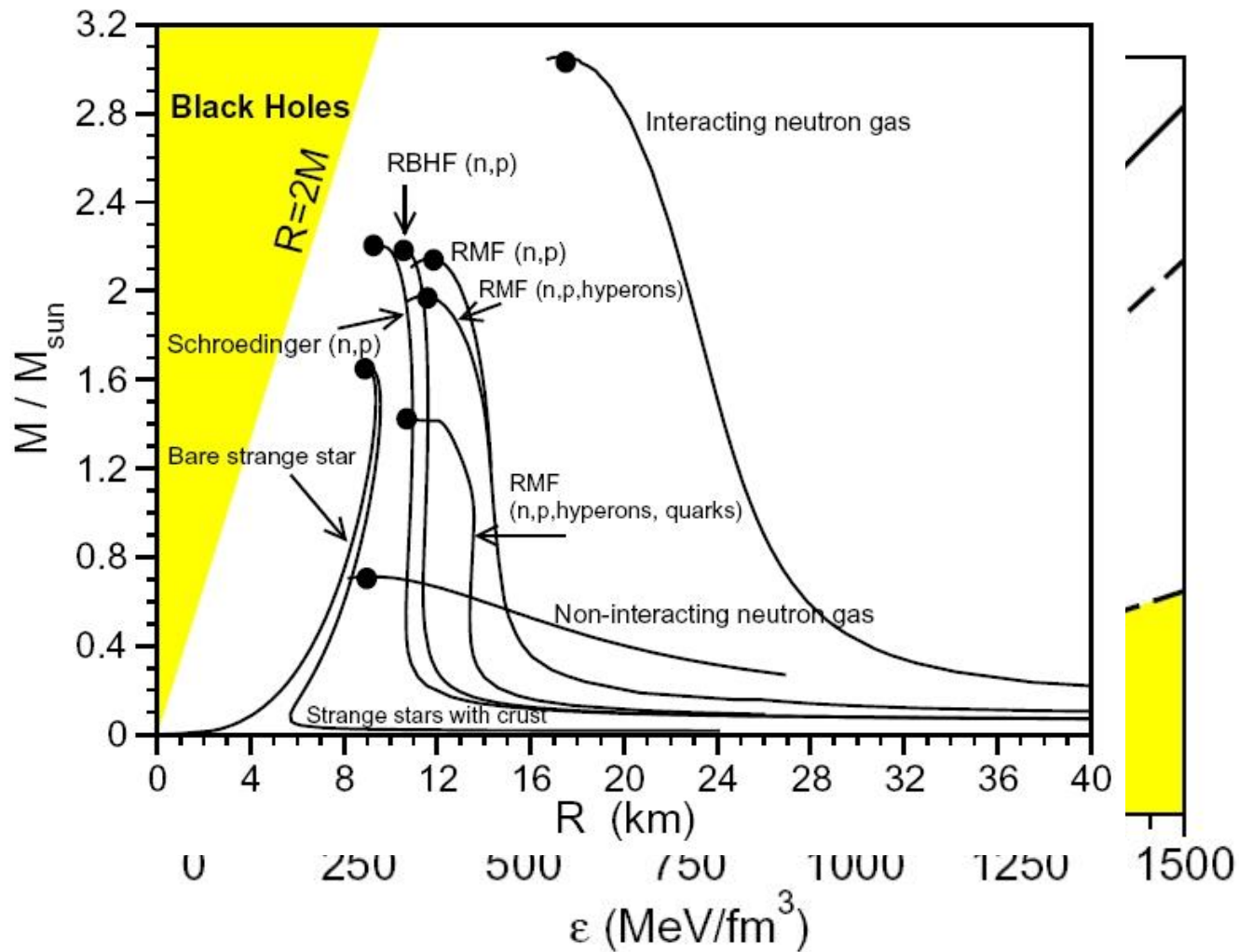
NS mass vs.  
central density  
(Weber et al.  
arXiv: 0705.2708)



Stable configurations  
for neutron stars and  
hybrid stars  
(astro-ph/0611595).



# EoS



(Weber et al. ArXiv: 0705.2708 )

# Mass-radius

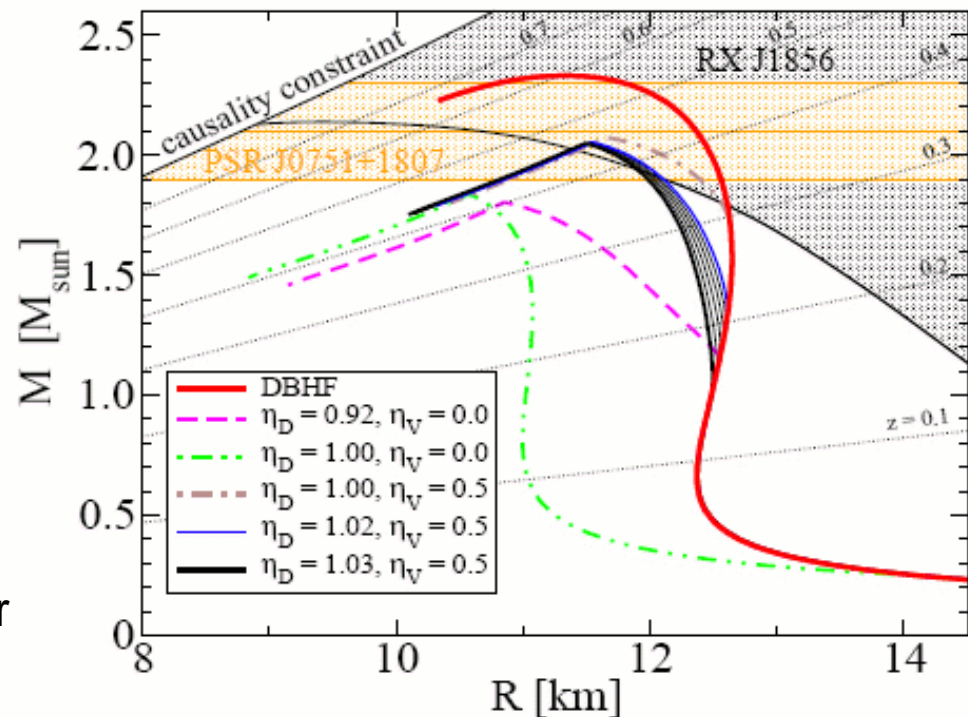
Mass and radius are macroscopical potentially measured parameters.

Thus, it is important to formulate EoS in terms of these two parameters.

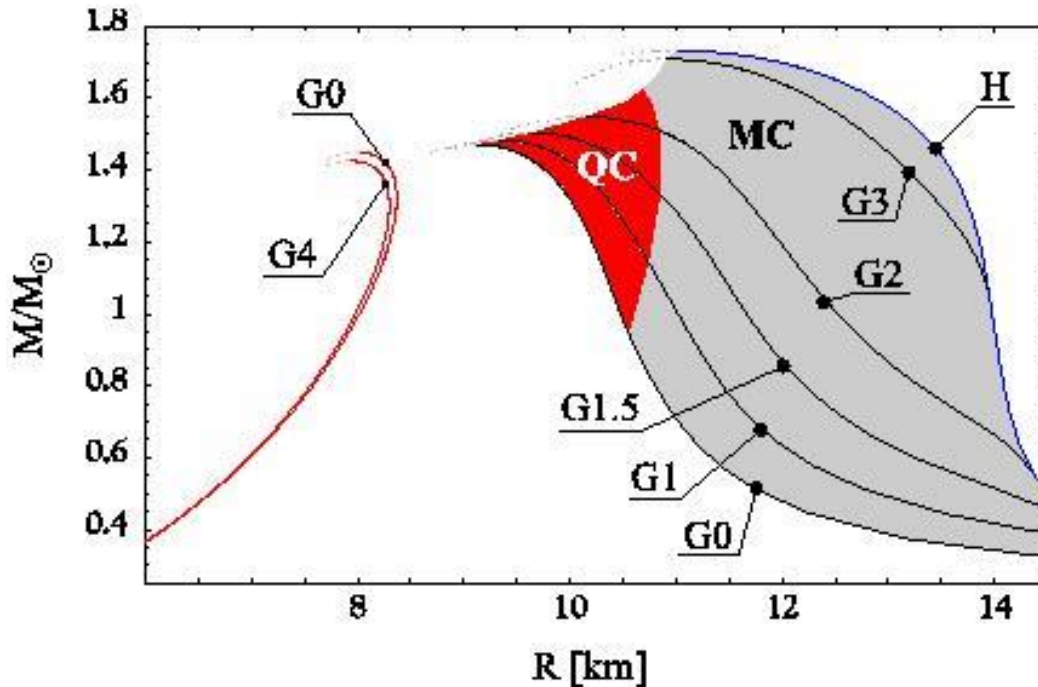
About hyperon stars see a review in 1002.1658.

About strange stars and some other exotic options – 1002.1793

Mass-radius relations for CSs with possible phase transition to deconfined quark matter.



# Mass-radius relation



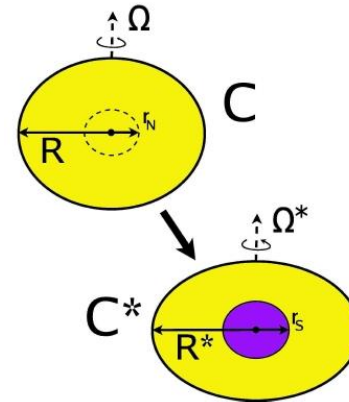
Rotation is neglected here.  
Obviously, rotation results in:

- larger max. mass
- larger equatorial radius

Spin-down can result in phase transition,  
as well as spin-up (due to accreted mass),  
see 1109.1179

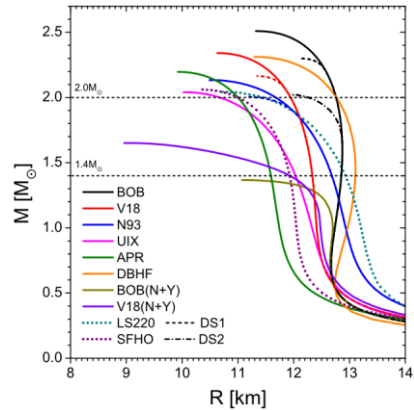
## Main features

- Max. mass
- Diff. branches (quark and normal)
- Stiff and soft EoS
- Small differences for realistic parameters
- Softening of an EoS with growing mass



Haensel, Zdunik  
astro-ph/0610549

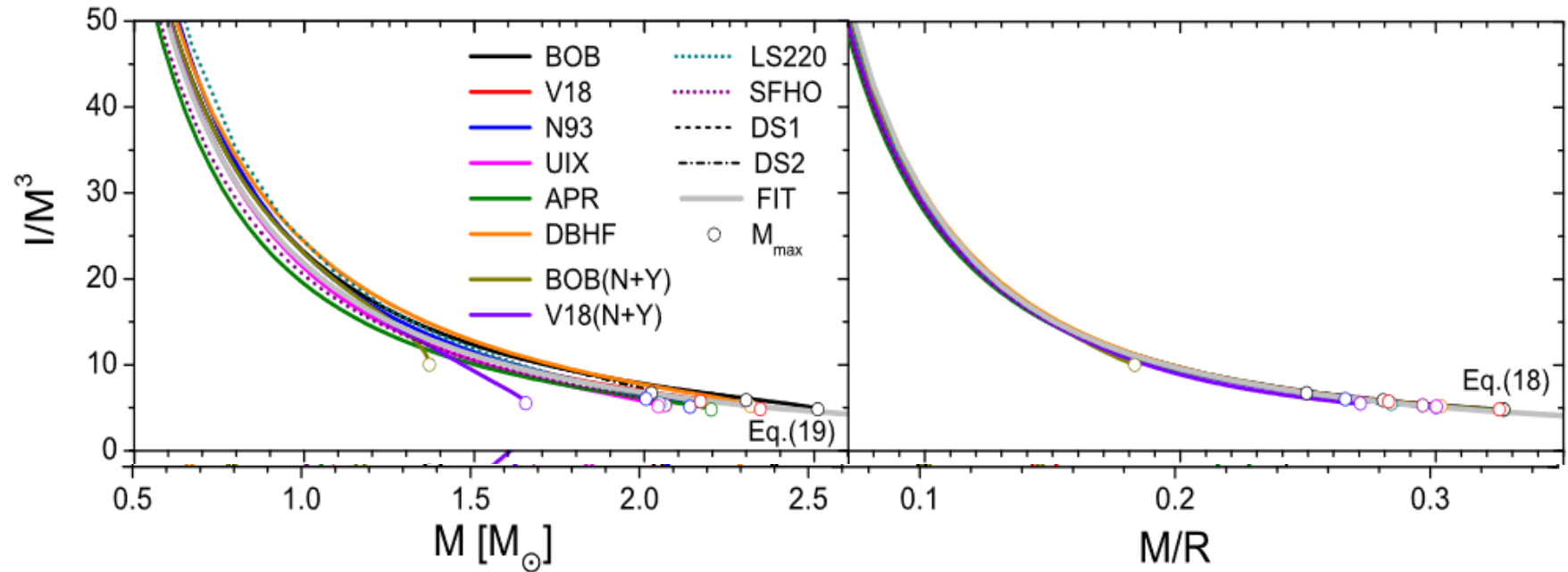
# Fitting formulae for moment of inertia



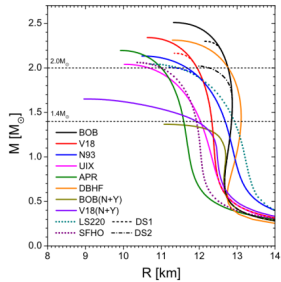
$$\frac{I}{M^3} \equiv 0.8134 \beta^{-1} + 0.2101 \beta^{-2} + 0.003175 \beta^{-3} - 0.0002717 \beta^{-4} \quad (18)$$

$$\frac{I}{M^3} \equiv 1.0334 M^{-1} + 30.7271 M^{-2} - 12.8839 M^{-3} + 2.8841 M^{-4} \quad (19)$$

$$\beta = Gm/(\bar{R}c^2)$$



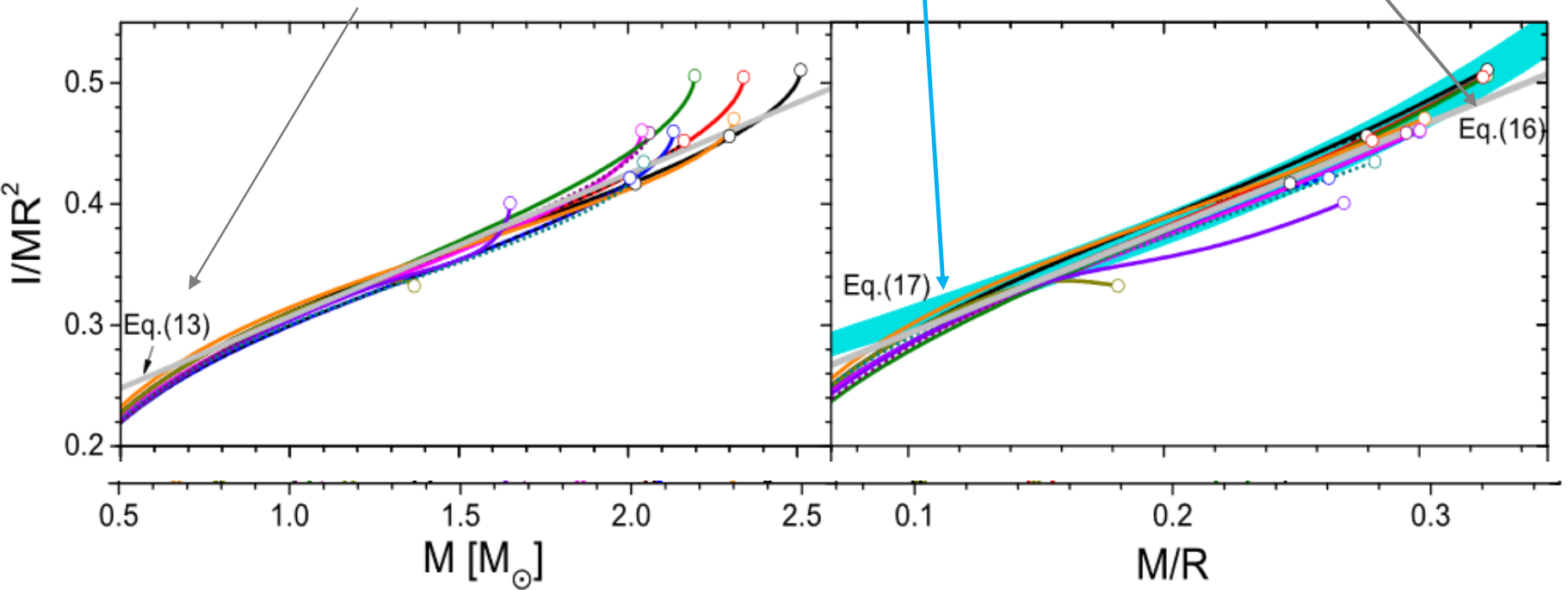
# Fits for $I/MR^2$



$$\frac{I}{MR^2} \approx 0.189 + 0.118 \frac{M}{M_{\odot}} \pm 0.016.$$

$$\frac{I}{MR^2} \approx (0.237 \pm 0.008)(1 + 2.844\beta + 18.91\beta^4)$$

$$\frac{I}{MR^2} \equiv 0.207 + 0.857\beta \pm 0.011$$

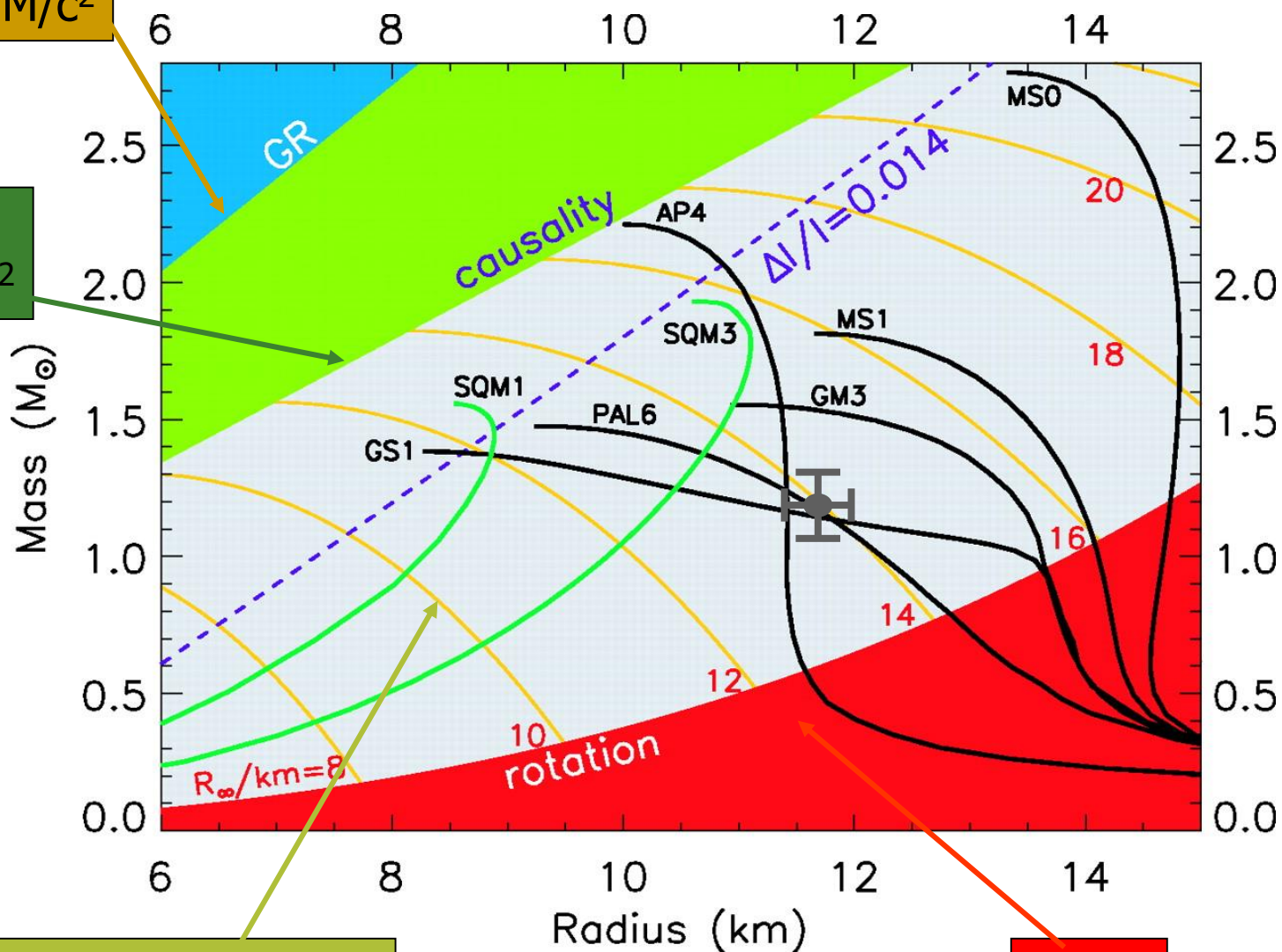




$$R=2GM/c^2$$

$$P=\rho$$

$$R\sim 3GM/c^2$$

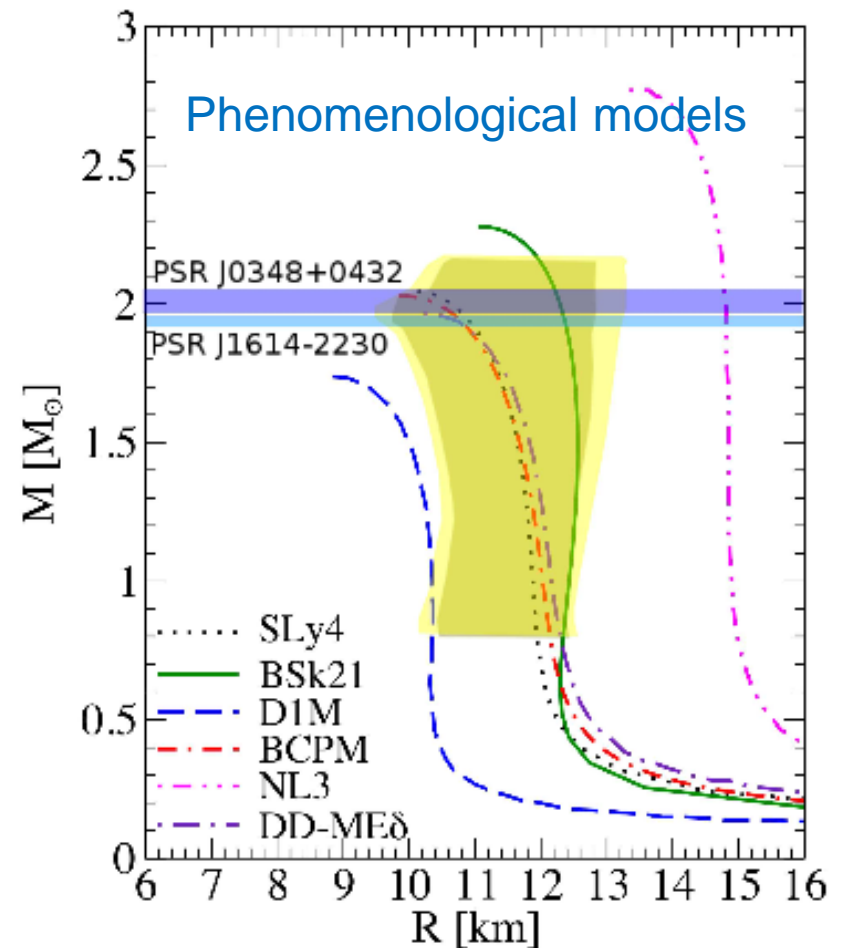
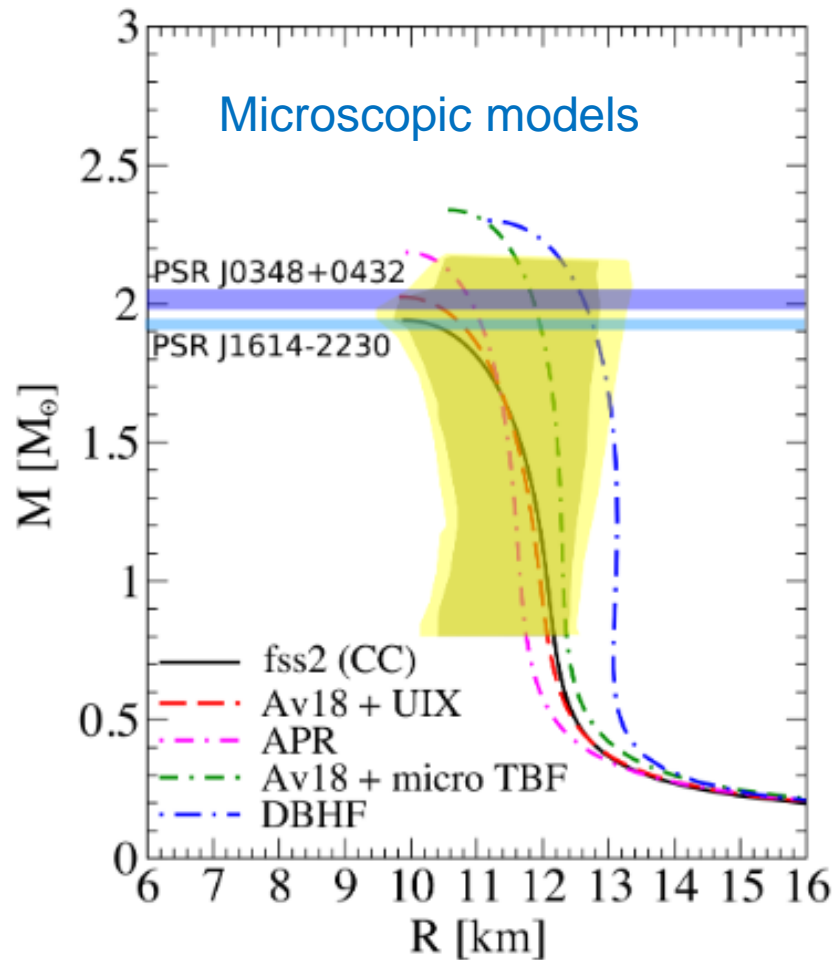


$$R_\infty=R(1-2GM/Rc^2)^{-1/2}$$

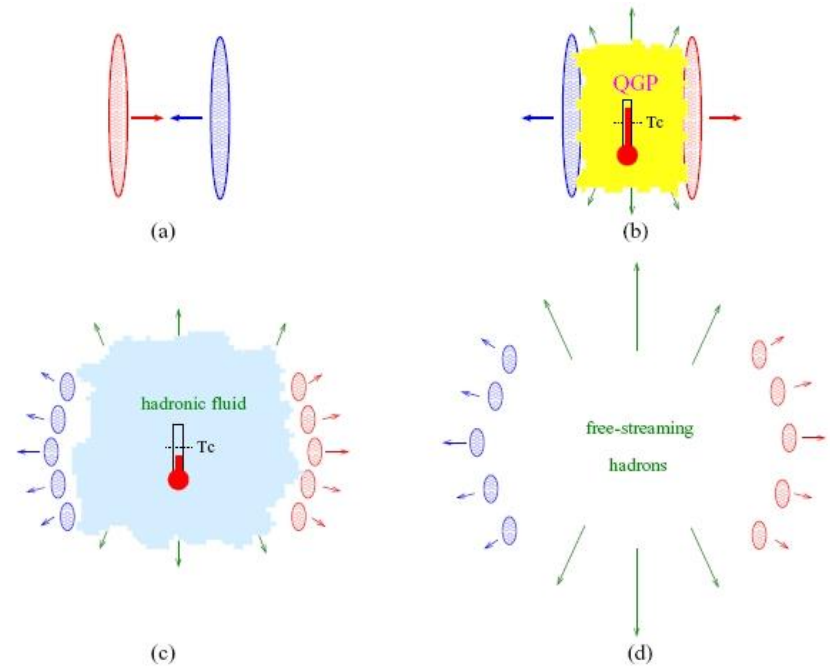
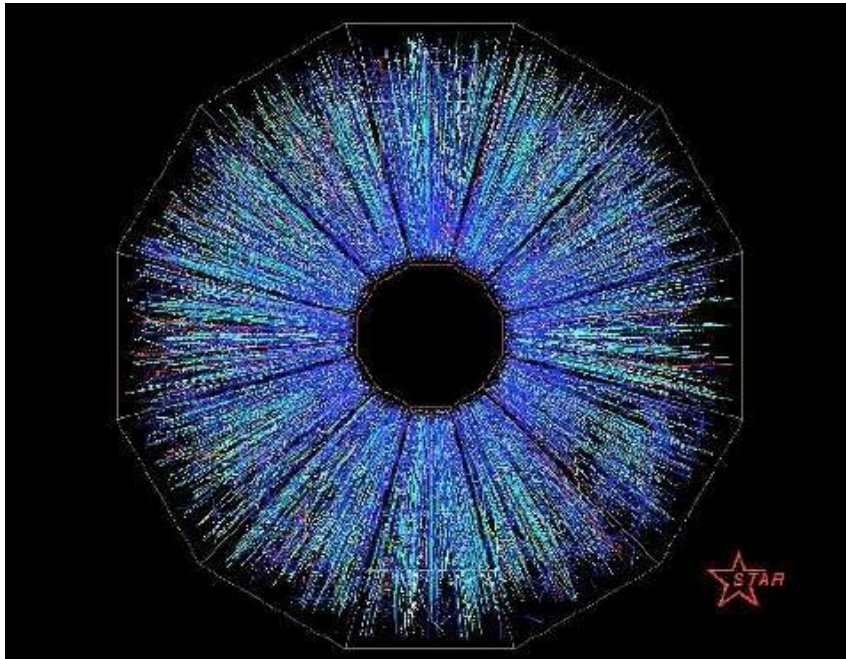
$$\omega=\omega_K$$

Lattimer & Prakash (2004)

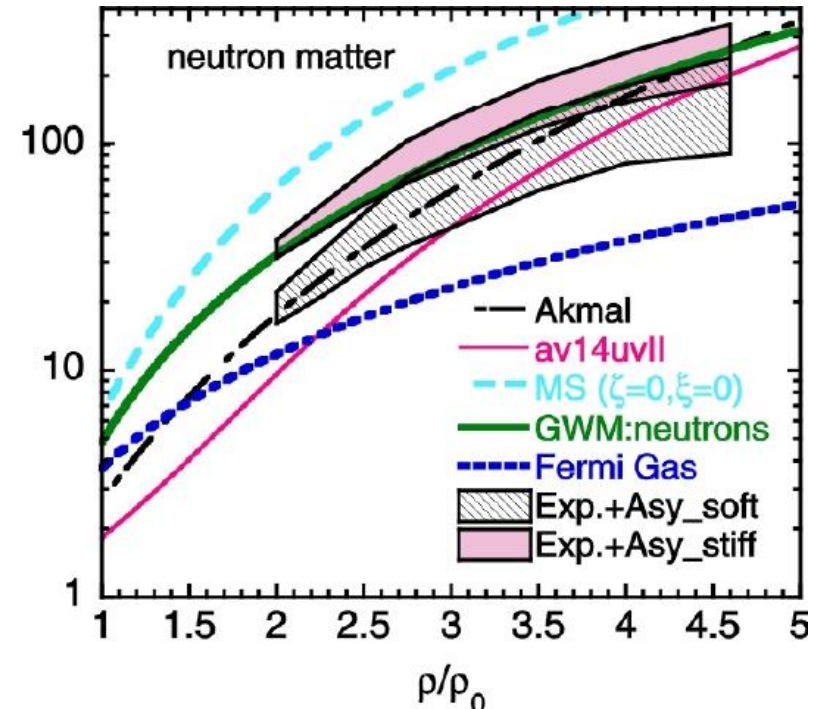
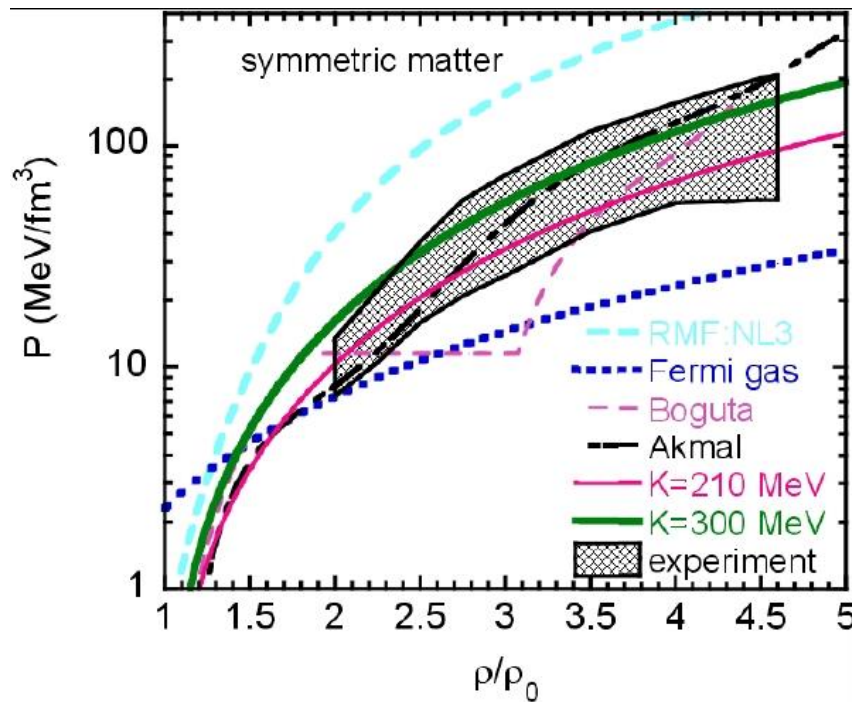
# Theory vs. observations



# Au-Au collisions



# Experimental results and comparison



$$1 \text{ MeV/fm}^3 = 1.6 \cdot 10^{32} \text{ Pa}$$

Danielewicz et al. nucl-th/0208016

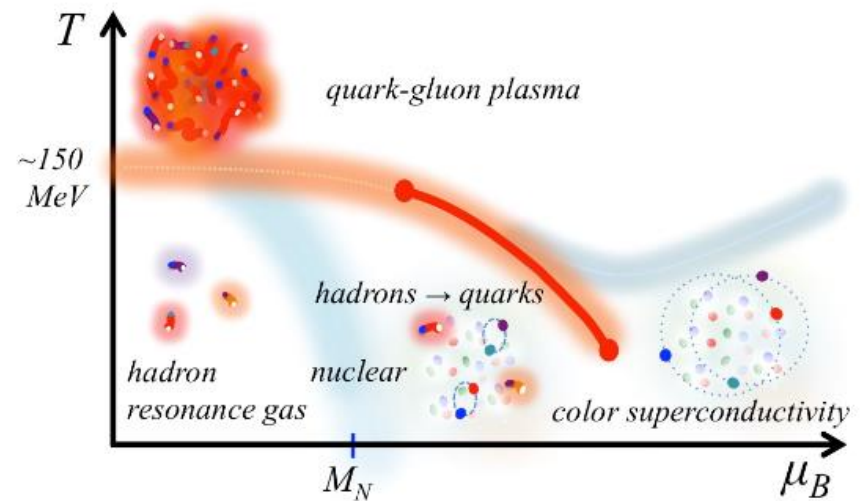
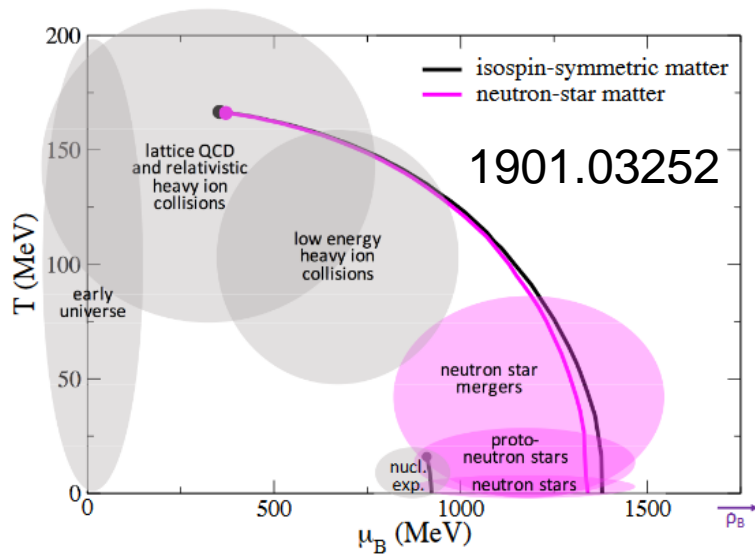
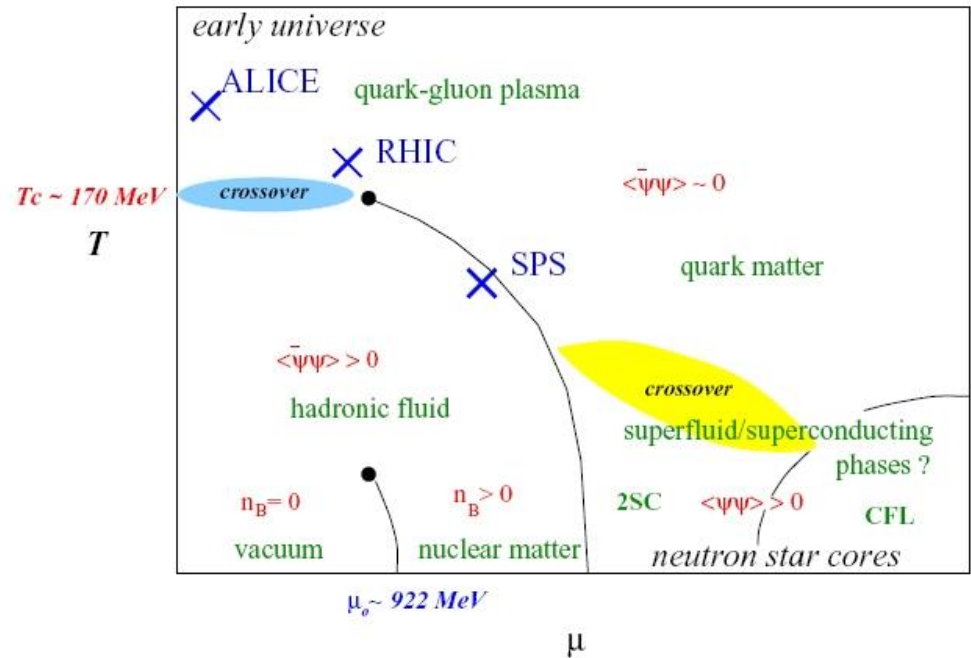
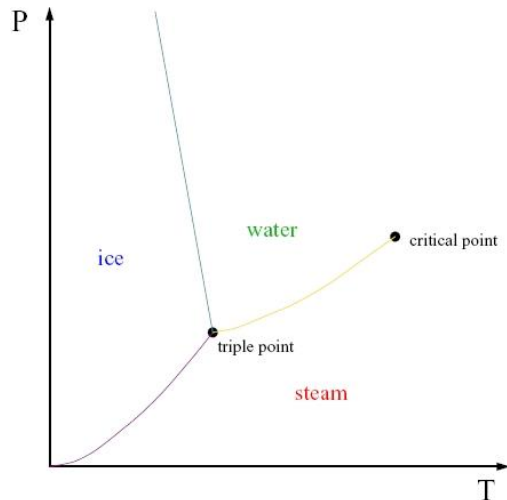
GSI-SIS and AGS data

New heavy-ion data and discussion: 1211.0427

Also laboratory measurements of lead nuclei radius can be important, see 1202.5701



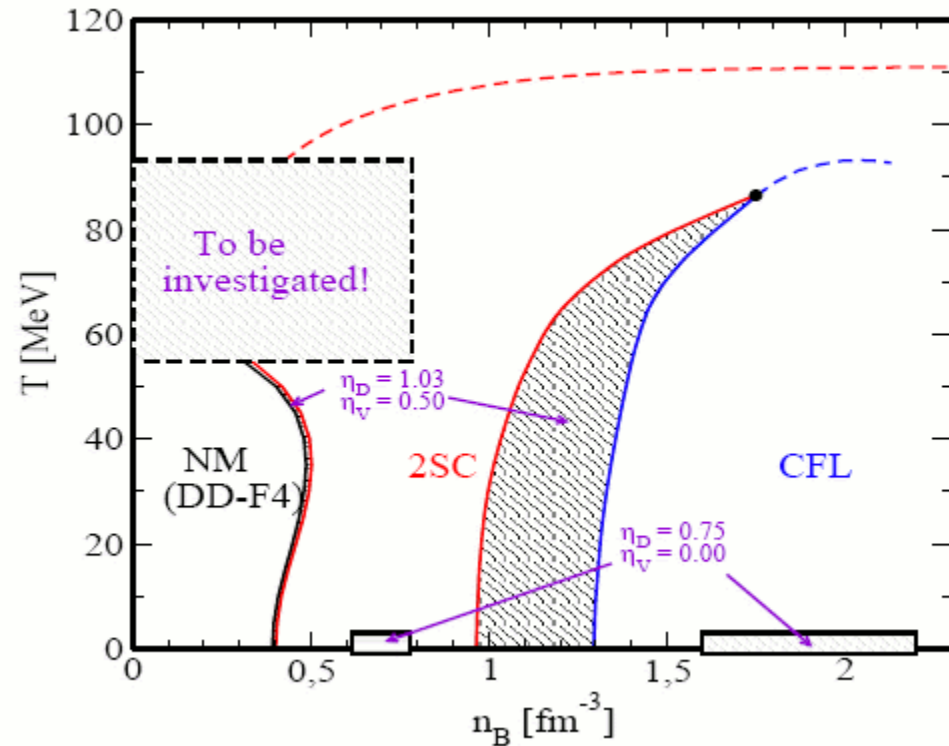
# Phase diagram



See 1803.01836

# Phase diagram

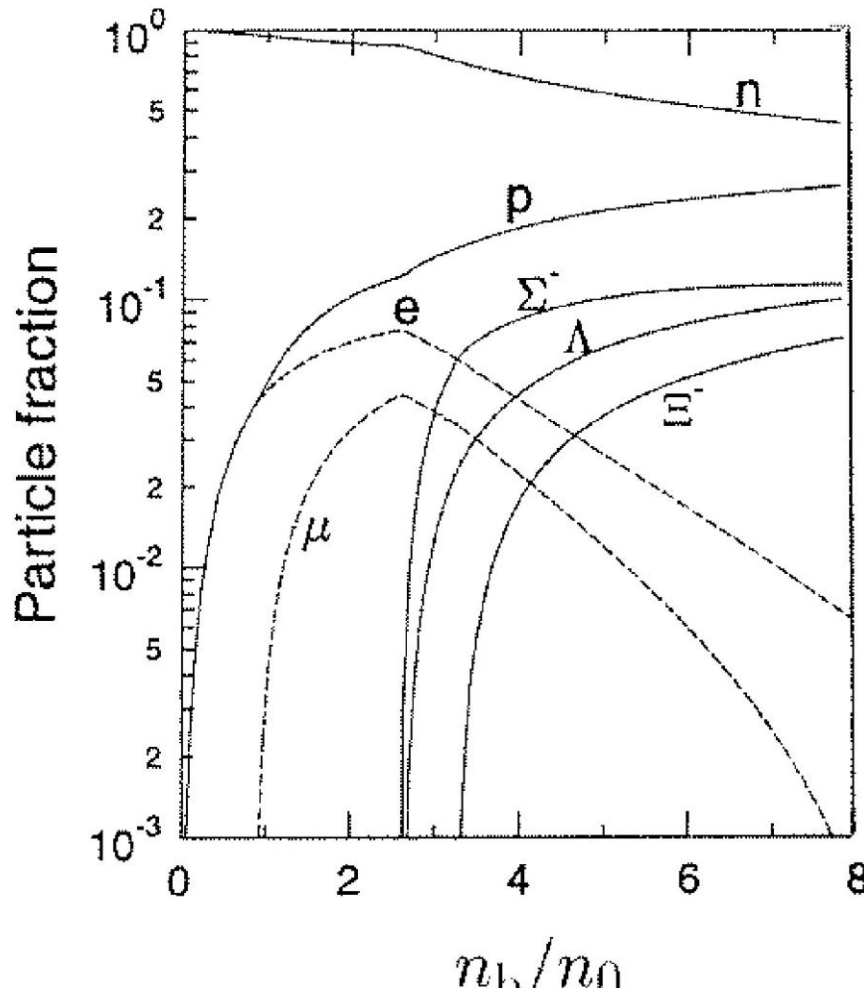
Phase diagram for isospin symmetry using the most favorable hybrid EoS studied in astro-ph/0611595.



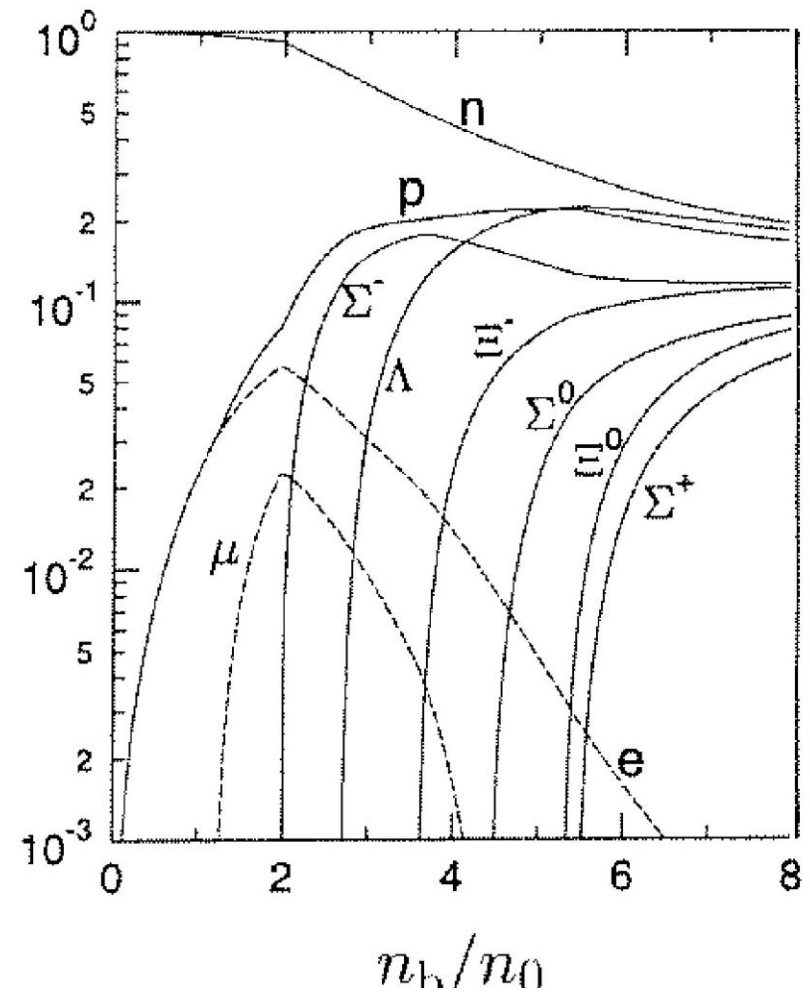
(astro-ph/0611595)



# Particle fractions

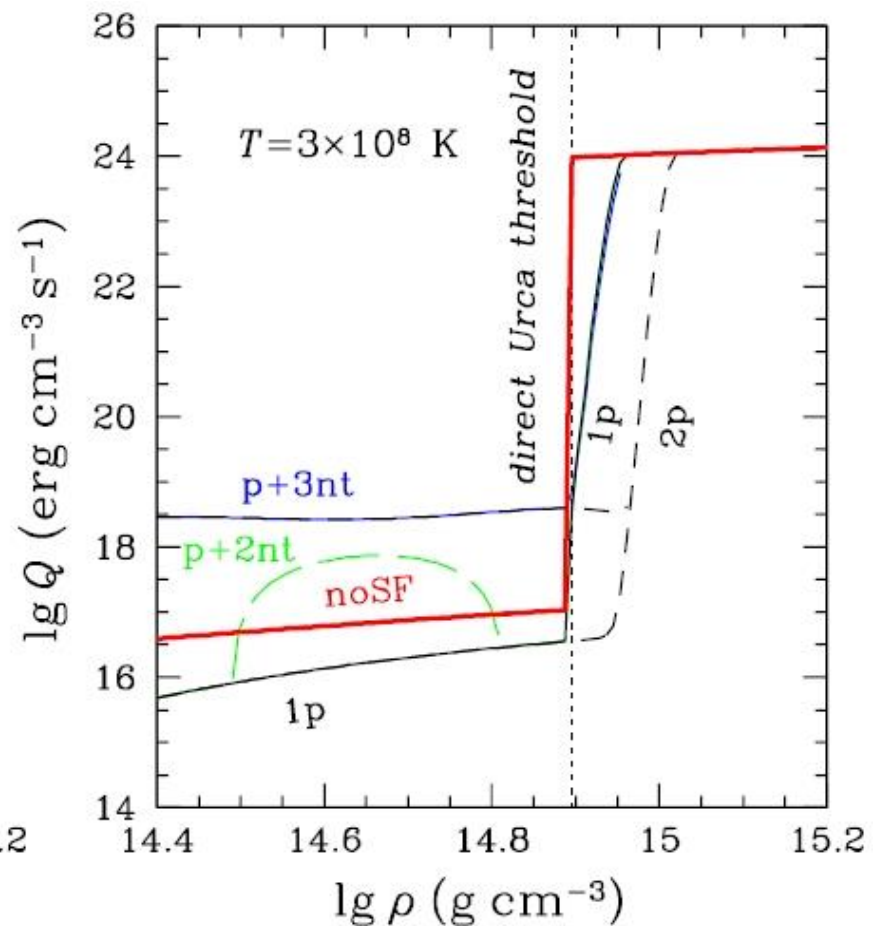
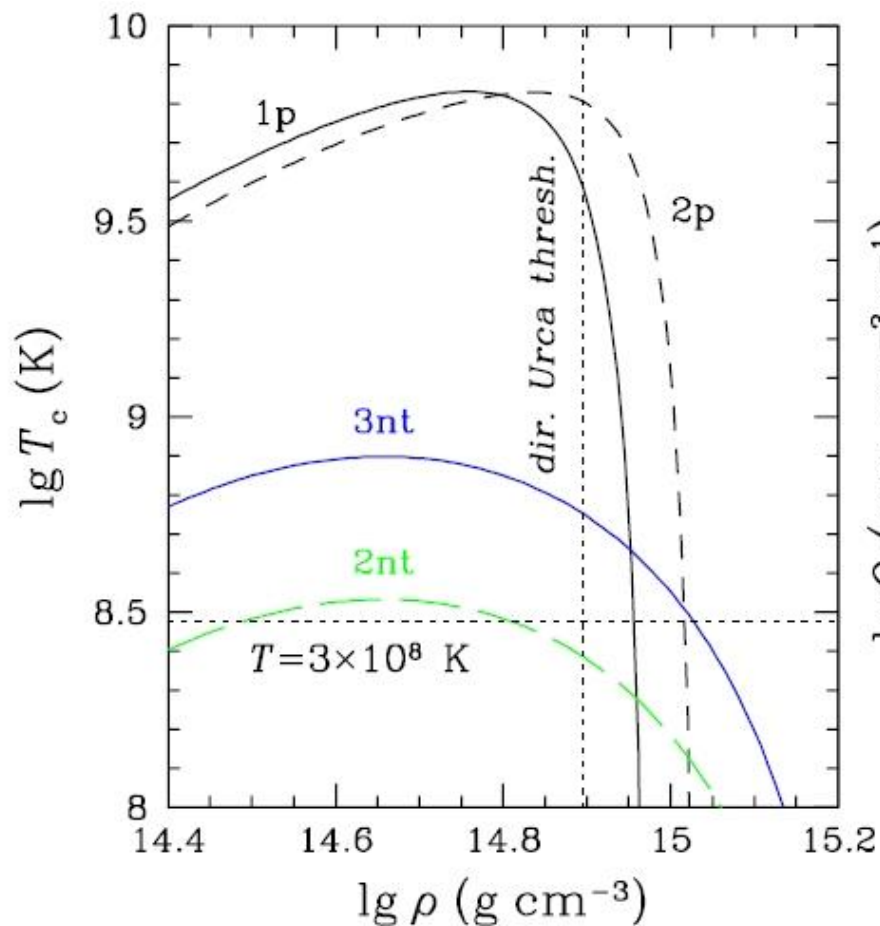


Effective chiral model of  
Hanauske et al. (2000)



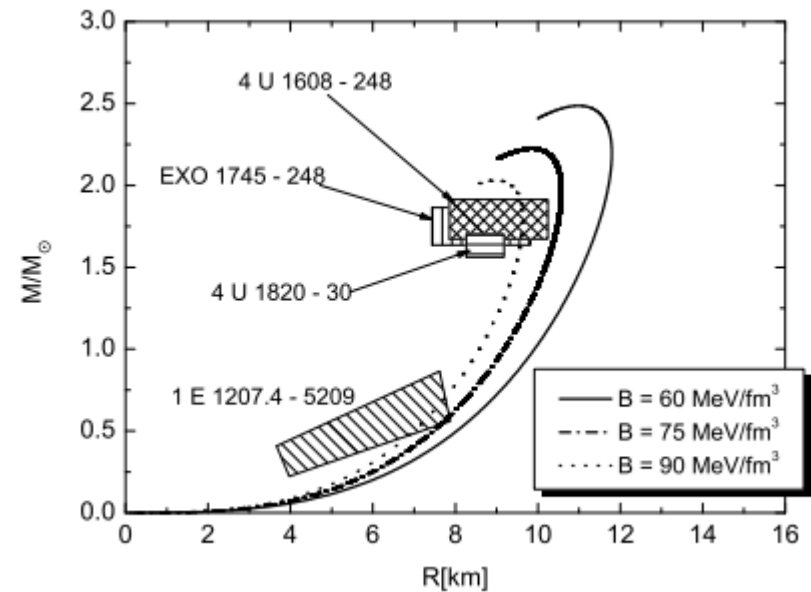
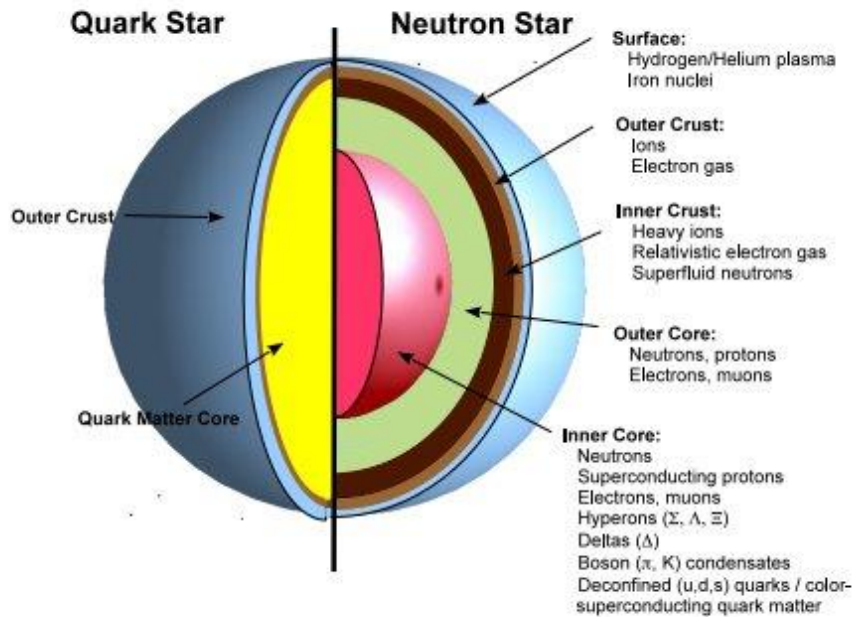
Relativistic mean-field model  
TM1 of Sugahara & Toki (1971)

# Superfluidity in NSs



(Yakovlev)

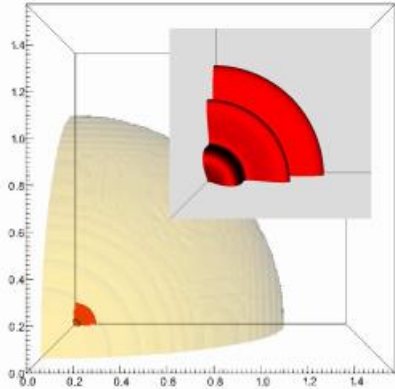
# Quark stars



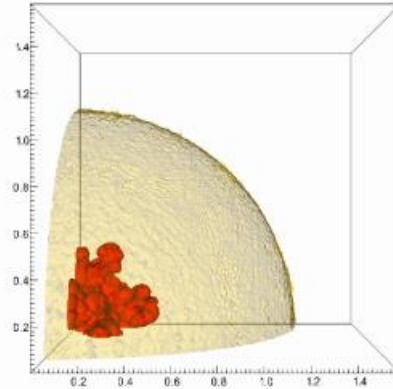
1210.1910

See also 1112.6430

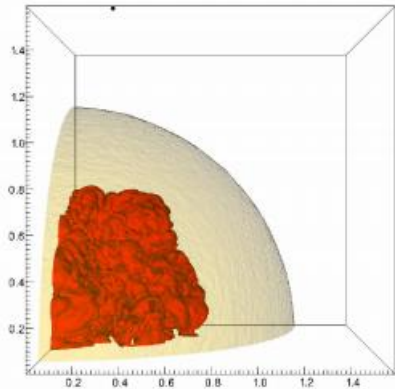
# Formation of quark stars



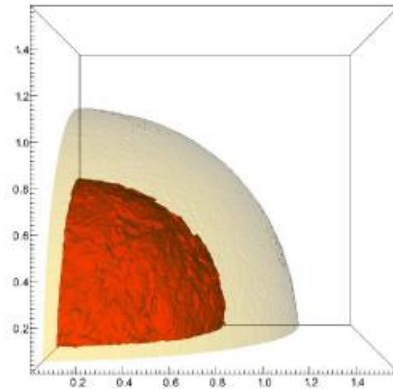
(a)  $t = 0$



(b)  $t = 0.7 \text{ ms}$



(c)  $t = 1.2 \text{ ms}$

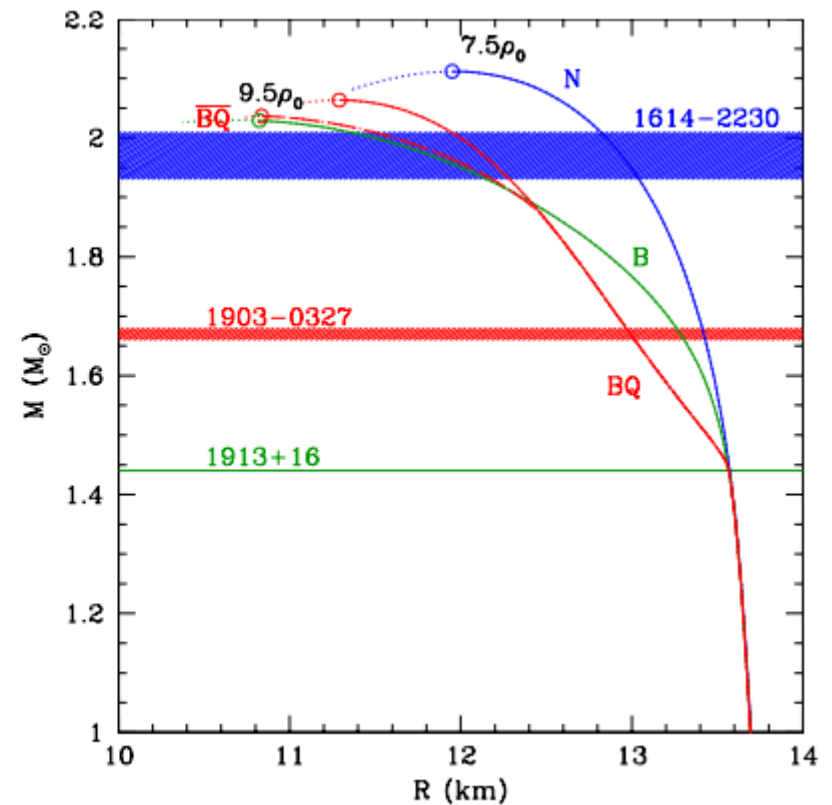
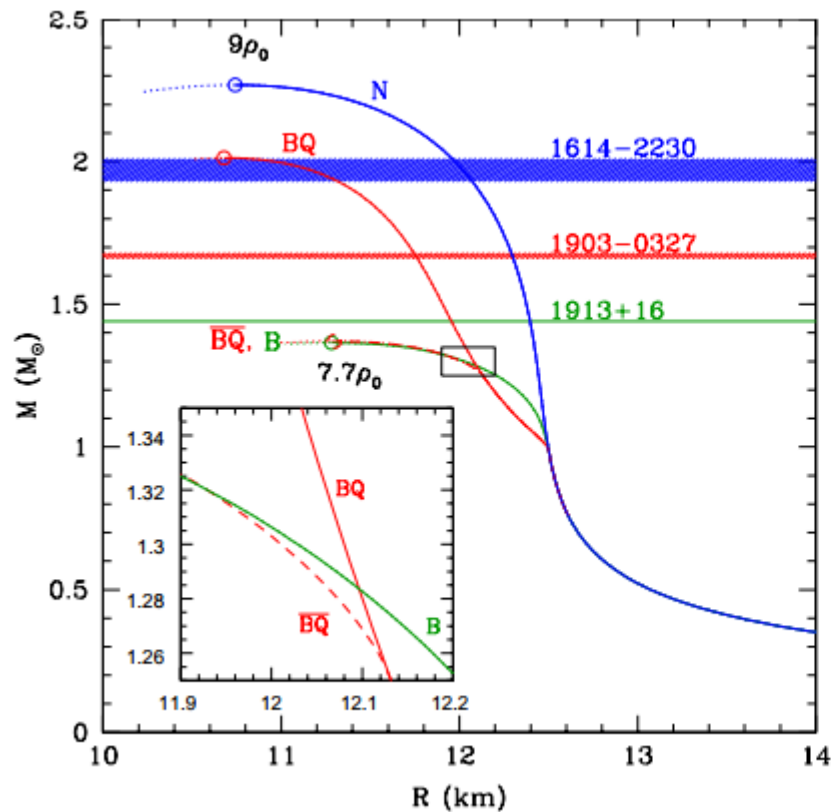


(d)  $t = 4.0 \text{ ms}$

Turbulent deflagration,  
as in SNIa.

Neutrino signal due to  
conversion of a NS into  
a quark star was calculated  
in 1304.6884

# Hybrid stars

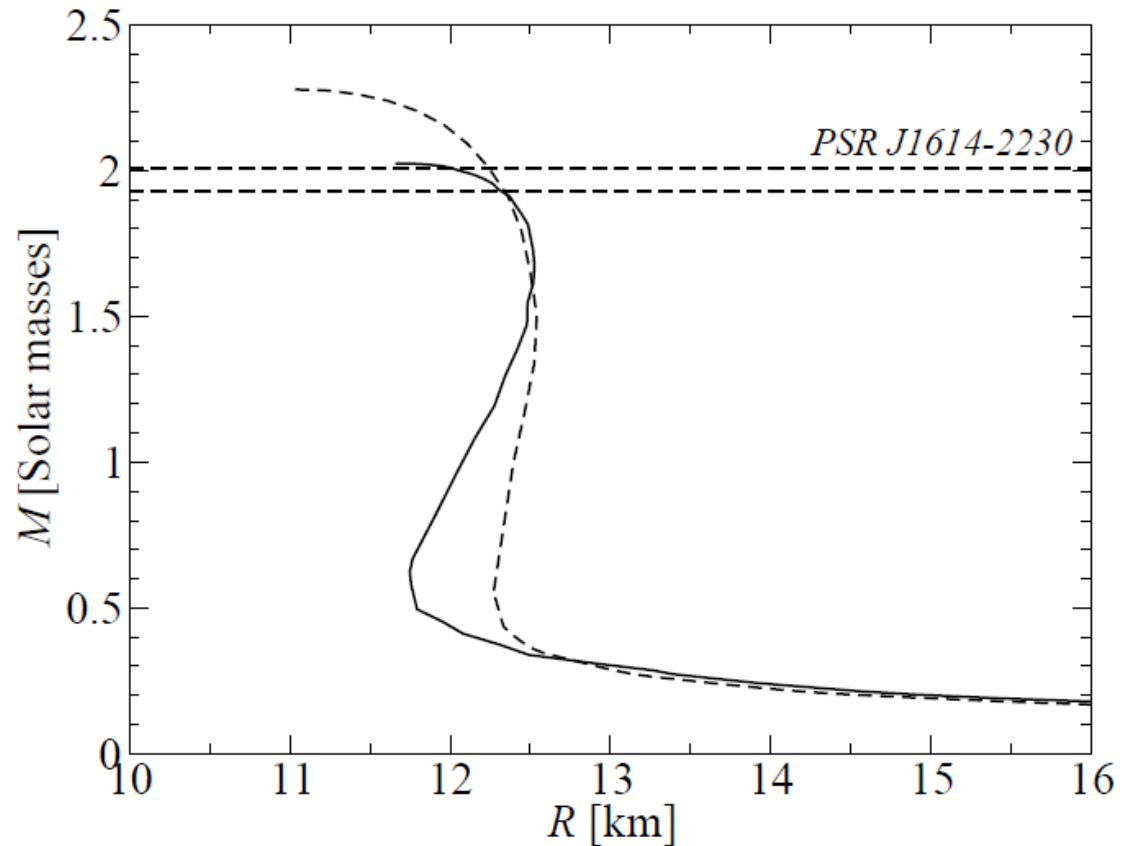


1211.1231

See also [1302.4732](#)

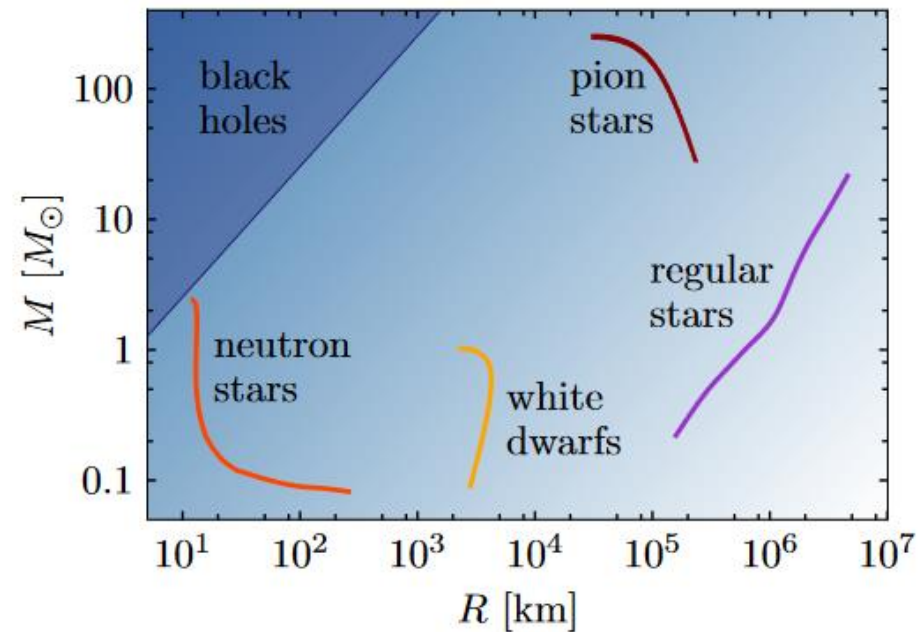
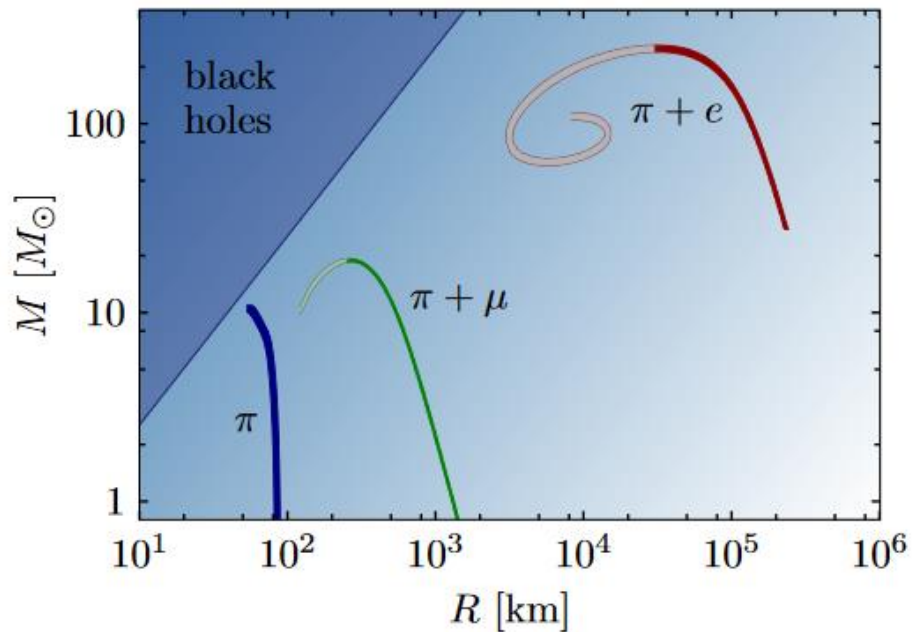
# Massive hybrid stars

Stars with quark cores can be massive, and so this hypothesis is compatible with existence of pulsars with  $M > 2$  Msolar





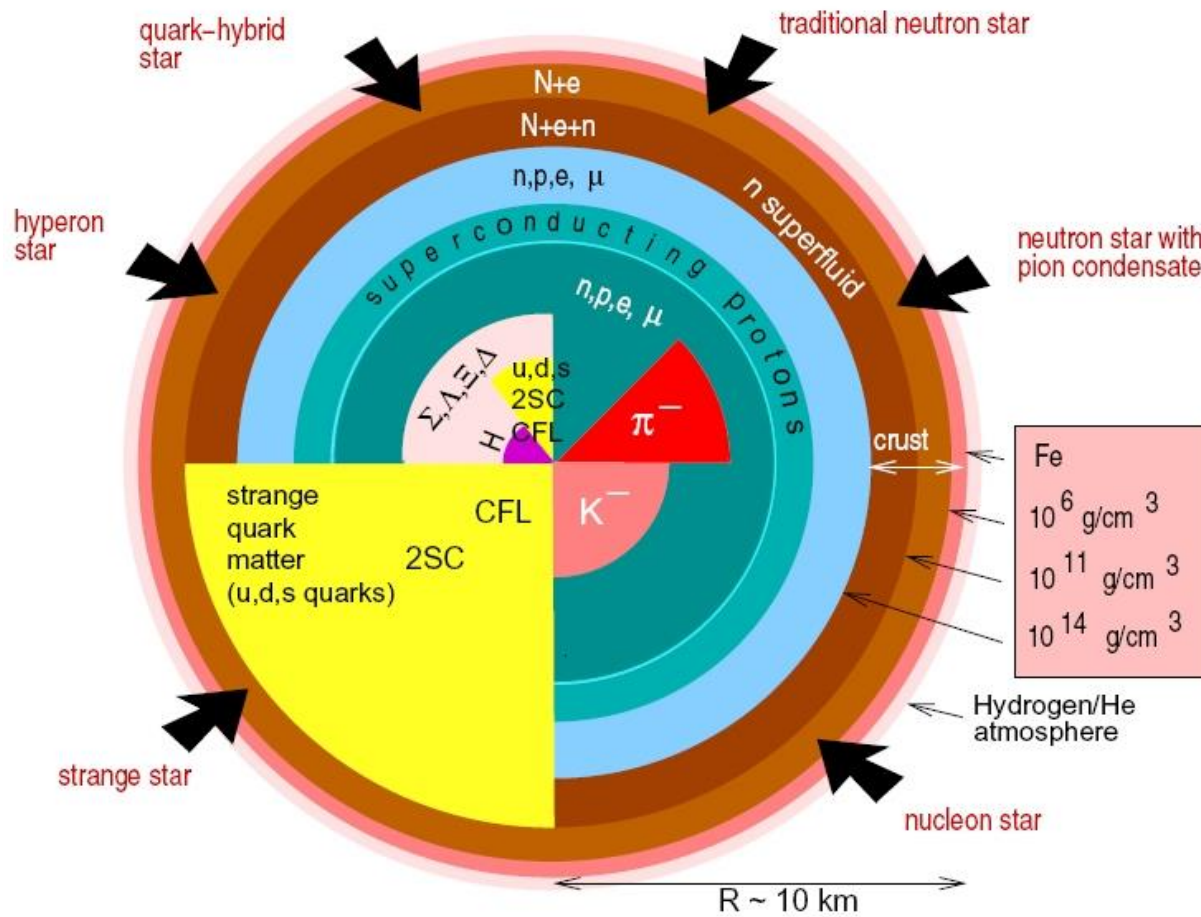
# Pion stars



New exotic solution.

It is not clear if it can be applied to any known type of sources.

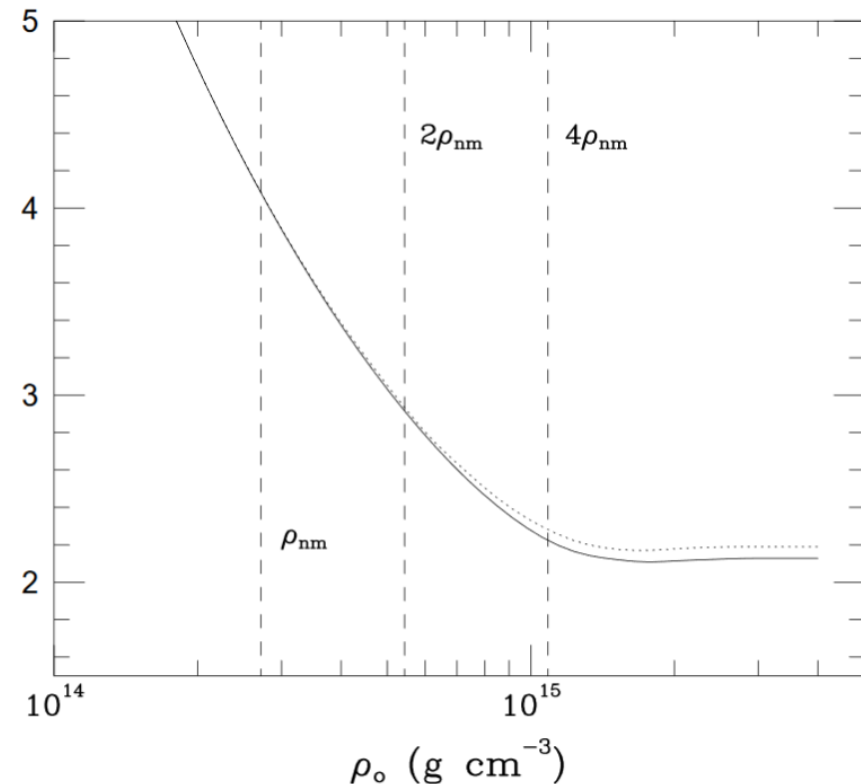
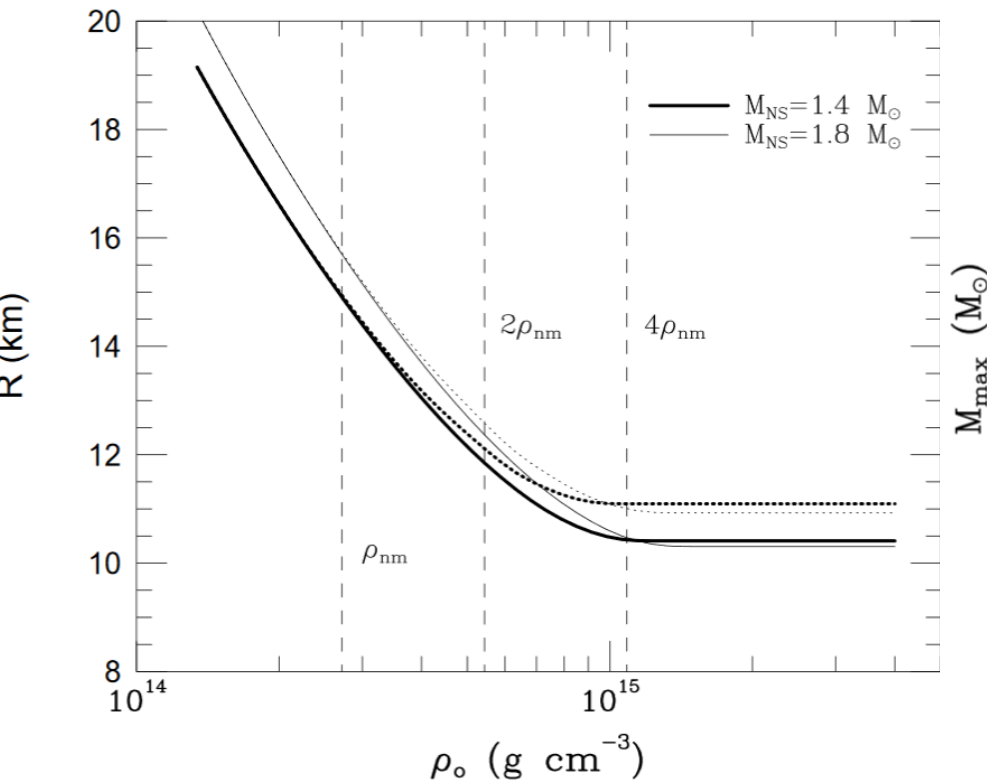
# NS interiors: resume



(Weber et al. ArXiv: 0705.2708)

# Maximum mass

Maximum mass of NSs depends on the EoS, however, it is possible to make calculations on the base of some fundamental assumptions.



astro-ph/9608059

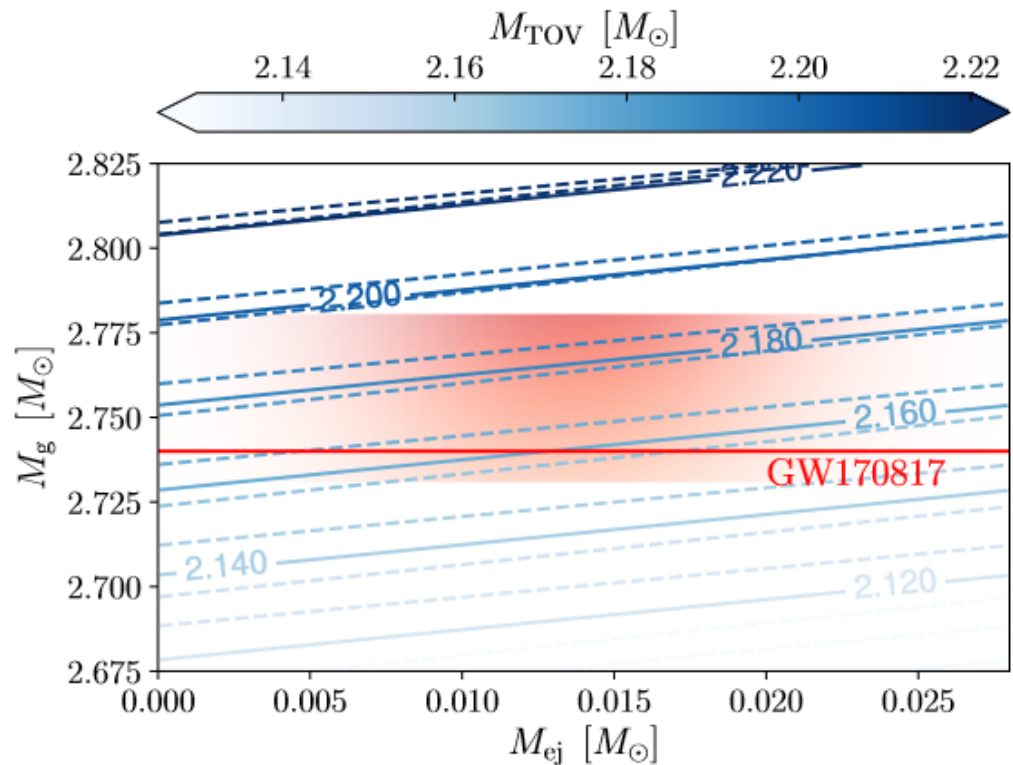
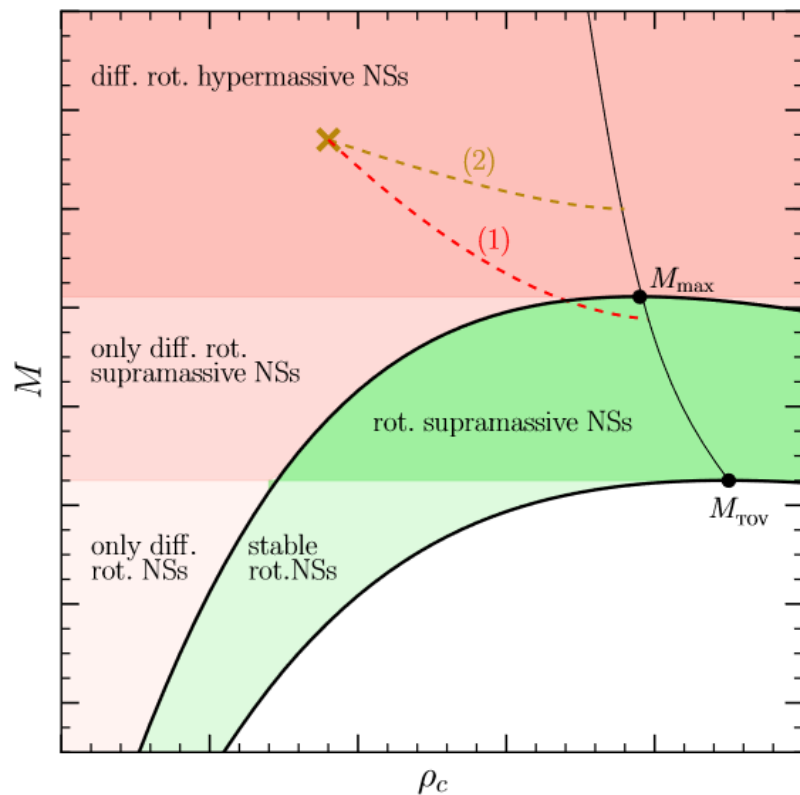
Seminal paper: Rhoades, Ruffini 1974

[http://prl.aps.org/abstract/PRL/v32/i6/p324\\_1](http://prl.aps.org/abstract/PRL/v32/i6/p324_1)

$$c_s^2 = \frac{dP}{d\rho} = c^2.$$

# Calculations based on recent data on NS-NS coalescence

What uniform rotation can give:  $M_{\max} = (1.20^{+0.02}_{-0.02}) M_{\text{TOV}}$  independently of the EOS



# Another constraint from GW170817

$$M_{\text{NSNS}} \approx 2.74 \lesssim M_{\text{thresh}} \approx \alpha M_{\text{max}}^{\text{sph}}. \quad \longleftarrow \text{As there was no prompt collapse}$$

Here  $\alpha \approx 1.3 - 1.7$  is the ratio of the HMNS threshold mass limit to the NS spherical maximum mass as gleaned from multiple numerical experiments of merging NSNSs

$$M_{\text{NSNS}} \approx 2.74 \gtrsim M_{\text{max}}^{\text{sup}} \approx \beta M_{\text{max}}^{\text{sph}},$$

where  $\beta \approx 1.2$  is the ratio of the uniformly rotating supra-massive NS limit to the nonrotating spherical maximum

$$M_{\text{max}}^{\text{sph}} = 4.8 \left( \frac{2 \times 10^{14} \text{ gr/cm}^3}{\rho_m/c^2} \right)^{1/2} M_{\odot},$$

$$M_{\text{max}}^{\text{sup}} = 6.1 \left( \frac{2 \times 10^{14} \text{ gr/cm}^3}{\rho_m/c^2} \right)^{1/2} M_{\odot},$$

$$\beta \approx 1.27.$$

$$2.74/\alpha \lesssim M_{\text{max}}^{\text{sph}} \lesssim 2.74/\beta$$

$$M_{\text{max}}^{\text{sph}} \lesssim 2.16. \quad \beta \approx 1.27.$$

$$M_{\text{max}}^{\text{sph}} \lesssim 2.28. \quad \beta = 1.2$$

# Papers to read

1. astro-ph/0405262 Lattimer, Prakash "Physics of neutron stars"
2. 0705.2708 Weber et al. "Neutron stars interiors and equation of state ..."
3. physics/0503245 Baym, Lamb "Neutron stars"
4. 0901.4475 Piekarewicz "Nuclear physics of neutron stars" (first part)
5. 0904.0435 Paerels et al. "The Behavior of Matter Under Extreme Conditions"
6. 1512.07820 Lattimer, Prakash "The EoS of hot dense matter ...."
7. 1001.3294 Schmitt "Dense matter in compact stars - A pedagogical introduction "
8. 1303.4662 Hebeler et al. "Equation of state and neutron star properties constrained by nuclear physics and observation "
9. 1210.1910 Weber et al. Structure of quark star
10. 1302.1928 Stone "High density matter "
11. 1707.04966 Baym et al. "From hadrons to quarks in neutron stars: a review"
12. 1804.03020. Burgio, Fantina.  
"Nuclear Equation of state for Compact Stars and Supernovae"
13. 1803.01836 Blaschke, Chamel. "Phases of dense matter in compact stars"

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+ the book by Haensel, Yakovlev, Potekhin



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# Lectures on the Web

Lectures can be found at my homepage:

<http://xray.sai.msu.ru/~polar/html/presentations.html>

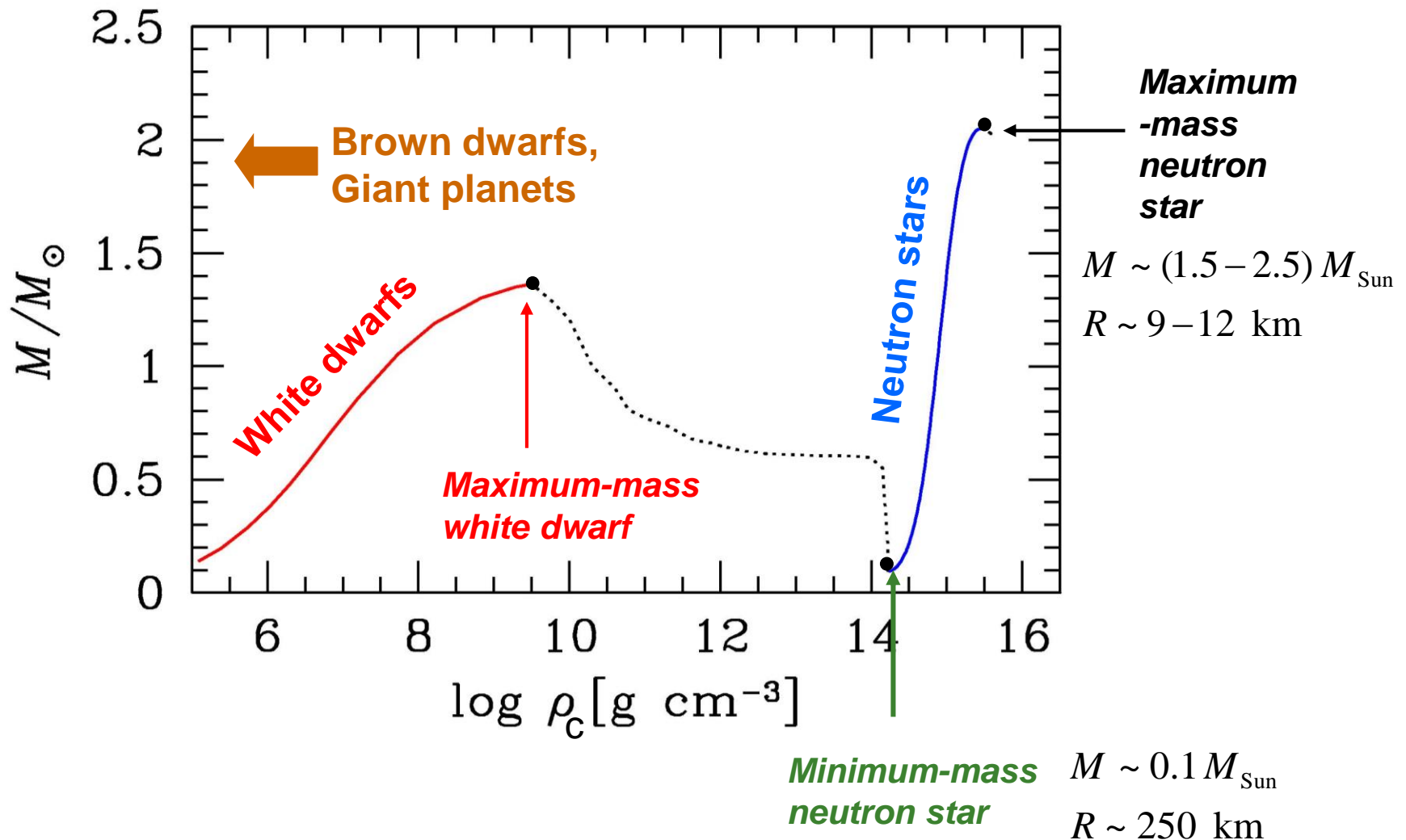
# Neutron Star masses and radii

# NS Masses

- Stellar masses are directly measured only in binary systems
- Accurate NS mass determination for PSRs in relativistic systems by measuring PK corrections
- Gravitational redshift may provide  $M/R$  in NSs by detecting a *known* spectral line,

$$E_{\infty} = E(1-2GM/Rc^2)^{1/2}$$

# Neutron stars and white dwarfs



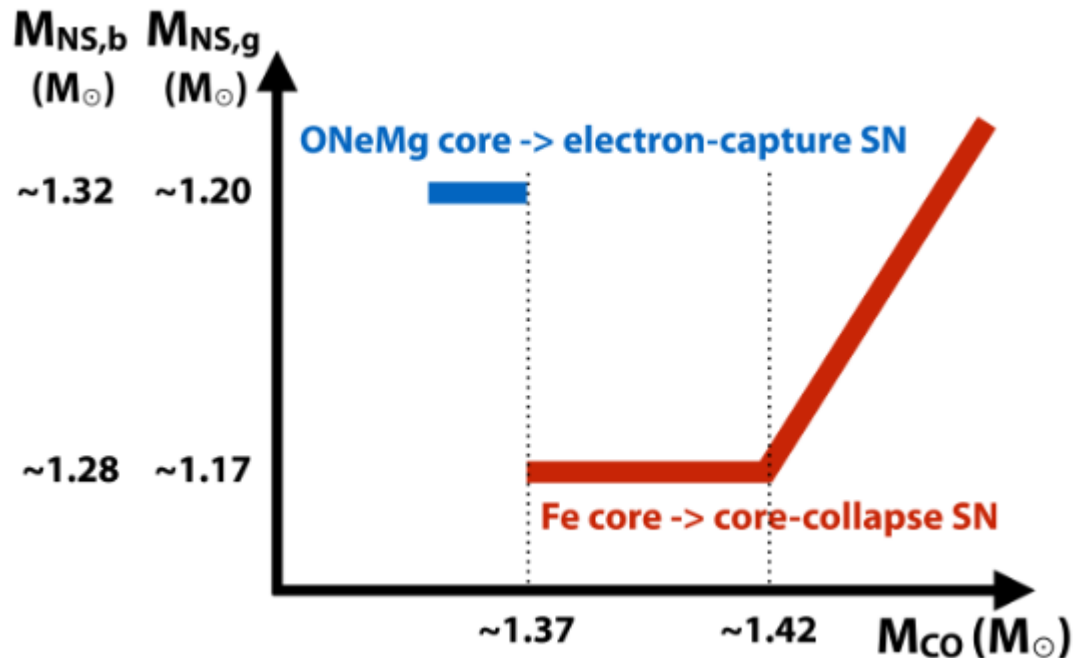
Remember about the difference between baryonic and gravitational masses in the case of neutron stars!

# Minimal mass

In reality, minimal mass is determined by properties of protoNSs.  
Being hot, lepton rich they have much higher limit: about 0.7 solar mass.

Stellar evolution does not produce  
NSs with baryonic mass  
less than about 1.1-1.2 solar.

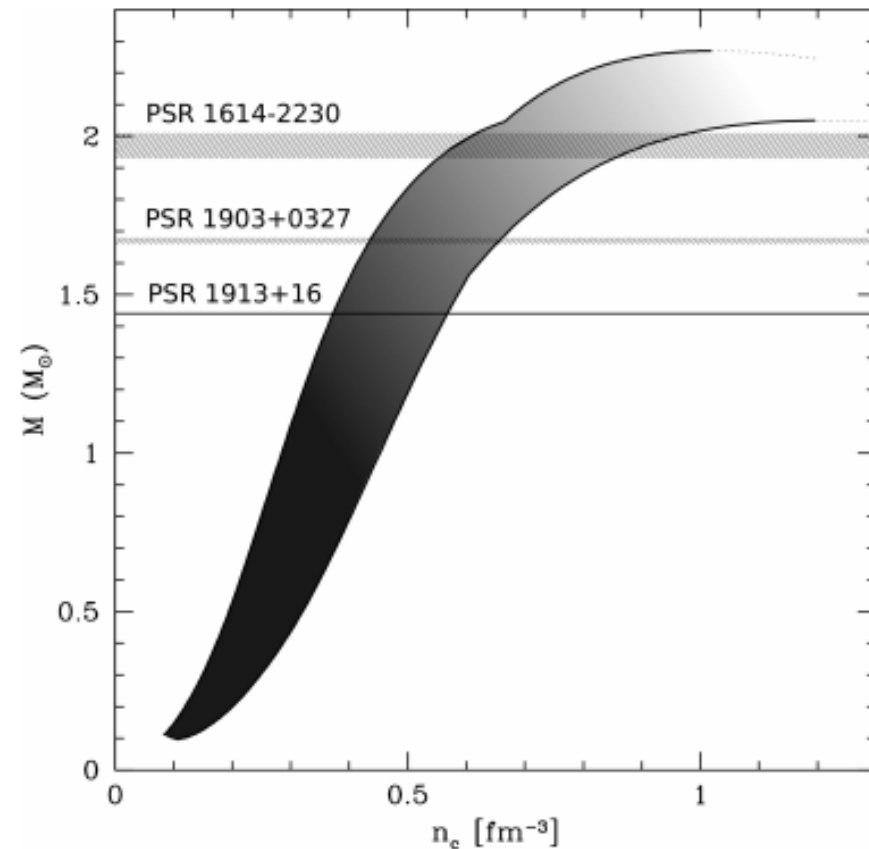
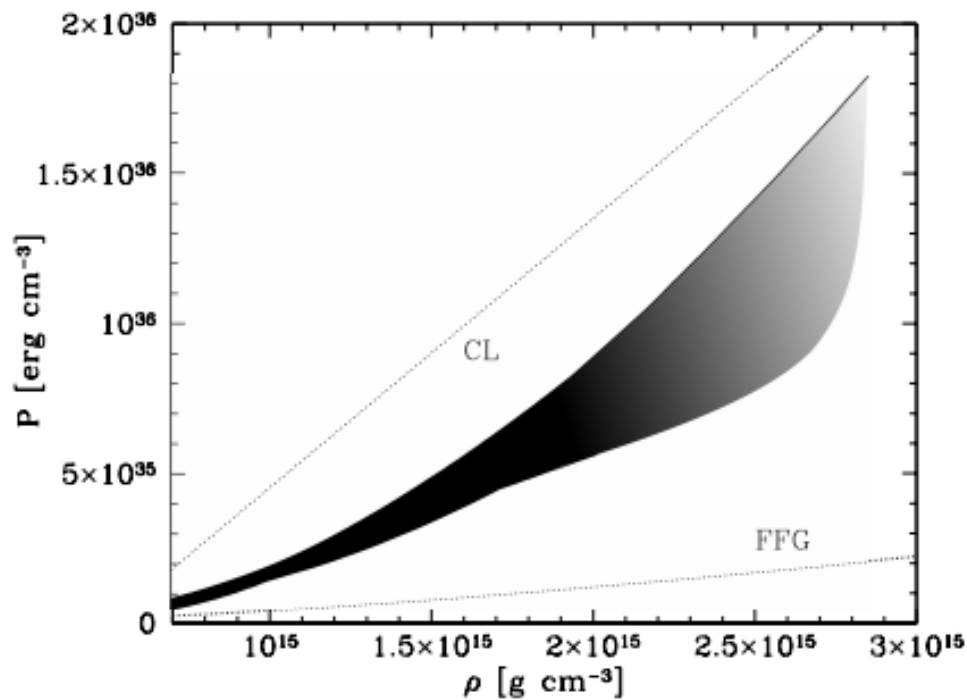
Fragmentation of a core  
due to rapid rotation  
potentially can lead  
to smaller masses,  
but not as small as  
the limit for cold NSs.



1808.02328

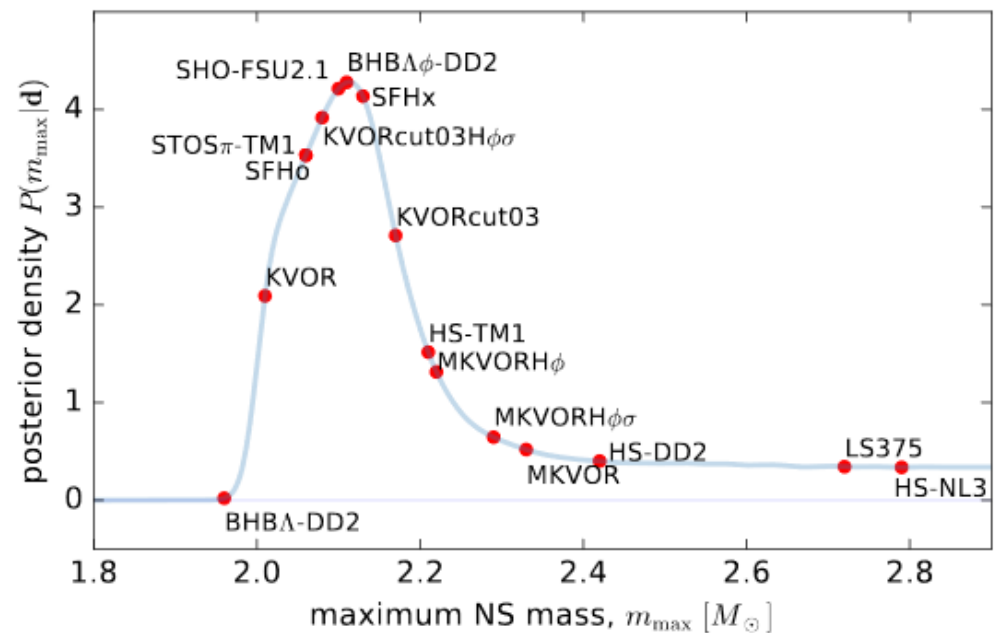
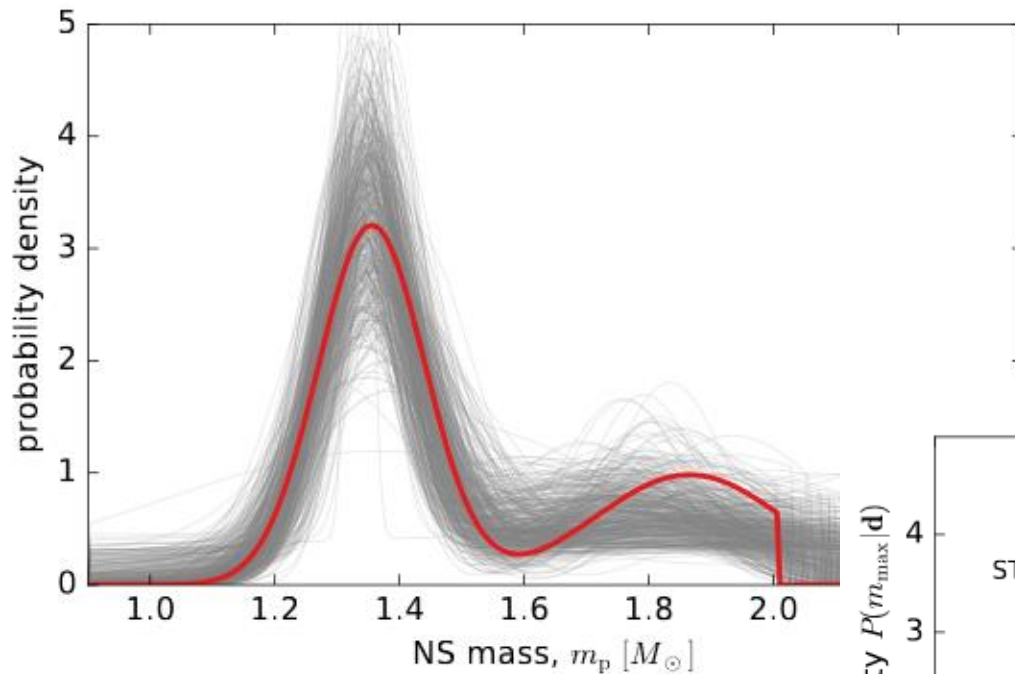
# Maximum mass

Detailed discussion about the maximum mass is given in 1307.3995

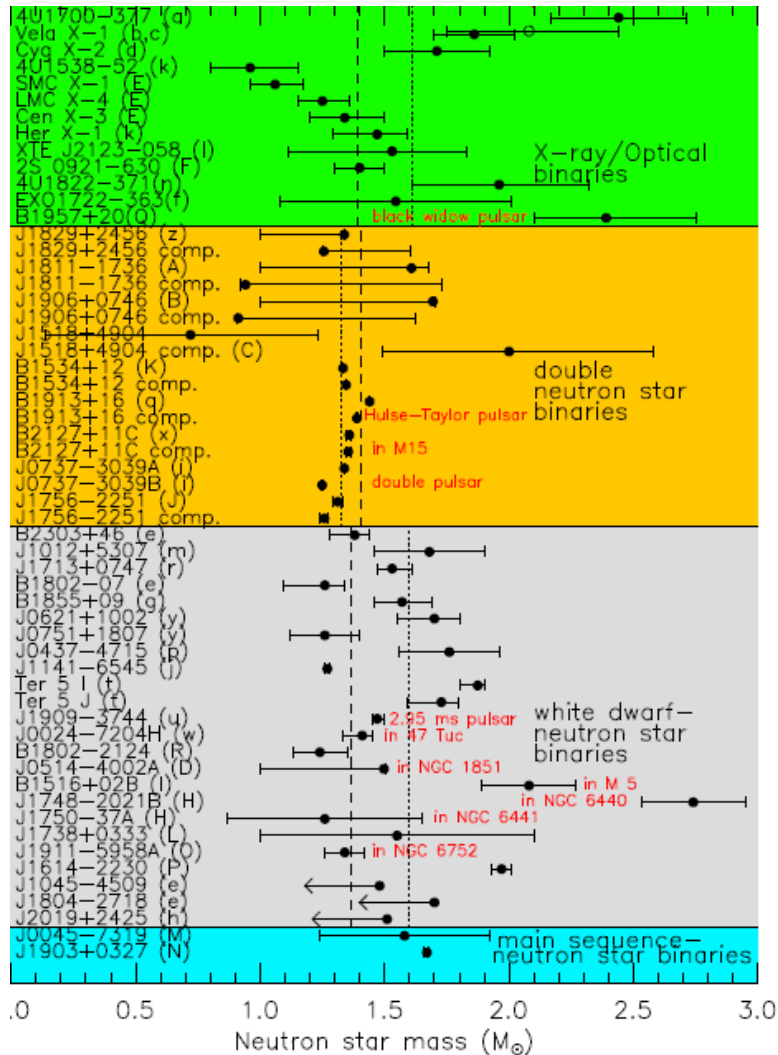




# Maximum mass and cut-off



# Neutron star masses



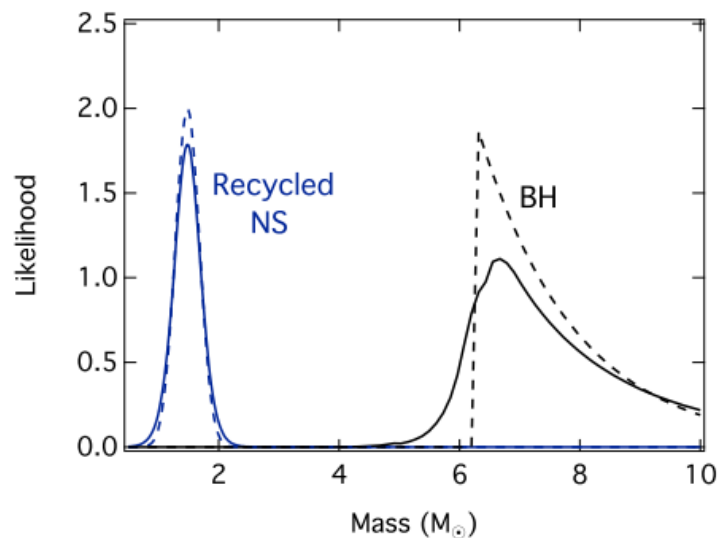
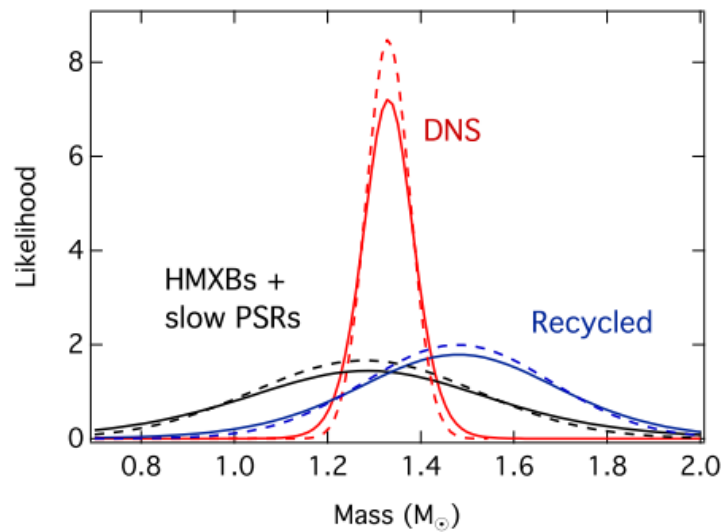
Object	Mass ( $M_{\odot}$ )	Reference	Object	Mass ( $M_{\odot}$ )	Reference
<i>X-Ray/Optical Binaries (mean = <math>1.609 M_{\odot}</math>, weighted mean = <math>1.393 M_{\odot}</math>)</i>					
4U1700-377	$2.44^{+0.27}_{-0.27}$	a (10)	Vela X-1	$1.86^{+0.16}_{-0.16}$	b, c (11) (12)
Cyg X-2	$1.71^{+0.21}_{-0.21}$	d (13)	4U1538-52	$0.96^{+0.19}_{-0.16}$	k (14)
SMC X-1	$1.06^{+0.11}_{-0.10}$	E (15)	LMC X-4	$1.25^{+0.11}_{-0.12}$	E (15)
Cen X-3	$1.34^{+0.18}_{-0.14}$	E (15)	Her X-1	$1.47^{+0.12}_{-0.18}$	k (14)
XTE J2123-058	$1.53^{+0.30}_{-0.42}$	l (16) (17)	2S 0921-630	$1.4^{+0.1}_{-0.1}$	F (18)
4U 1822-371	$1.96^{+0.36}_{-0.32}$	n (19)	EXO 1722-363	$1.545^{+0.465}_{-0.465}$	f (47)
B1957+20	$2.39^{+0.32}_{-0.29}$	Q (49)			
<i>Neutron Star - Neutron Star Binaries (mean = <math>1.325 M_{\odot}</math>, weighted mean = <math>1.403 M_{\odot}</math>)</i>					
J1829+2456	$1.338^{+0.002}_{-0.002}$	z (20)	J1829+2456 (c)	$1.256^{+0.346}_{-0.003}$	z (20)
J1811-1736	$1.608^{+0.066}_{-0.068}$	A (21)	J1811-1736 (c)	$0.941^{+0.787}_{-0.021}$	A (21)
J1906+07	$1.694^{+0.012}_{-0.012}$	B (22)	J1906+07 (c)	$0.912^{+0.710}_{-0.004}$	B (22)
J1518+4904	$0.72^{+0.58}_{-0.58}$	C (23)	J1518+4904 (c)	$2.00^{+0.51}_{-0.51}$	C (23)
1534+12	$1.3332^{+0.0010}_{-0.0010}$	K (24)	1534+12 (c)	$1.3452^{+0.0010}_{-0.0010}$	K (24)
1913+16	$1.4398^{+0.0002}_{-0.0002}$	q (25)	1913+16 (c)	$1.3886^{+0.0002}_{-0.0002}$	q (25)
2127+11C	$1.358^{+0.010}_{-0.010}$	x (26)	2127+11C (c)	$1.354^{+0.010}_{-0.010}$	x (26)
J0737-3039A	$1.3381^{+0.0007}_{-0.0007}$	i (27)	J0737-3039B	$1.2489^{+0.0007}_{-0.0007}$	i (27)
J1756-2251	$1.312^{+0.017}_{-0.017}$	J (28)	J1756-2251 (c)	$1.258^{+0.017}_{-0.017}$	J (28)
<i>Neutron Star - White Dwarf Binaries (mean = <math>1.599 M_{\odot}</math>, weighted mean = <math>1.362 M_{\odot}</math>)</i>					
B2303+46	$1.38^{+0.06}_{-0.10}$	e (29)	J1012+5307	$1.68^{+0.22}_{-0.22}$	m (30)
J1713+0747	$1.53^{+0.08}_{-0.08}$	r (31) (51)	B1802-07	$1.26^{+0.08}_{-0.08}$	e (29)
B1855+09	$1.57^{+0.04}_{-0.12}$	g (32) (51)	J0621+1002	$1.70^{+0.10}_{-0.17}$	y (33)
J0751+1807	$1.26^{+0.14}_{-0.14}$	y (33)	J0437-4715	$1.76^{+0.20}_{-0.20}$	p (34)
J1141-6545	$1.27^{+0.01}_{-0.01}$	j (35)	Ter 5 I	$1.874^{+0.32}_{-0.068}$	t (36)
Ter 5 J	$1.728^{+0.066}_{-0.136}$	t (36)	J1909-3744	$1.47^{+0.02}_{-0.02}$	u (37)
J0024-7204H	$1.41^{+0.04}_{-0.08}$	w (38)	B1802-2124	$1.24^{+0.11}_{-0.11}$	R (50)
J0514-4002A	$1.497^{+0.008}_{-0.497}$	D (39)	B1516+02B	$2.08^{+0.19}_{-0.19}$	I (40)
J1748-2021B	$2.74^{+0.21}_{-0.21}$	H (41)	J1750-37A	$1.26^{+0.39}_{-0.39}$	H (41)
J1738+0333	$1.55^{+0.55}_{-0.55}$	L (42)	J1911-5958A	$1.34^{+0.08}_{-0.08}$	O (43)
J1614-2230	$1.97^{+0.04}_{-0.04}$	P (48)	J1045-4509	$< 1.48$	e (29)
J1804-2718	$< 1.70$	e (29)	J2019+2425	$< 1.51$	h (44)
<i>Neutron Star - Main Sequence Binaries</i>					
J0045-7319	$1.58^{+0.34}_{-0.34}$	M (45)	J1903+0327	$1.67^{+0.01}_{-0.01}$	N (46)

Follow updates at <https://stellarcollapse.org/nsmasses>

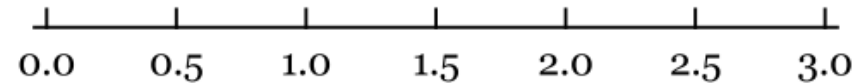
arXiv: 1012.3208

# Update - 2012

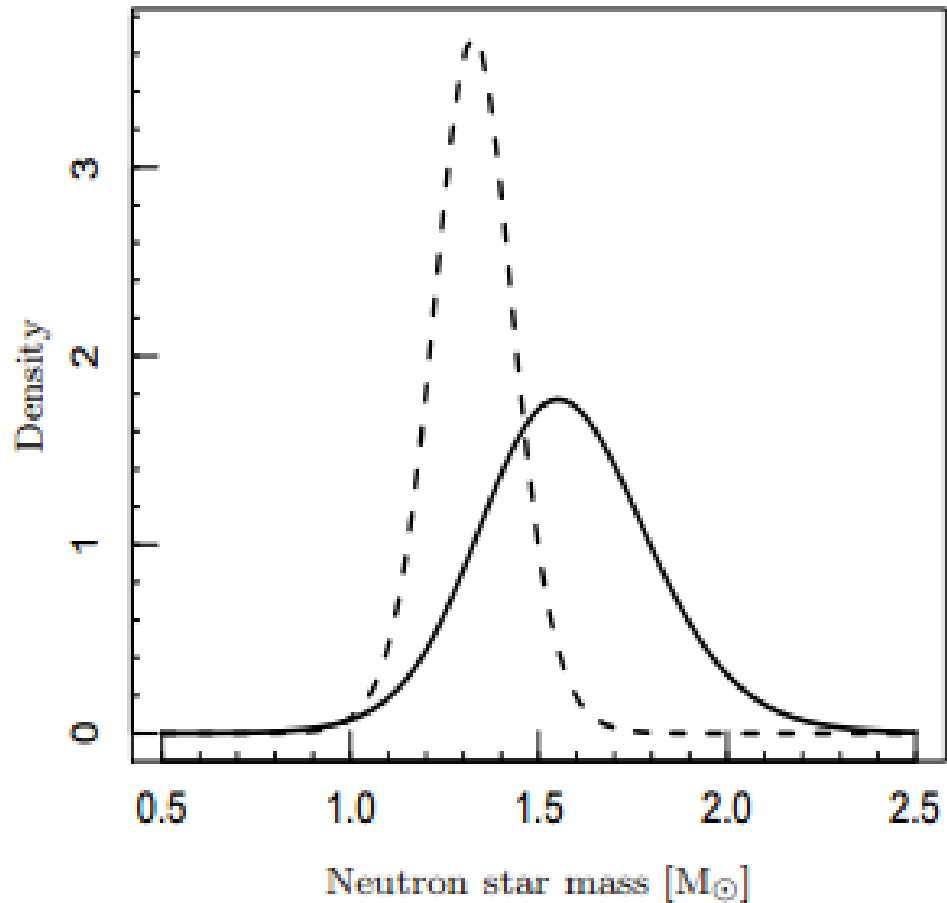
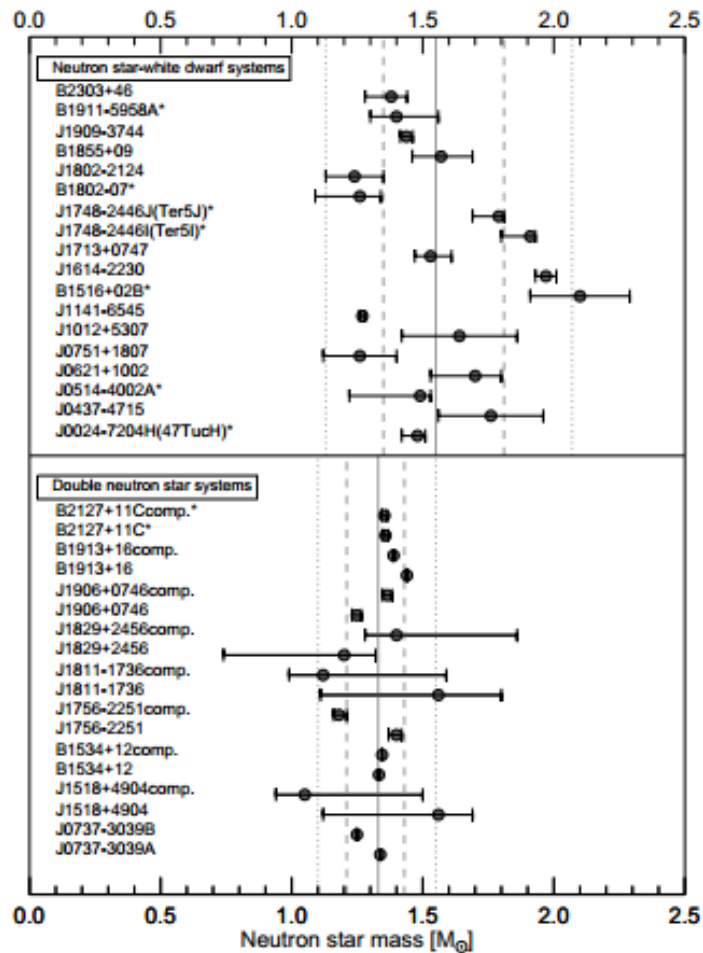
Neutron Stars



J0737-3039B  
J1756-2251c  
J1906+0746c  
J1756-2251  
J1906+0746  
B1534+12  
J0737-3039A  
B1534+12c  
B2127+11Cc  
B2127+11C  
B1913+16c  
B1913+16  
J1829+2456  
J1811-1736  
J1518+4904  
J1518+4904c  
J1811-1736c  
J1829+2456c  
4U1538-52  
SMC X-1  
Her X-1  
LMC X-4  
Cen X-3  
Vela X-1  
J1802-2124  
J0751+1807  
J1141-6545  
J1713+0747  
J1909-3744  
B1855+09  
J1903+0327  
J0437-4715  
J1614-2230  
J1750-37A  
B1802-07  
J1824-2452C  
J0024-7204H  
B2303+46  
J0514-4002A  
J0621+1002  
J1748-2446J  
J1748-2446I  
B1516+02B  
J1748-2021B  
B1911-5958A  
J1012+5307  
KS 1731-260  
Cyg X-2  
4U 1820-30  
4U 1735-345  
4U 1608-52



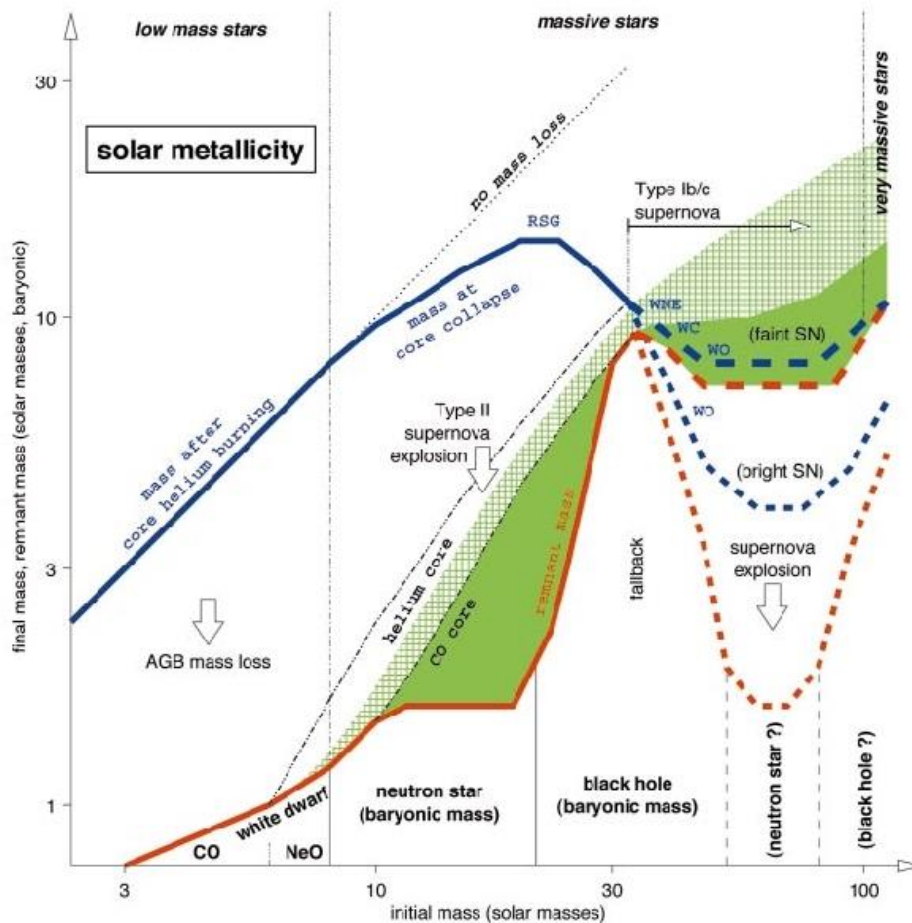
# Update - 2013



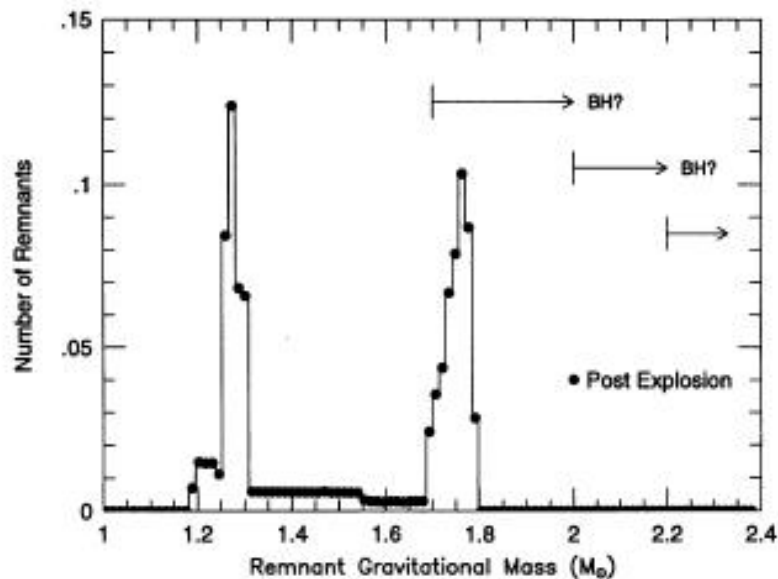
# Compact objects and progenitors.

## Solar metallicity.

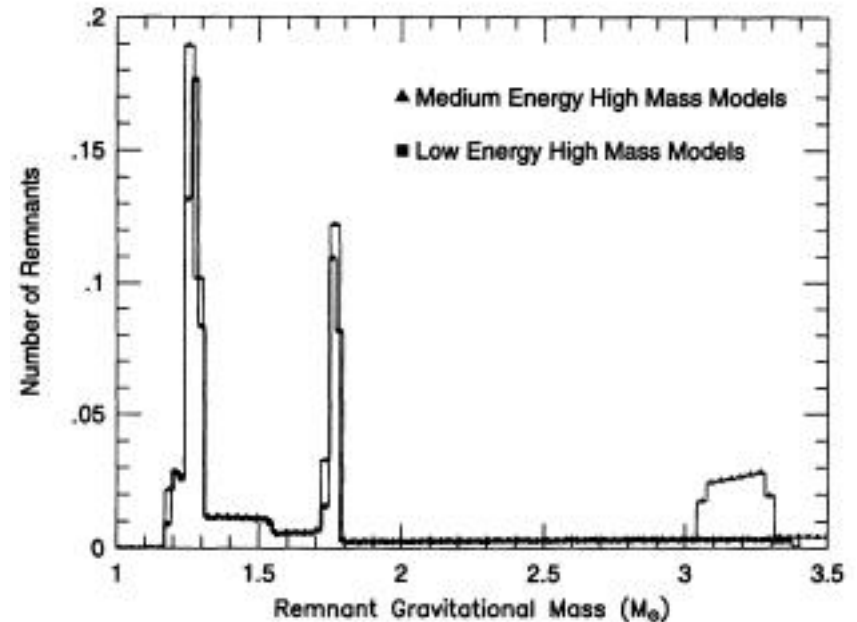
There can be a range of progenitor masses in which NSs are formed, however, for smaller and larger progenitors masses BHs appear.



# Mass spectrum of compact objects

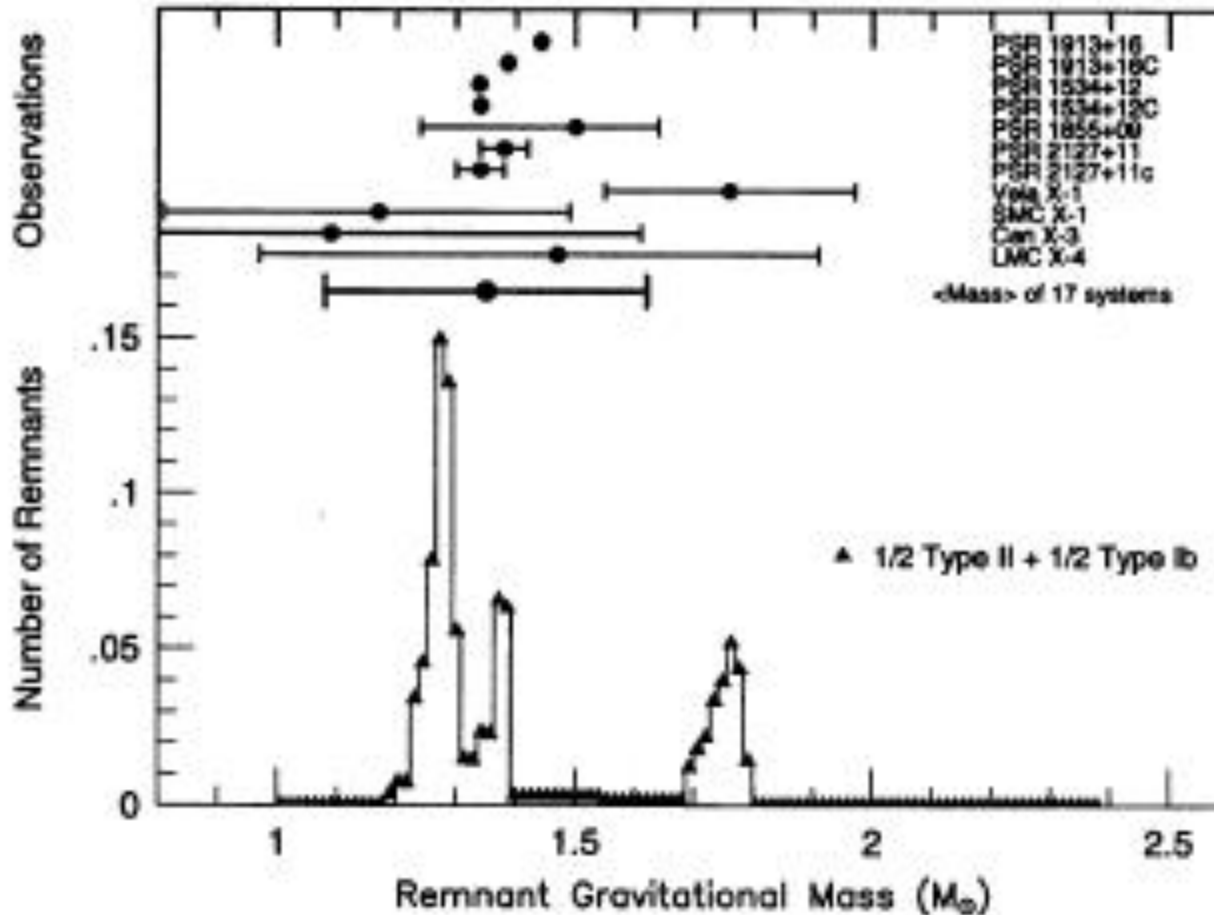


Results of calculations  
(depend on the assumed model  
of explosion)





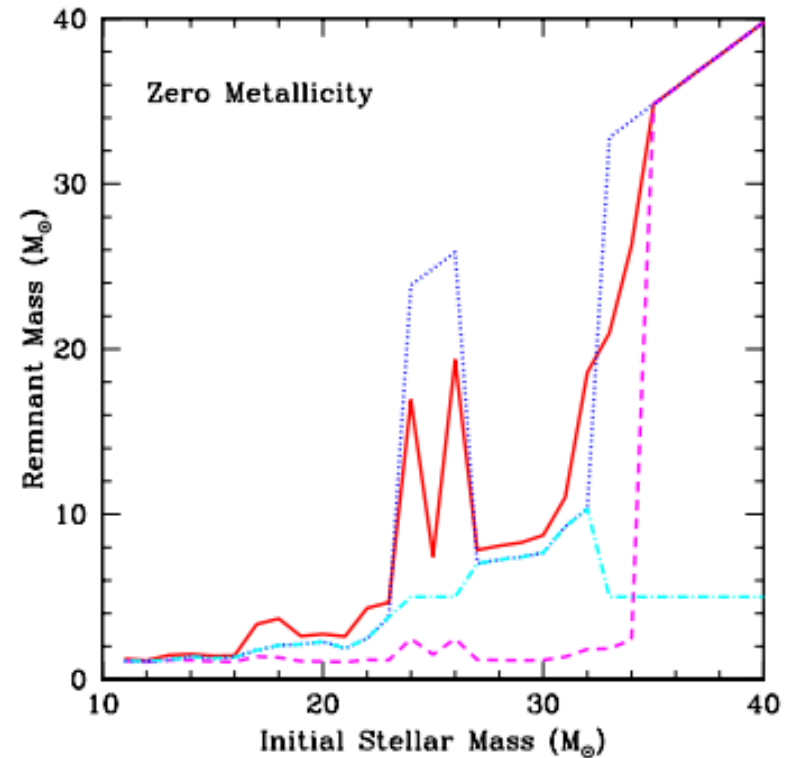
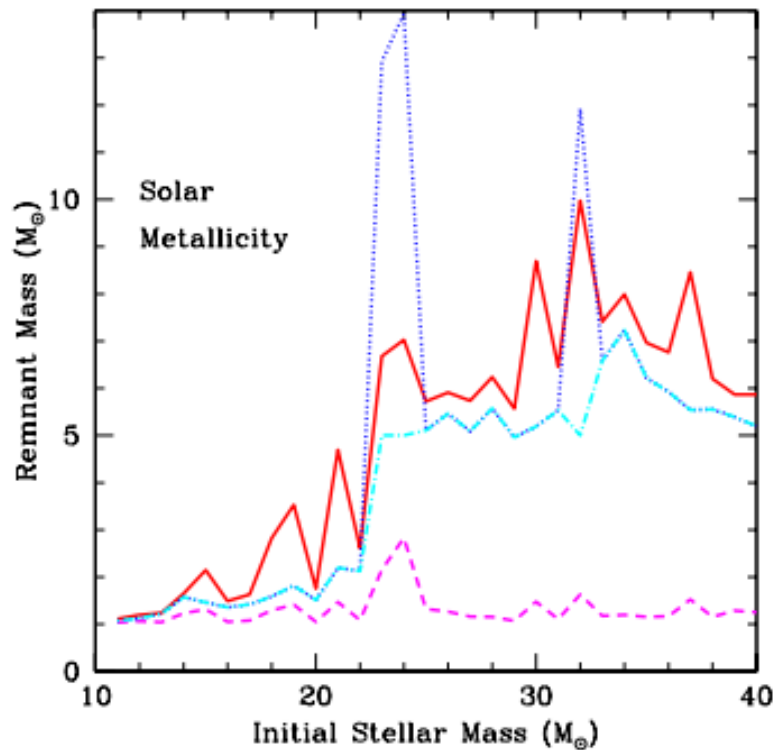
# Mass spectrum of compact objects



Comparison of one of the model with observations.

However, selection effects can be important as observed NSs are all in binaries.

# Newer calculations of the mass spectrum

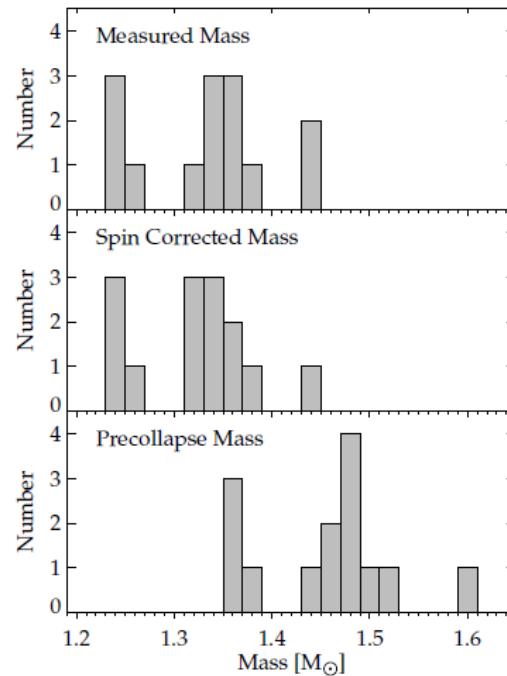
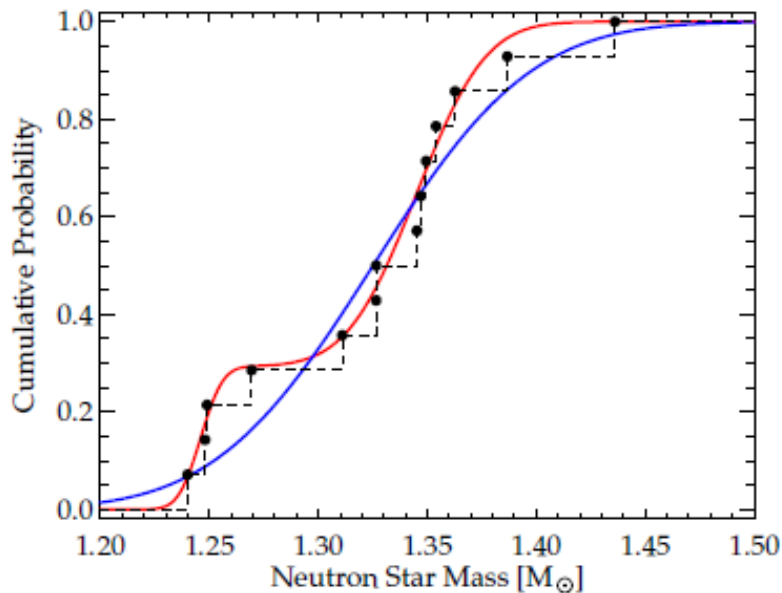


Different curves are plotted for different models of explosion:  
dashed – with a magnetar

# Bi-modal mass spectrum?

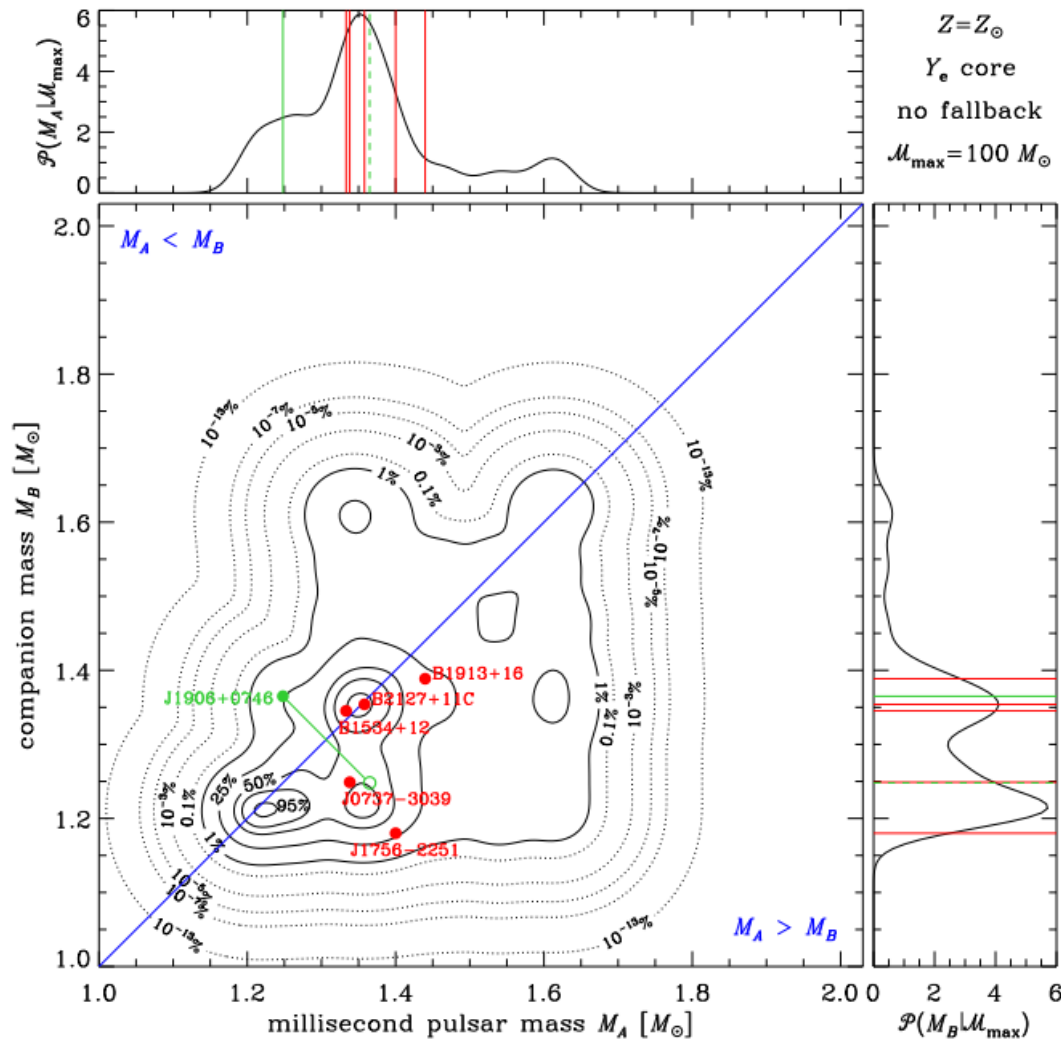
Pulsar Name	Mass of Recycled Neutron Star ( $M_{\odot}$ )	Mass of Young Neutron Star ( $M_{\odot}$ )	$P_{\text{orb}}$ (hours)	Eccentricity	Pulse Period (ms)	Reference
J0737-3039A/B	$1.3381 \pm 0.0007$	$1.2489 \pm 0.0007$	2.4	0.088	23	Kramer et al. (2006)
B1534+12	$1.3332 \pm 0.0010$	$1.3452 \pm 0.0010$	10.1	0.273	38	Stairs et al. (2002)
J1756-2251	$1.32 \pm 0.02$	$1.24 \pm 0.02$	7.67	0.18	28	Stairs (2008)
J1906+0746	$1.365 \pm 0.018$	$1.248 \pm 0.018$	3.98	0.085	144 <sup>†</sup>	Kasian (2008)
B1913+16	$1.4414 \pm 0.0002$	$1.3867 \pm 0.0002$	7.92	0.617	59	Weisberg & Taylor (2005)
B2127+11C	$1.358 \pm 0.010$	$1.354 \pm 0.010$	8.05	0.681	30	Jacoby et al. (2006)
J1909-3744	$1.438 \pm 0.024$	white dwarf	36.7	$\lesssim 10^{-6}$	2.9	Jacoby et al. (2005)
J1141-6545	white dwarf	$1.27 \pm 0.01$	4.74	0.172	393 <sup>†</sup>	Bhat et al. (2008)

The low-mass peak the authors relate to e-capture SN.



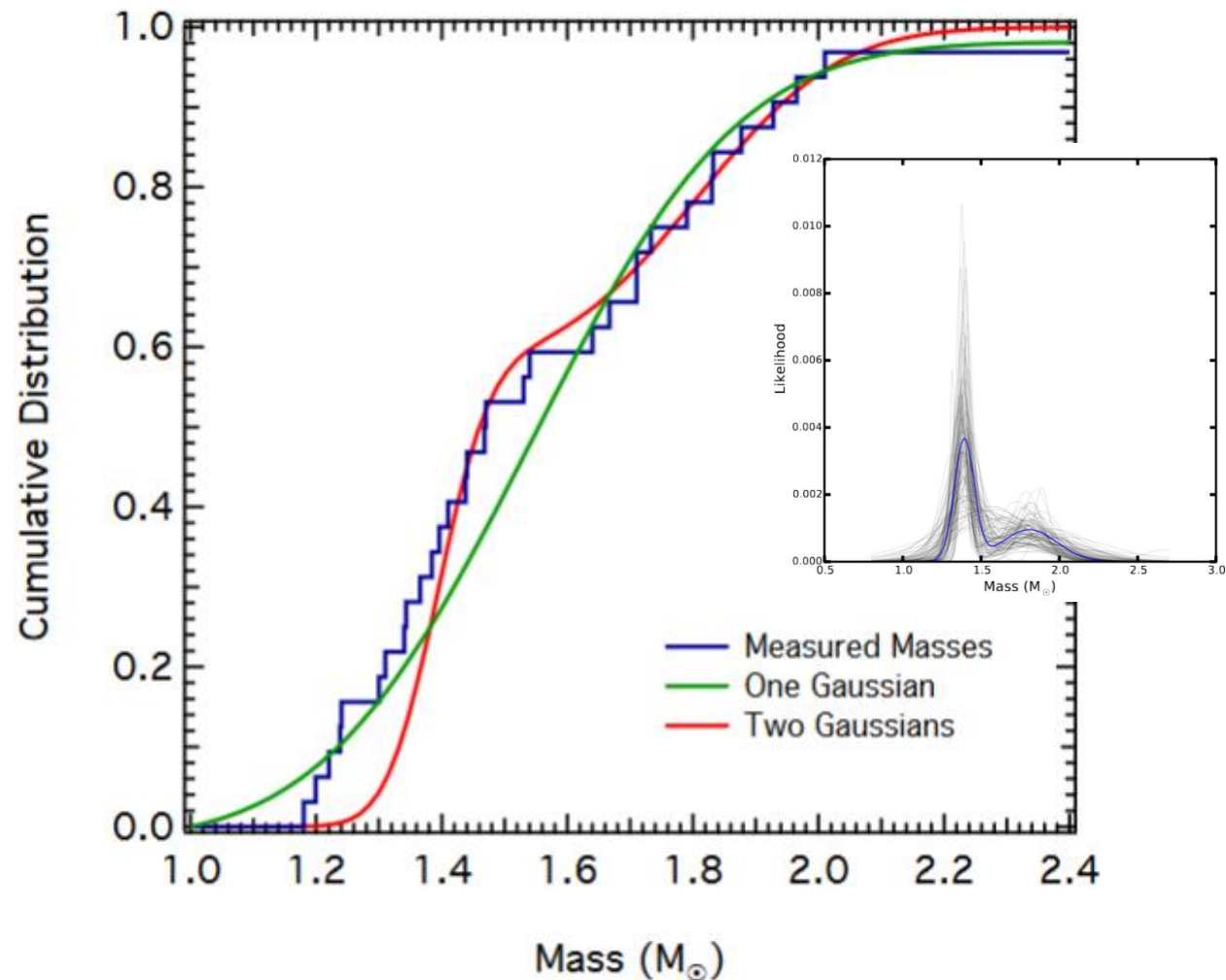
Based on 14 observed systems

# Comparison of observations with theory



# Bimodality in mPSR mass distribution

#	PSR Name	Mass [ $M_{\odot}$ ]
1	J0337+1715	1.4378(13)
2	J0348+0432	2.01(4)
3	J0437-4715	1.44(7)
4	J0751+1807	1.64(15)
5	J1012+0507	1.83(11)
6	J1023+0038	1.71(16)
7	J1614-2230	1.928(17)
8	J1713+0747	1.31(11)
9	J1738+0333	1.47(7)
10	J1802-2124	1.24(11)
11	J1807-2500B	1.3655(21)
12	B1855+09	1.30(11)
13	J1903+0327	1.667(7)
14	J1909-3744	1.540(27)
15	J1910-5959A	1.34(8)
16	J1918-0642	1.18(11)
17	J1946+3417	1.832(13)
18	J2234+0611	1.396(11)



+ 14 PSR with less precisely  
determined masses

The bimodality reflects birth properties?

1605.01665

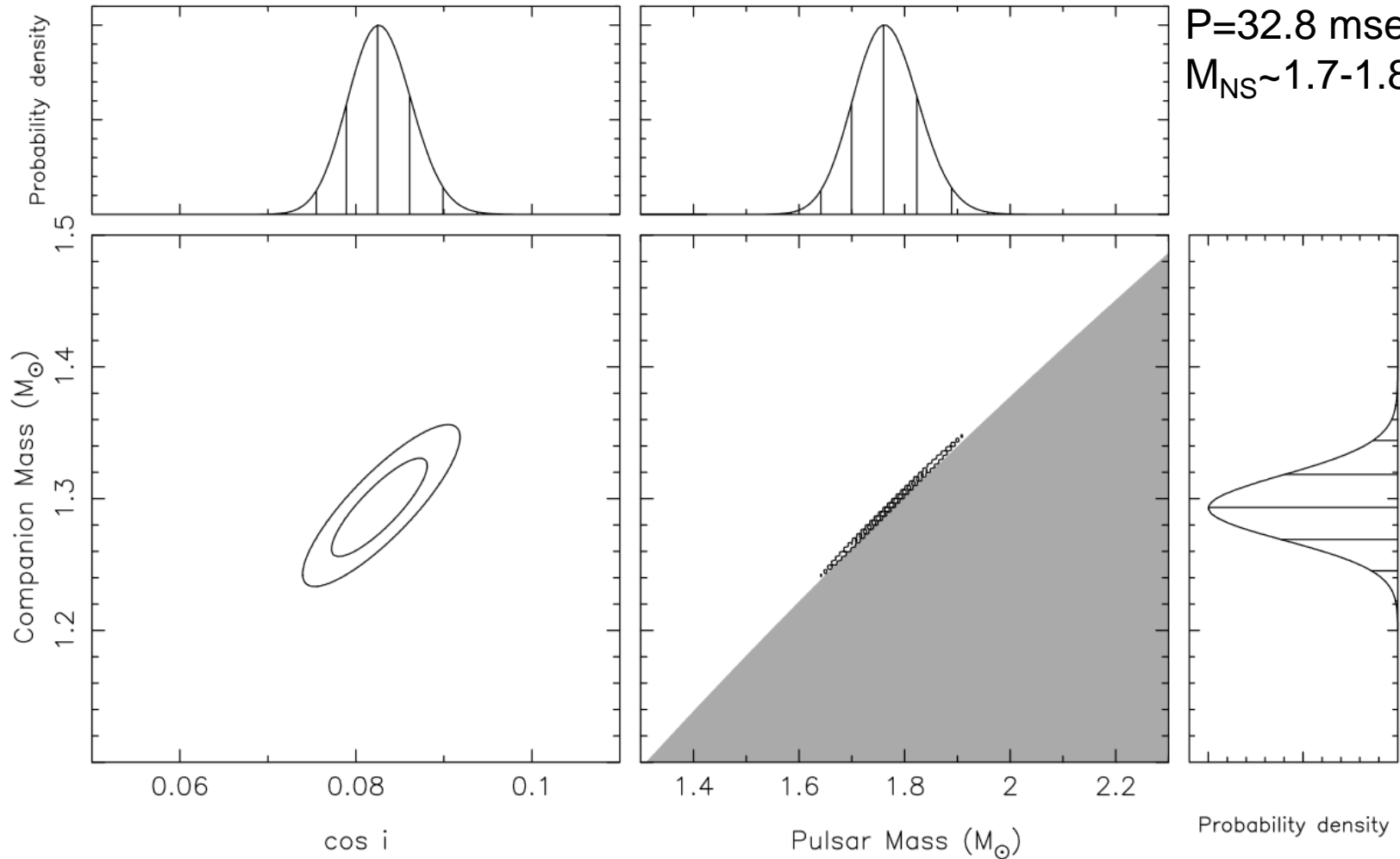
# Massive born NS

PSR J2222-0137

WD companion

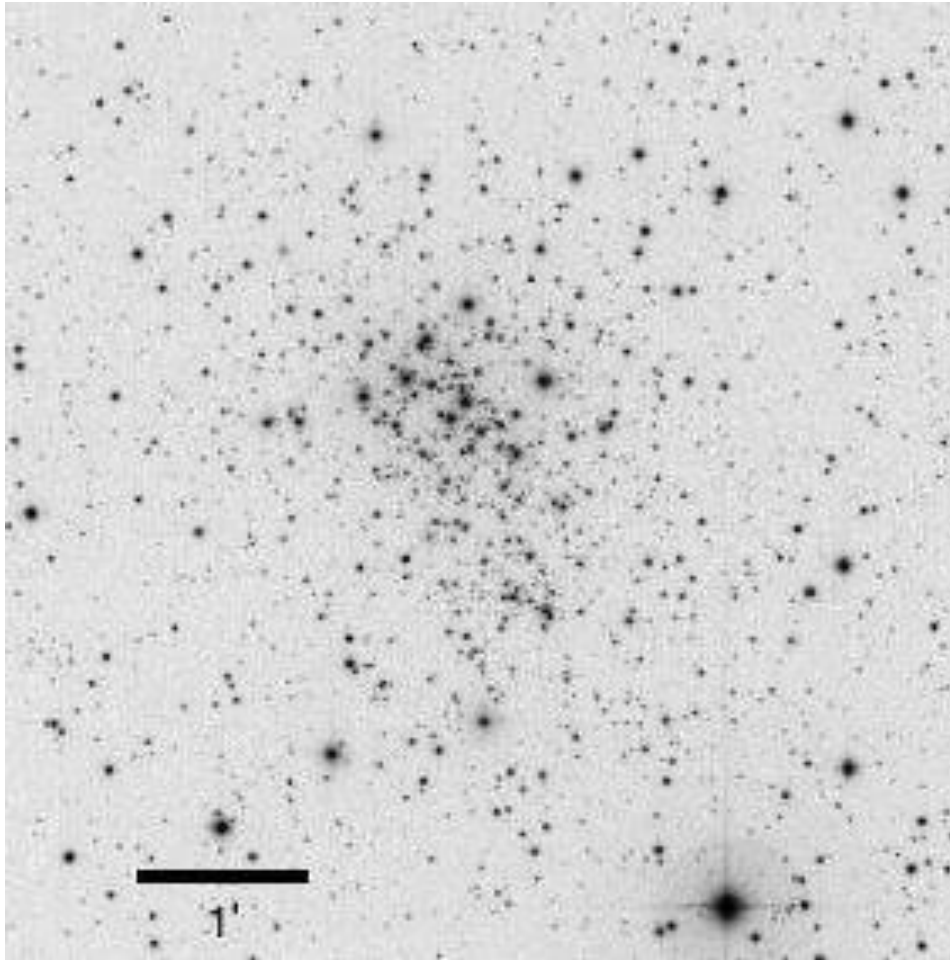
$P=32.8$  msec

$M_{\text{NS}} \sim 1.7-1.8$





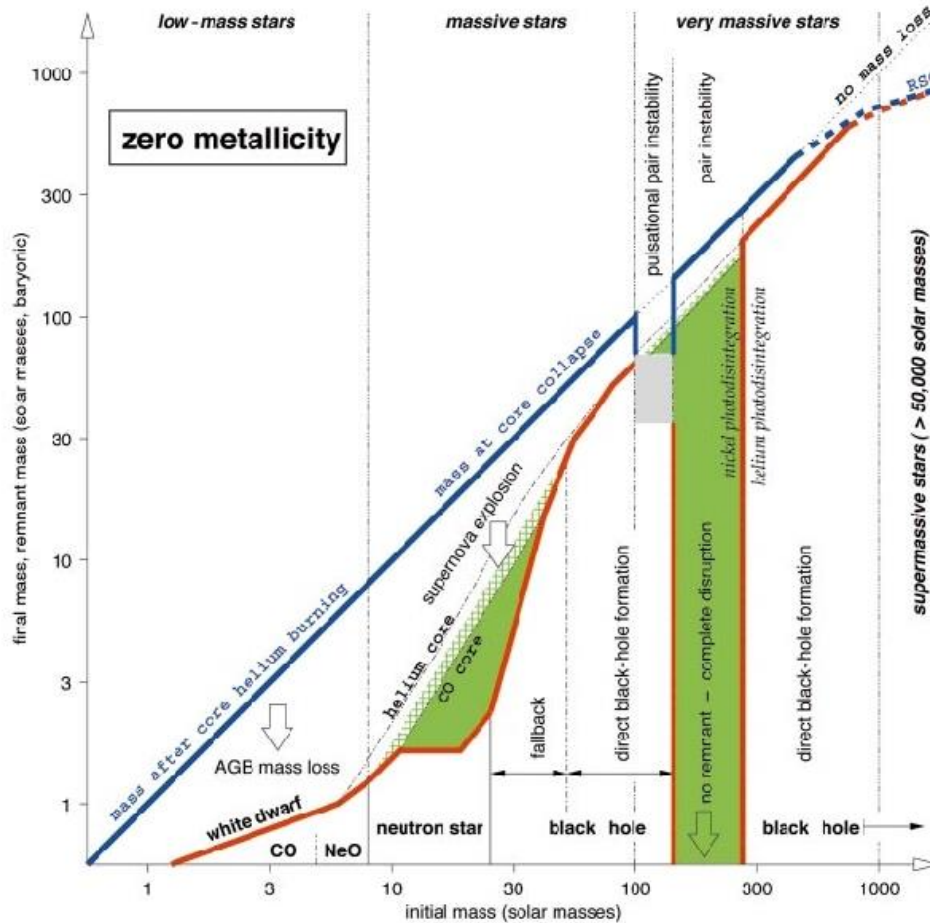
# A NS from a massive progenitor



Anomalous X-ray pulsar  
in the association

Westerlund1 most probably has  
a very massive progenitor,  $>40 M_{\odot}$ .

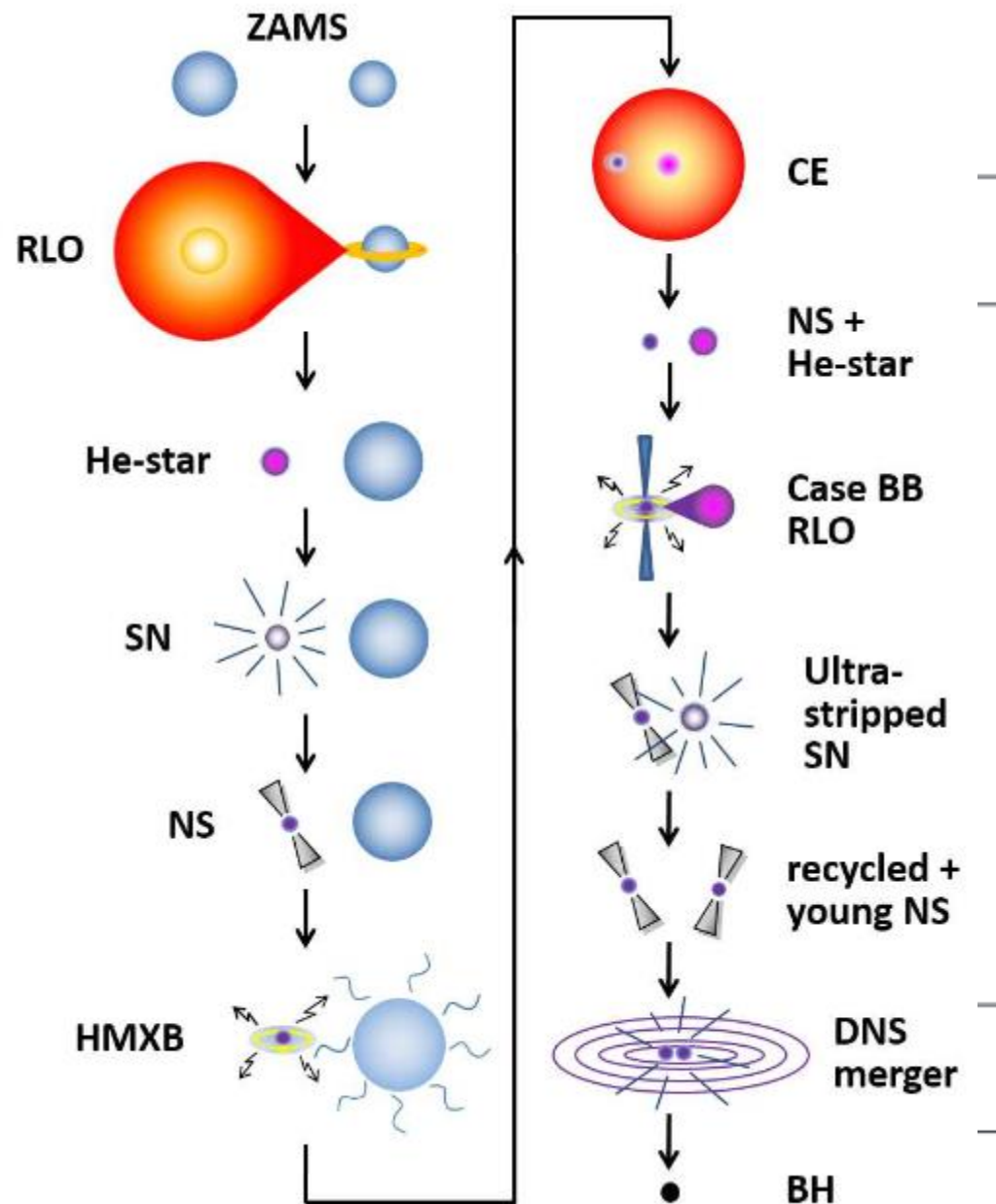
# The case of zero metallicity



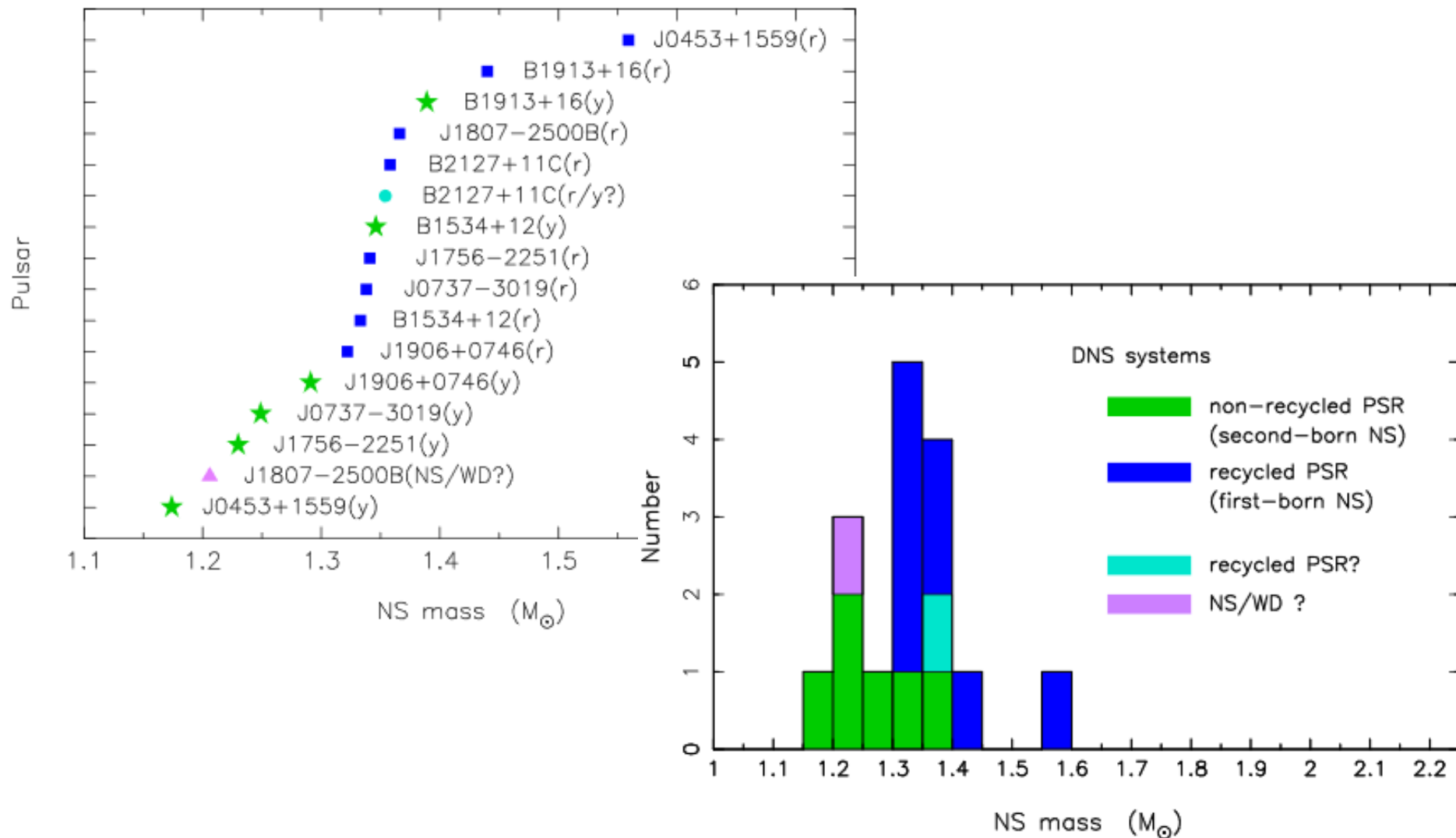
No intermediate mass range for NS formation.

# DNS

Radio Pulsar	Type	$P$ (ms)
J0453+1559 <sup>a</sup>	recycled	45.8
J0737–3039A <sup>b</sup>	recycled	22.7
J0737–3039B <sup>b</sup>	young	2773.5
J1518+4904 <sup>c</sup>	recycled	40.9
B1534+12 <sup>d</sup>	recycled	37.9
J1753–2240 <sup>e</sup>	recycled	95.1
J1755–2550 <sup>f*</sup>	young	315.2
J1756–2251 <sup>g</sup>	recycled	28.5
J1811–1736 <sup>h</sup>	recycled	104.2
J1829+2456 <sup>i</sup>	recycled	41.0
J1906+0746 <sup>j*</sup>	young	144.1
J1913+1102 <sup>k</sup>	recycled	27.3
B1913+16 <sup>l</sup>	recycled	59.0
J1930–1852 <sup>m</sup>	recycled	185.5
J1807–2500B <sup>n*</sup>	GC	4.2
B2127+11C <sup>p</sup>	GC	30.5



# Mass distribution



# PSR J1518+4904

[Janssen et al. arXiv: 0808.2292]

## Surprising results !!!

Mass of the recycled pulsar is  
<1.17 solar masses

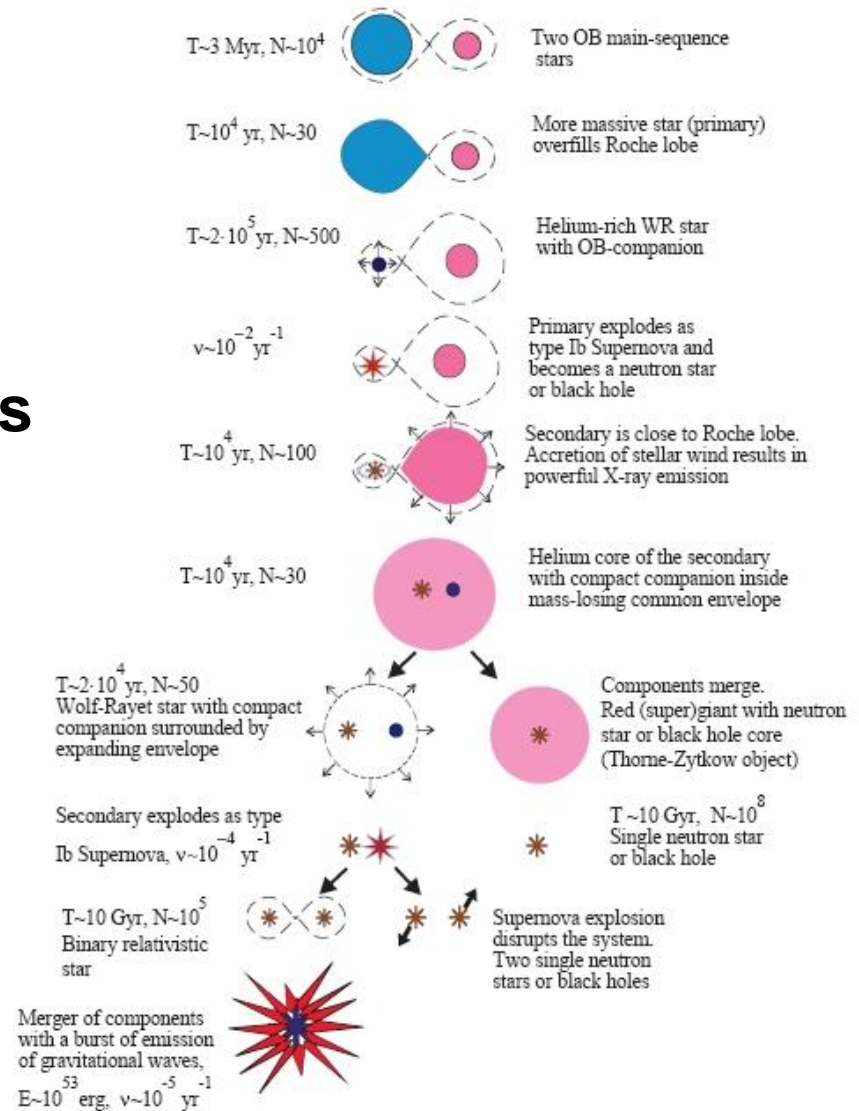
Mass of its component is  
>1.55 solar masses

Central values are even more shocking:

$0.72^{+0.51}_{-0.58}$  and  $2.00^{+0.58}_{-0.51}$

$V \sim 25$  km/s,  $e \sim 0.25$

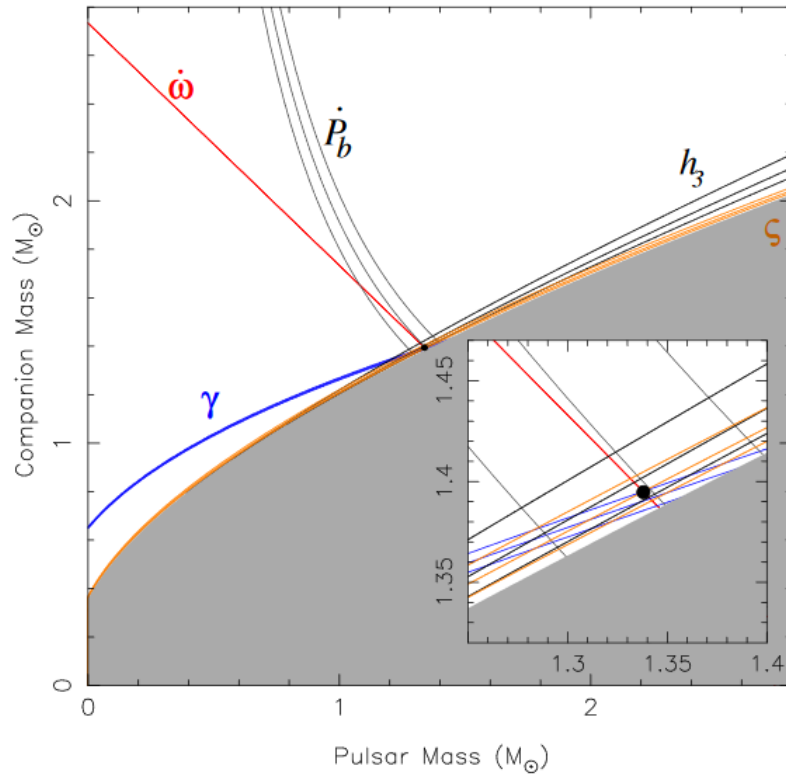
The second SN was e<sup>-</sup>-capture?



New measurements show less extreme values, see table 1 in 1603.02698:  
<1.768 and >0.95 solar masses. Total mass is the same 2.7183 solar masses.

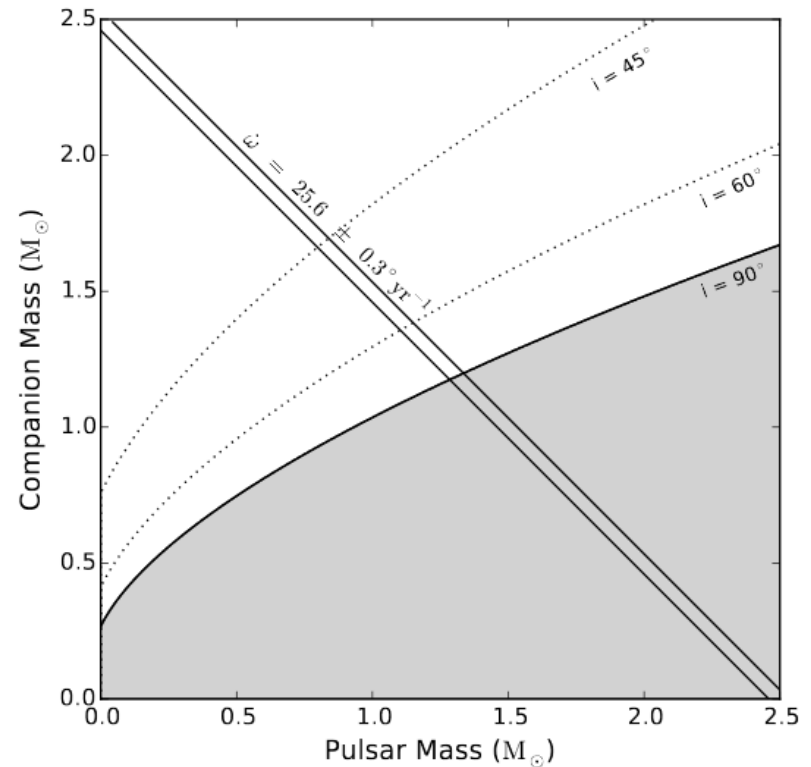
# Recent discoveries with records

PSR J1757–1854  
4.4 hours



$$\dot{\omega} \simeq 10.37^{\circ} \text{ yr}^{-1},$$

PSR J1946+2052  
1.88 hours



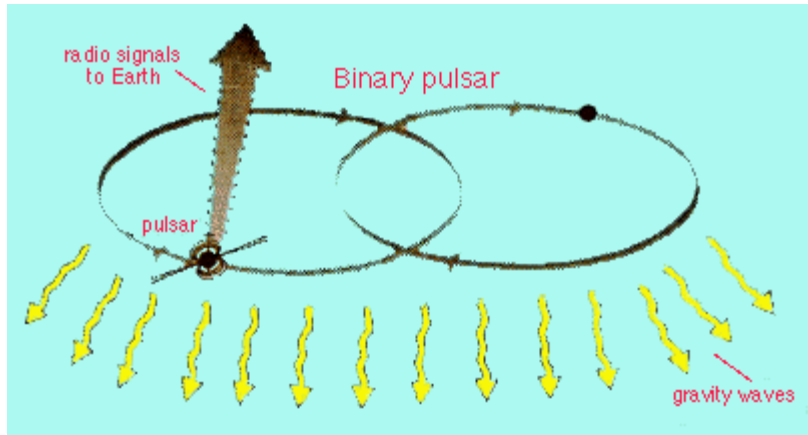
$$\dot{\omega} = 25.6 \pm 0.3 \text{ deg yr}^{-1},$$

1711.07697

1802.01707



# Binary pulsars



$$\frac{d\Delta_{E\odot}}{dt} = \sum_i \frac{Gm_i}{c^2 r_i} + \frac{v_{\oplus}^2}{2c^2} - \text{constant} .$$

$$\Delta_{S\odot} = -\frac{2GM_{\odot}}{c^3} \log (1 + \cos \theta) ,$$

$$\begin{aligned} T = & t_{\text{obs}} - t_0 + \Delta_C - D/f^2 + \Delta_{R\odot}(\alpha, \delta, \mu_{\alpha}, \mu_{\delta}, \pi) \\ & + \Delta_{E\odot} - \Delta_{S\odot}(\alpha, \delta) \\ & - \Delta_R(x, e, P_b, T_0, \omega, \dot{\omega}, \dot{P}_b) - \Delta_E(\gamma) - \Delta_S(r, s) \end{aligned}$$

See 1502.05474 for a recent detailed review

# Relativistic corrections and measurable parameters

$$\begin{aligned}\dot{\omega} &= 3 \left[ \frac{P_b}{2\pi} \right]^{-5/3} (T_{\odot} M)^{2/3} (1-e^2)^{-1} , \\ \gamma &= e \left[ \frac{P_b}{2\pi} \right]^{1/3} T_{\odot}^{2/3} M^{-4/3} m_2 (m_1 + 2m_2) , \\ \dot{P}_b &= -\frac{192\pi}{5} \left[ \frac{P_b}{2\pi} \right]^{-5/3} \left[ 1 + \frac{73}{24}e^2 + \frac{37}{96}e^4 \right] \\ &\quad \times (1-e^2)^{-7/2} T_{\odot}^{5/3} m_1 m_2 M^{-1/3} ,\end{aligned}$$

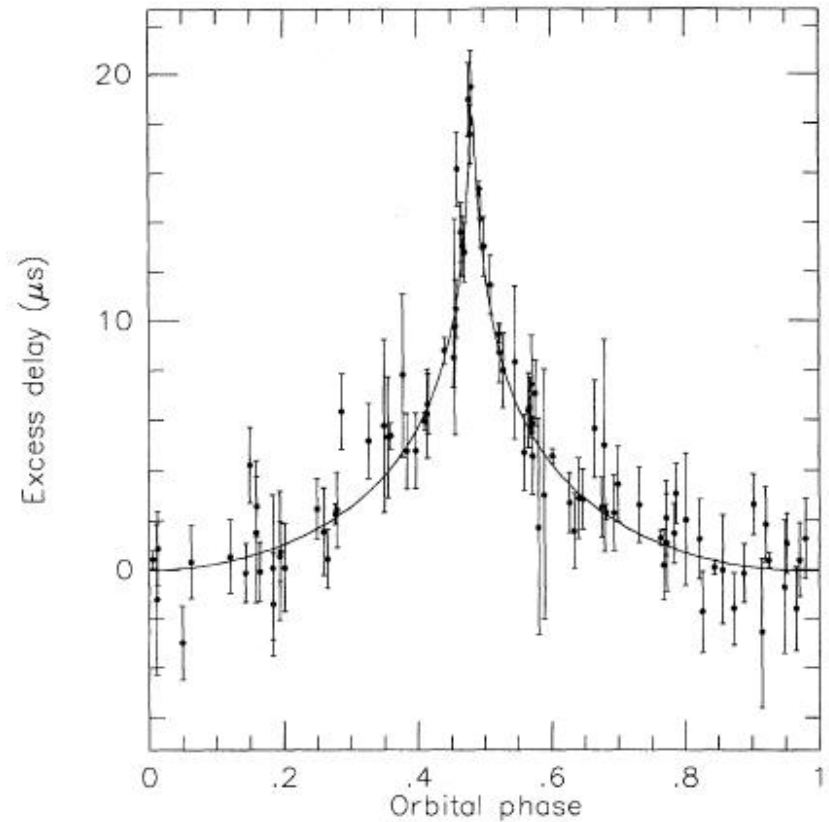
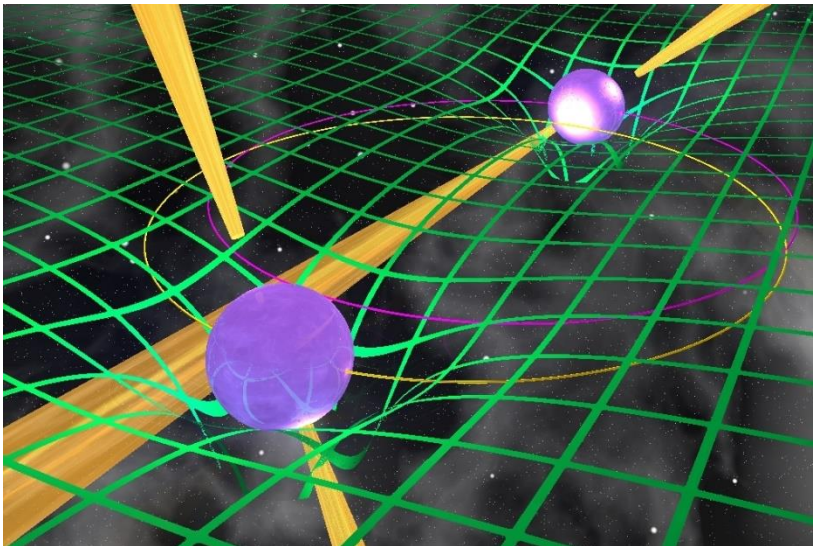
$$r = T_{\odot} m_2 ,$$

$$s = x \left[ \frac{P_b}{2\pi} \right]^{-2/3} T_{\odot}^{-1/3} M^{2/3} m_2^{-1} .$$

For details see  
Taylor, Weisberg 1989  
ApJ 345, 434

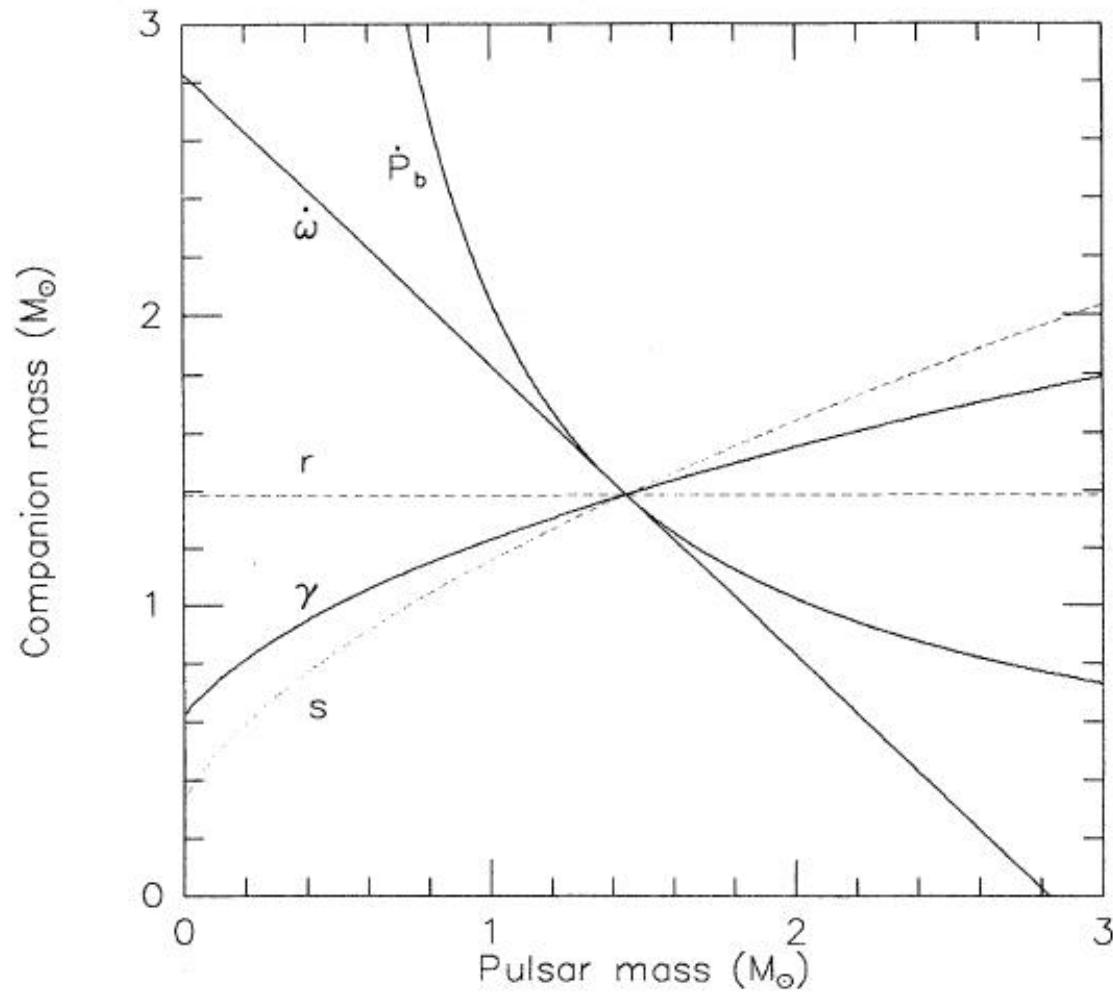
# Shapiro delay

$$\Delta_S = -2r \log(1 - s \cos[2\pi(\phi - \phi_0)])$$



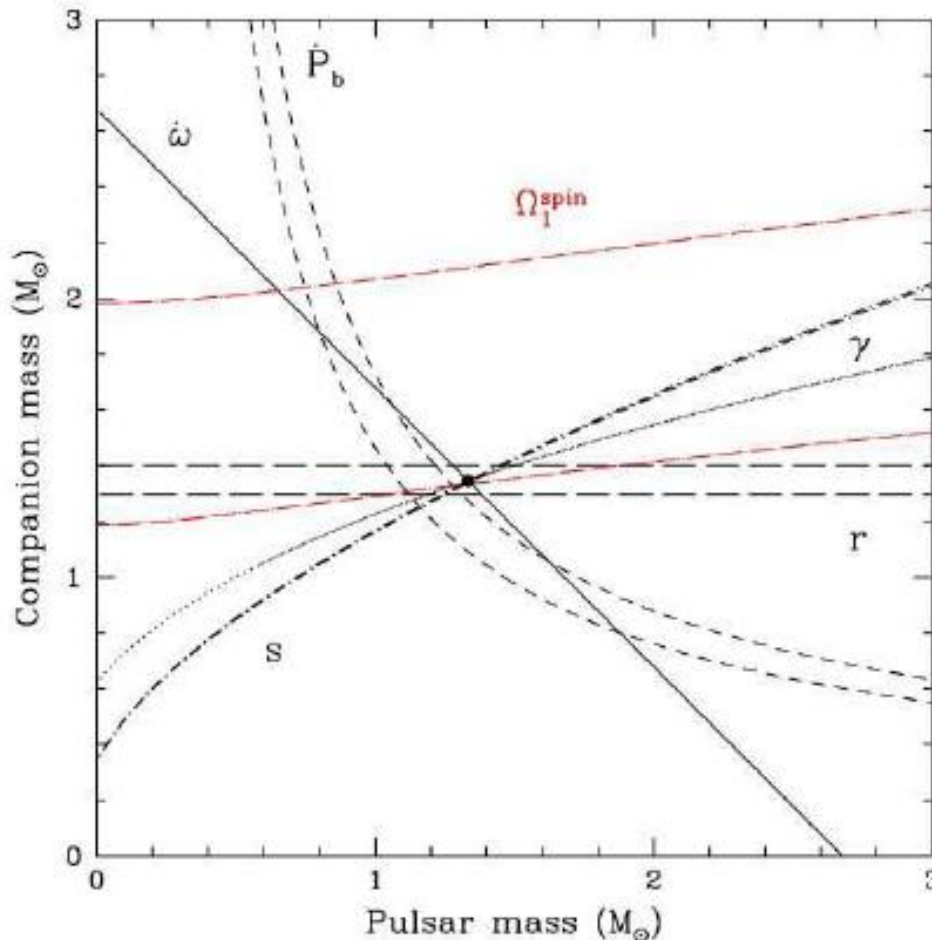
PSR 1855+09 (Taylor, Nobel lecture)

# Mass measurements



PSR 1913+16

# Uncertainties and inverse problems

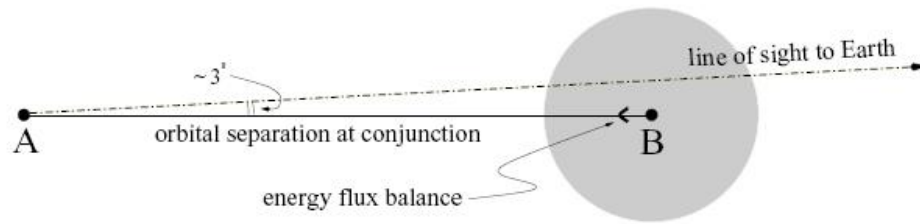
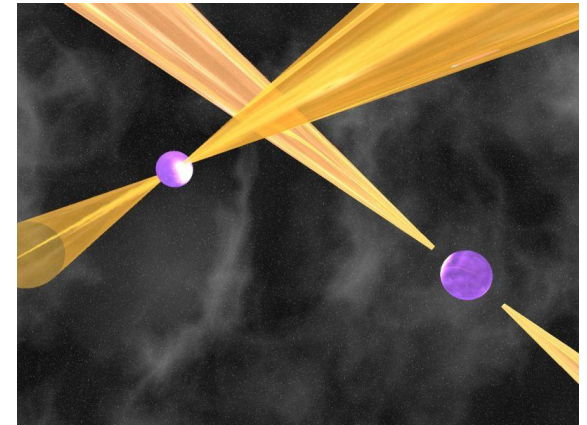
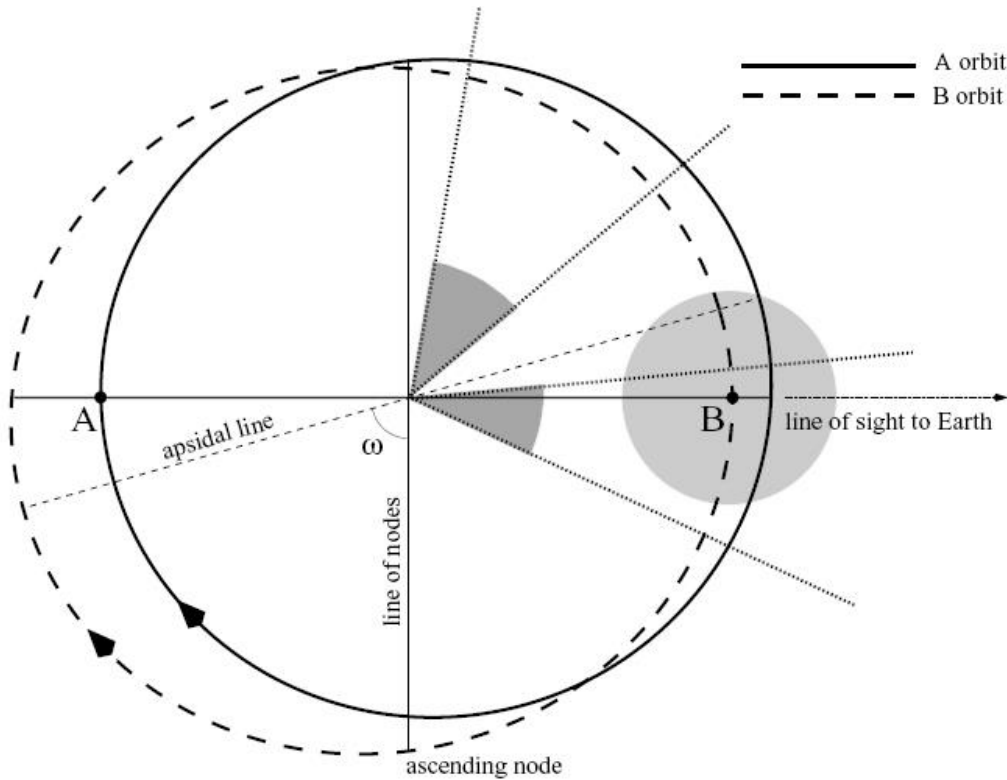


$\dot{P}_b$  depends on the Shklovskii effect. So, if distance is not certain, it is difficult to have a good measurement of this parameter.

It is possible to invert the problem. Assuming that GR is correct, one can improve the distance estimate for the given source.

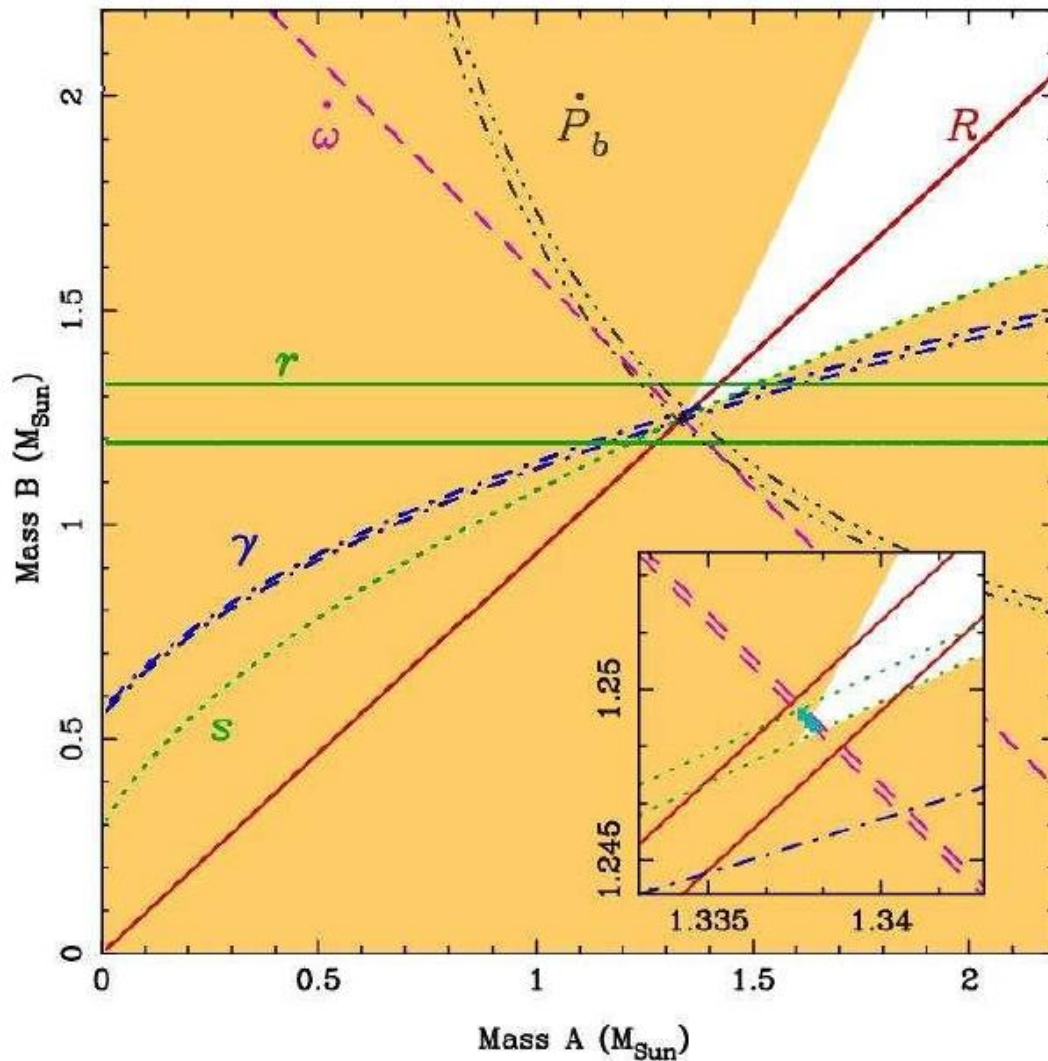
PSR B1534+12.

# Double pulsar J0737-3039





# Masses for PSR J0737-3039

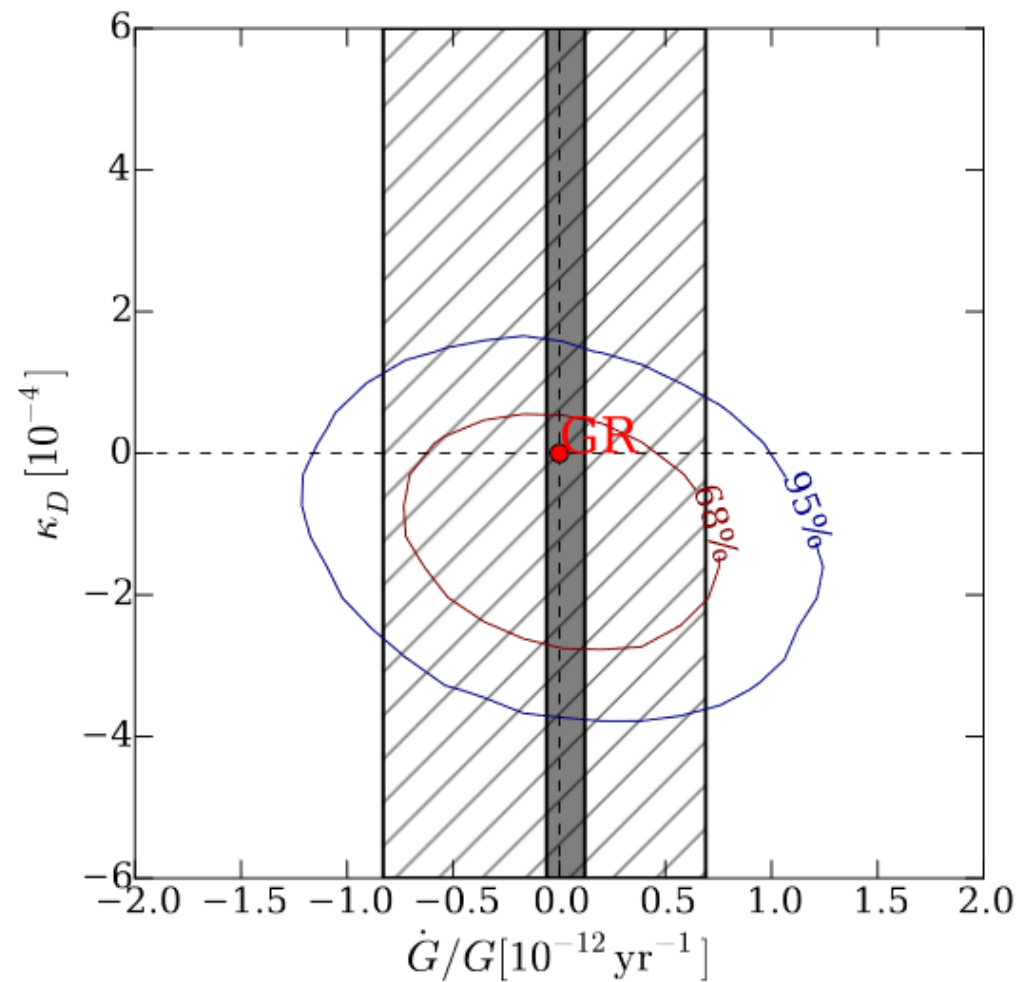
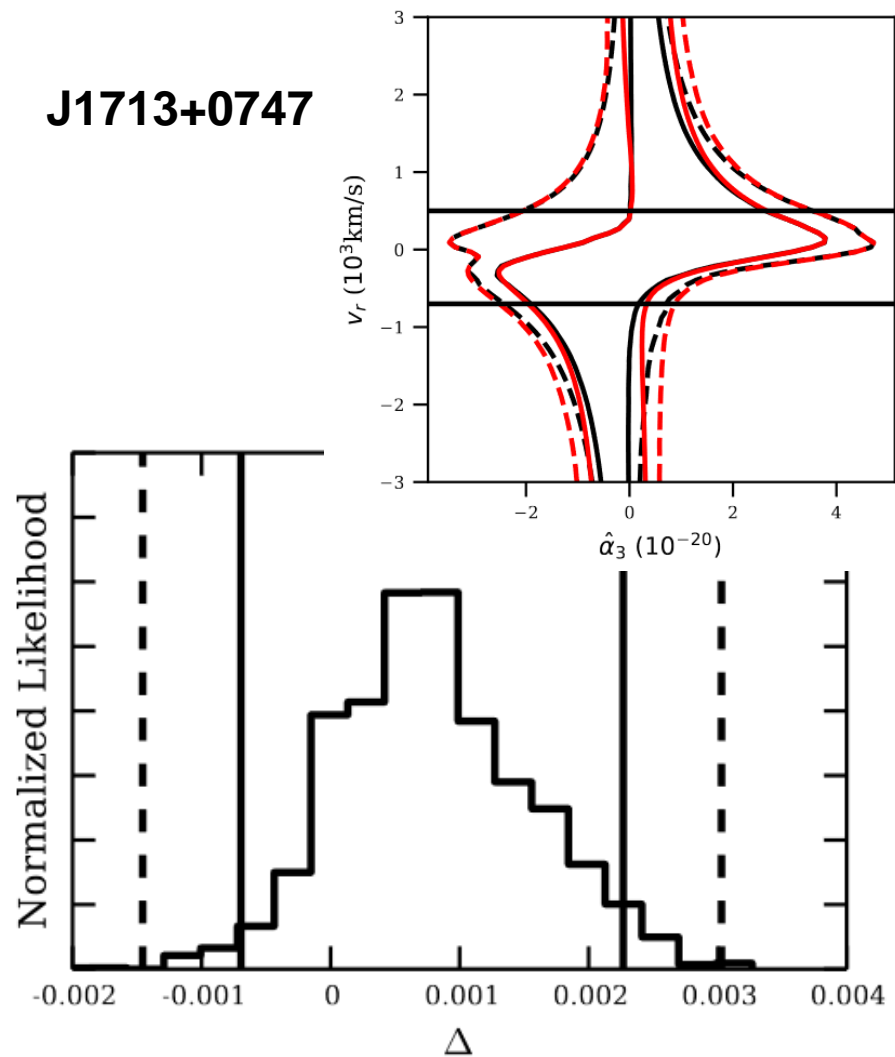


The most precise values.

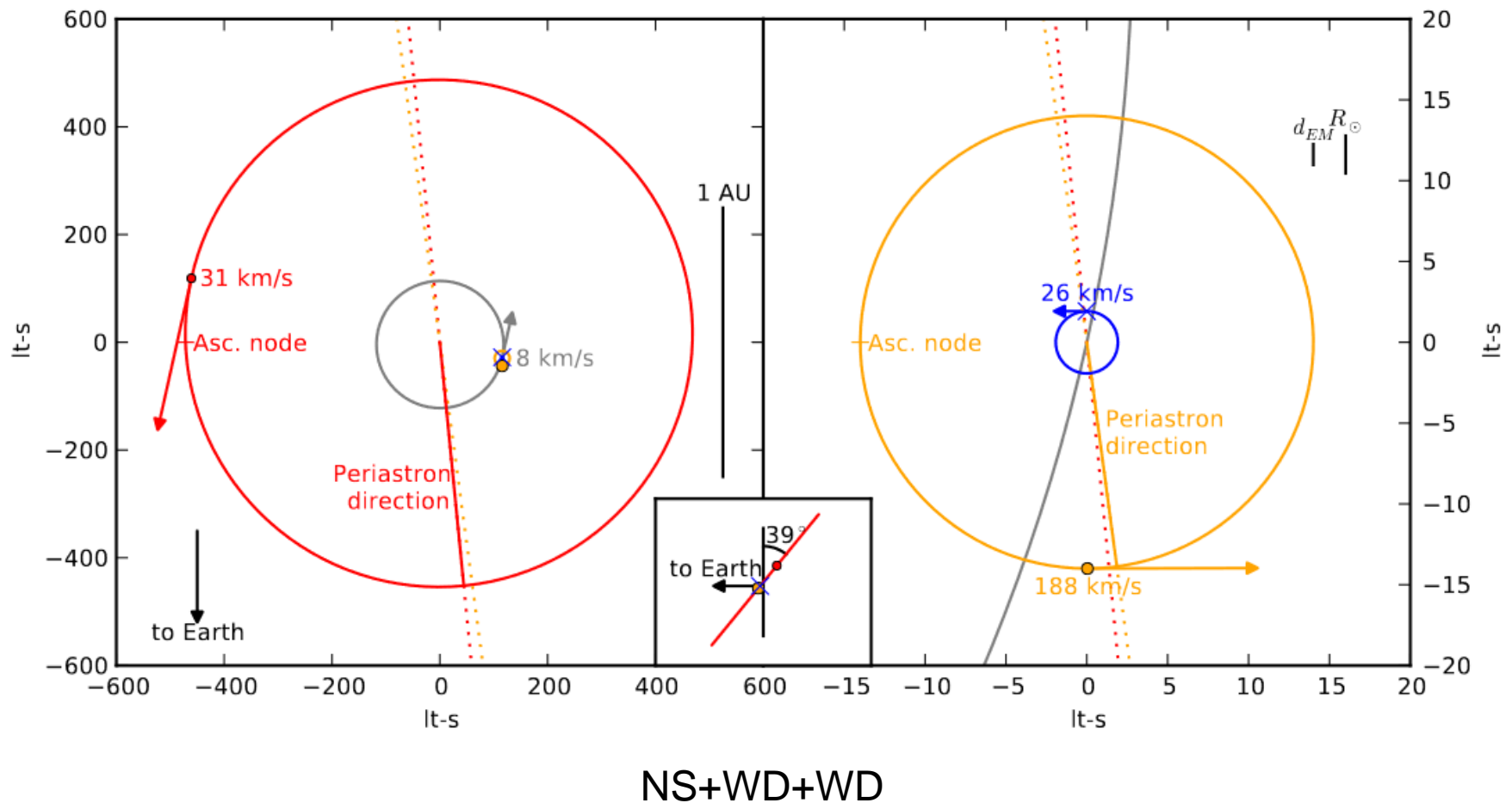
New mass estimates  
have uncertainties  $<0.001$

# Tests of theories of gravity

**J1713+0747**



# Testing strong equivalence principle with triple pulsar PSR J0337+1715



# NS+WD binaries

## Some examples

PSR J0437-4715. WD companion [[0801.2589](#), [0808.1594](#) ].  
The closest millisecond PSR.  $M_{\text{NS}}=1.76\pm0.2$  solar.

The case of PSR J0751+1807.

Initially, it was announced that it has a mass  $\sim 2.1$  solar [astro-ph/0508050].

However, then in 2007 at a conference the authors announced that the result was incorrect. Actually, the initial value was  $2.1\pm0.2$  (1 sigma error).

New result:  $1.26 \pm 0.14$  solar

[Nice et al. 2008, Proc. of the conf. “40 Years of pulsars”]

It is expected that most massive NSs get their additional “kilos” due to accretion from WD companions [[astro-ph/0412327](#) ].

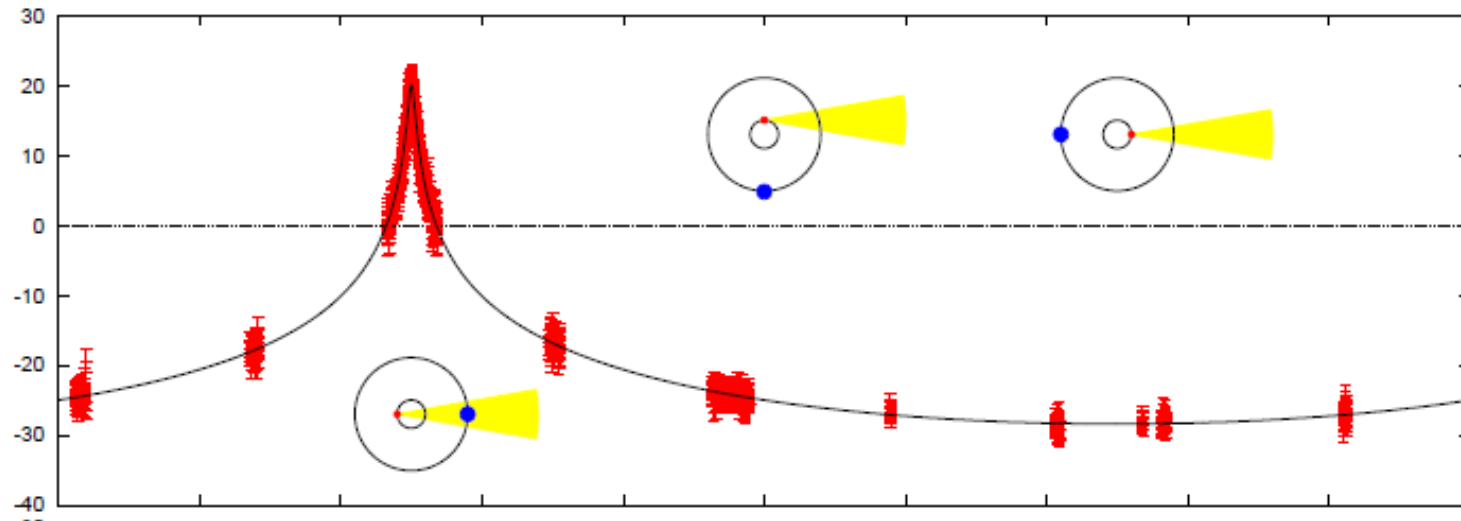
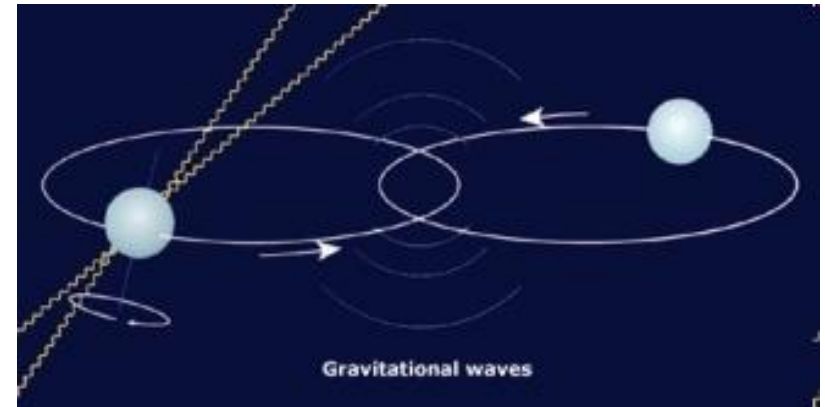
# Very massive neutron star

Binary system: pulsar + white dwarf  
PSR 1614-2230

Mass  $\sim 2$  solar

About the WD see 1106.5497.

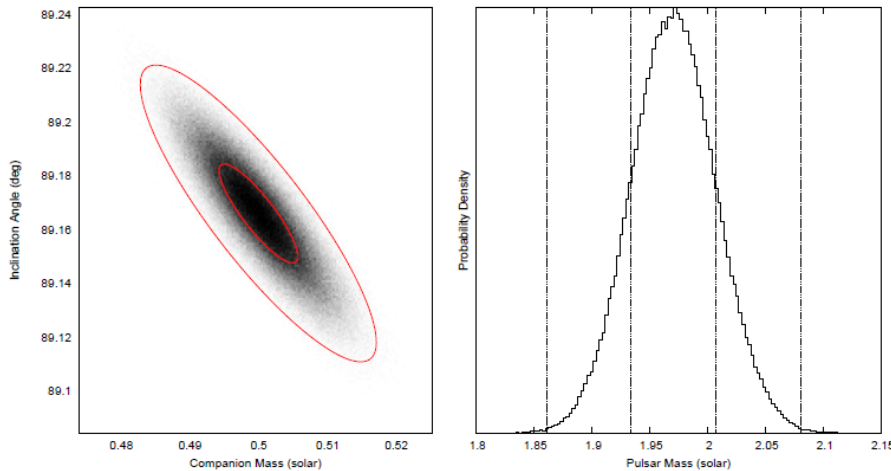
The object was identified in optics.



arXiv: 1010.5788

About formation of this objects see 1103.4996

# Why is it so important?



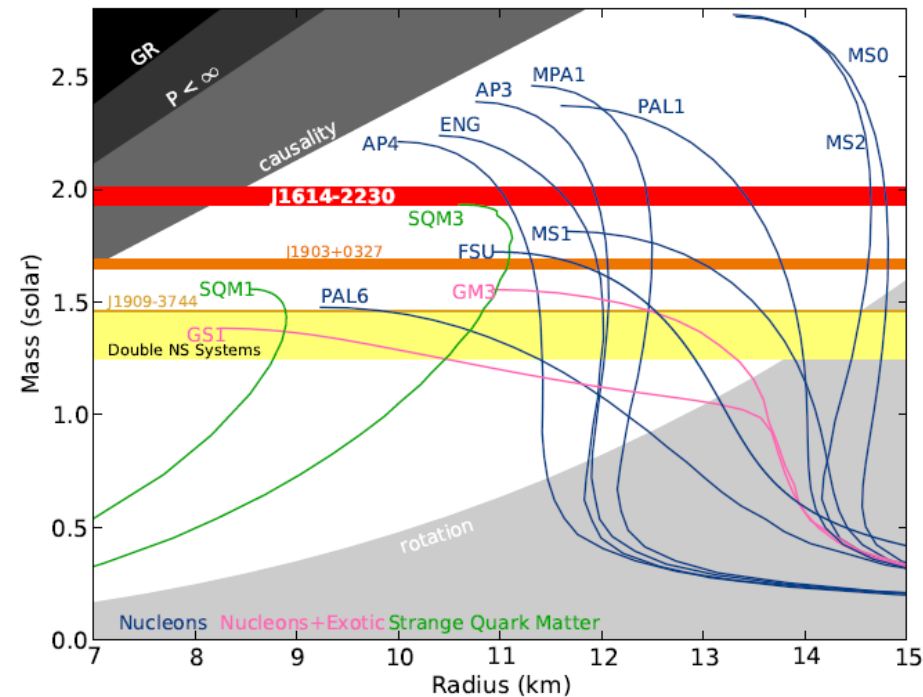
Collapse happens earlier for softer EoSs, see however, 1111.6929 about quark and hybrid stars to explain these data.

Interestingly, it was suggested that just  $<0.1$  solar masses was accreted (1210.8331)

arXiv: 1010.5788

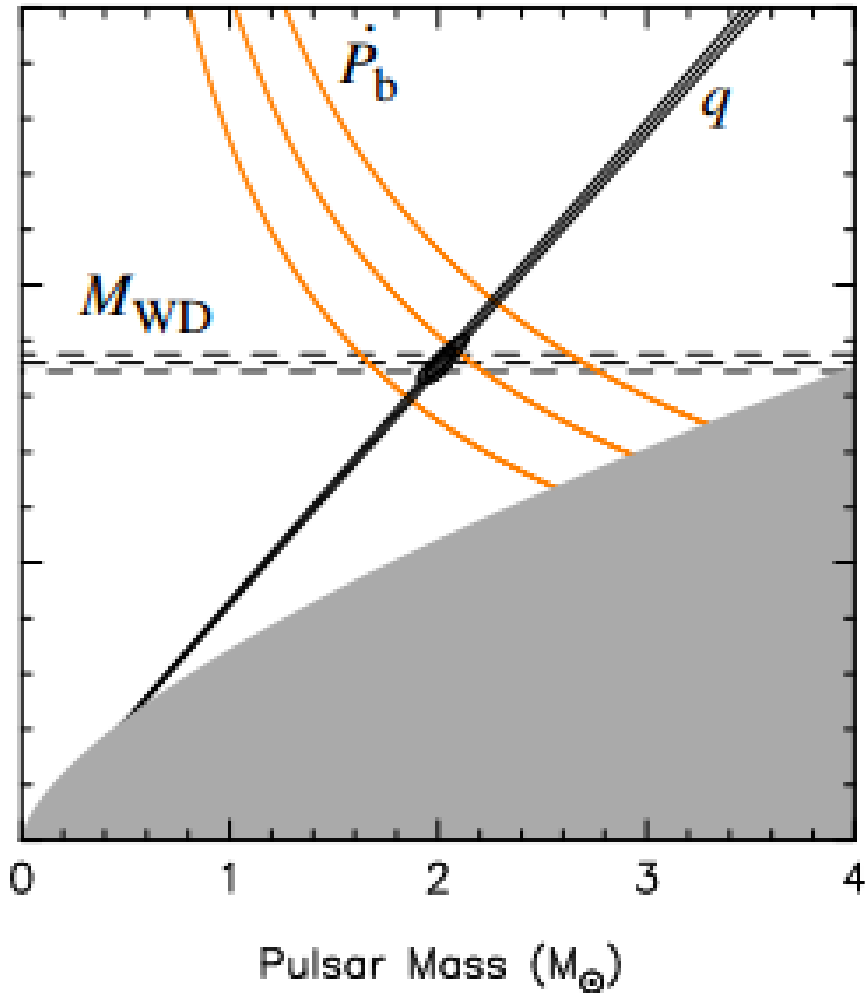
In the future specific X-ray sources (eclipsing msec PSR like SWIFT J1749.4–2807) can show Shapiro delay and help to obtain masses for a different kind of systems, see 1005.3527 , 1005.3479 .

The maximum mass is a crucial property of a given EoS





## 2.01 solar masses NS

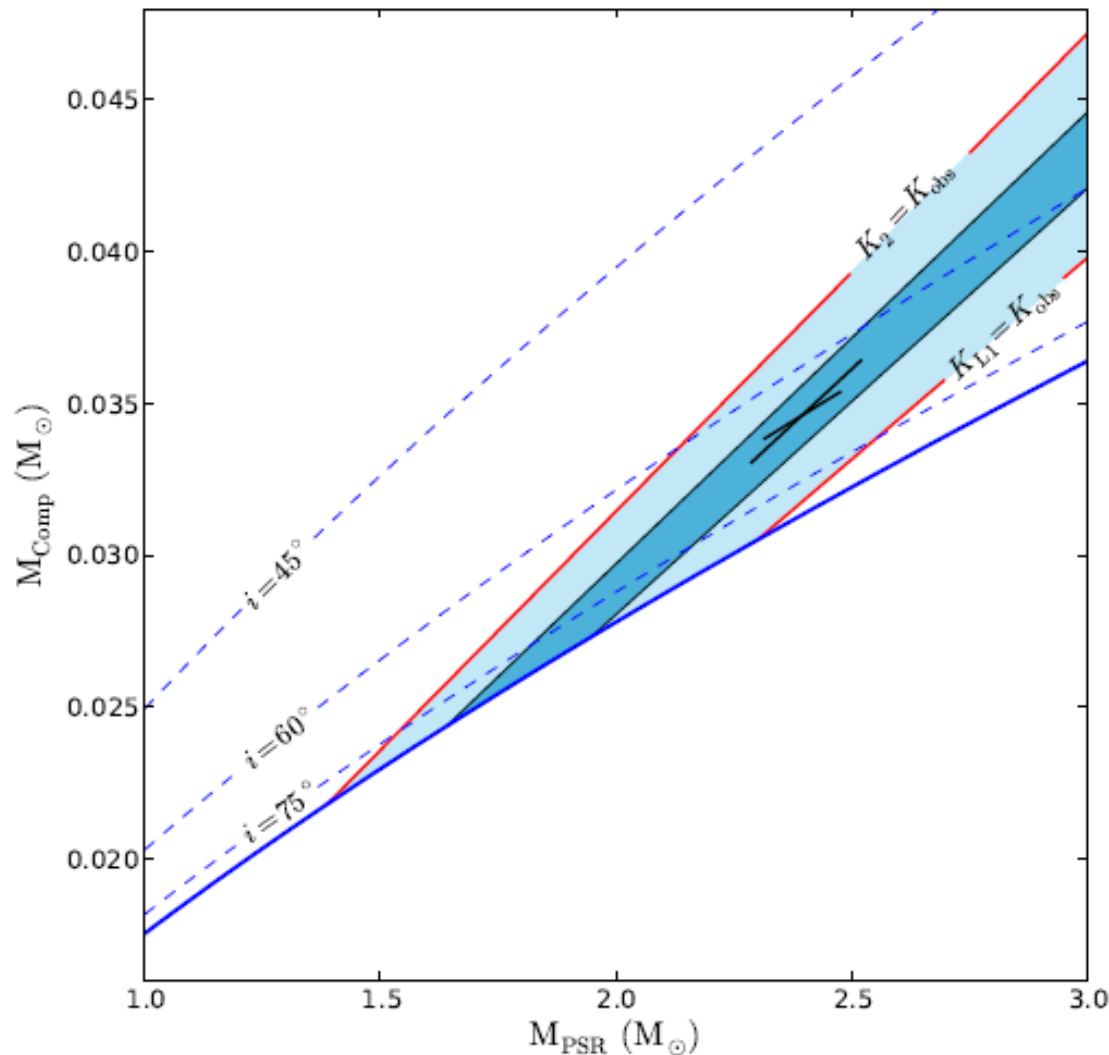


PSR J0348+0432  
39 ms, 2.46 h orbit  
WD companion

The NS mass is estimated to be:  
1.97 – 2.05 solar mass at 68.27%  
1.90 – 2.18 solar mass at 99.73%  
confidence level.

System is perfect for probing  
theories of gravity as it is very compact.

# The most extreme (but unclear) example



BLACK WIDOW PULSAR  
PSR B1957+20

2.4 $\pm$ 0.12 solar masses

# More measurements

PSR J1738+0333 NS+WD

arXiv: 1204.3948

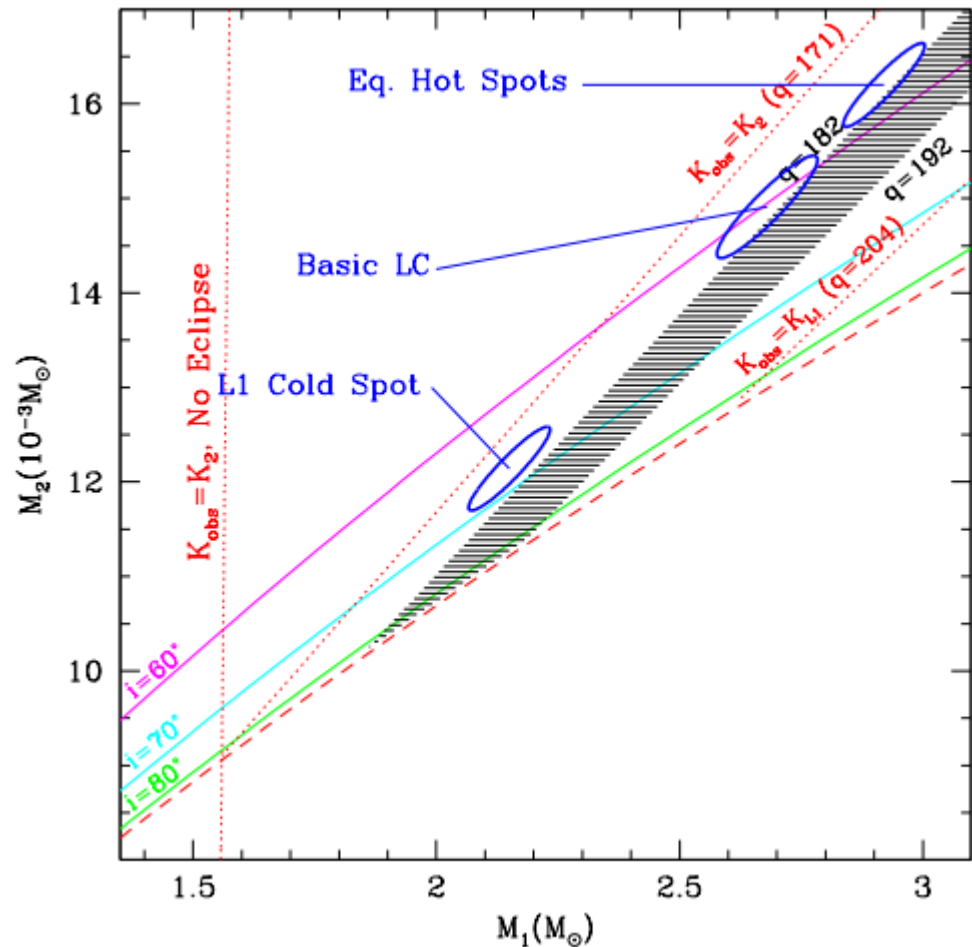
$$M_{\text{WD}} = 0.181 \pm 0.007 \pm 0.005 M_{\odot}$$

$$M_{\text{PSR}} = 1.47 \pm 0.07 \pm 0.06 M_{\odot}$$

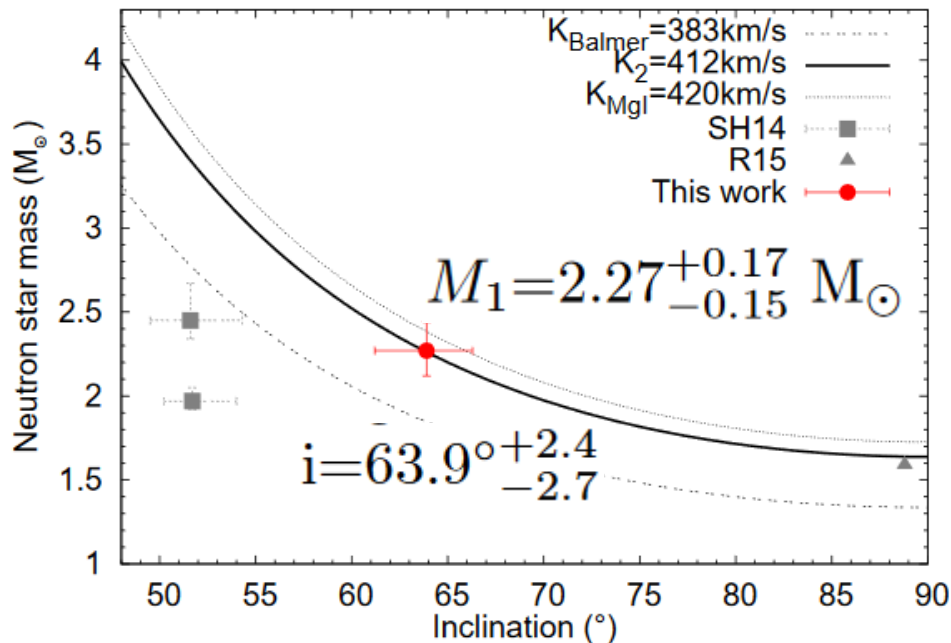
PSR J1311-3430

arXiv: 1210.6884

MPSR > 2.1 at least!



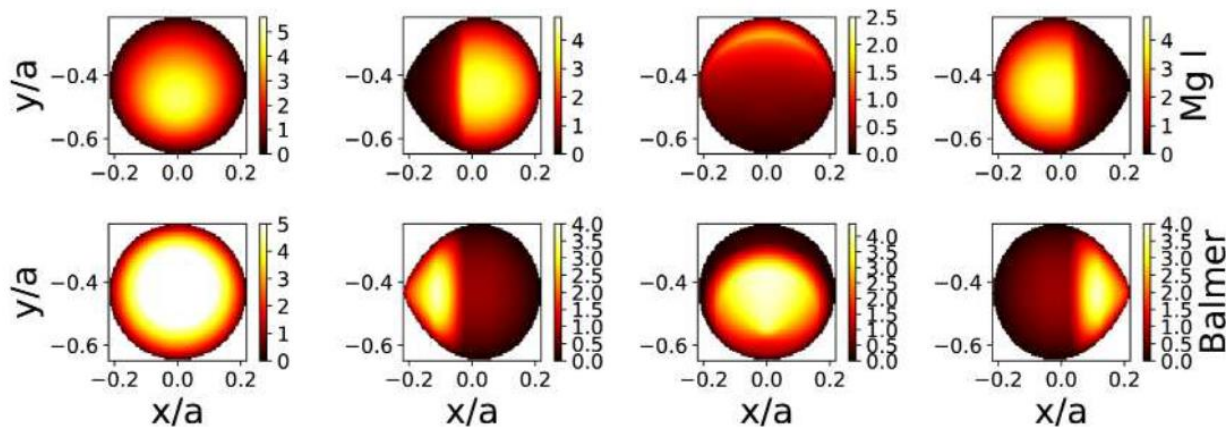
# A massive NS in PSR J2215+5135



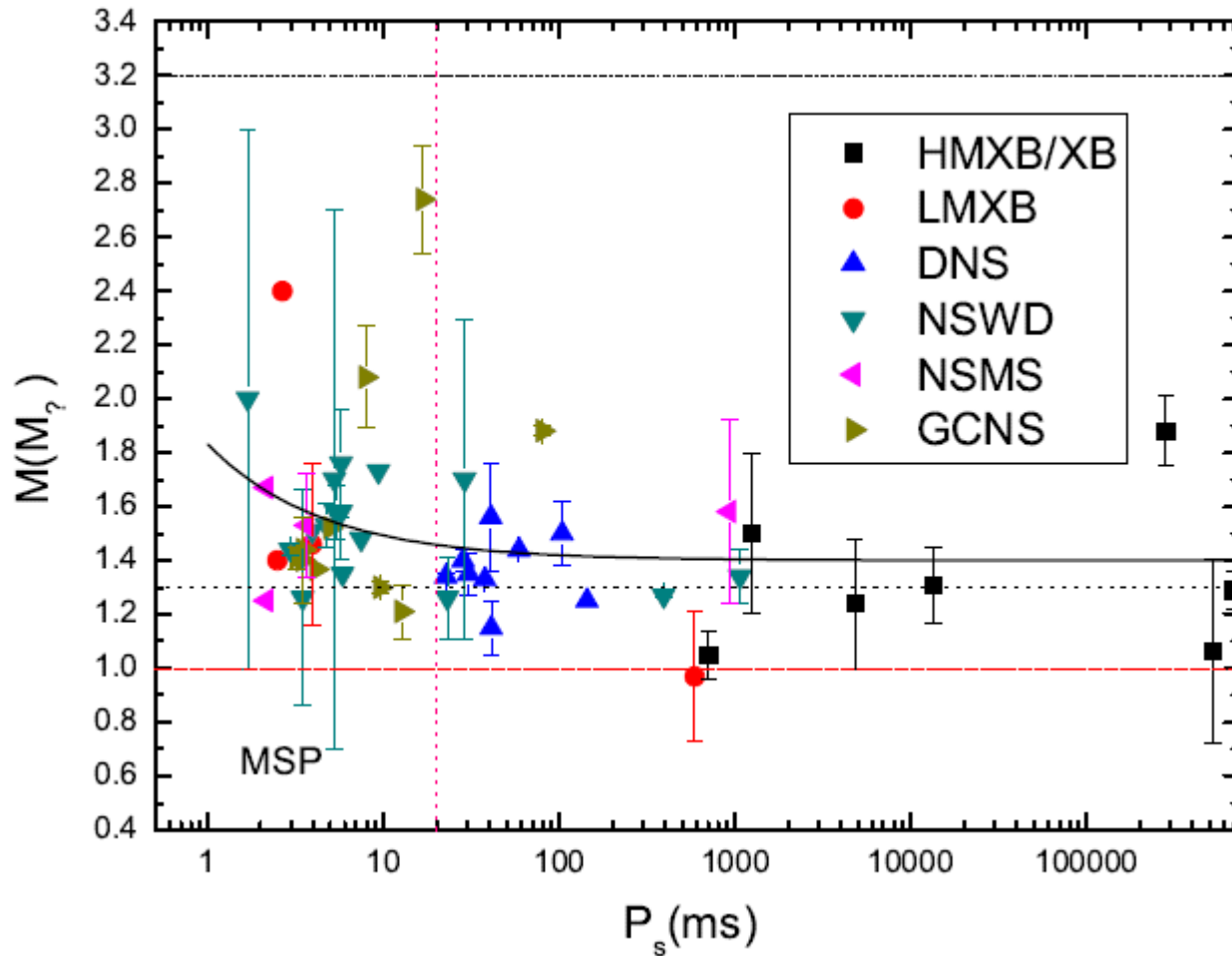
Different lines provide different velocity as they are emitted from different sides of the companion.

Different sides of the companion move with different velocity.

Thus, a correct model provides new mass determination.



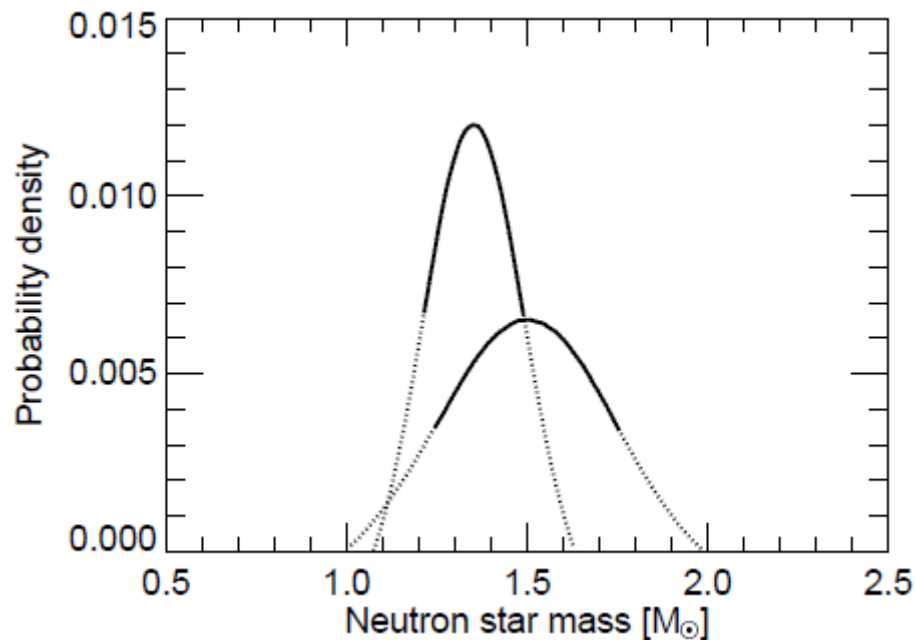
# How much do PSRs accrete?



$$M = 1.4 + 0.43(P/\text{ms})^{-2/3}$$

Millisecond pulsars are  
~0.2 solar masses more  
massive than the rest ones.

# DNS and NS+WD binaries



1.35 $\pm$ 0.13 and 1.5 $\pm$ 0.25

Cut-off at  $\sim 2.1$  solar masses  
can be mainly due to evolution  
in a binary, not due to nuclear  
physics (see 1309.6635)



# Neutron stars in binaries

Study of close binary systems gives an opportunity to obtain mass estimate for progenitors of NSs (see for example, Ergma, van den Heuvel 1998 A&A 331, L29).

For example, an interesting estimate was obtained for GX 301-2.

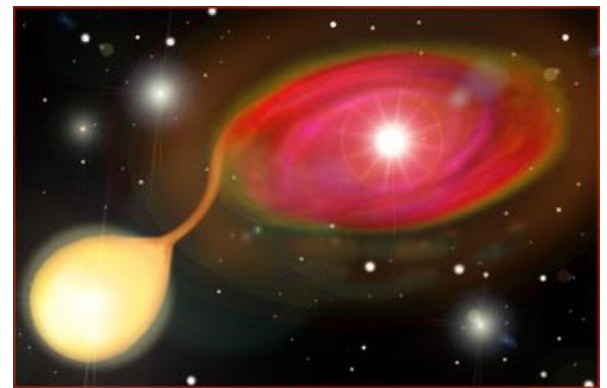
The progenitor mass is  $>50$  solar masses.

On the other hand, for several other systems with both NSs and BHs progenitor masses are smaller: from 20 up to 50.

Finally, for the BH binary LMC X-3 the progenitor mass is estimated as  $>60$  solar.

So, the situation is tricky.

Most probably, in some range of masses, at least in binary systems, stars can produce both types of compact objects: NSs and BHs.



# Mass determination in binaries: mass function

$$f_v(m) \frac{m_x^3 \sin^3 i}{(m_x + m_v)^2} = 1,038 \cdot 10^{-7} K_v^3 P (1 - e^2)^{3/2},$$

$m_x, m_v$  - masses of a compact object and of a normal star (in solar units),  
 $K_v$  – observed semi-amplitude of line of sight velocity of the normal star (in km/s),  
 $P$  – orbital period (in days),  $e$  – orbital eccentricity,  $i$  – orbital inclination  
(the angle between the orbital plane and line of sight).

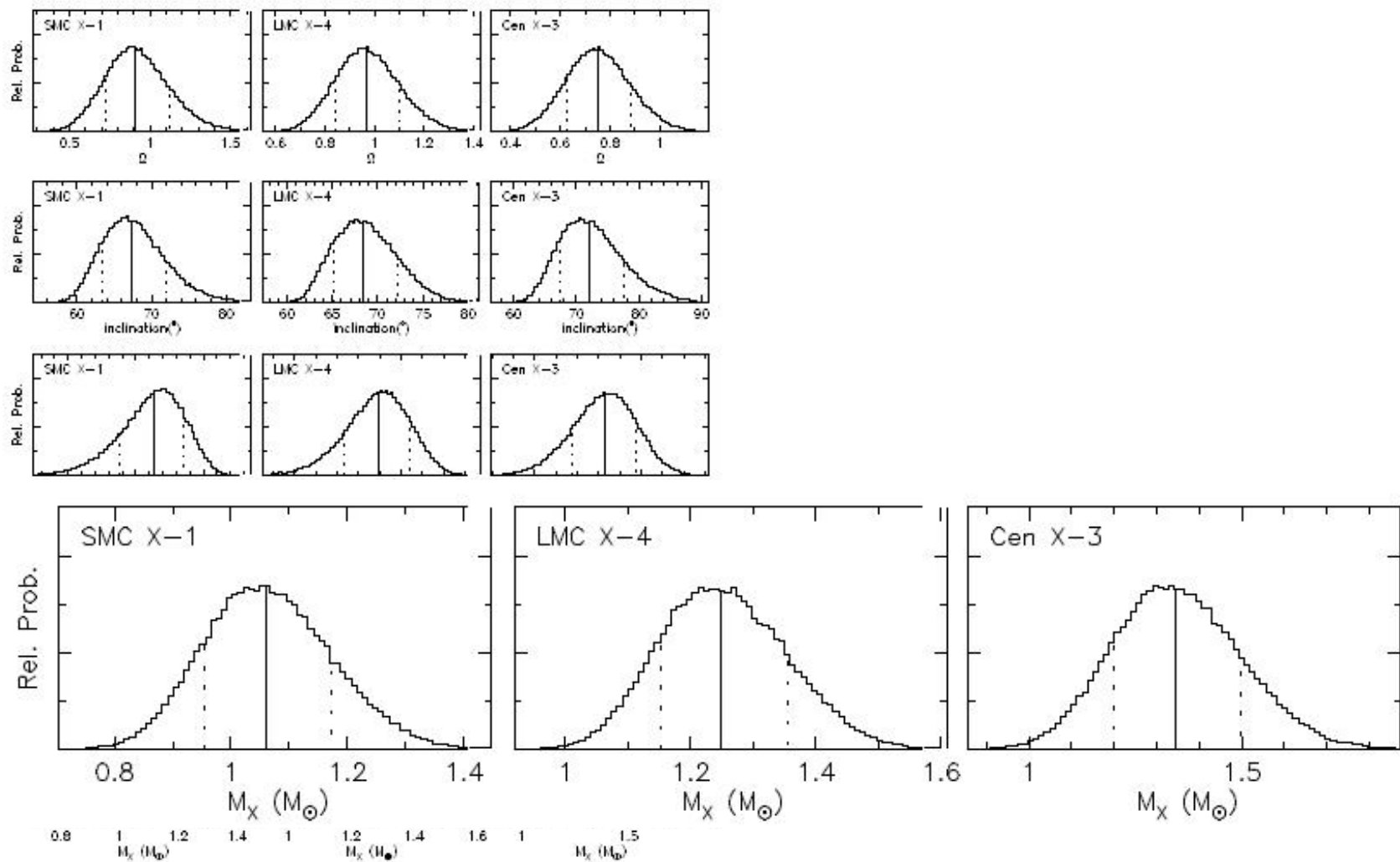
One can see that the mass function is the lower limit for the mass of a compact star.

The mass of a compact object can be calculated as:

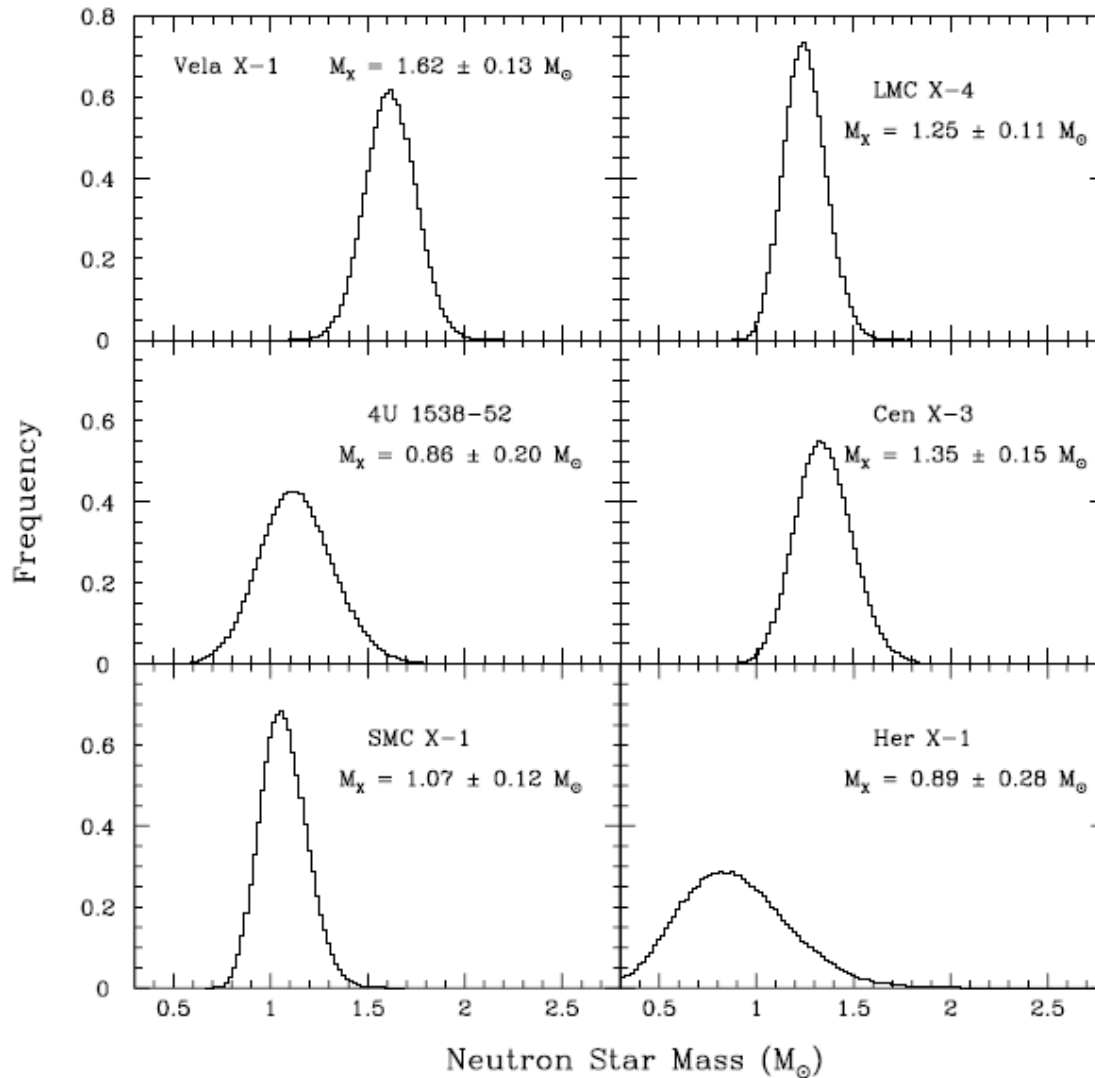
$$m_x = f_v(m) \left(1 + \frac{m_v}{m_x}\right)^2 \frac{1}{\sin^3 i}.$$

So, to derive the mass it is necessary to know (besides the line of sight velocity) independently two more parameters: mass ratio  $q = m_x/m_v$ , and orbital inclination  $i$ .

# Some mass estimates

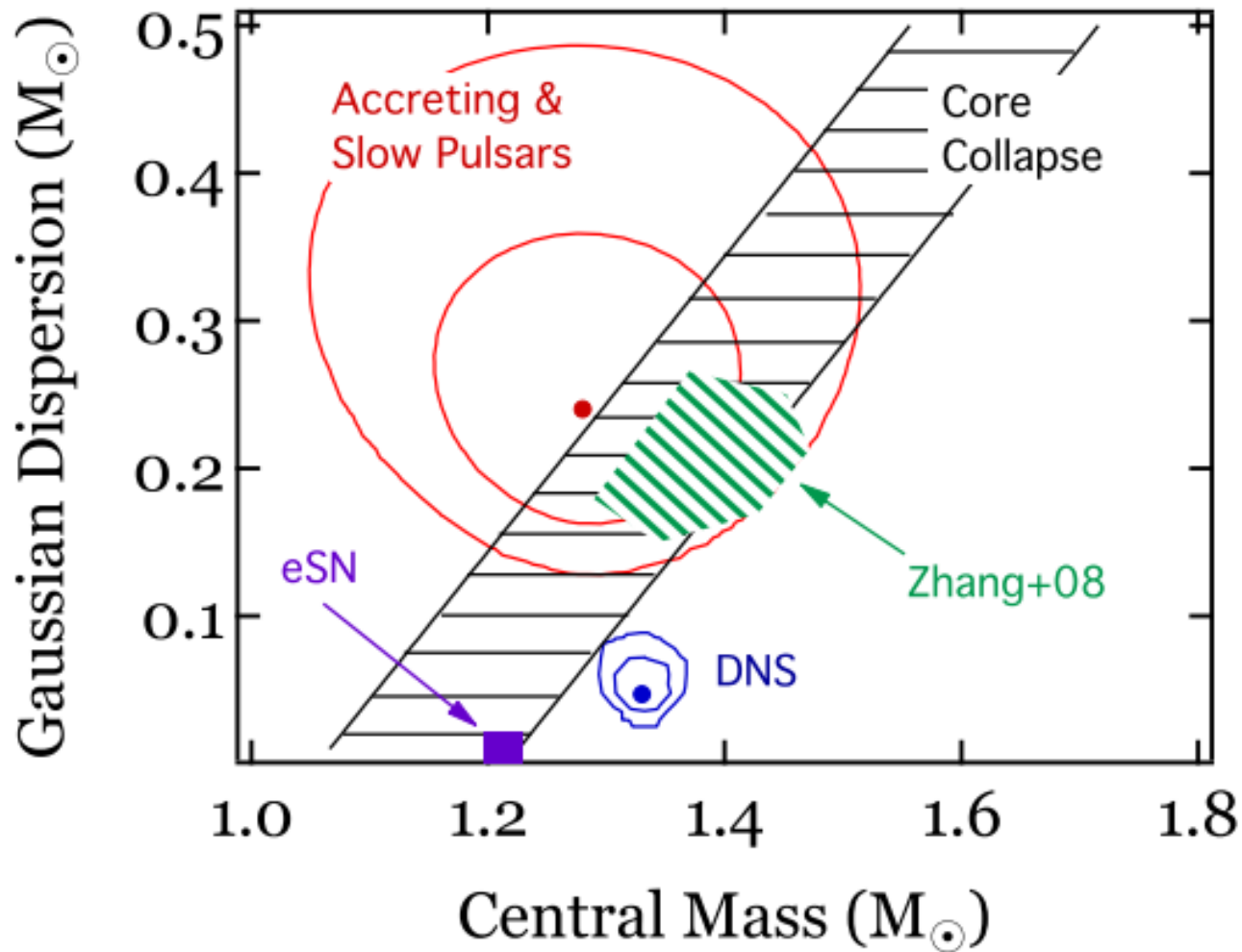


# More measurements



Six X-ray binary systems.  
All are eclipsing pulsars.

# Altogether

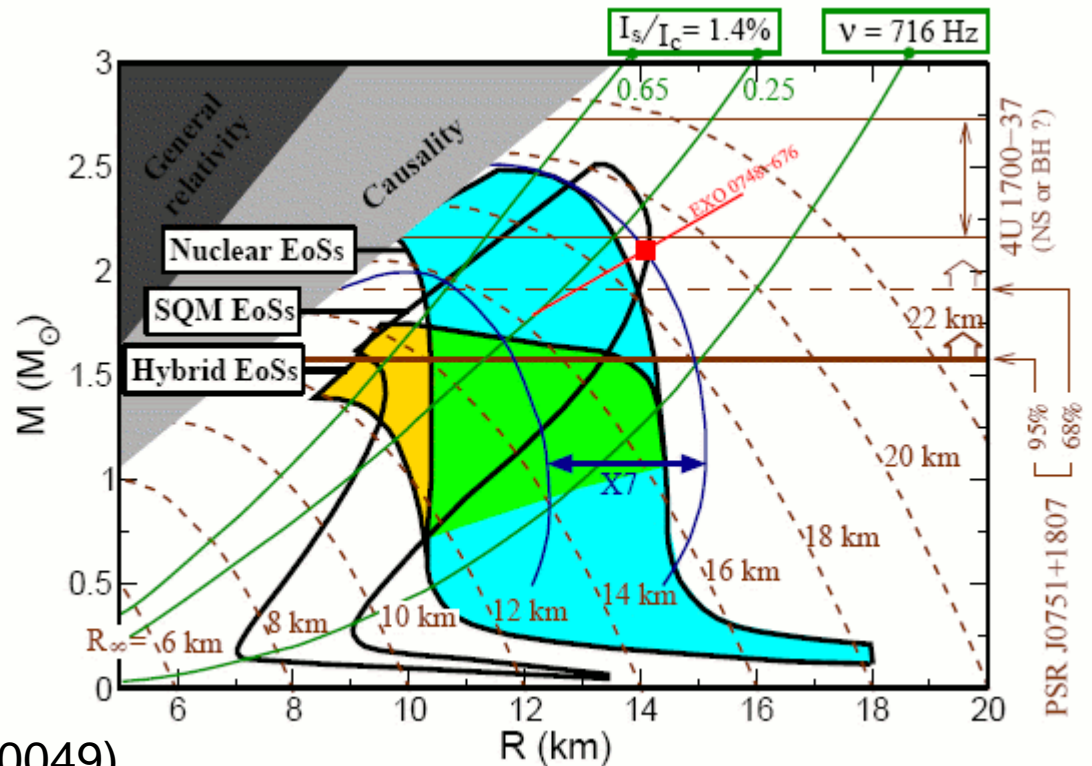


# Mass-radius diagram and constraints

Unfortunately, there are no good data on independent measurements of masses and radii of NSs.

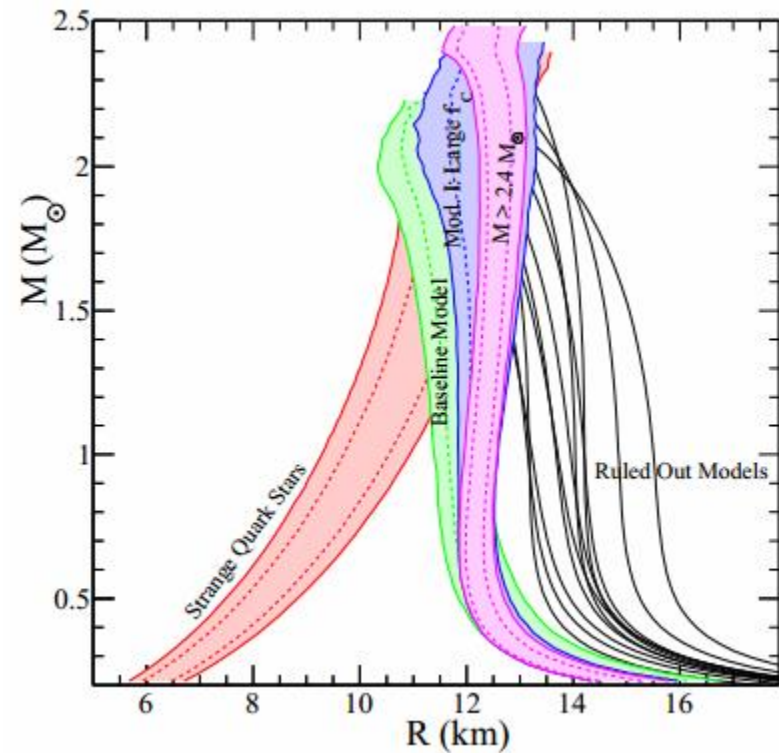
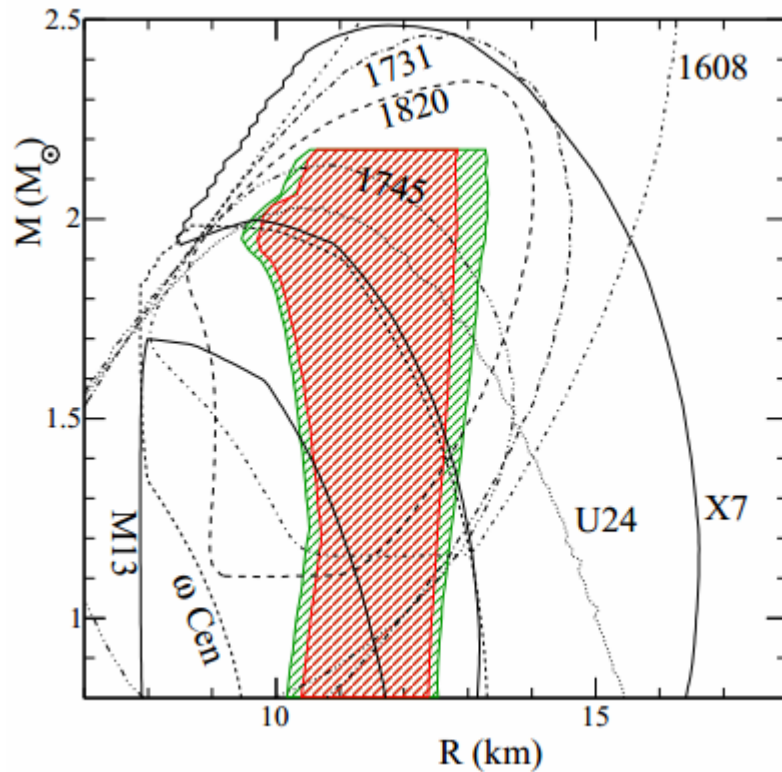
Still, it is possible to put important constraints. Most of recent observations favour stiff EoS.

Useful analytical estimates for EoS can be found in 1310.0049).





# Observations vs. data



1205.6871

Some newer results by the same group are presented in 1305.3242

# Mass and radius for a pulsar!

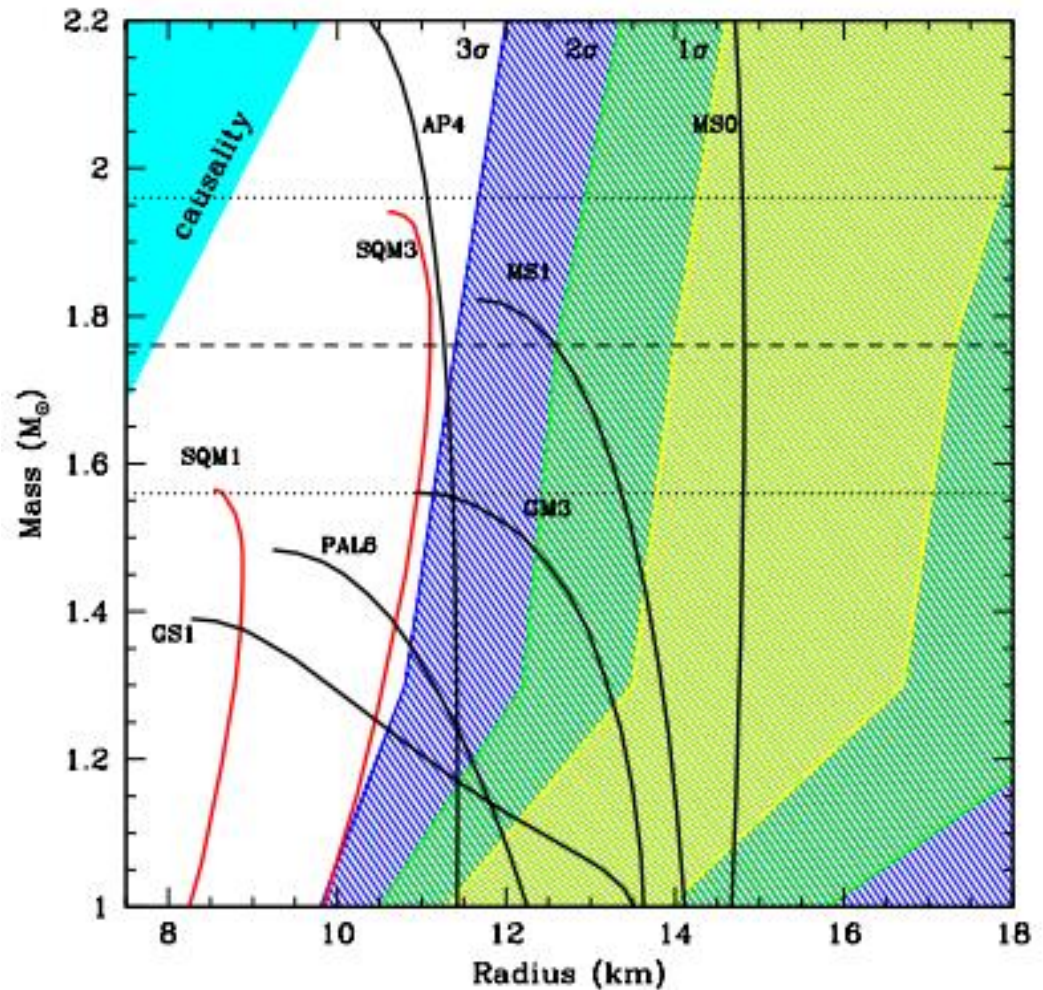
PSR J0437–4715 NS+WD

The nearest known mPSR  
155-158 pc

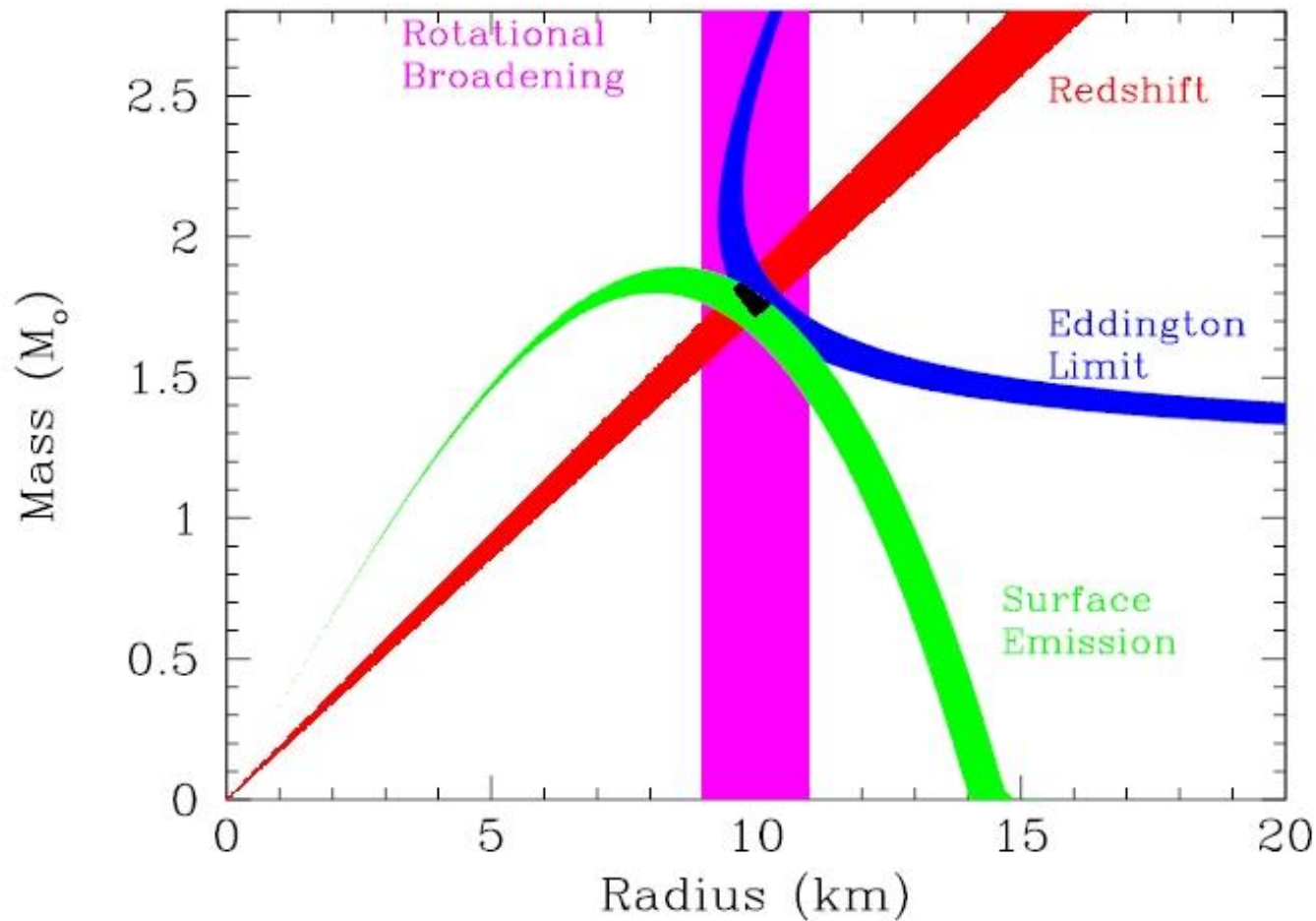
XMM-Newton observations  
showed thermal emission.

H-atmosphere model fits.

Hot caps are non-antipodal.

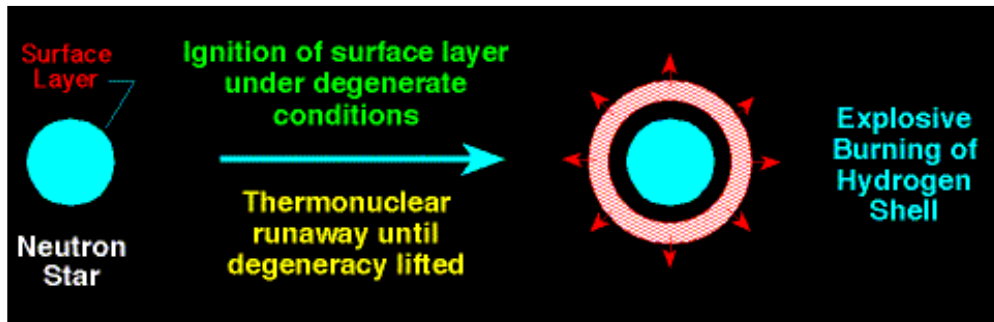


# Combination of different methods



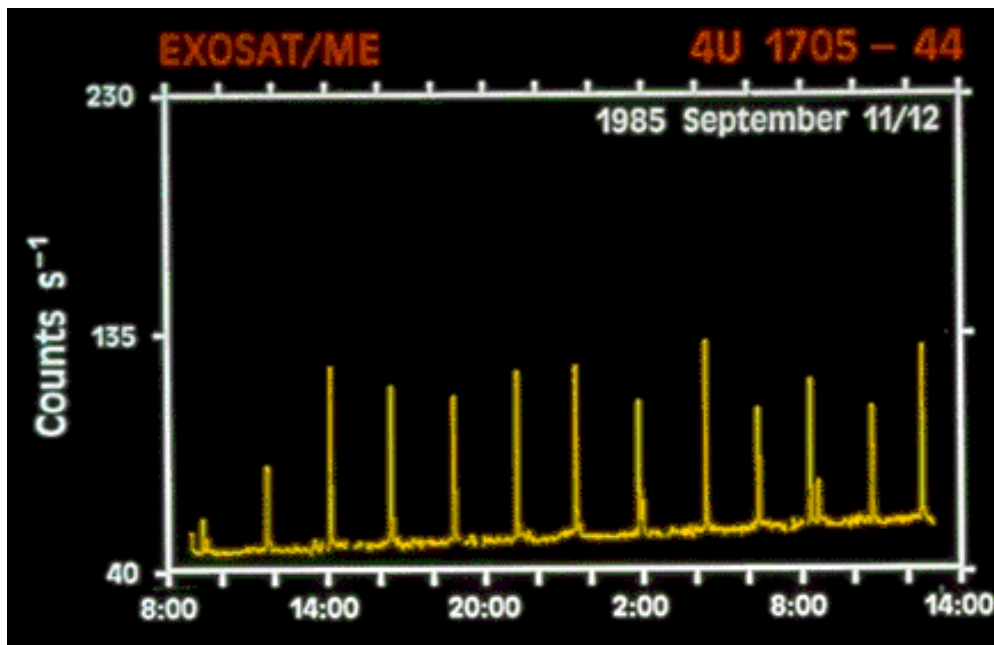
EXO 0748-676

# Radius determination in bursters



Explosion with a  $\sim$  Eddington luminosity.

Modeling of the burst spectrum and its evolution.



See, for example,  
Joss, Rappaport 1984,  
Haberl, Titarchuk 1995

---

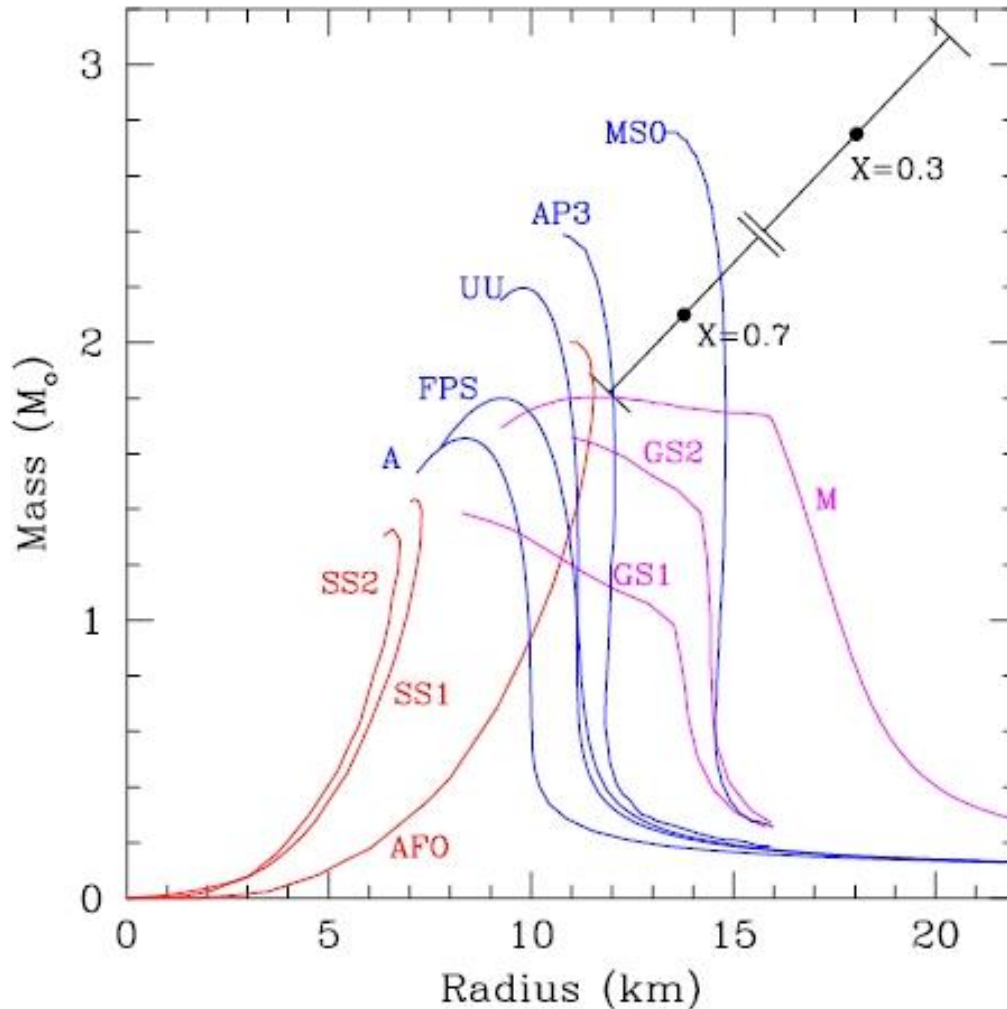
# More measurements

Continuously new measurements, critics and discussion appears

- 1104.2602 Systematic Uncertainties in the Spectroscopic Measurements of Neutron-Star Masses and Radii from Thermonuclear X-ray Bursts. II. Eddington Limit
  - 1104.5027 The Mass and Radius of the Neutron Star in the Bulge Low-Mass X-ray Binary KS 1731-260
  - 1103.5767 Systematic Uncertainties in the Spectroscopic Measurements of Neutron-Star Masses and Radii from Thermonuclear X-ray Bursts. I. Apparent Radii
  - 1105.1525 Mass and radius estimation for the neutron star in X-ray burster 4U 1820-30
  - 1105.2030 New Method for Determining the Mass and Radius of Neutron Stars
  - 1106.3131 Constraints on the Mass and Radius of the Neutron Star XTE J1807-294
  - 1111.0347 Constraints on neutron star mass and radius in GS 1826-24 from sub-Eddington X-ray bursts
  - 1201.1680 On the consistency of neutron-star radius measurements from thermonuclear bursts
  - 1204.3627 Constraints on the mass and radius of the accreting neutron star in the Rapid Burster
  - 1301.0831 The mass and the radius of the neutron star in the transient low mass X-ray binary SAX J1748.9-2021
-



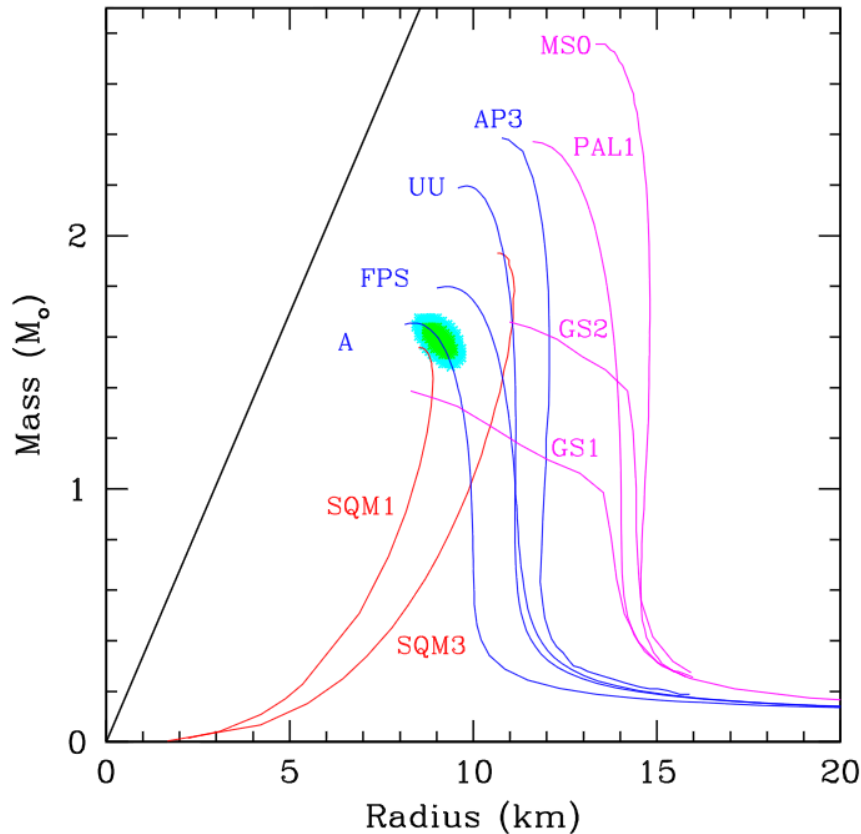
# Limits on the EoS from EXO 0748-676



Stiff EoS are better.  
Many EoS for strange matter are rejected.  
But not all! (see discussion in Nature).

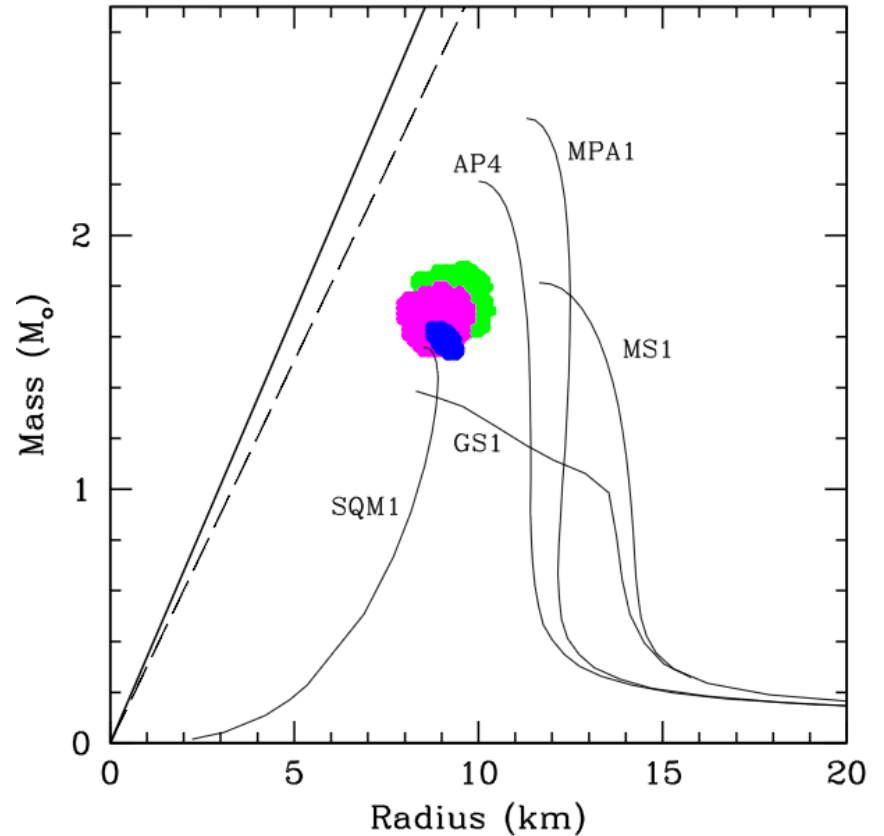
X- hydrogen fraction  
in the accreted material

# Some optimistic estimates



4U 1820-30

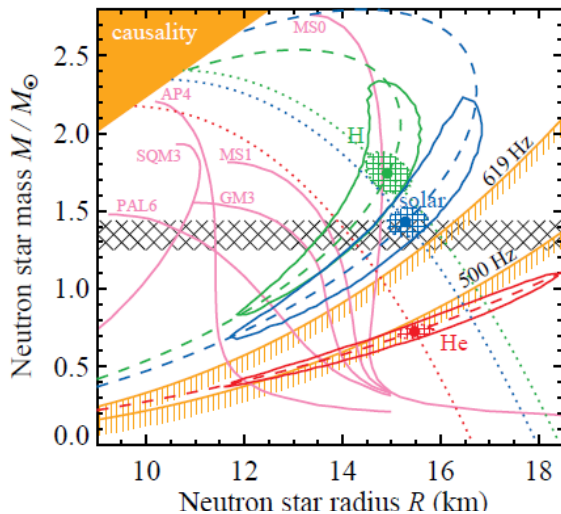
1002.3825



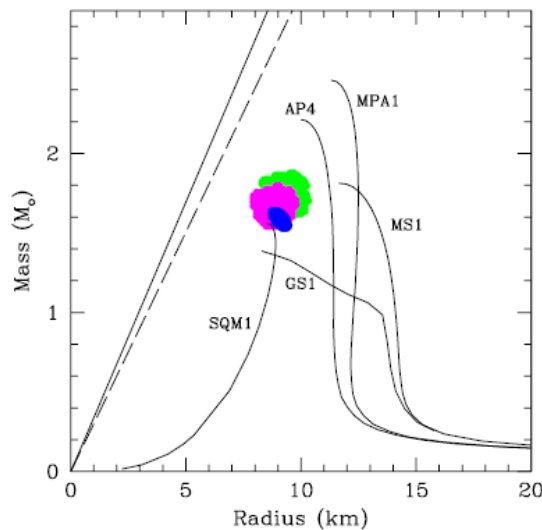
1002.3153



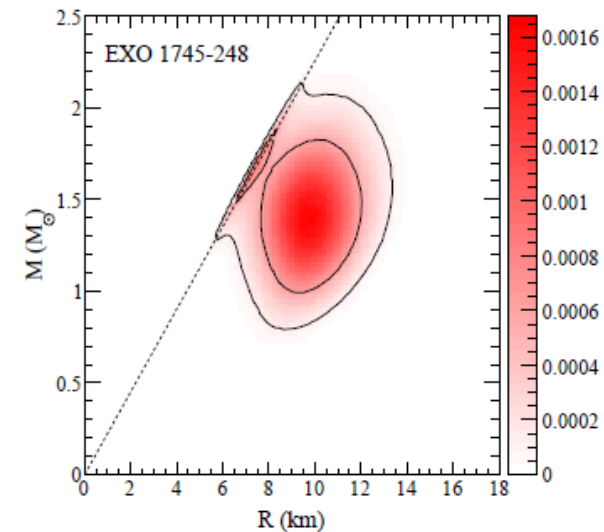
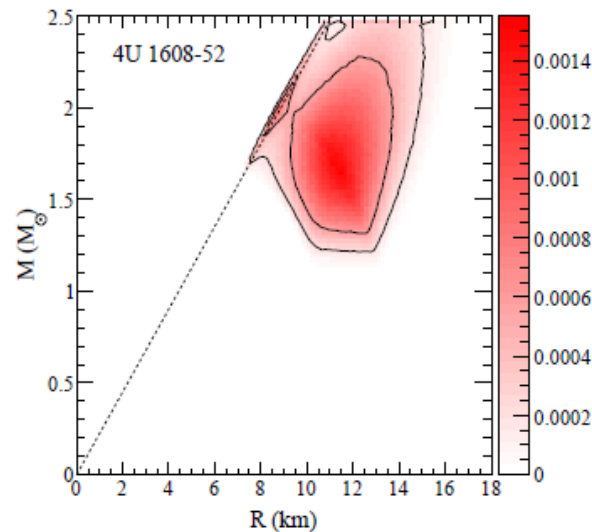
# Pessimistic estimates



1004.4871



1005.0811



It seems that Ozel et al. underestimate different uncertainties and make additional assumptions.

1002.3153

# Radius measurement

Fitting X-ray spectrum of a low-mass X-ray binary in quiescent state.

Mostly sources in globular clusters.

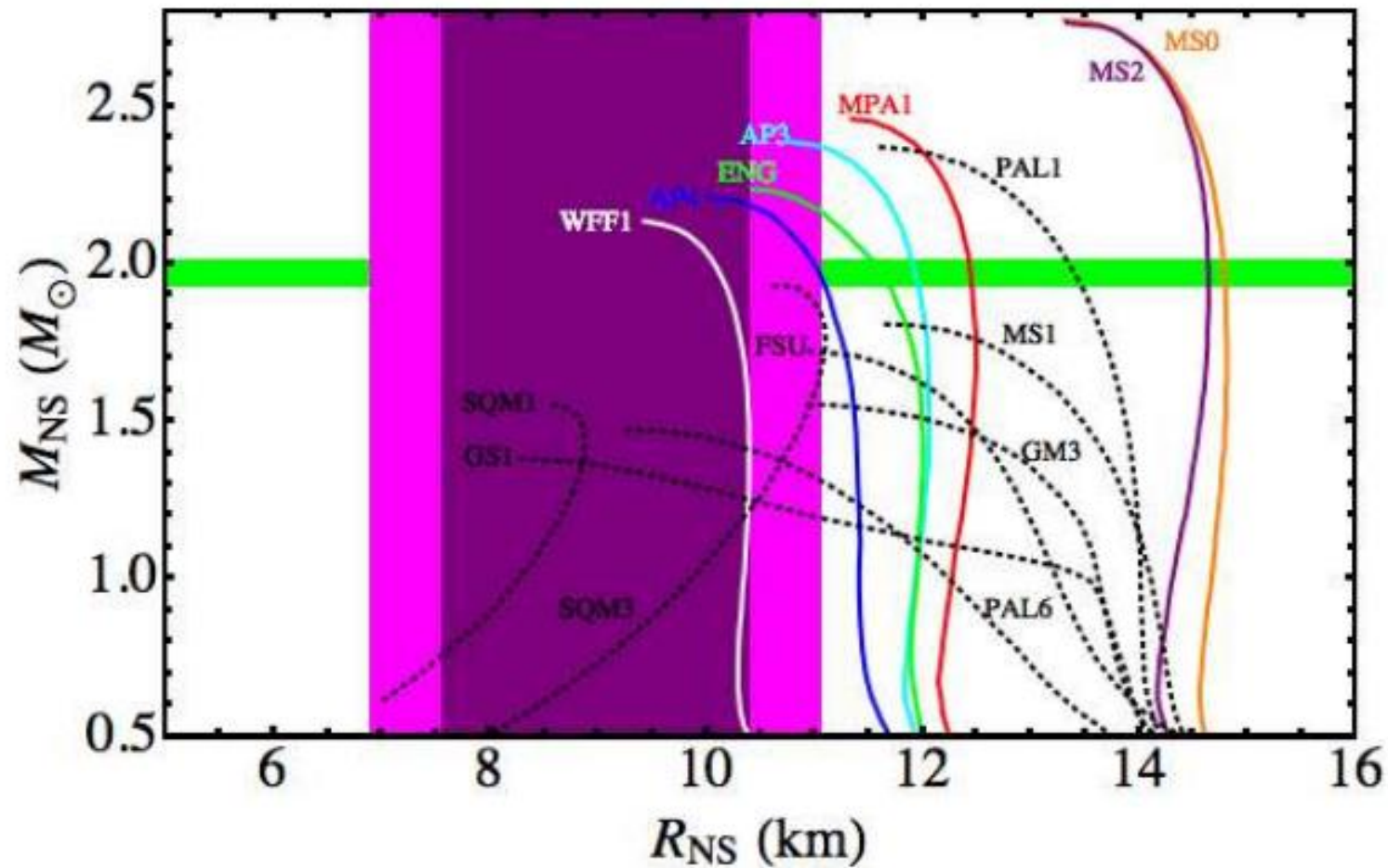
For 4 objects ~10% precision. But this is for fixed mass.

For U24 in NGC 6397  $R_{\text{NS}} = 8.9^{+0.9}_{-0.6}$  km for 1.4 solar masses.

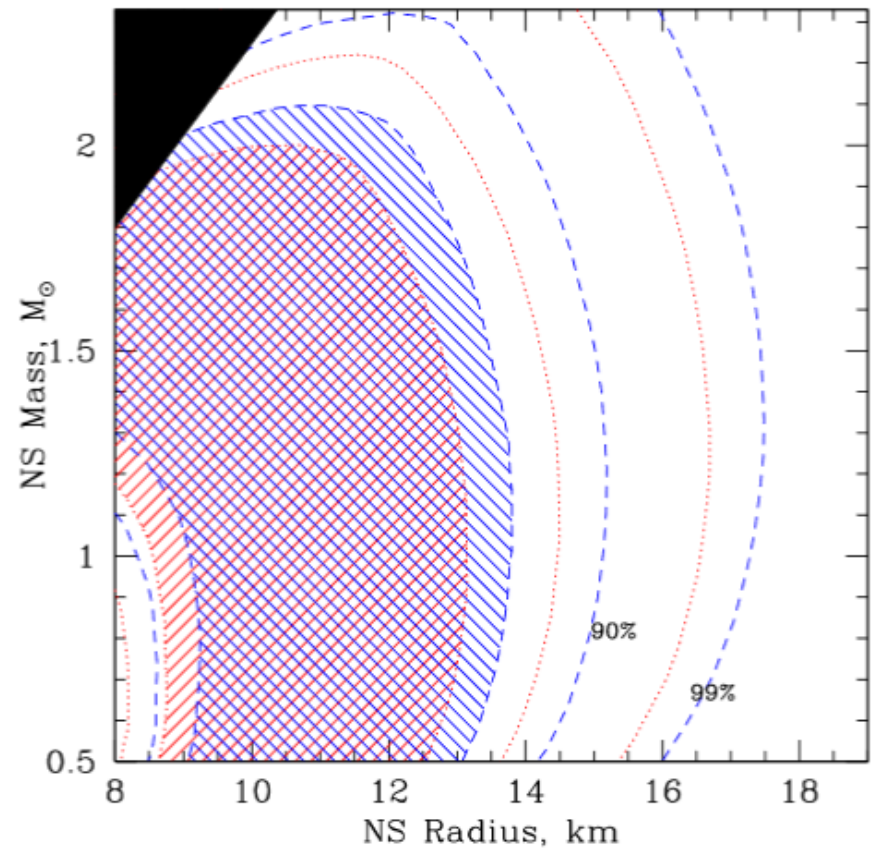
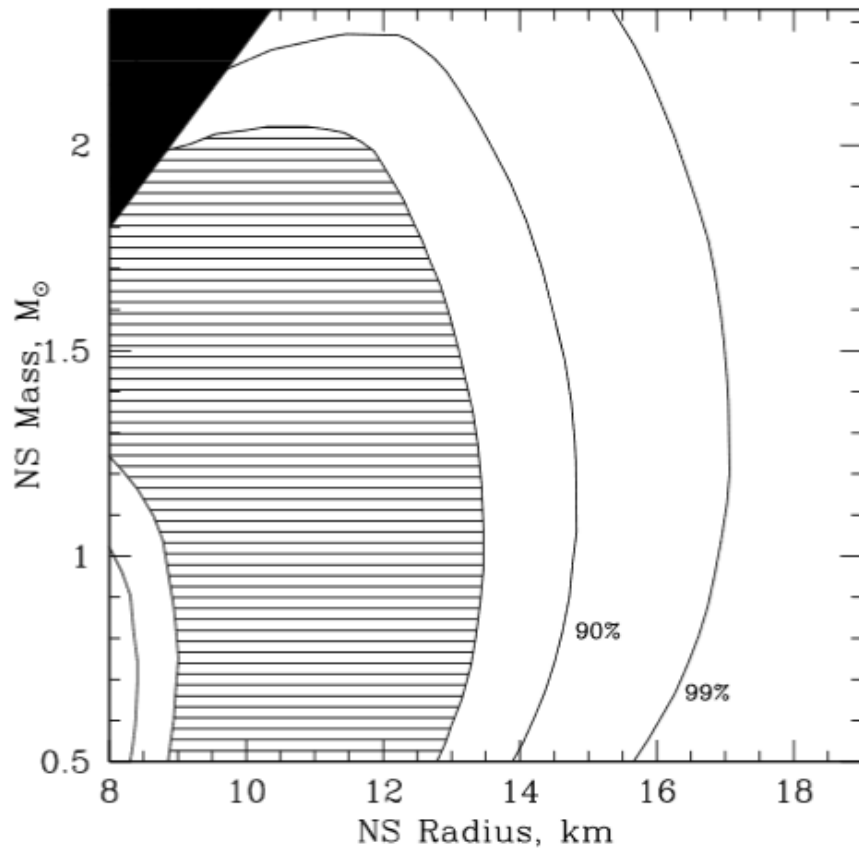
For the radius observed from infinity:  $11.9^{+2.2}_{-2.5}$  km

# Radii measurements for qLMXBs in GCs

5 sources

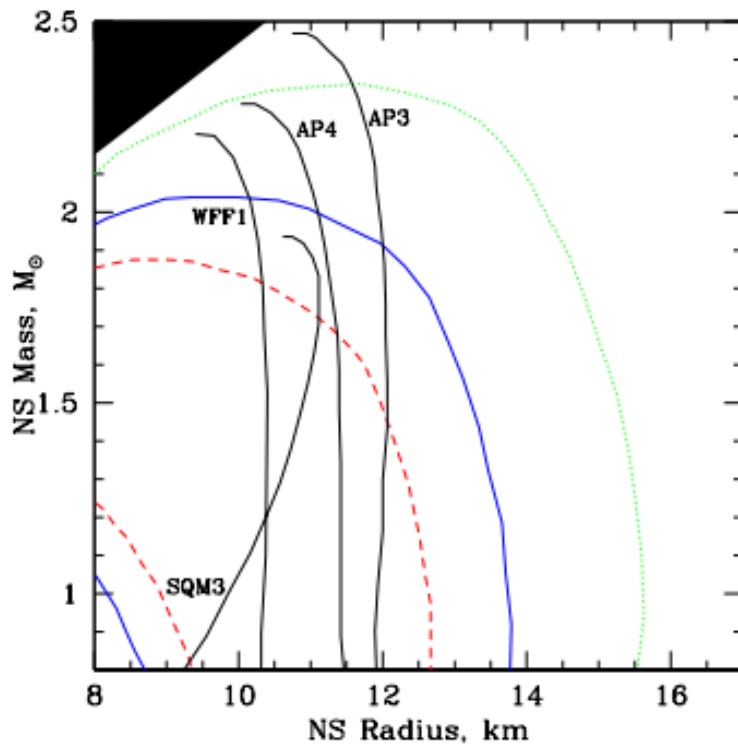


# Distance uncertainty

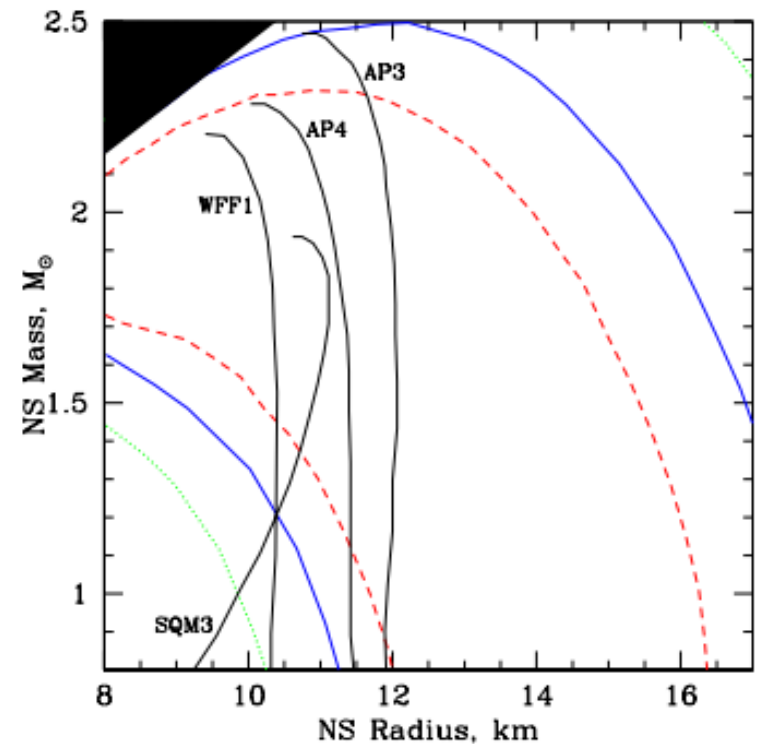


# Atmospheric uncertainties

qLMXB in M13

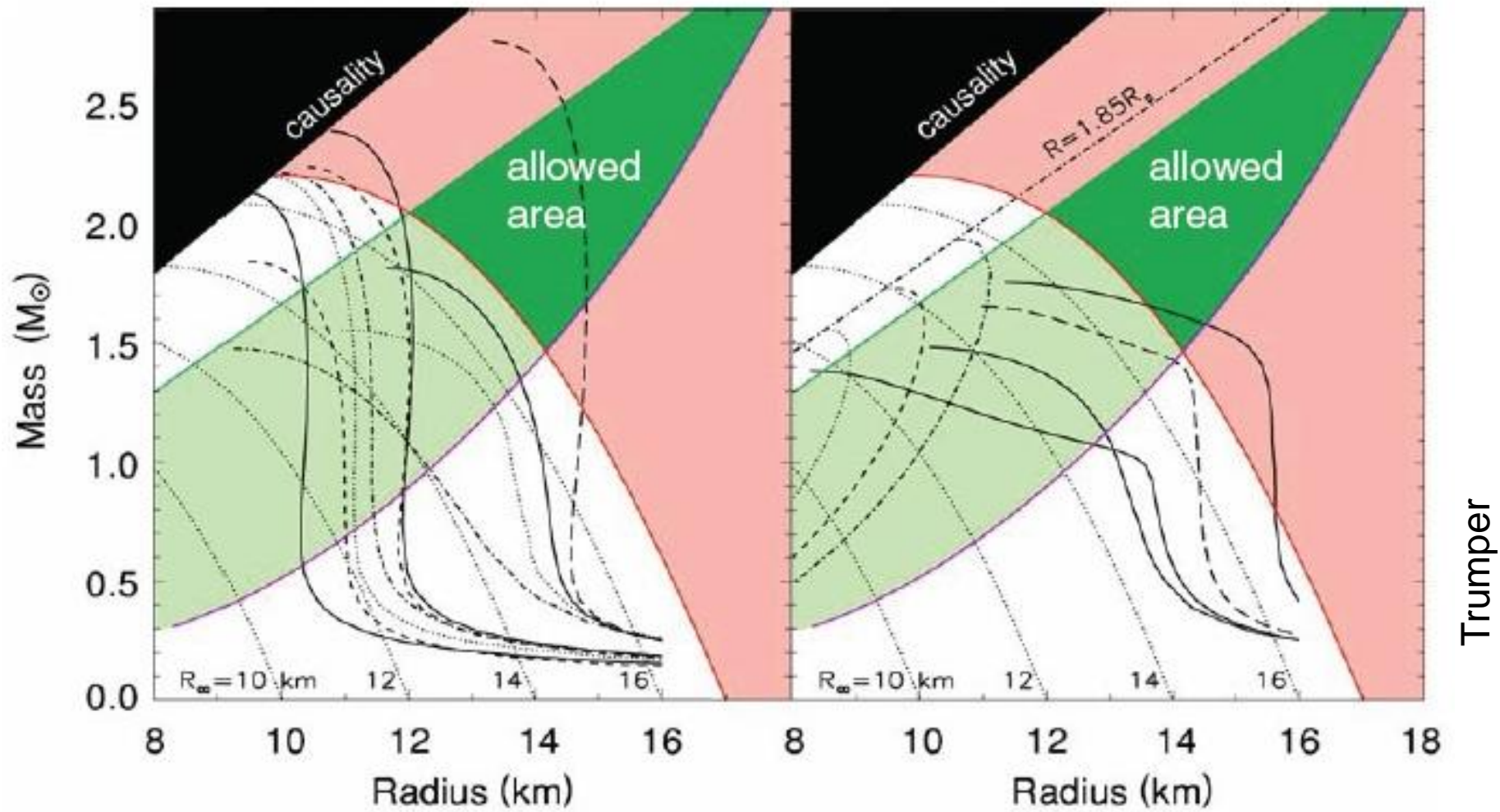


Hydrogene



Helium

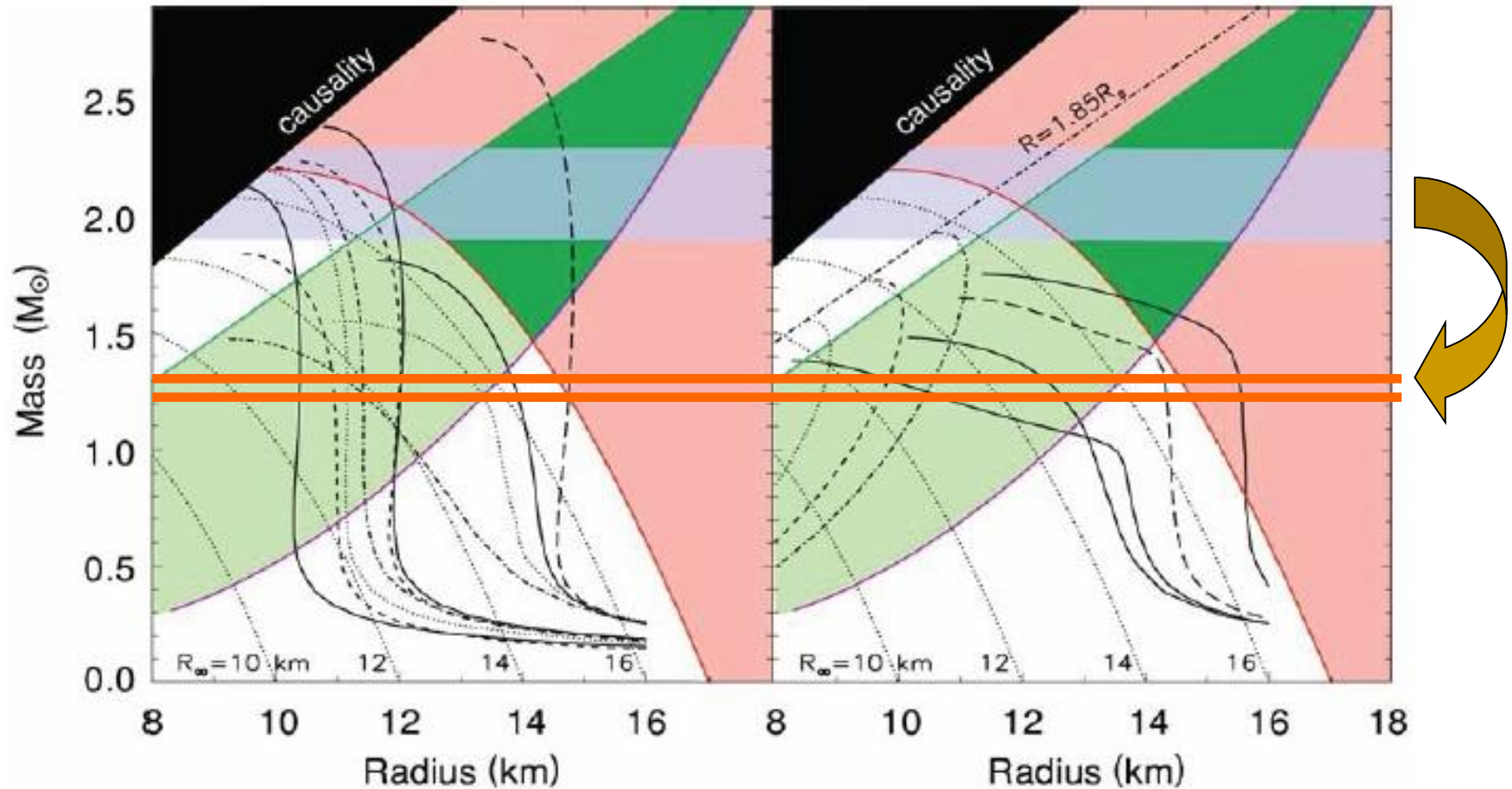
# Limits from RX J1856



About M7 for constraints on the EoS see 1111.0447



# PSR 0751+1807



Massive NS:  $2.1 \pm 0.3$  solar masses – Now shown to be wrong (!)  
[see Nice et al. 2008]

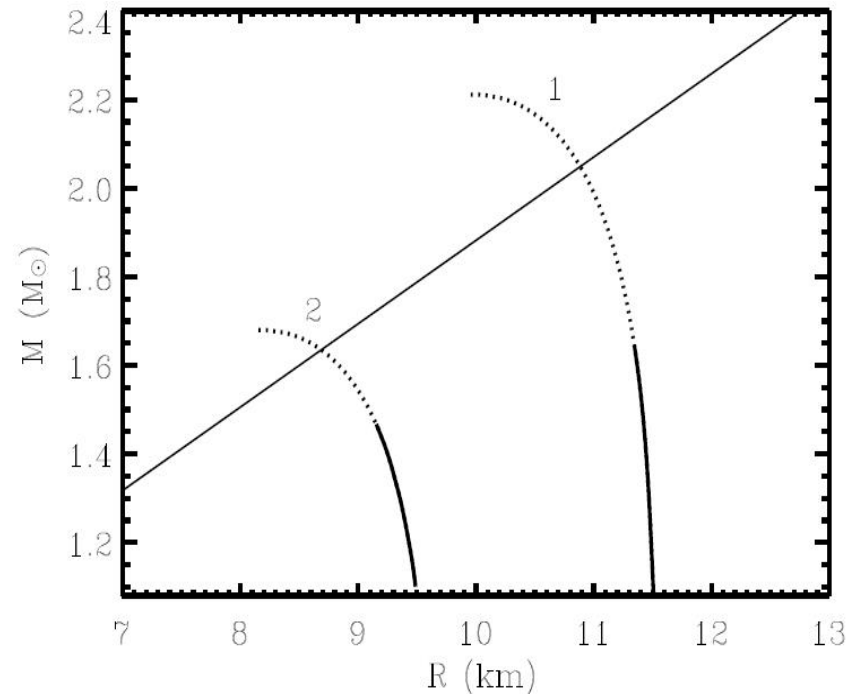
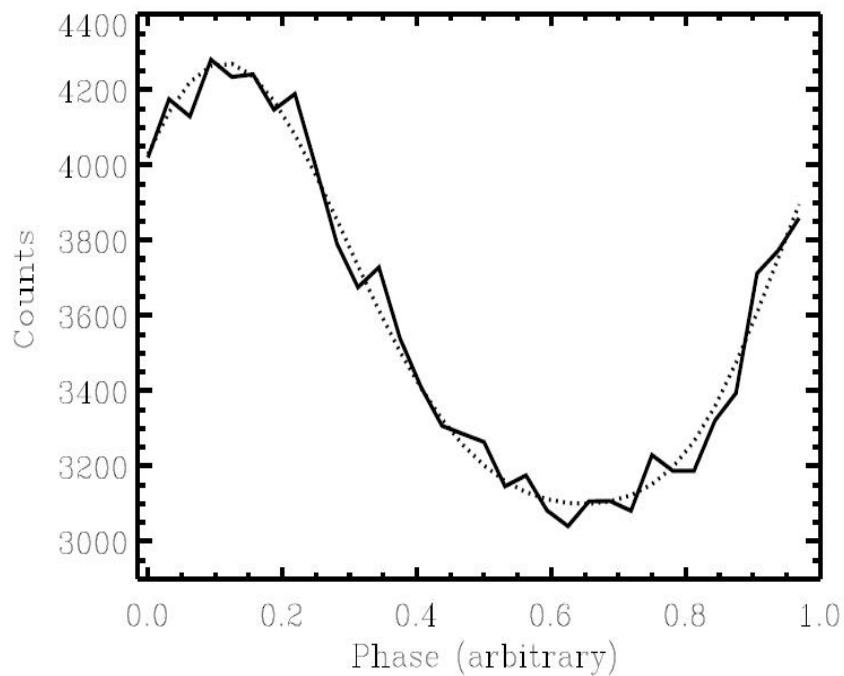
Trumper

# Burst oscillations

Fitting light curves of X-ray bursts.

Oscillations due to rotation of a NS.

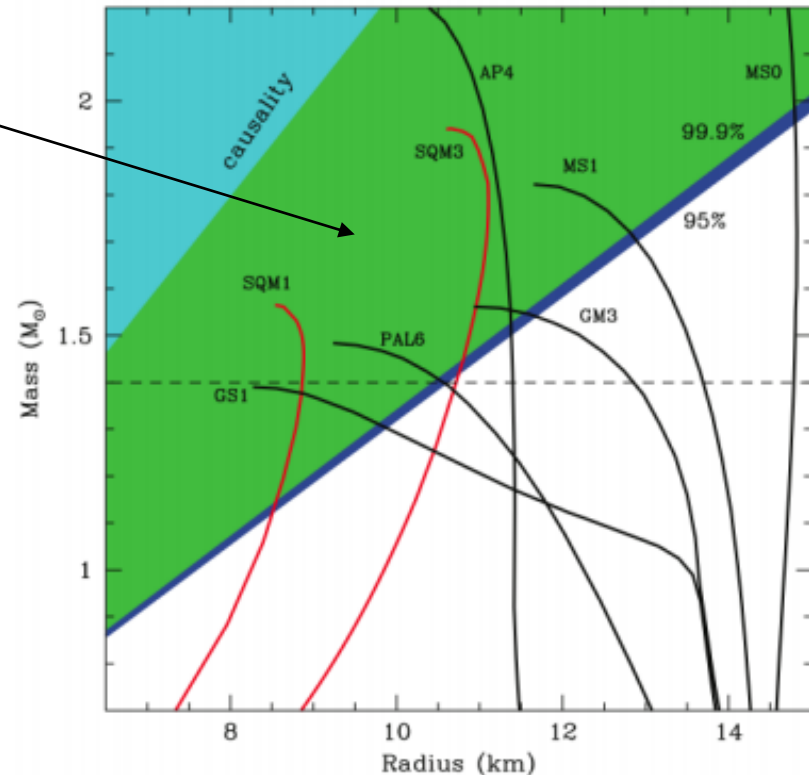
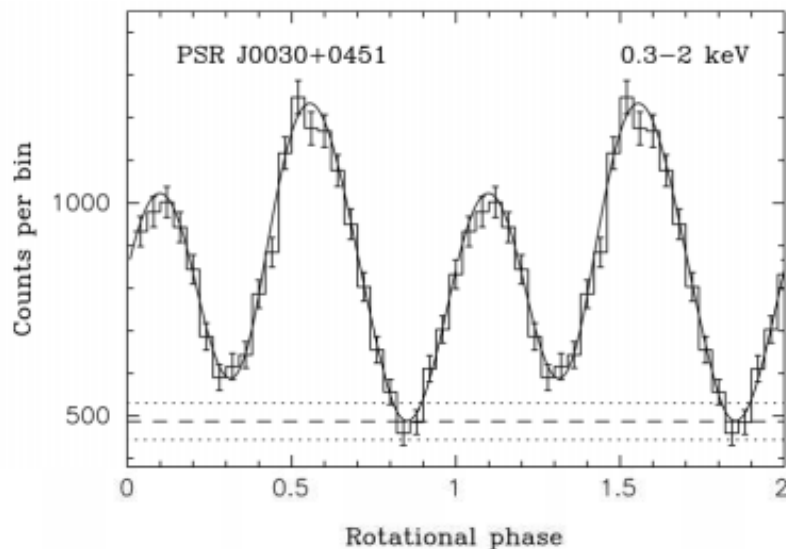
$Rc^2/GM > 4.2$  for the neutron star in XTE J1814-338.



# Pulse profile constraints

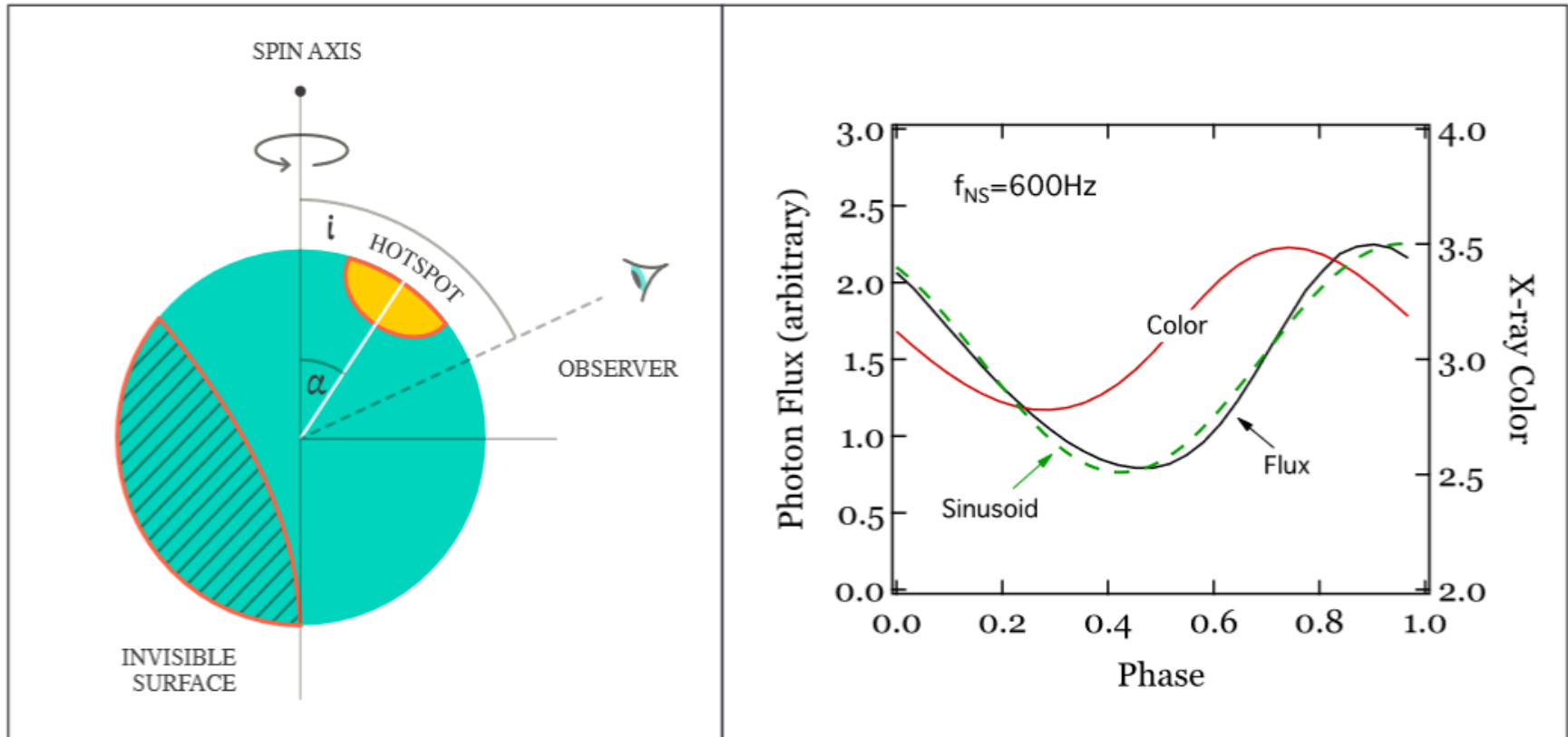
The idea is that: sharp pulses are possible only in the case of a large star

Green – excluded region



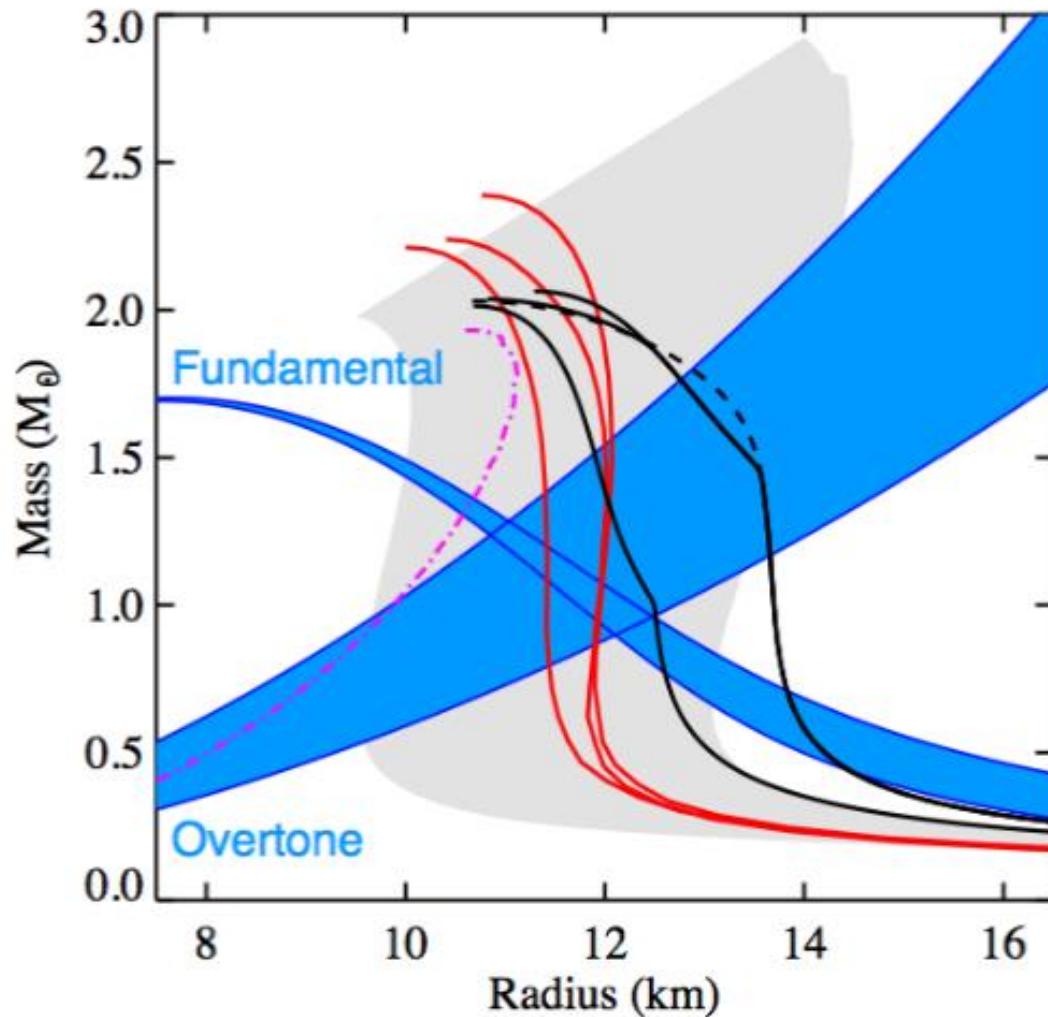
Based on Bogdanov, Grindlay 2009

# Hot spots and pulse profiles



As the neutron star rotates, emission from a surface hotspot generates a pulsation. The figure shows observer inclination  $i$ , and hotspot inclination  $\alpha$ . The invisible surface is smaller than a hemisphere due to relativistic light-bending.

# Astroseismology

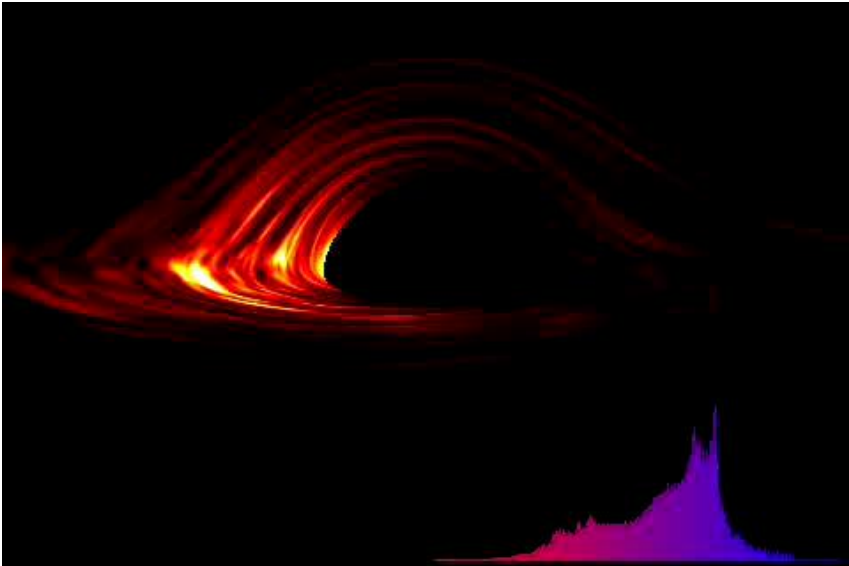


M – R diagram showing the seismological constraints for the soft gamma-ray repeater SGR 1806–20 using the relativistic torsional crust oscillation model of Samuelsson and Andersson (2007), in which the 29 Hz QPO is identified as the fundamental and the 625 Hz QPO as the first radial overtone. The neutron star lies in the box where the constraints from the two frequency bands overlap.

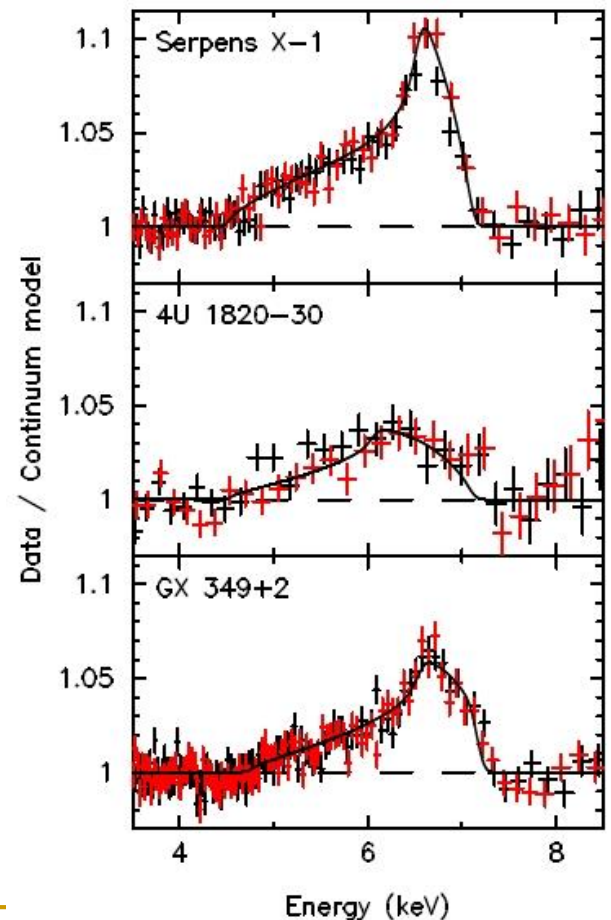
This is a simplified model. More detailed are in progress.

# Fe K lines from accretion discs

Measurements of the inner disc radius provide upper limits on the NS radius.



Ser X-1       $<15.9 \pm 1$   
4U 1820-30    $<13.8^{+2.9}_{-1.4}$   
GX 349+2     $<16.5 \pm 0.8$   
(all estimates for 1.4 solar mass NS)  
[Cackett et al. arXiv: 0708.3615]

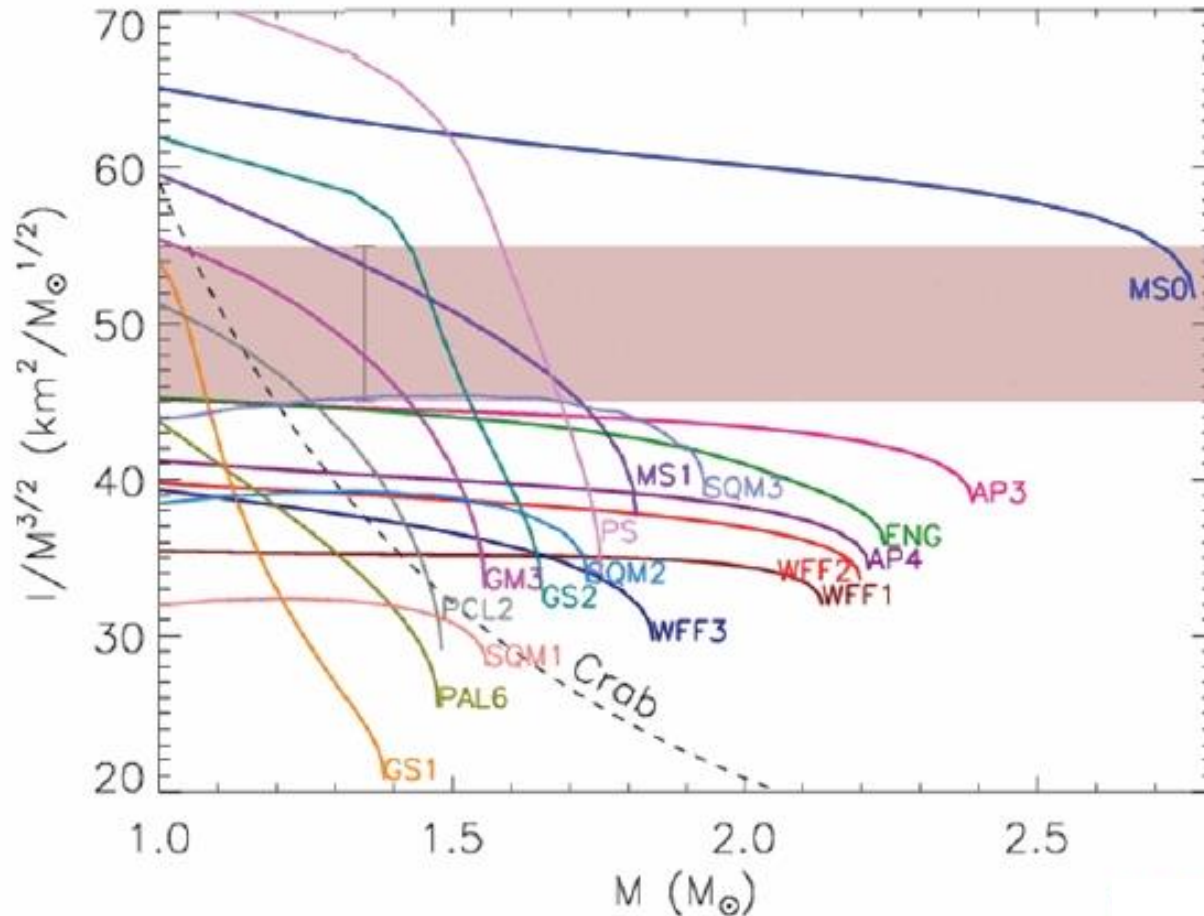


Suzaku observations

See also Papitto et al. arXiv: 0812.1149,  
a review in Cackett et al. 0908.1098, and theory in 1109.2068.



# Limits on the moment of inertia



Spin-orbital interaction

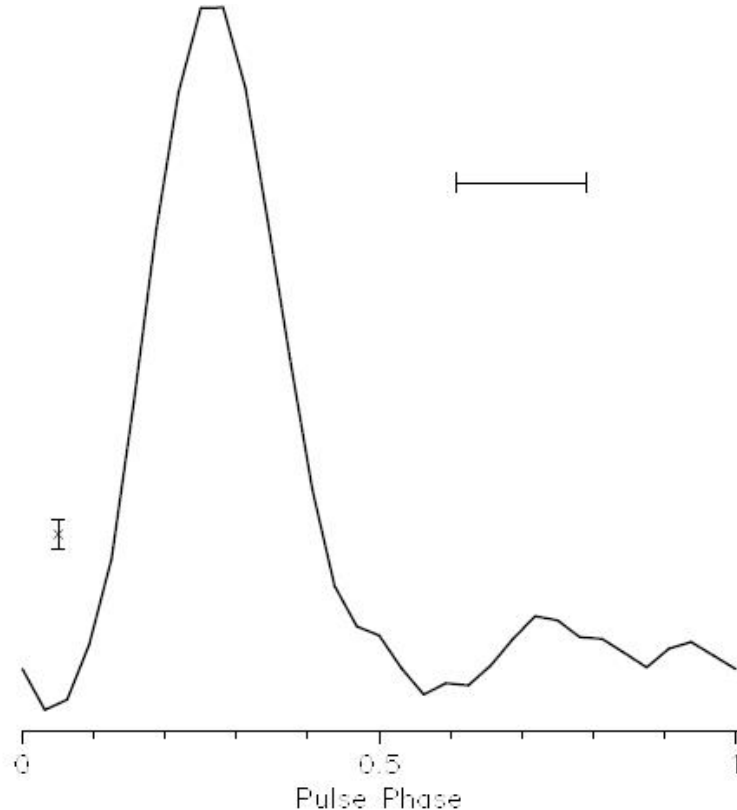
PSR J0737-3039  
(see Lattimer, Schutz  
astro-ph/0411470)

The band refers to a  
*hypothetical* 10% error.  
This limit, hopefully,  
can be reached in  
several years of observ.

See a more detailed  
discussion in 1006.3758

# Most rapidly rotating PSR

716-Hz eclipsing binary radio pulsar in the globular cluster Terzan 5

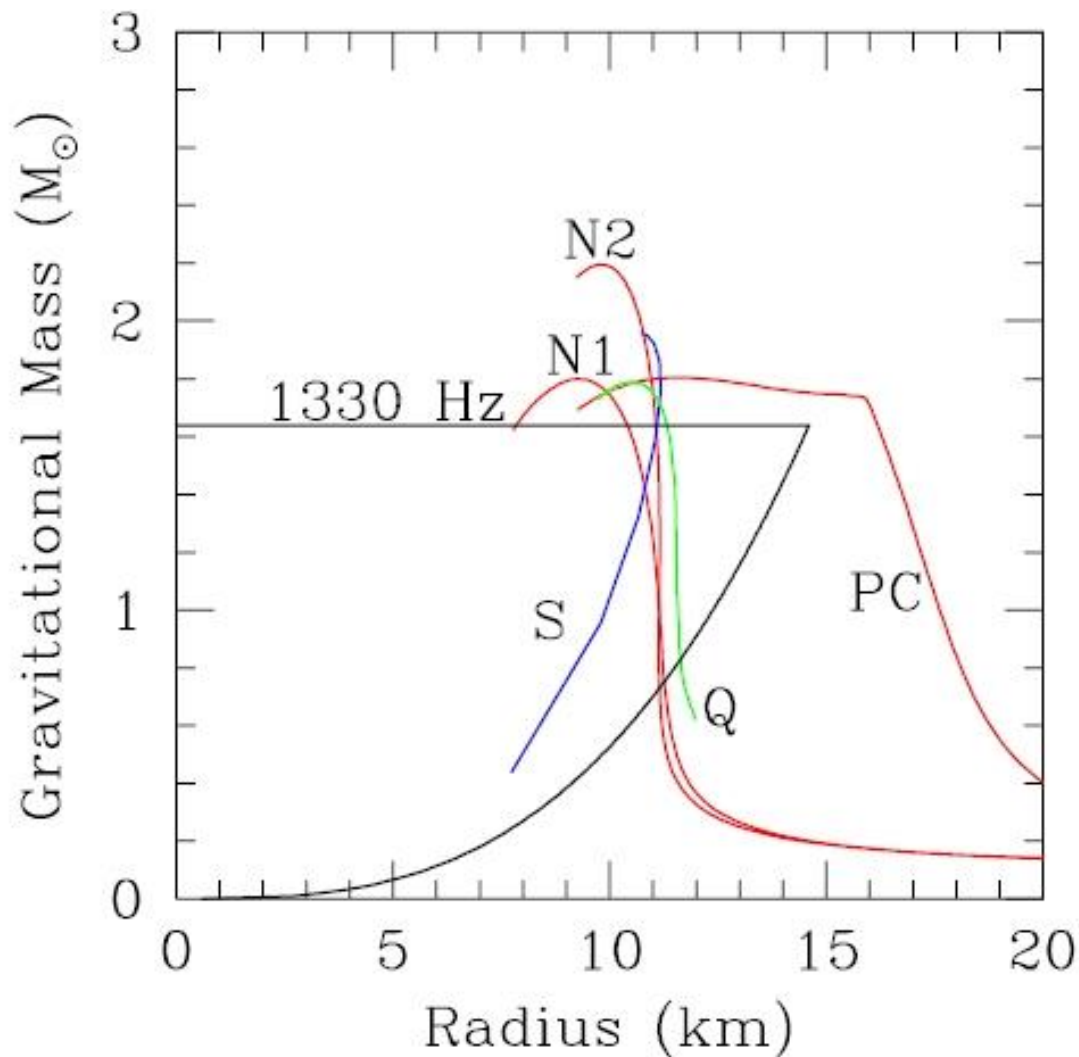


Previous record  
(642-Hz pulsar B1937+21)  
survived for more than 20 years.

**Interesting calculations  
for rotating NS have been  
performed recently by Krastev et al.  
arXiv: 0709.3621**

Rotation starts to be important  
from periods  $\sim 3$  msec.

# QPO and rapid rotation



XTE J1739-285

1122 Hz

[P. Kaaret](#) et al.

[astro-ph/0611716](#)

1330 Hz – one of the highest QPO frequency

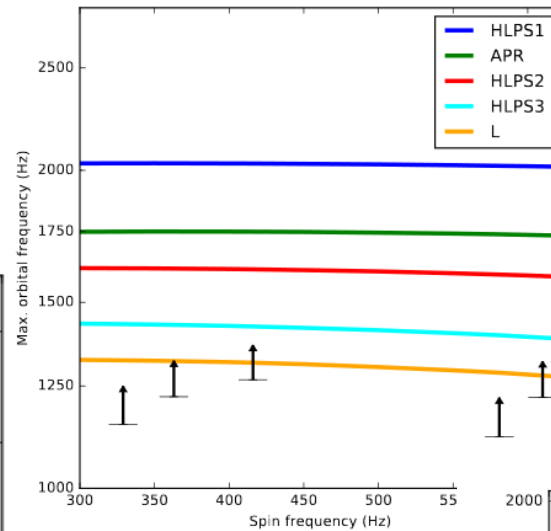
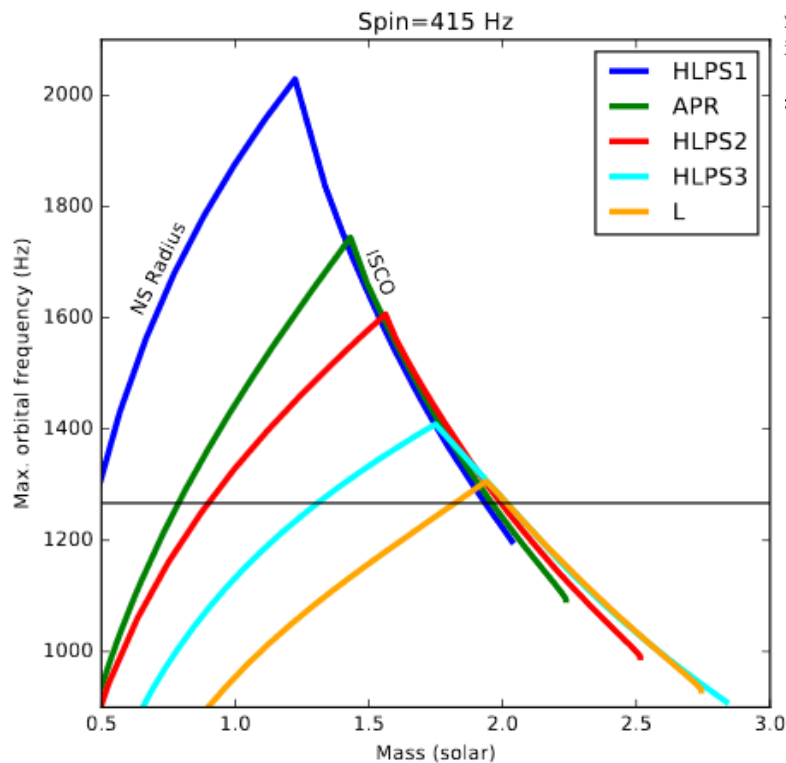
The line corresponds to the interpretation, that the frequency is that of the last stable orbit,  $6GM/c^2$

# New measurements for 4U 0614+09

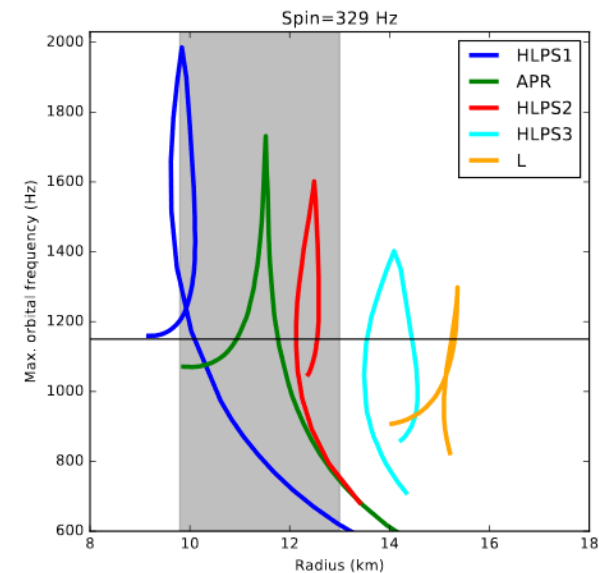
$$M \leq 2.2(v_{orb}/1000 \text{ Hz})^{-1}(1+0.75j)M_{\odot}$$

$$R \leq 19.5(v_{orb}/1000 \text{ Hz})^{-1}(1+0.2j)\text{km}$$

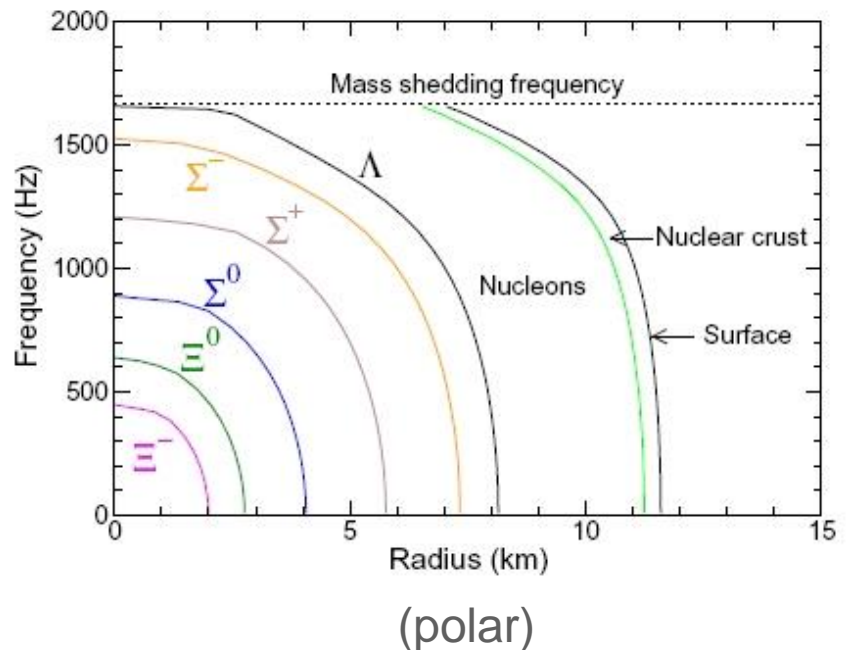
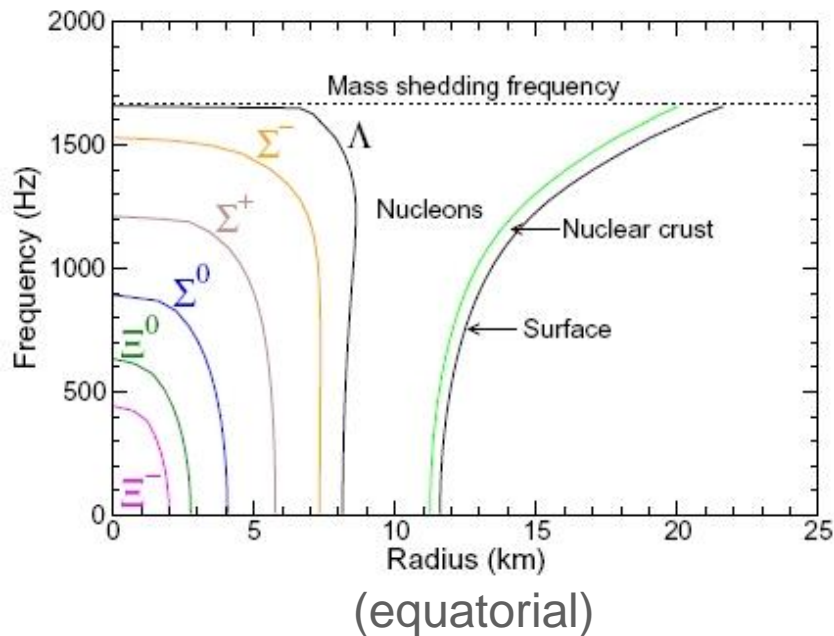
$$j \equiv cJ/GM^2$$



1267 Hz



# Rotation and composition

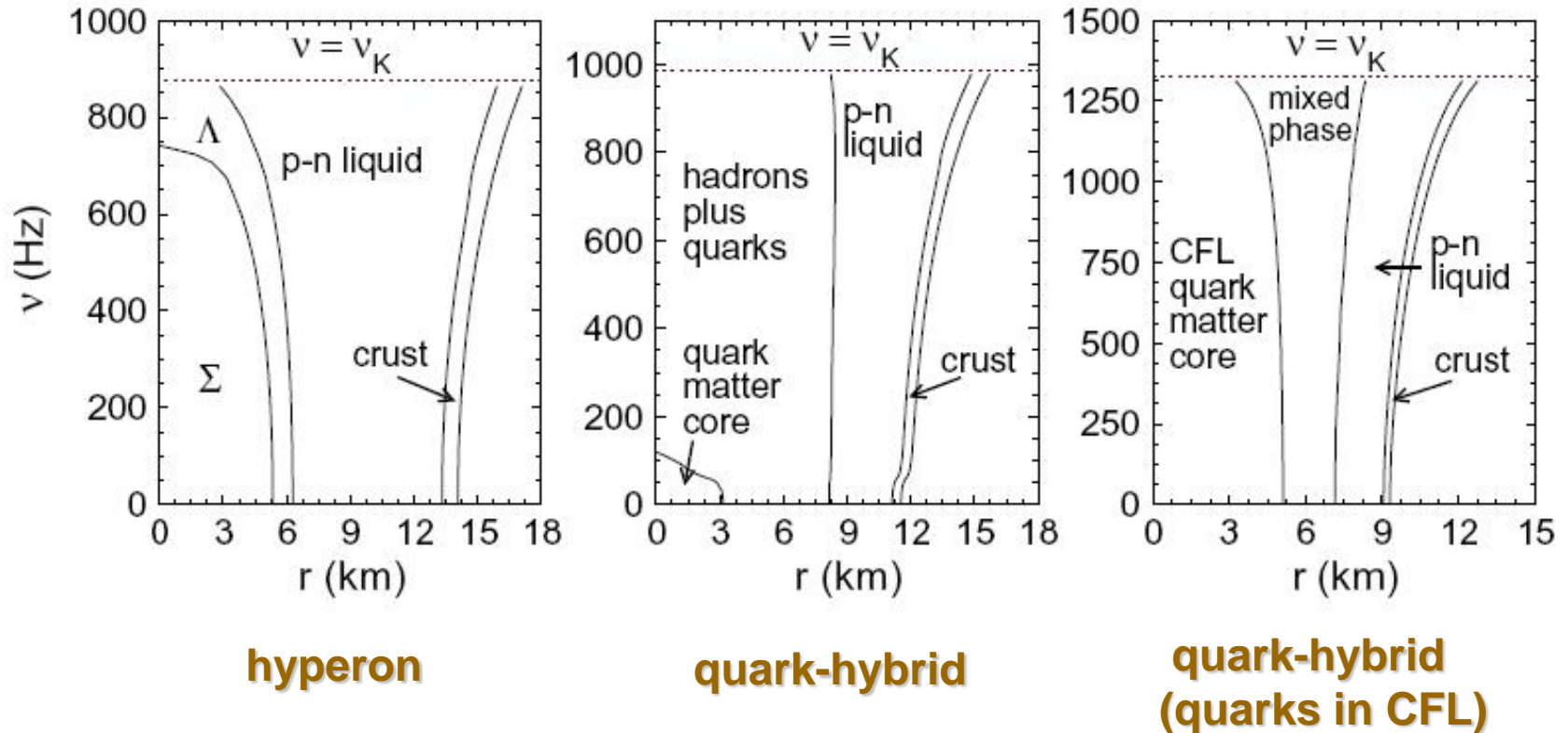


Computed for a particular model:  
density dependent relativistic Brueckner-Hartree-Fock (DD-RBHF)

(Weber et al. arXiv: 0705.2708)

Detailed study of the influence of rotation onto structure and composition is  
given in 1307.1103

# Rotation and composition



(Weber et al. arXiv: 0705.2708)

1.4 solar mass NS (when non-rotating)

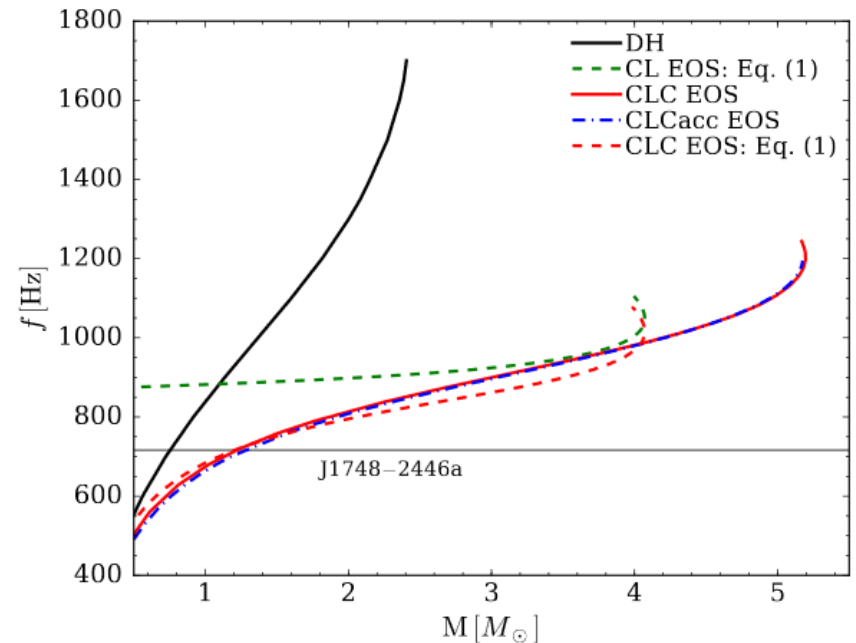
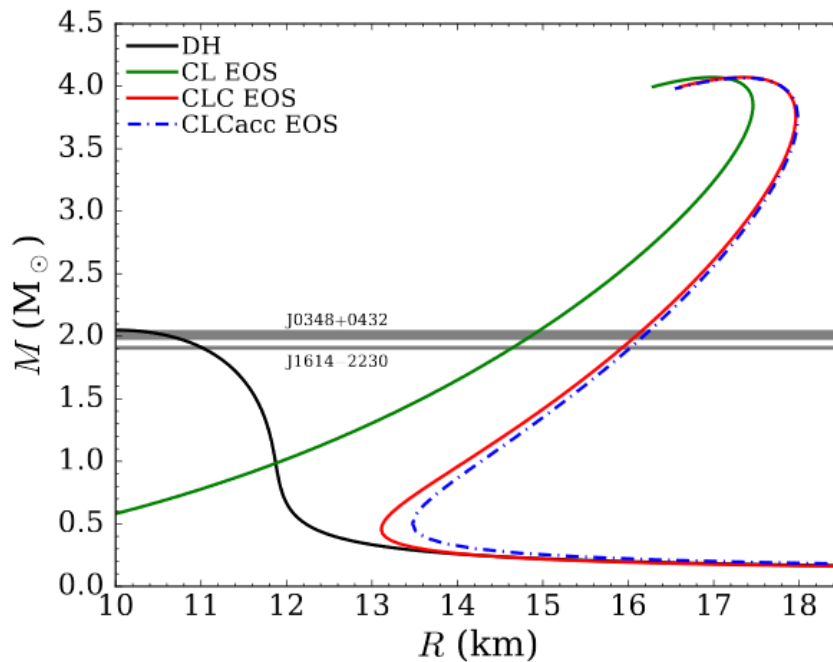


# Limiting rotation

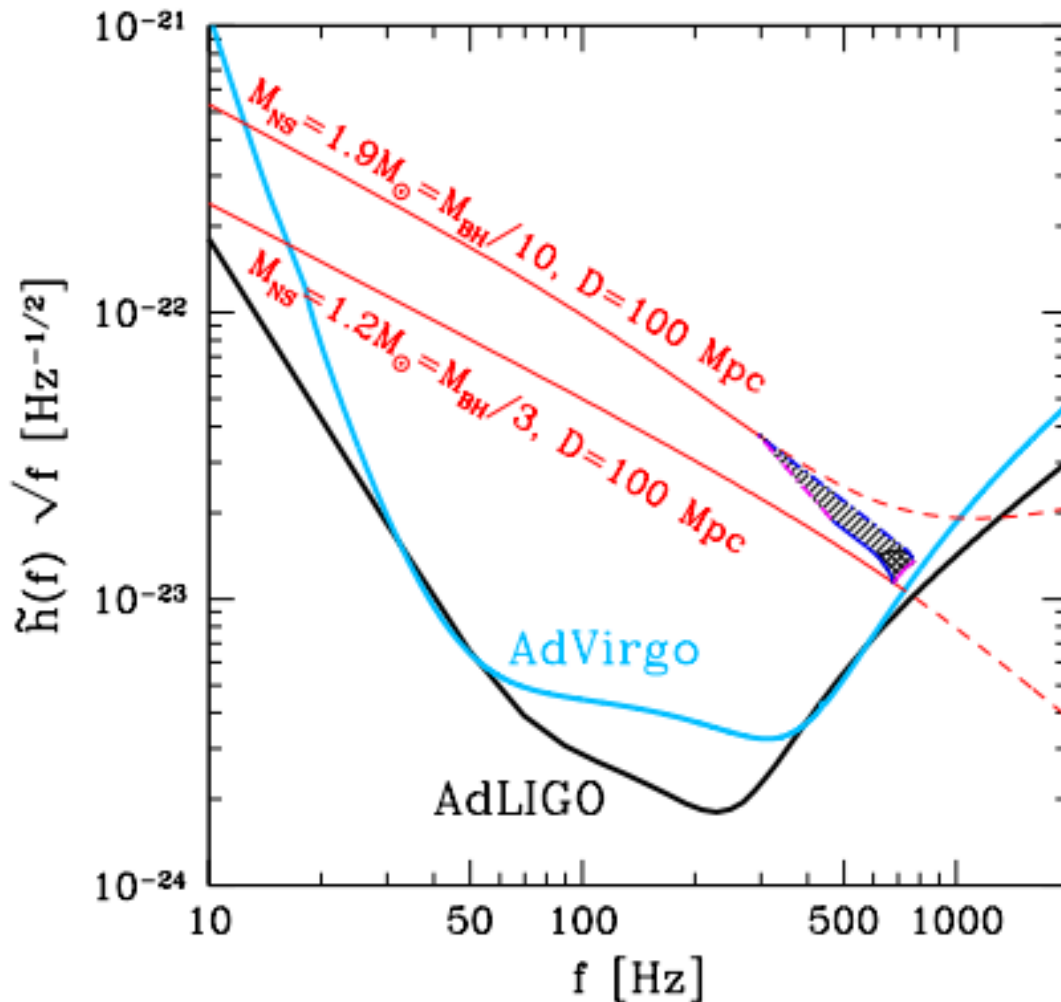
$$f_{\max}^{\text{EOS}} = C_{\max} \left( \frac{M_{\max}^{\text{stat}}}{M_{\odot}} \right)^{1/2} \left( \frac{R_{M_{\max}^{\text{stat}}}^{\text{stat}}}{10 \text{ km}} \right)^{-3/2}$$

$$C_{\max} = 1.22 \text{ kHz}$$

Without additional assumptions for realistic EoS it is expected that NS can rotate faster than  $f=716 \text{ Hz}$  for masses close to the limiting value.

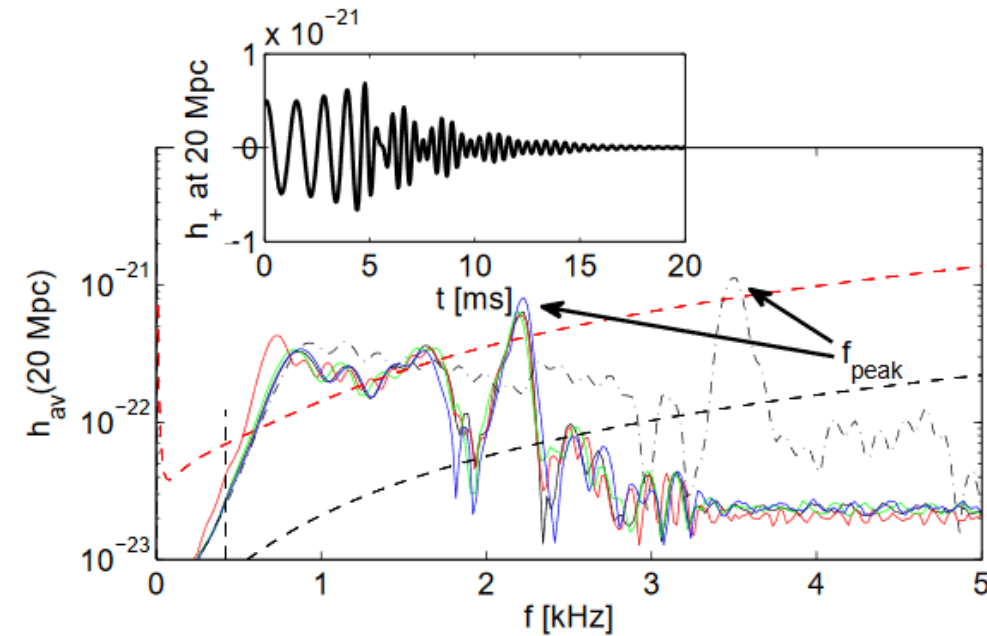


# Limits on the EoS from GW observations

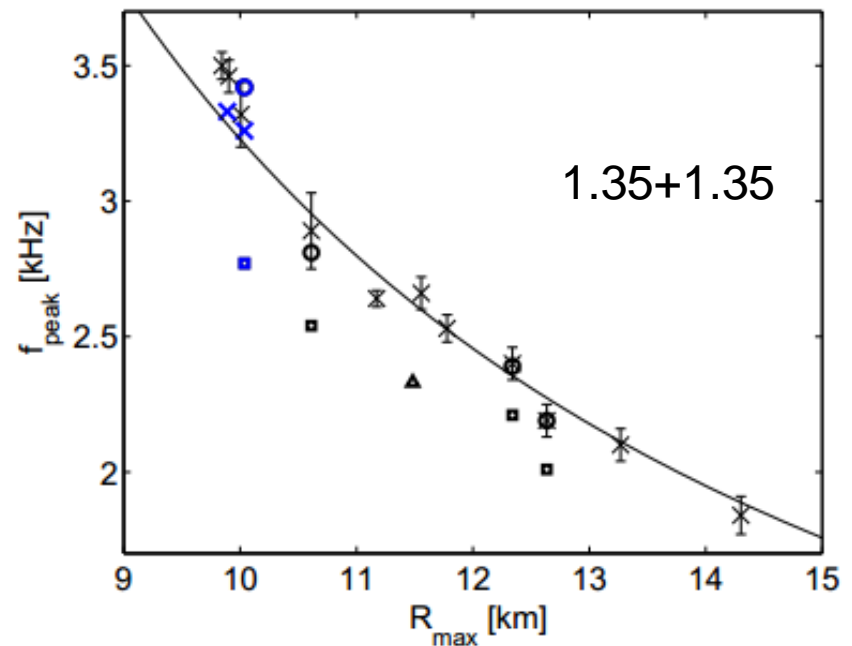


For stiff EoS  
AdLIGO and AdVIRGO  
can detect signatures  
in the GW signal  
during BH-NS mergers.

# Another constraint



Measuring NS-NS mergers  
one can better constraint the EoS.



# GW170817: deformability $\Lambda$

Many papers are published based on detection of GW signal from GW170817: 1803.00549, 1804.08583, 1805.09371, 1805.11579, 1805.11581, 1901.04138.

$$\tilde{\Lambda} = \frac{16}{13} \frac{(12q + 1)\Lambda_1 + (12 + q)q^4\Lambda_2}{(1 + q)^5},$$

$$q = m_2/m_1 \leq 1$$

$$\Lambda_{1,2} = \frac{2}{3} k_2 \left( \frac{R_{1,2} c^2}{G m_{1,2}} \right)^5$$

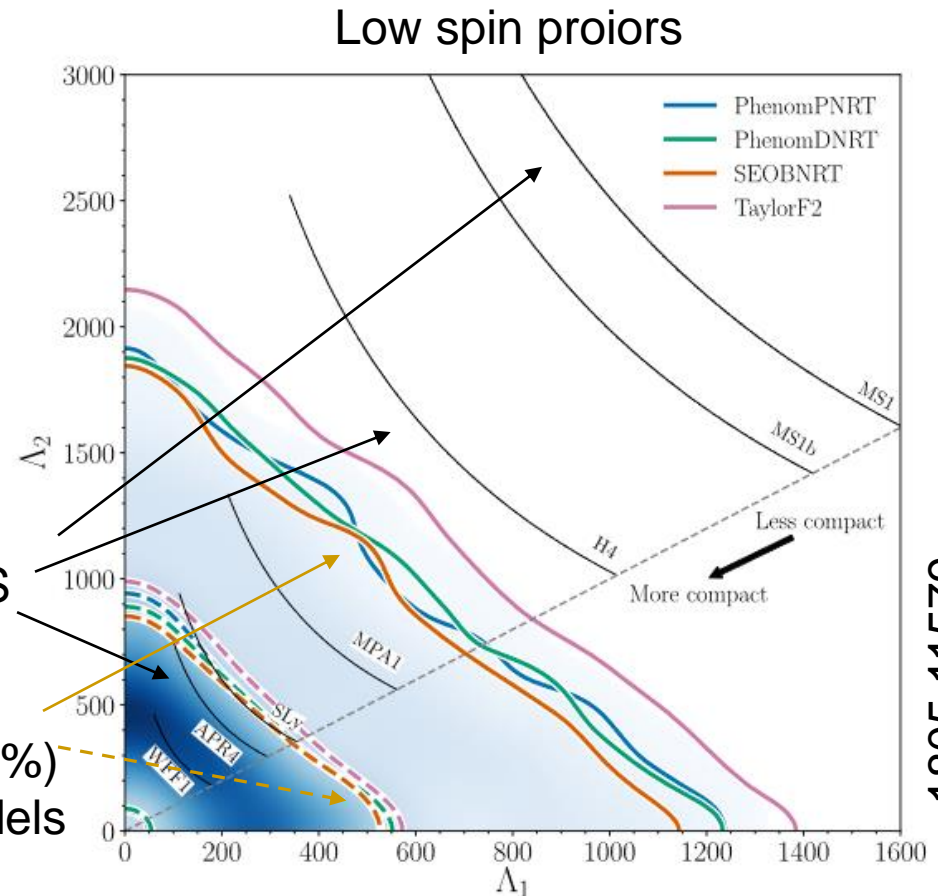
$$\beta = Gm/(\dot{R}c^2)$$

$$k_2 \sim \beta^{-1}$$

$$\Lambda \sim \beta^{-6}$$

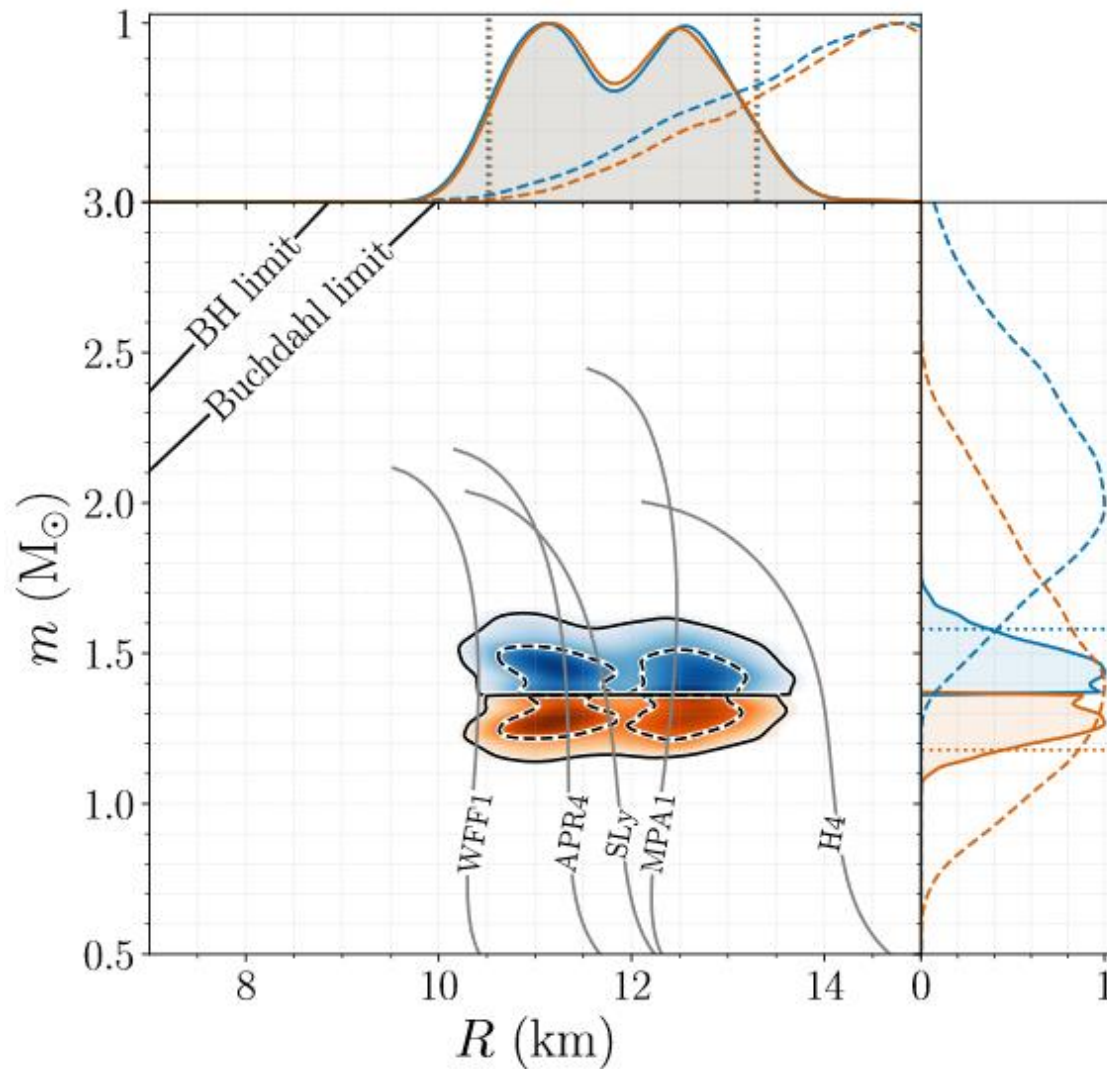
Solid – theoretical EoS

Colored – limits  
(dashed 50%, solid 90%)  
for four waveform models



Collapse to a BH after ~1 sec? (1901.04138)

# GW170817: M-R

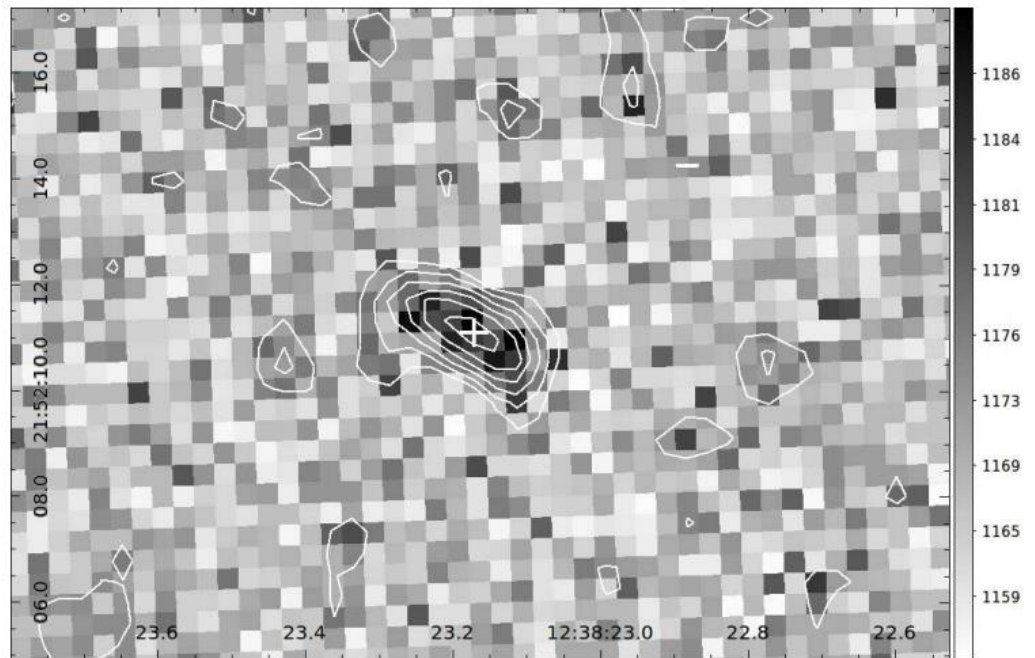


$$R_1 = 11.9^{+1.4}_{-1.4} \text{ km}$$

$$R_2 = 11.9^{+1.4}_{-1.4} \text{ km}$$

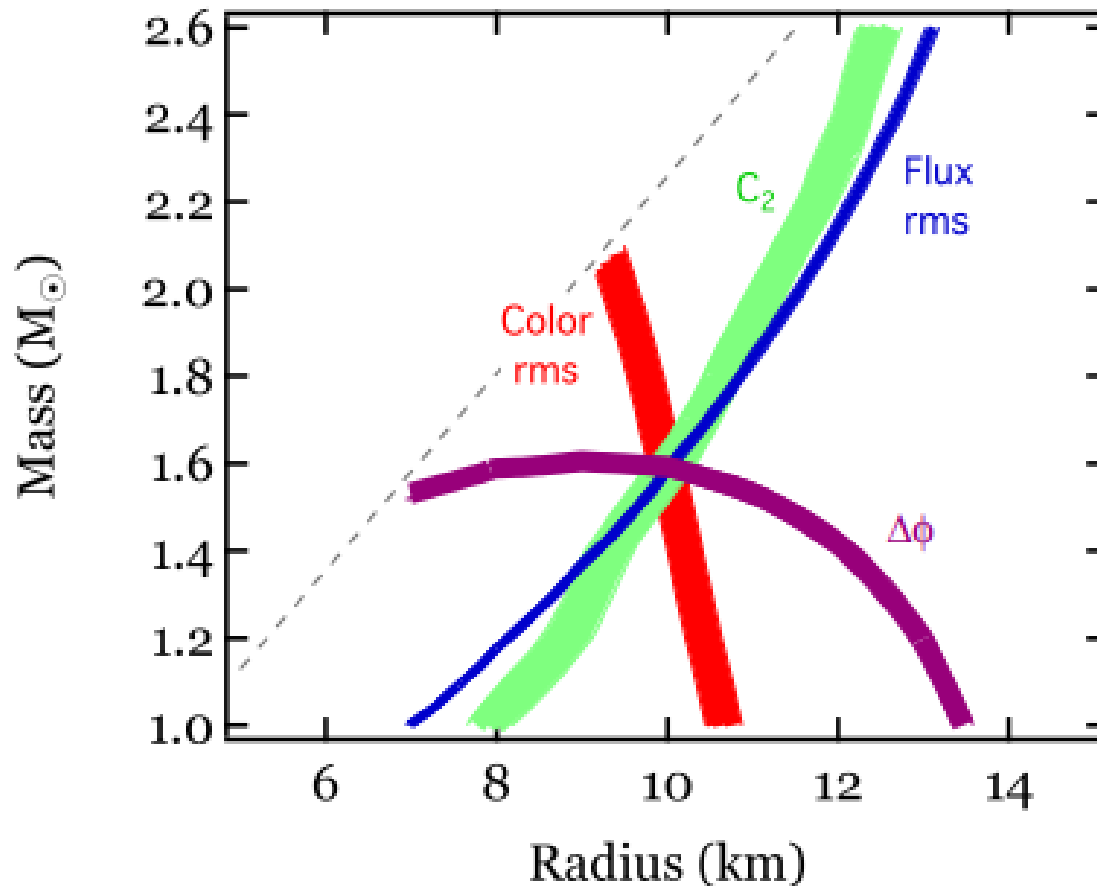
# Microlensing and weak lensing

In the future (maybe already with Gaia)  
it can possible to determine NS mass with lensing.  
Different techniques can be discussed:  
photometric (normal) microlensing (1009.0005),  
astrometric microlensing, weak lensing (1209.2249).





# Future X-ray measurements



Valid for future observations aboard NICER and LOFT space projects.

Data based on pulse profile.

The idea is to observe X-ray pulsars with spin periods ~few msec and to collect about  $10^6$  counts.

It allows to derive from the pulse profile a lot of info about a NS.

---

# References

- Observational Constraints on Neutron Star Masses and Radii 1604.03894  
The review is about X-ray systems
  - Mass, radii and equation of state of neutron stars 1603.02698  
The review about different kinds of measurements, including radio pulsars.  
Recent lists of mass measurements for different NSs.
  - Measuring the neutron star equation of state using X-ray timing 1602.01081  
The review about EoS and X-ray measurements
  - The masses and spins of neutron stars and stellar-mass black holes 1408.4145  
The review covers several topics. Good brief description of radio pulsar mass measurements.
  - Properties of DNS systems. 1706.09438  
The review covers all aspects of observations, formation and evolution.
  - Testing the equation of state of neutron stars with electromagnetic observations. 1806.02833  
The review describes observational tests of the EoS.
-

# NS+NS binaries

Secondary companion in double NS binaries can give a good estimate of the initial mass if we can neglect effects of evolution in a binary system.

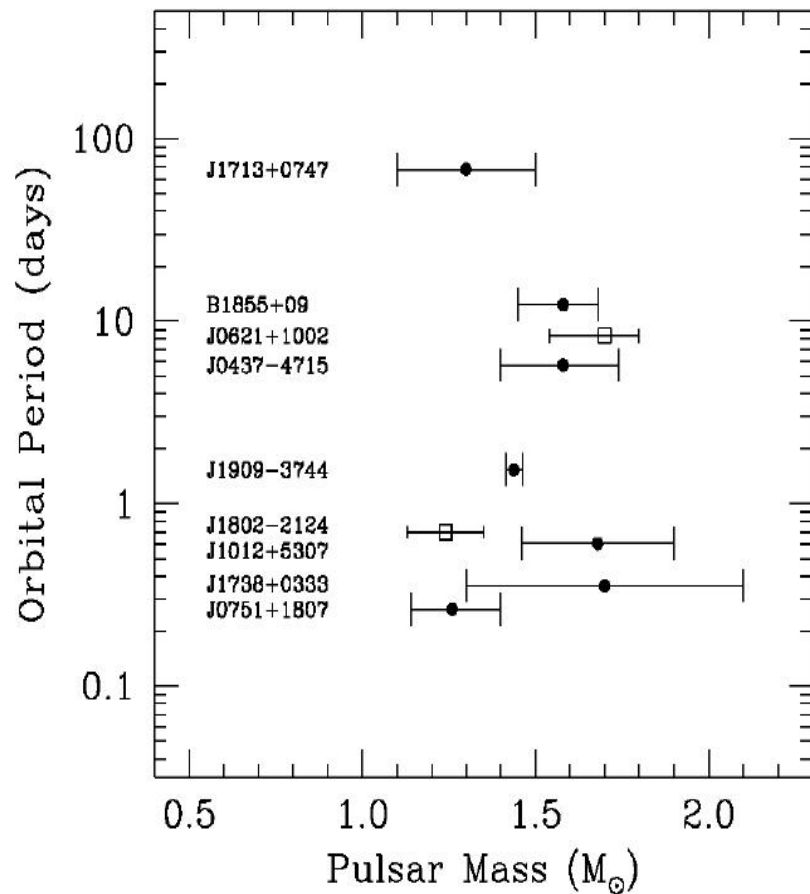
	Pulsar	Pulsar mass	Companion mass
GC →	B1913+16	1.44	1.39
	B2127+11C	1.36	1.35
	B1534+12	1.33	1.35
	J0737-3039	1.34	1.25
	J1756-2251	1.40	1.18
Non-recycled →	J1518+4904	<1.17	>1.55 → 0808.2292
	J1906+0746	1.25	1.37
	J1811-1736	1.56	1.12
	J1829+2456	1.2	1.4

Also there are candidates, for example  
PSR J1753-2240  
[arXiv:0811.2027](#)

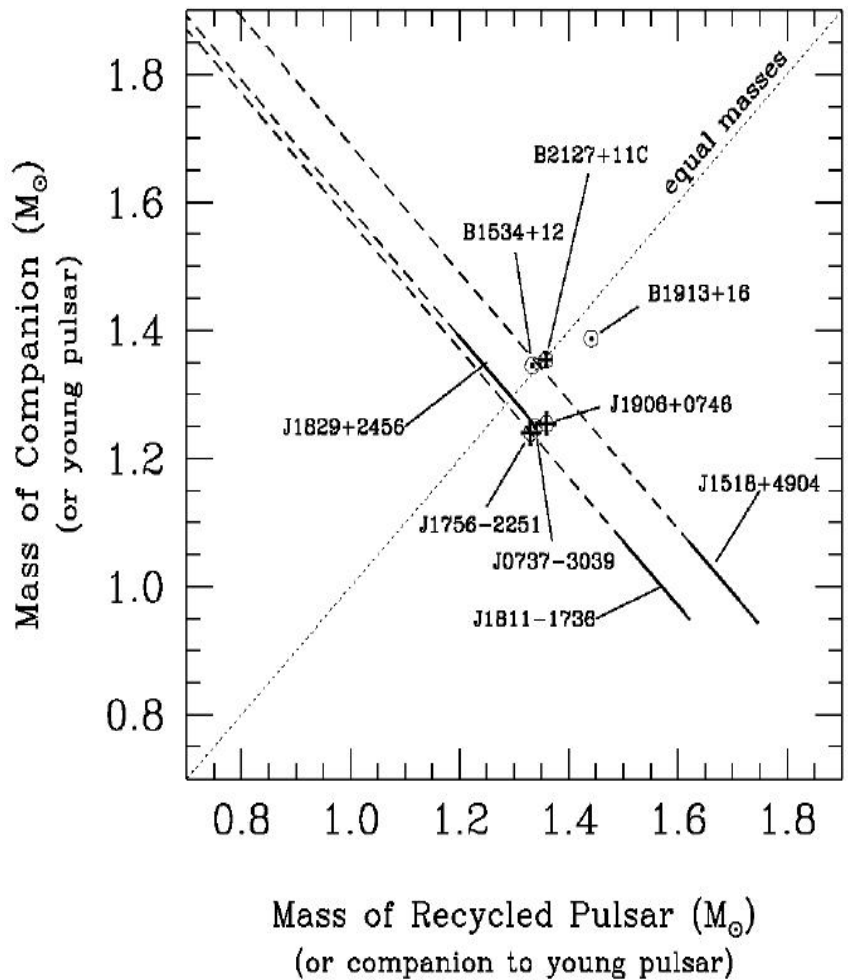
In NS-NS systems we can neglect all tidal effects etc.

See a review on formation and evolution of  
DNS binaries in 1706.09438

# Pulsar masses



With WD companions

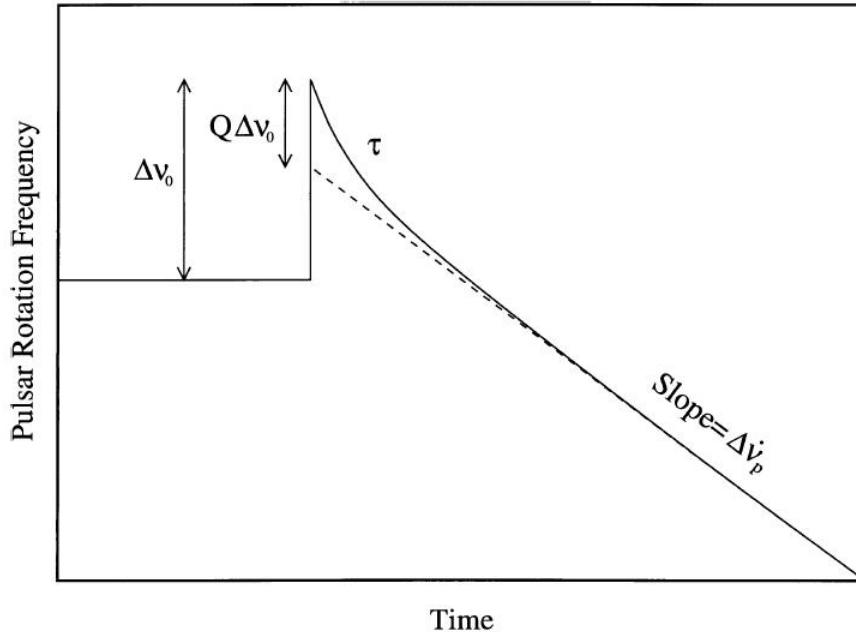


With NS companions

# Glitches and precession

---

# What is a glitch?



A sudden increase of rotation rate  
(limits are down to <40 sec in Vela).

ATNF catalogue gives >130 normal PSRs  
with glitches.

The most known: Crab and Vela

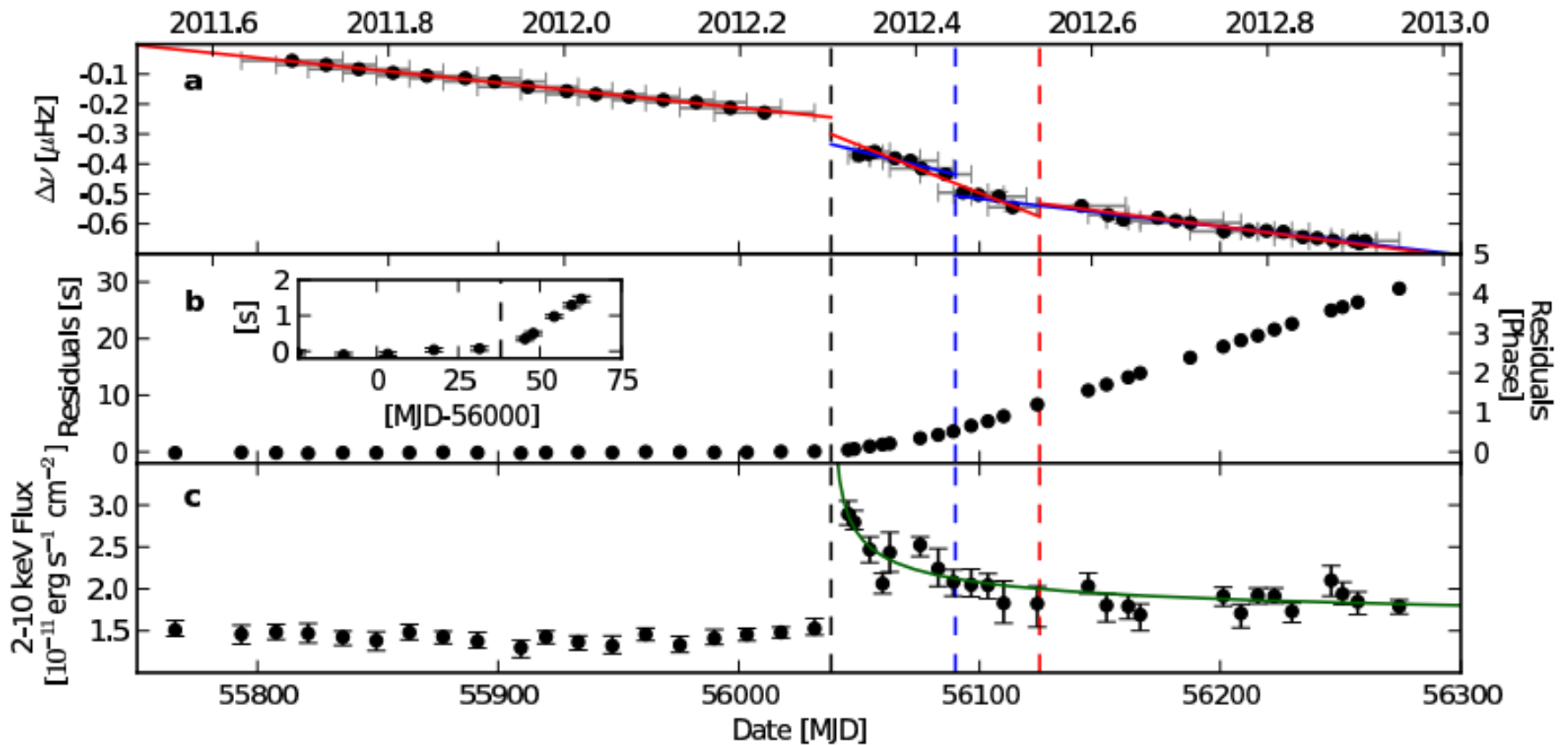
$$\Delta\Omega/\Omega \sim 10^{-9} - 10^{-6}$$

Spin-down rate can change after a glitch.  
Vela is spinning down faster after a glitch.

**Starquakes or/and vortex lines unpinning -  
new configuration or transfer of angular momentum**

Glitches are important because they probe internal structure of a NS.

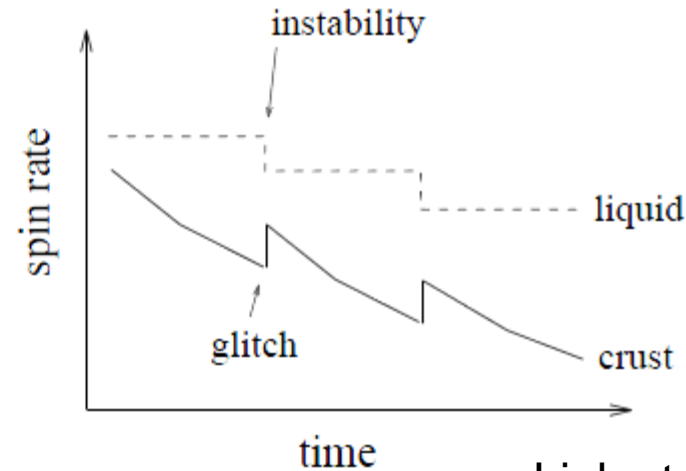
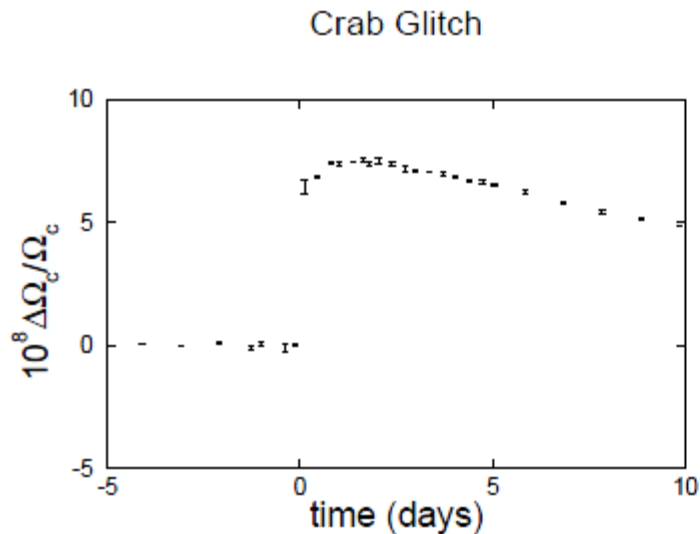
# Anti-glitch of a magnetar



AXP 1E 2259+586

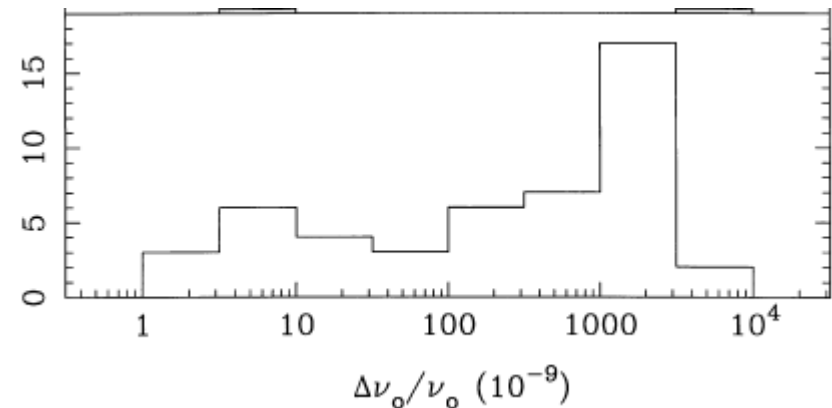


# Crab glitch and the general idea



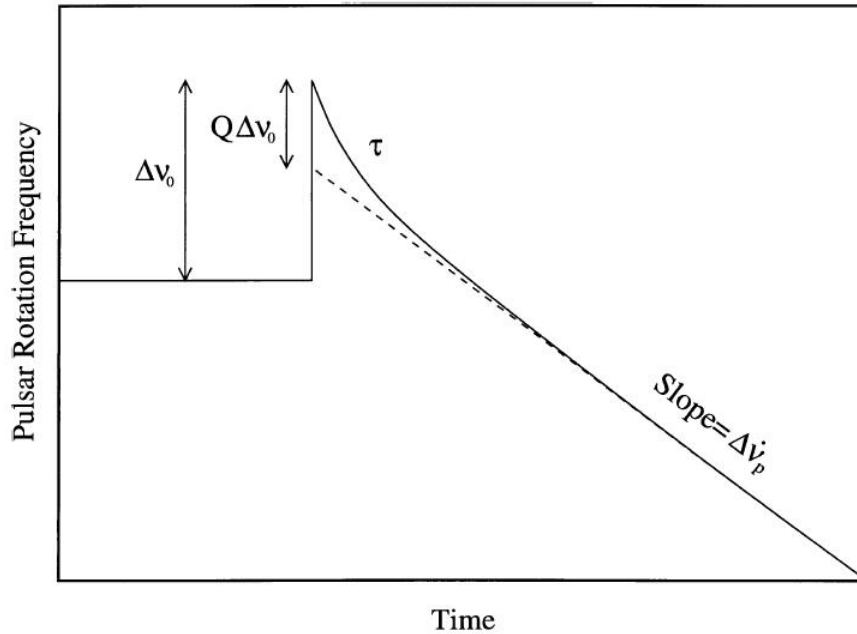
Link et al. (2000)

While the crust we see (and all coupled to it) is slowing down, some component of a star is not. Then suddenly (<40 sec) an additional momentum stored in such a “reservoir” is released and given to the crust. The crust spins-up, up the internal reservoir – down.



Lyne et al. (2000)

# Glitches

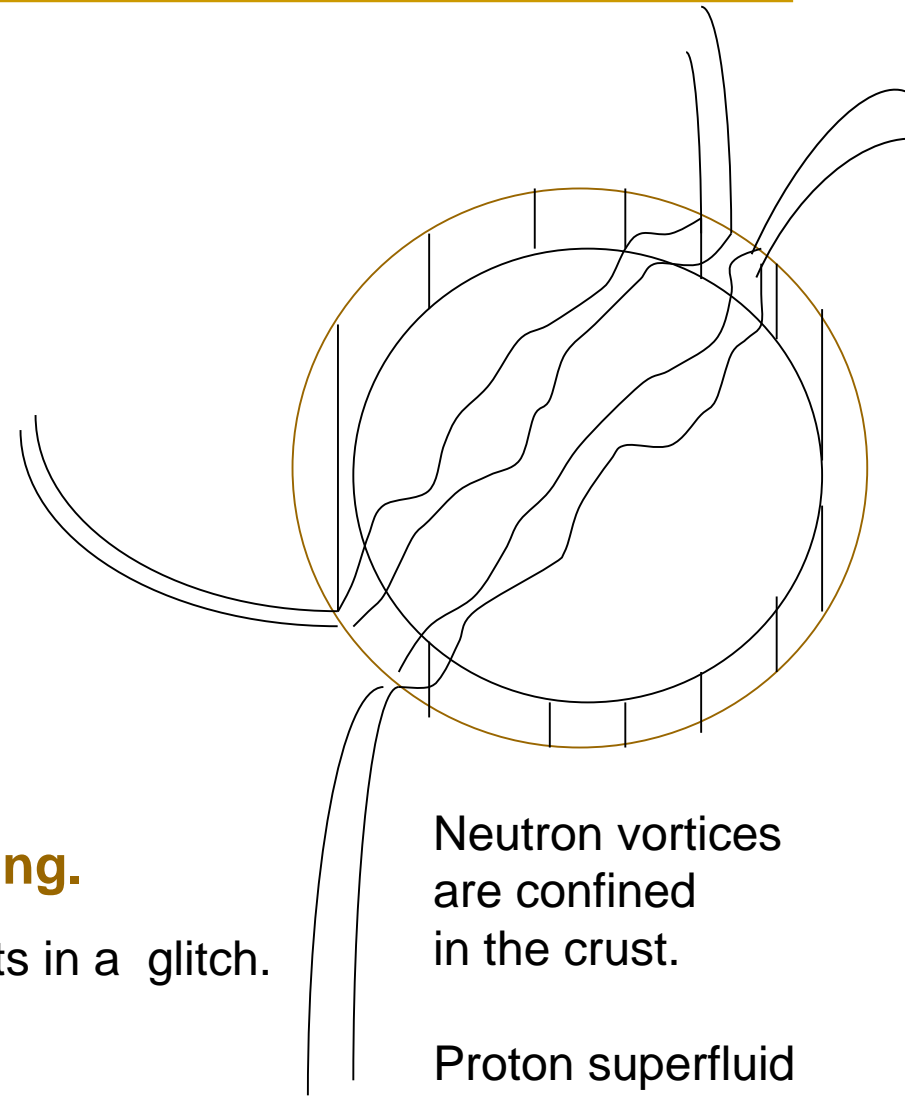


## Starquakes or vortex lines unpinning.

Unpinning of superfluid vortex lines results in a glitch.

Vortex density is about  $10^4 \text{ cm}^{-2} \text{ P}^{-1}$

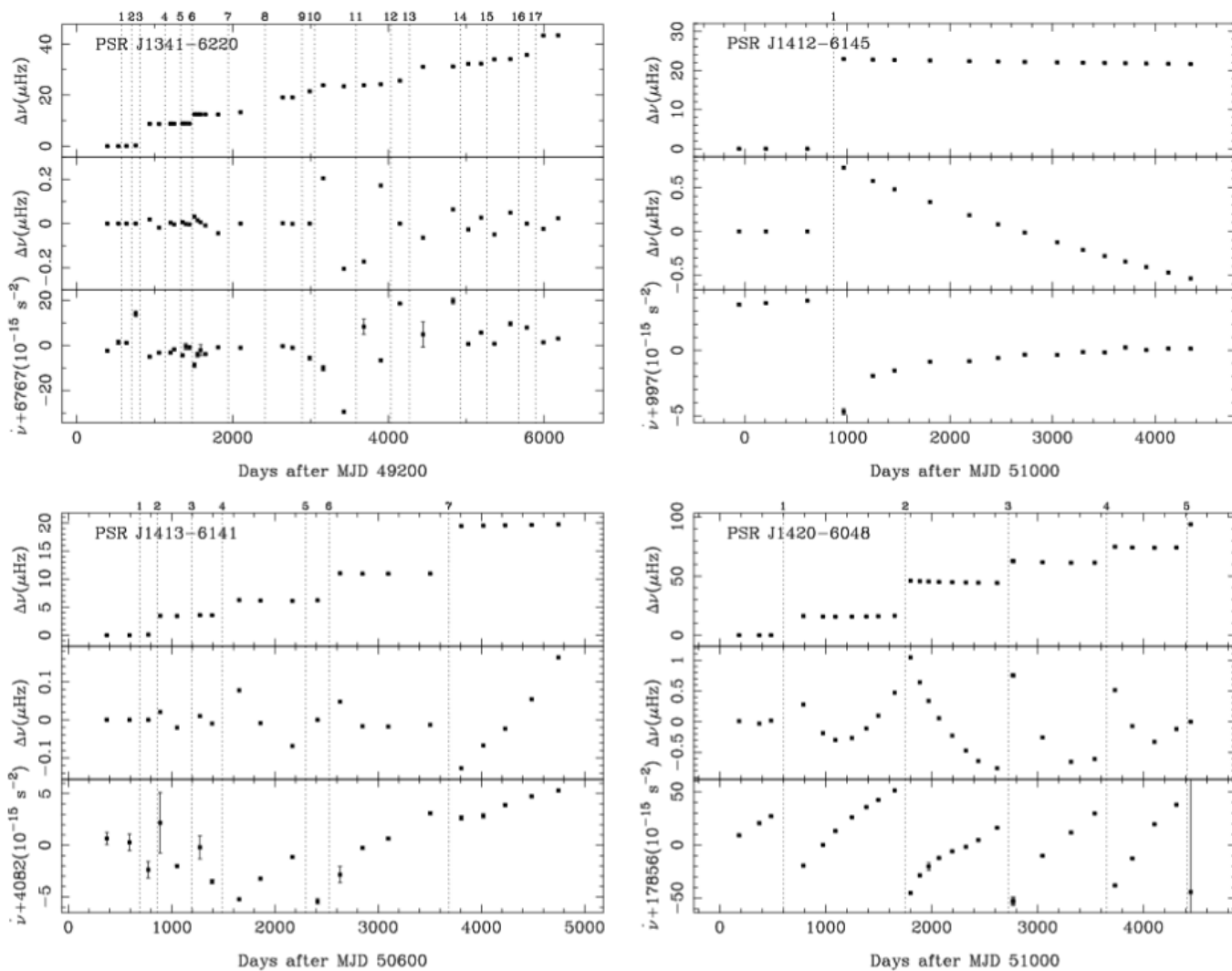
Flux lines density is  $5 \cdot 10^{18} \text{ B}_{12} \text{ cm}^{-2}$



Neutron vortices are confined in the crust.

Proton superfluid is strongly coupled to the crust.

# Glitch discovery and observations



# The largest glitch of the Crab pulsar

2017 November 8

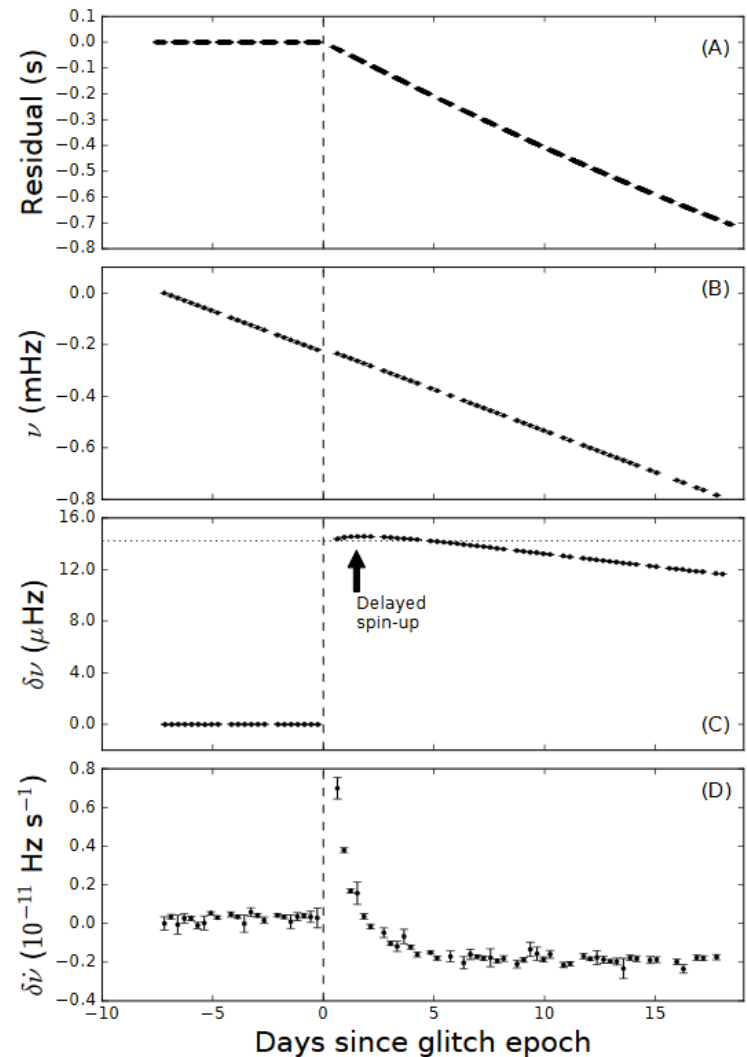
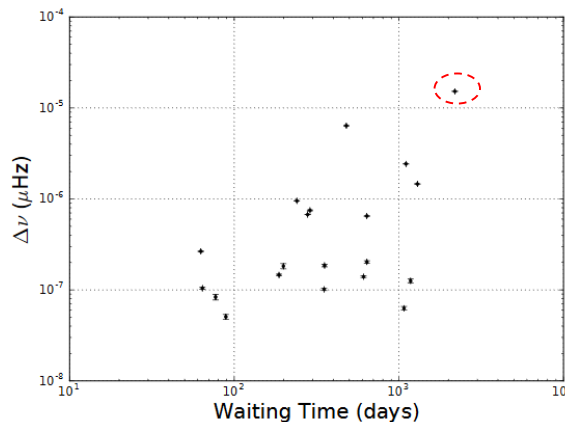
$$\Delta\nu = 1.530 \times 10^{-5} \text{ Hz}$$

$$\Delta\nu/\nu = 0.516 \times 10^{-6}$$

$$\Delta\dot{\nu}/\dot{\nu} = 7 \times 10^{-3}$$

The glitch occurred after the longest period of glitch inactivity – 6 years, – since beginning of daily monitoring in 1984.

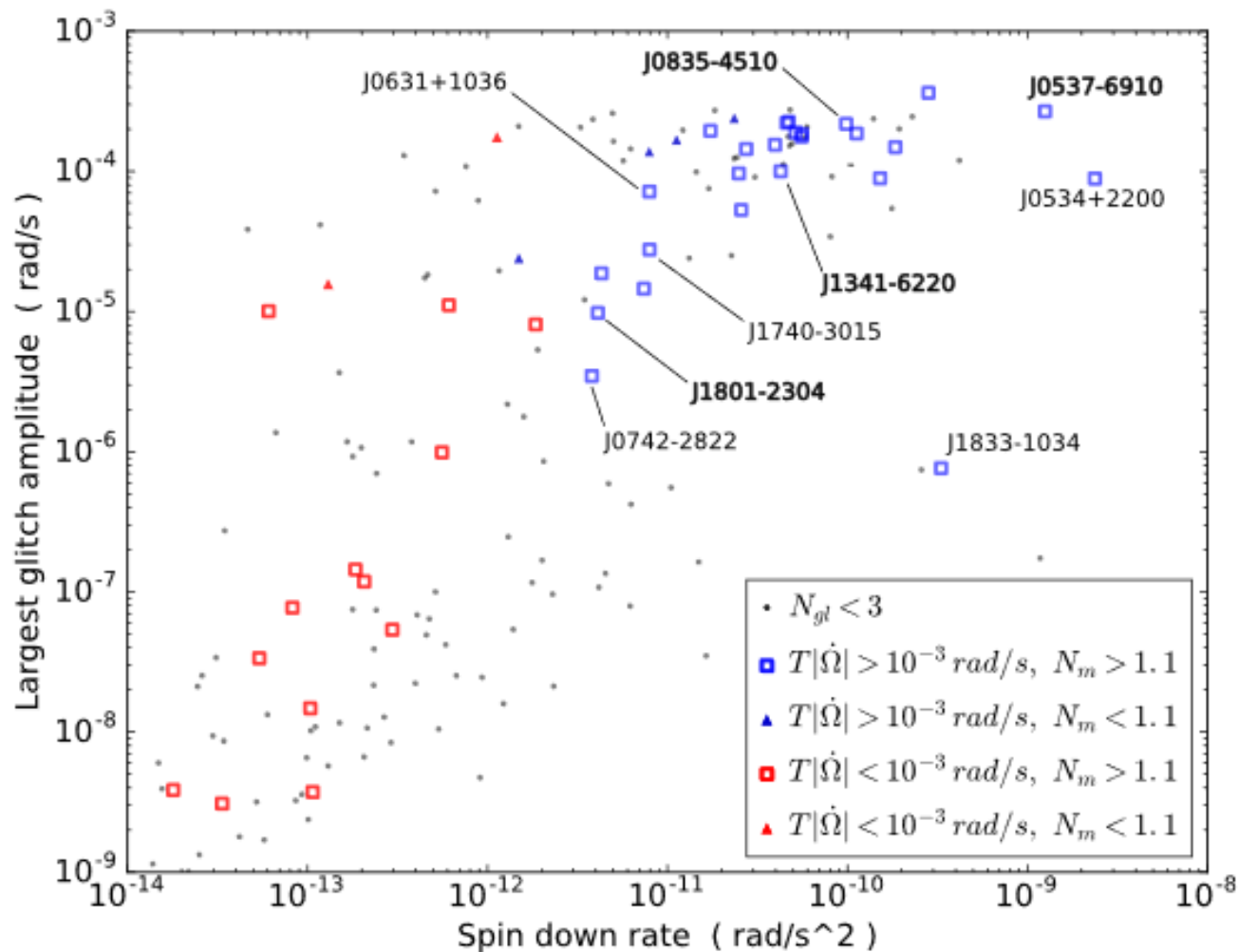
No changes in the shape of the pulse profile, no changes in the X-ray flux.



1805.05110

(See theoretical discussion in 1806.10168)

# Glitch size – spin down rate correlation



# Phenomenology and the Vela pulsar

$$\Delta J_i = I_c \Delta \Omega_i,$$

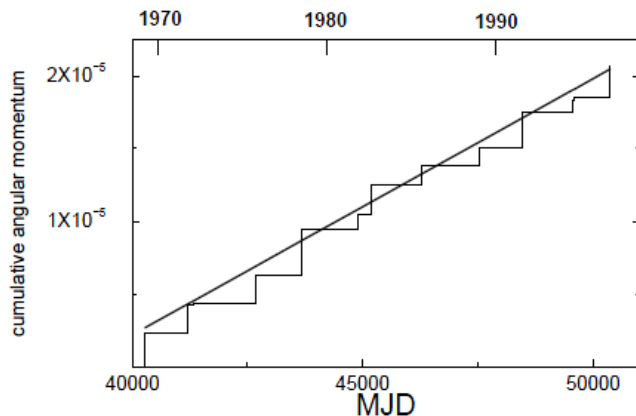
Glitches are driven by the portion of the liquid interior that is differentially rotating with respect to the crust.

$$J(t) = I_c \bar{\Omega} \sum_i \frac{\Delta \Omega_i}{\bar{\Omega}},$$

$I_c$  – crust + everything coupled with (i.e., nearly all the star, except superfluid neutrons).

The average rate of angular momentum transfer associated with glitches is  $I_c \bar{\Omega} A$ ,

$$A = (6.44 \pm 0.19) \times 10^{-7} \text{ yr}^{-1}. \quad - \text{Pulsar activity parameter}$$



Vela glitches are not random, they appear every ~840 days.

$A$  – the slope of the straight line in the figure.

*(Values are for the Vela PSR)*

In Vela glitches can be related also to the outer core 1806.10168

# General features of the glitch mechanism

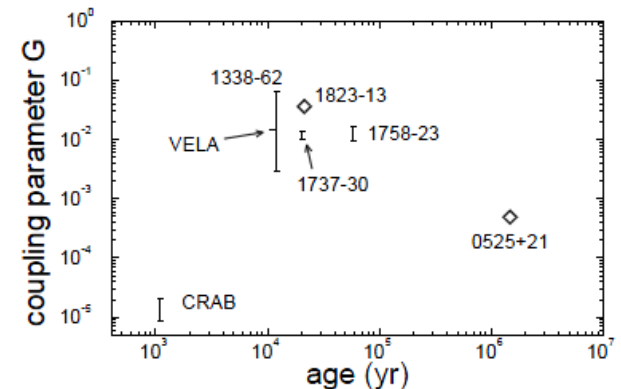
Glitches appear because some fraction (unobserved directly) rotates faster than the observed part (crust plus charged parts), which is decelerated (i.e., which is spinning-down).

$\dot{J}_{\text{res}} \leq I_{\text{res}} |\dot{\Omega}|$ ,      The angular momentum is “collected” by the reservoir, related to differentially rotating part of a star (SF neutrons)

$\frac{I_{\text{res}}}{I_c} \geq \frac{\bar{\Omega}}{|\dot{\Omega}|} A \equiv G$ ,      G – the coupling parameter. It can be slightly different in different sources.

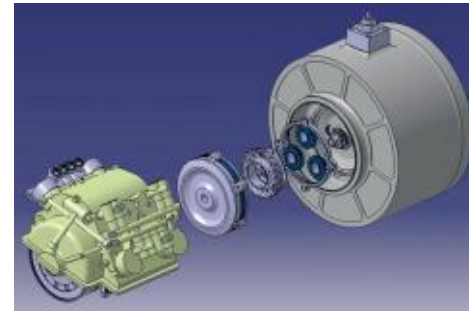
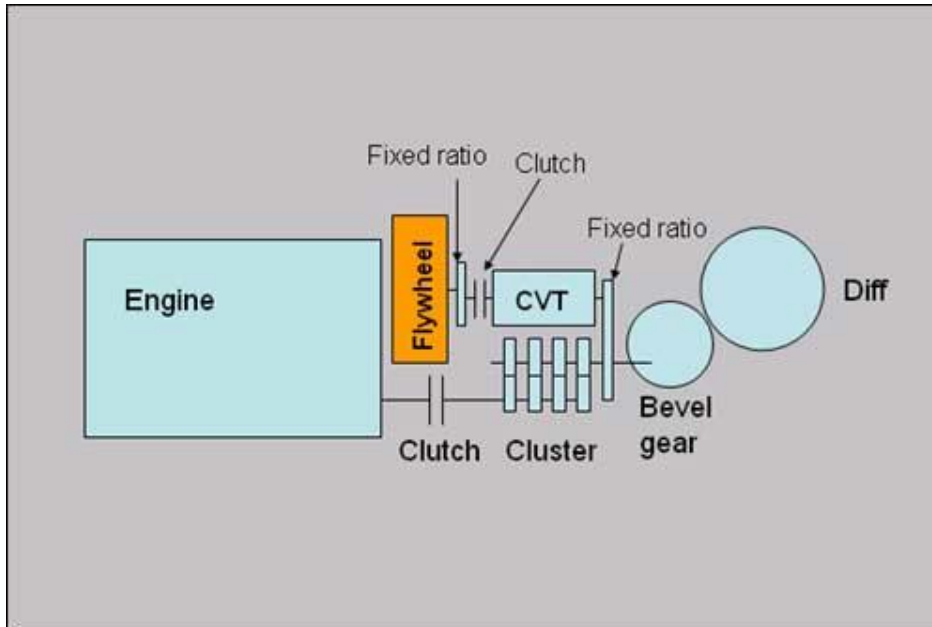
$\frac{I_{\text{res}}}{I_c} \geq G_{\text{Vela}} = 1.4\%$ .      Glitch statistics for Vela provide an estimate for G.

Superfluid is a good candidate to form a “reservoir” because relaxation time after a glitch is very long (~months) which points to very low viscosity.

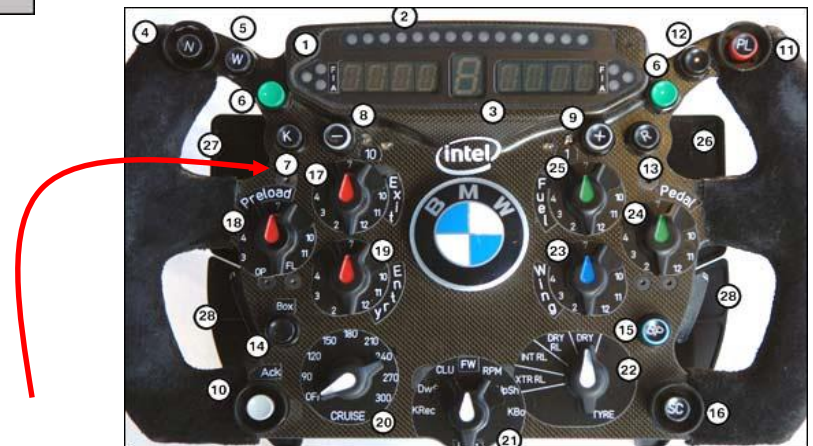




# KERS



Williams-F1 used mechanical KERS. Energy is stored in a flywheel.



# Critical velocity difference

In most popular models glitches appear when the difference in angular velocity between the crust and the superfluid reaches some critical value.

$$I_{\text{super}}/I_{\text{crust}} \sim 10^{-2}$$

$$\Delta\Omega/\Omega \sim 10^{-6}$$

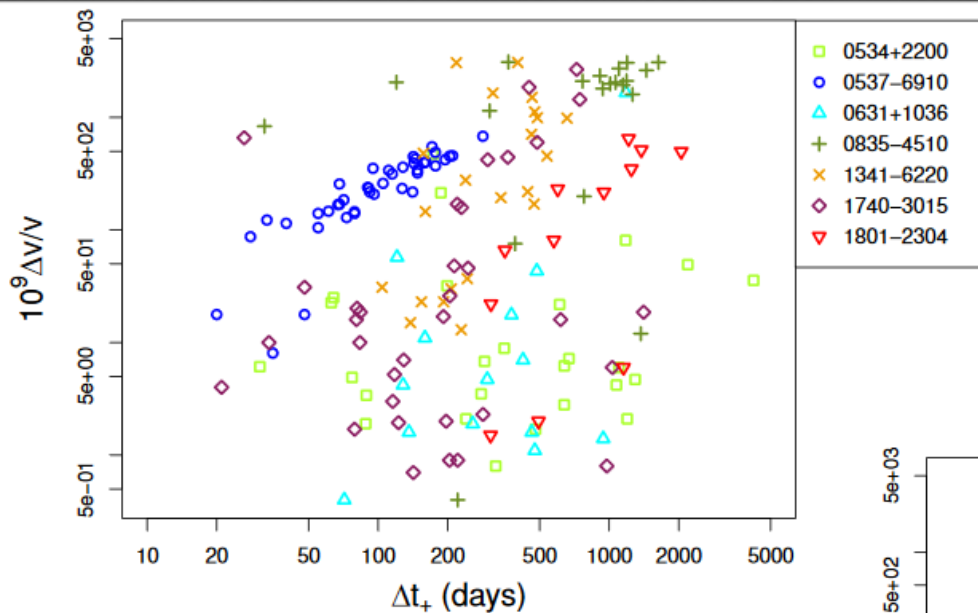
$\Delta\Omega$  – is for the crust (we see it!)

$$\Delta\Omega I_{\text{crust}} = \Delta\Omega_{\text{super}} I_{\text{super}}$$

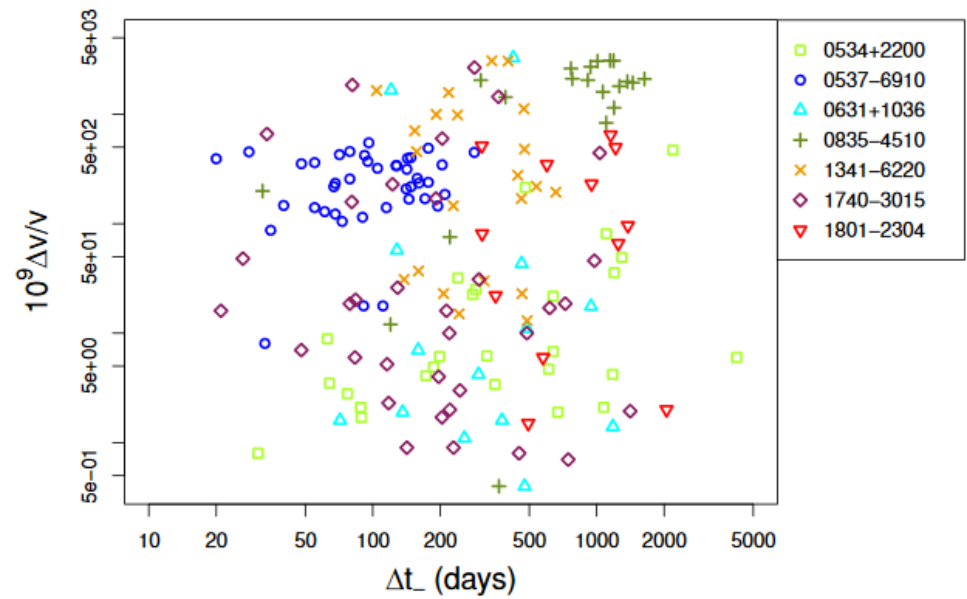
$$\Delta\Omega_{\text{super}} = \Delta\Omega I_{\text{crust}}/I_{\text{super}} = \Omega 10^{-6} 10^2 = 10^{-4} \Omega$$

---

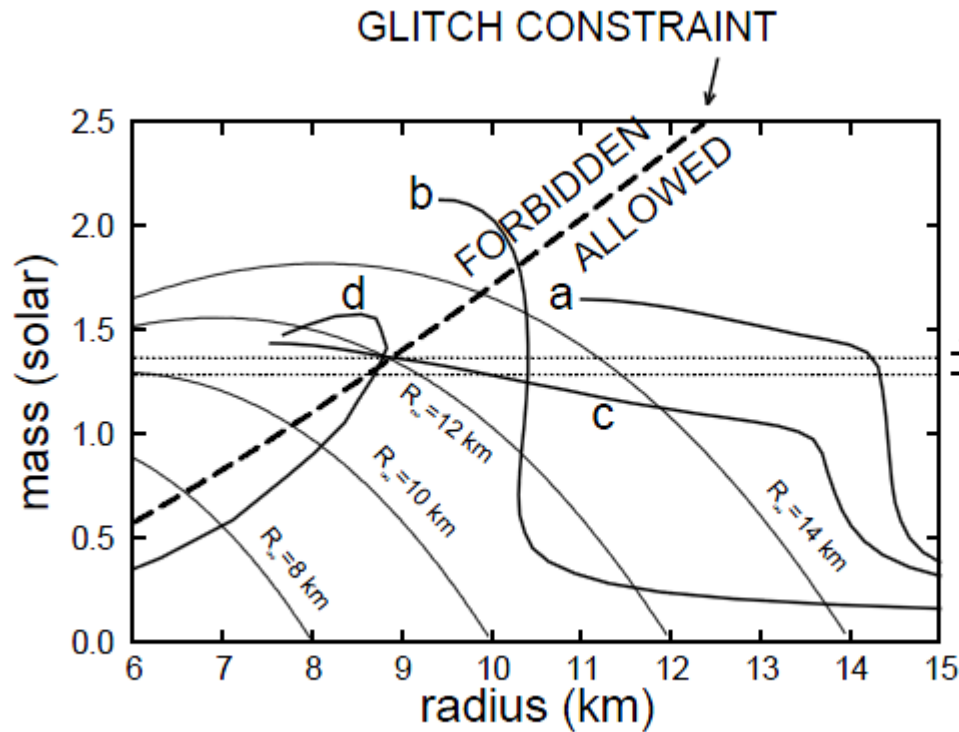
# Glitch size – waiting time correlation



No correlation of a glitch size with time since the previous glitch, or with time before the next one.



# EoS and glitches



$$R = 3.6 + 3.9M/M_{\odot}.$$

↓ maximum  
likelihood  
mass range  
↑

$P_t = 0.65 \text{ MeV fm}^{-3}$   
 $n_t = 0.075 \text{ fm}^{-3}$   
 pressure and density  
 on the core-crust boundary.

$$\Lambda \equiv (1 - 2GM/Rc^2)^{-1}$$

$$\frac{\Delta I}{I} \simeq \frac{28\pi}{3} \frac{P_t R^4}{GM^2} \left[ 1 + \frac{8P_t}{n_t m_n c^2} \frac{4.5 + (\Lambda - 1)^{-1}}{\Lambda - 1} \right]^{-1}$$

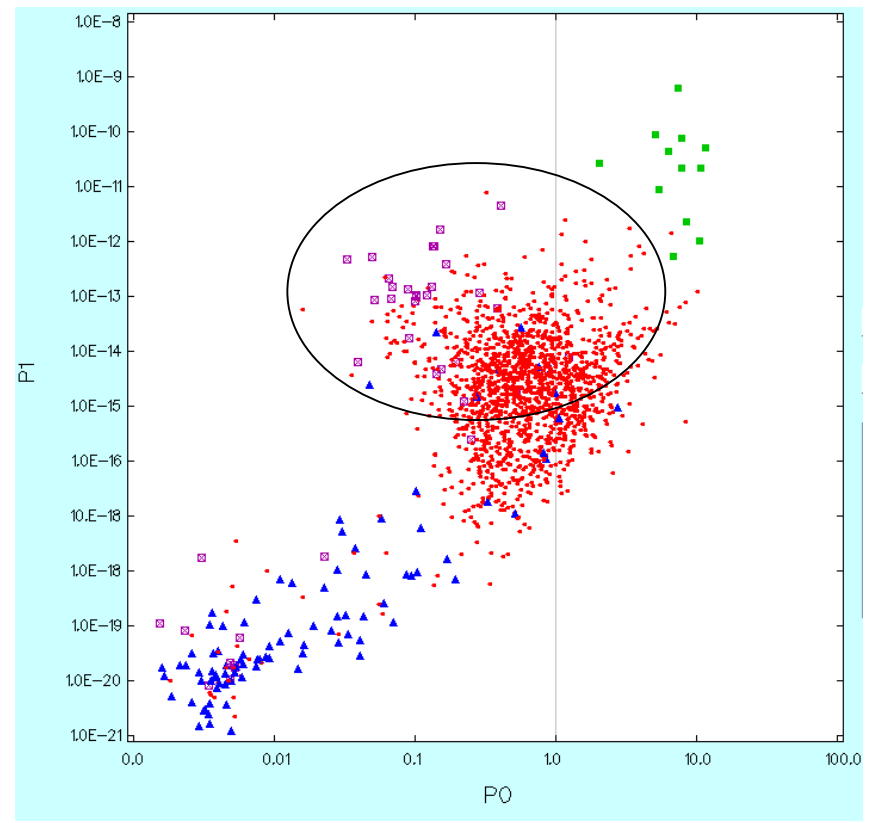
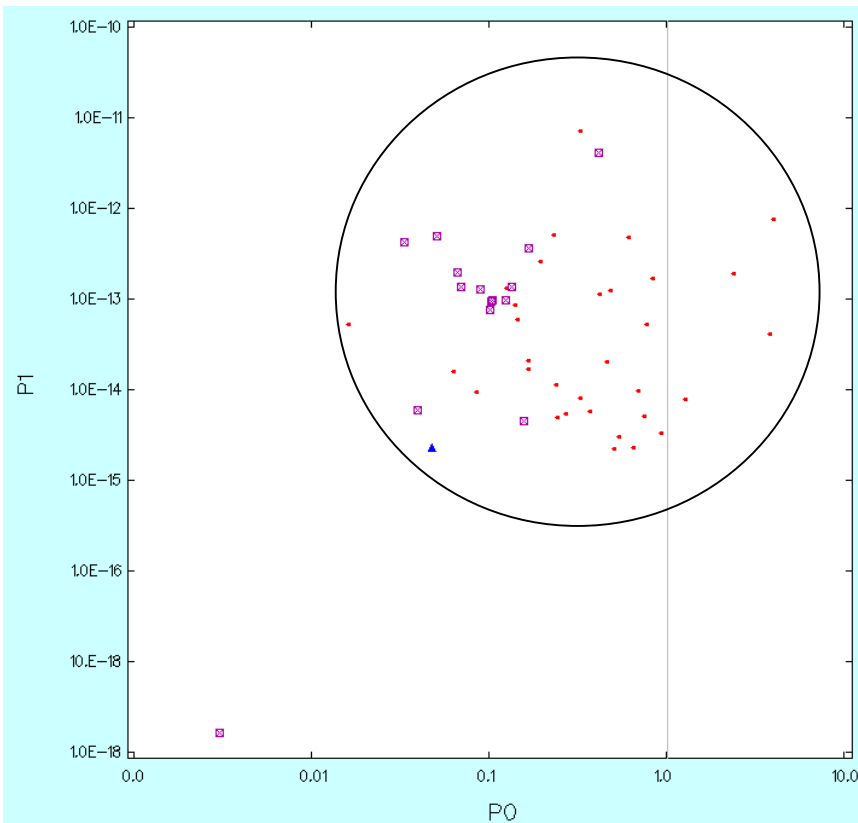
$$\Delta I / (I - \Delta I) \geq \Delta I / I_c \geq I_{\text{res}} / I_c \geq 0.014.$$

Link et al. 0001245

See some critics in 1207.0633 “Crust is not enough” and 1210.8177  
 Further discussion – in 1404.2660, 1809.07834.

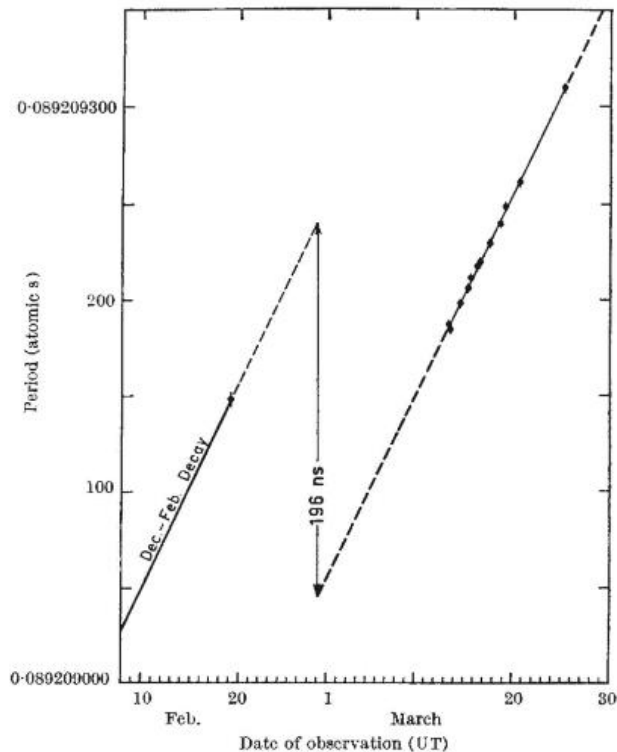
# Which PSRs do glitch?

On average young pulsars with larger spin-down glitch more frequently

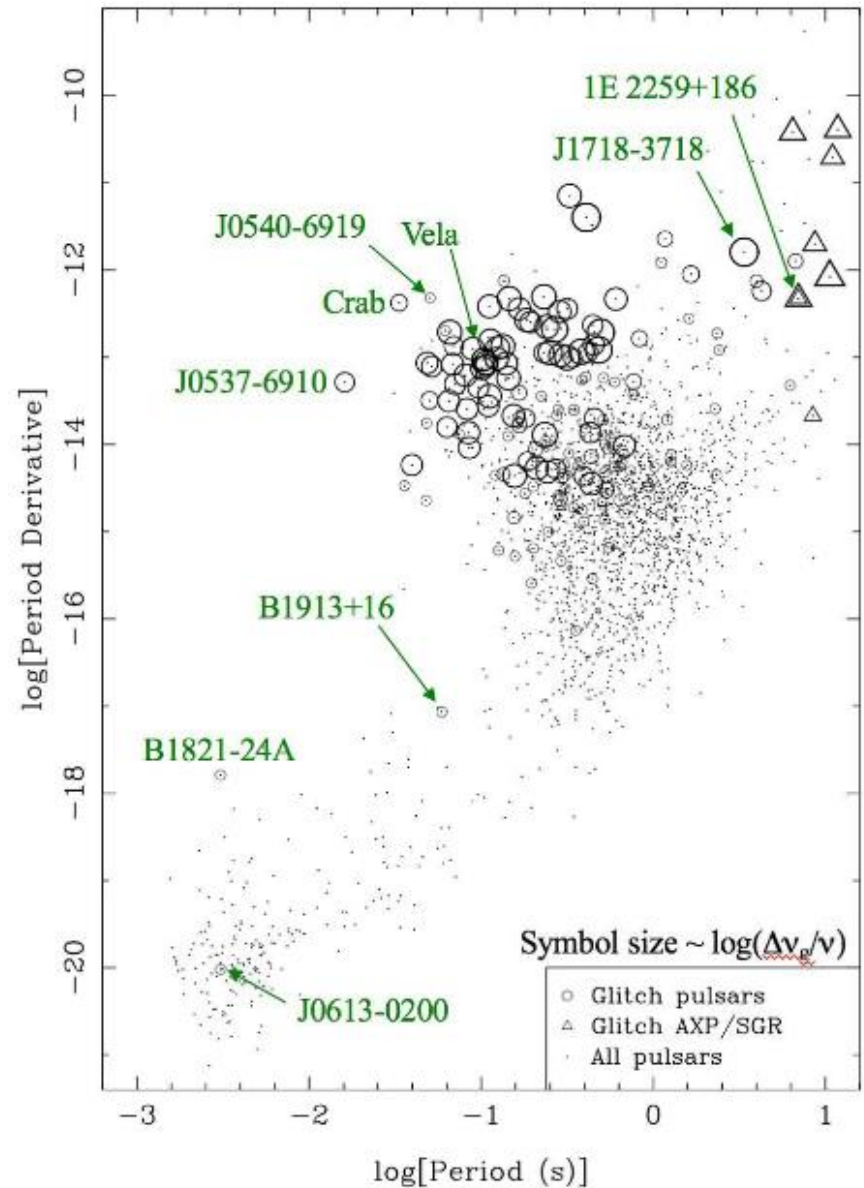


# P-Pdot

>520 glitches  
in >180 PSRs

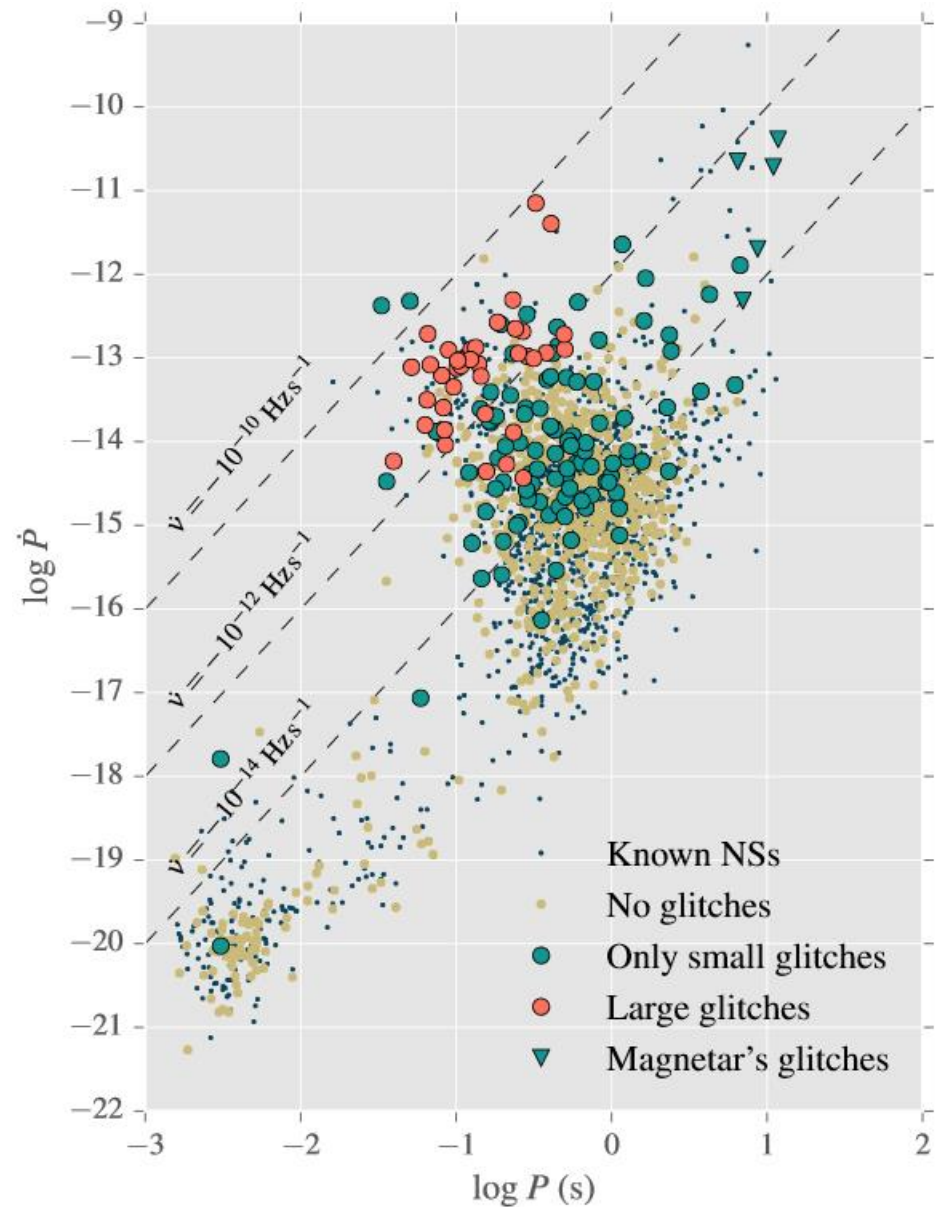
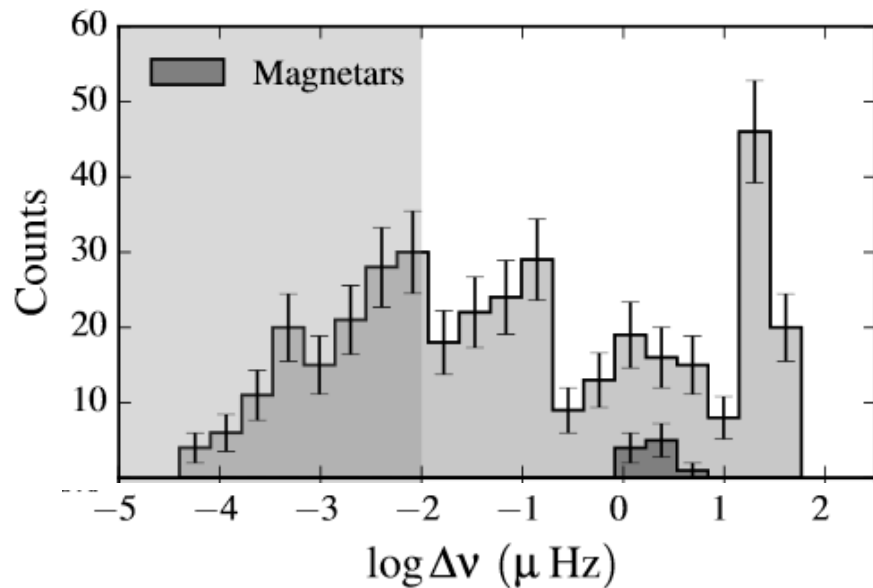


Vela glitch in 1969



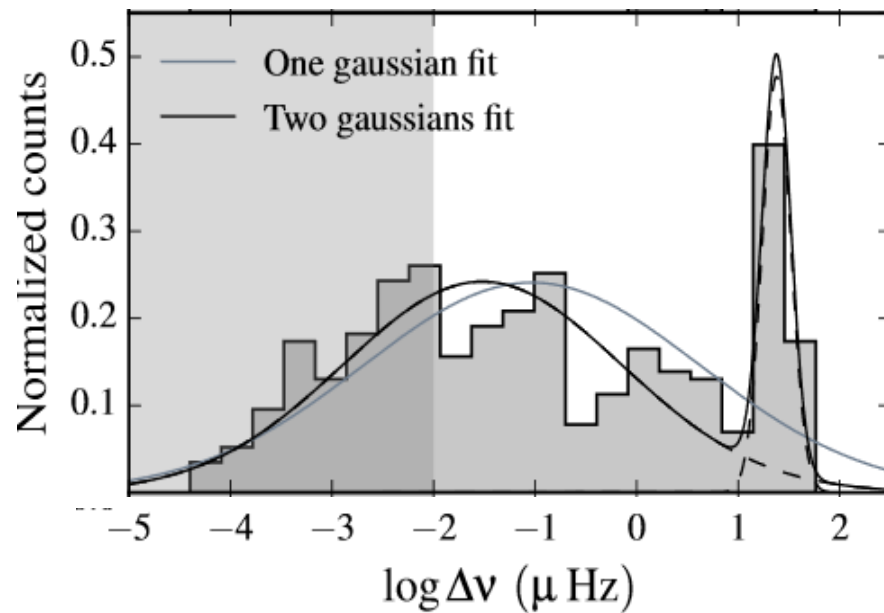
# Statistics

384 glitches in 141 NSs

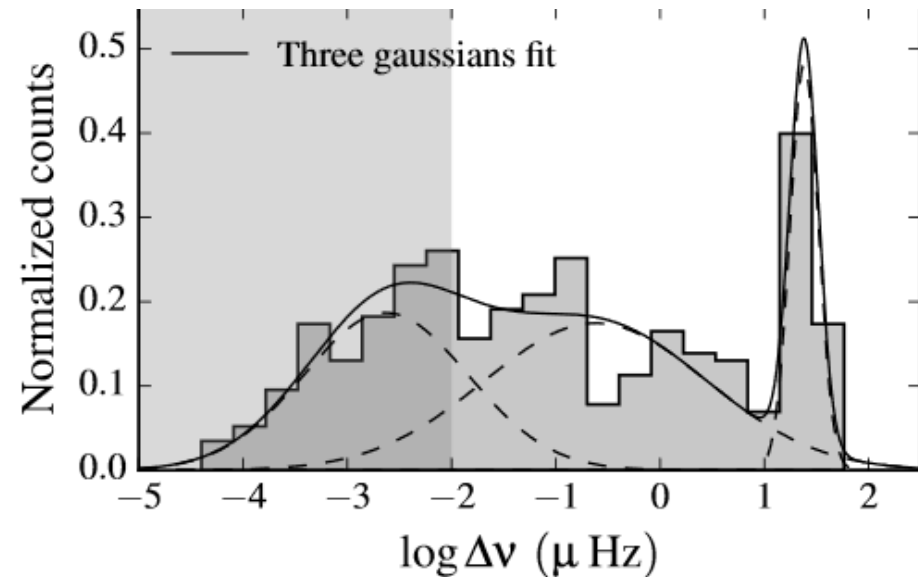




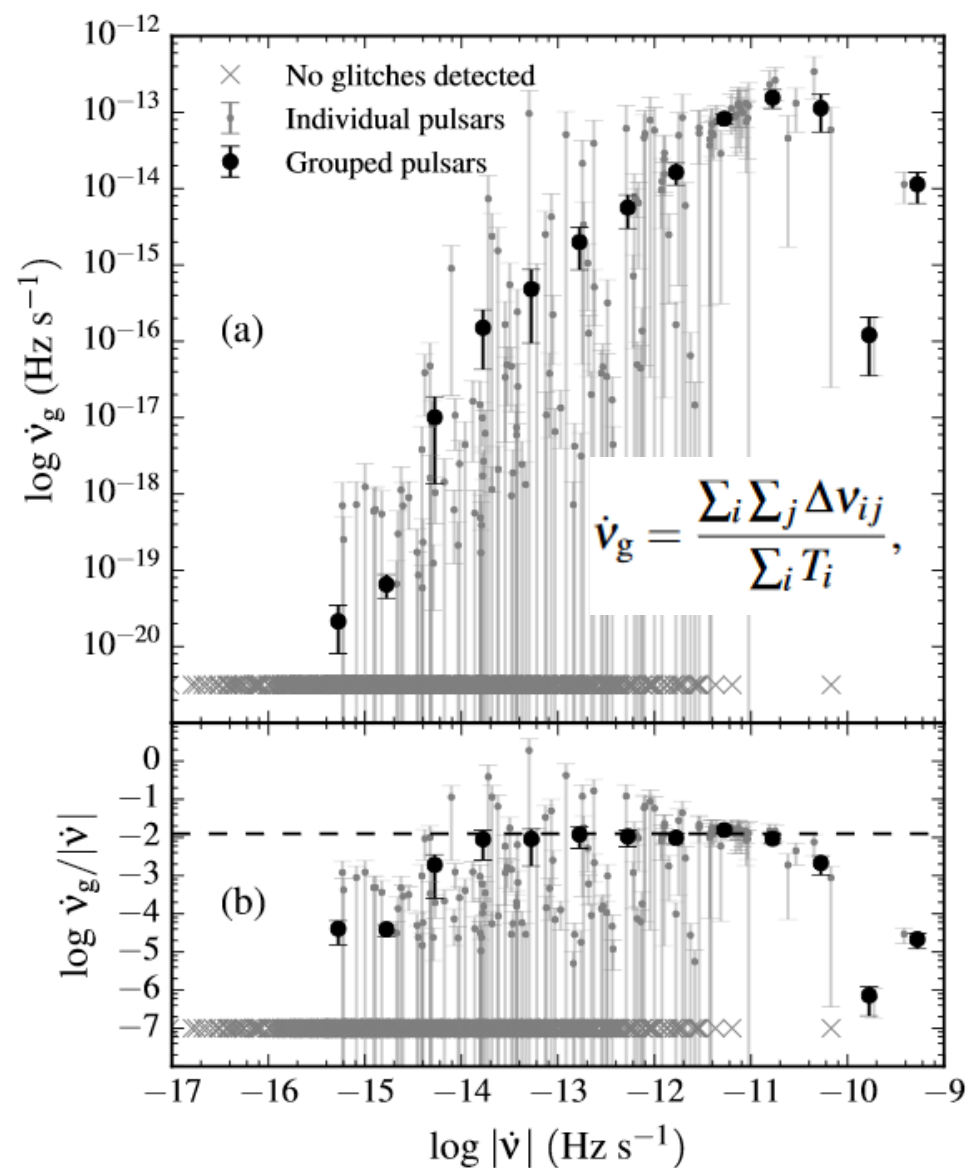
# Glitches properties

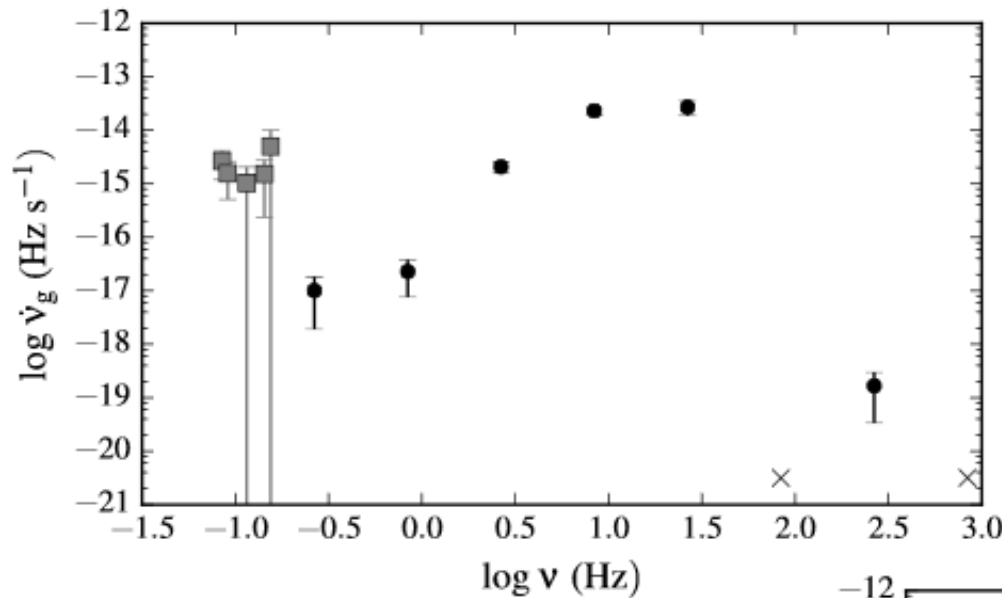


384 glitches in 141 NSs



# bin	$\log  \dot{\nu} $ (Hz s <sup>-1</sup> )	$\sum T_i$ (yr)	$N_\ell$	$N_t$	$N_{\text{pg}}$	$N_p$
1	-16.75	117	0	0	0	7
2	-16.25	430	0	0	0	25
3	-15.75	1233	0	0	0	70
4	-15.25	2478	0	3	3	139
5	-14.75	2675	0	11	8	142
6	-14.25	1973	0	25	16	105
7	-13.75	2083	0	35	20	113
8	-13.25	1706	1	29	18	105
9	-12.75	1312	3	26	14	81
10	-12.25	745	4	38	15	48
11	-11.75	493	8	74	15	33
12	-11.25	357	37	78	18	20
13	-10.75	66	13	19	5	5
14	-10.25	44	4	8	2	3
15	-9.75	16	0	2	1	1
16	-9.25	46	0	25	1	1

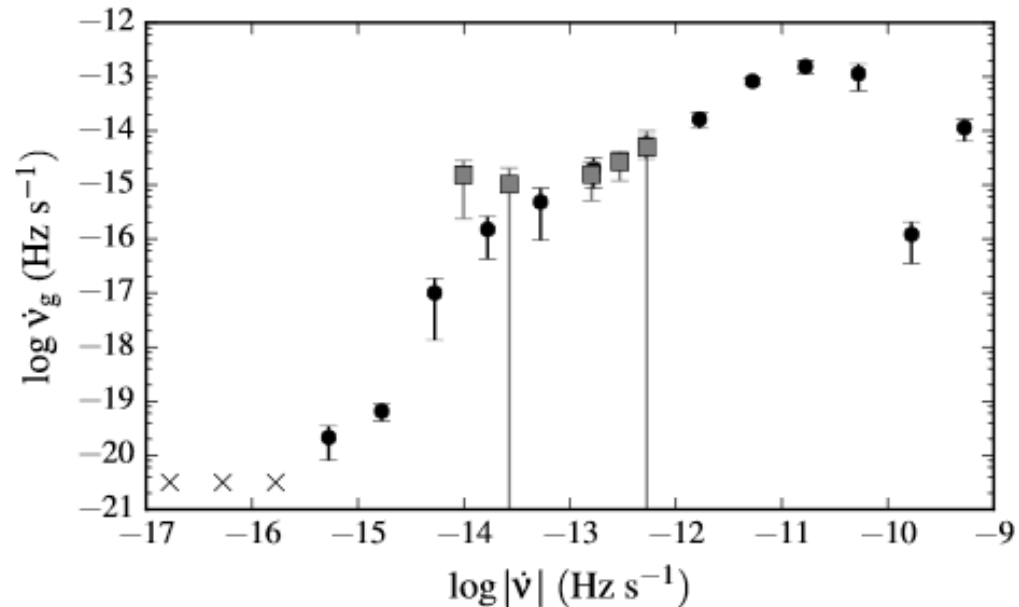




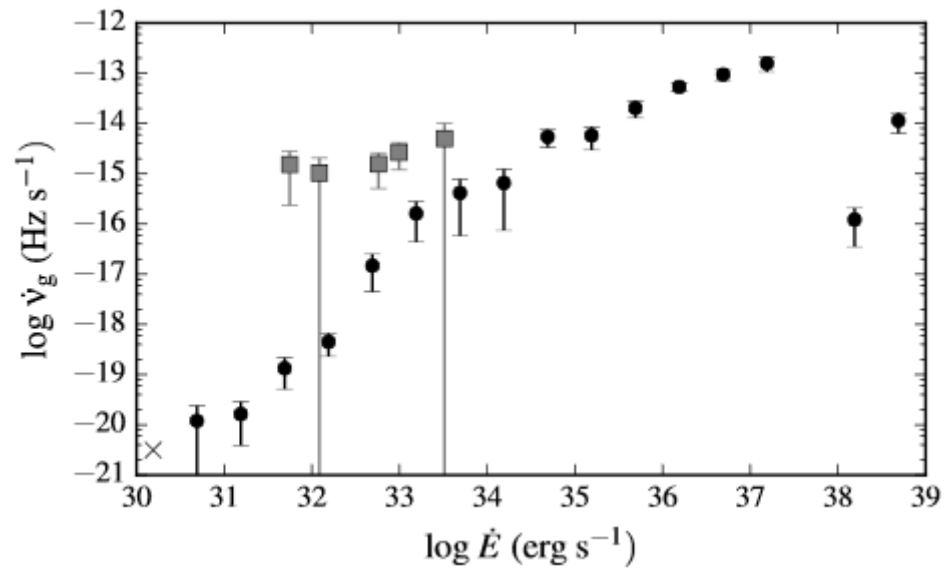
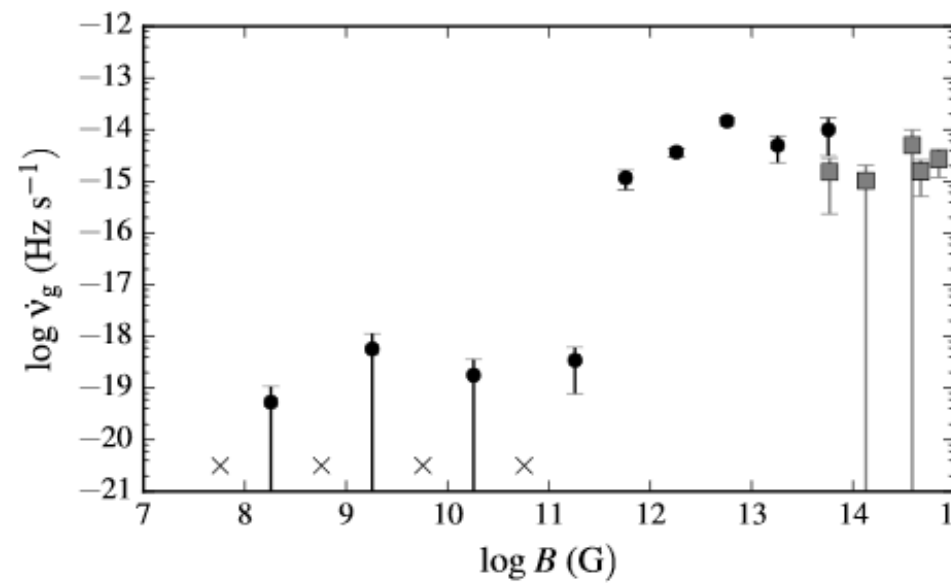
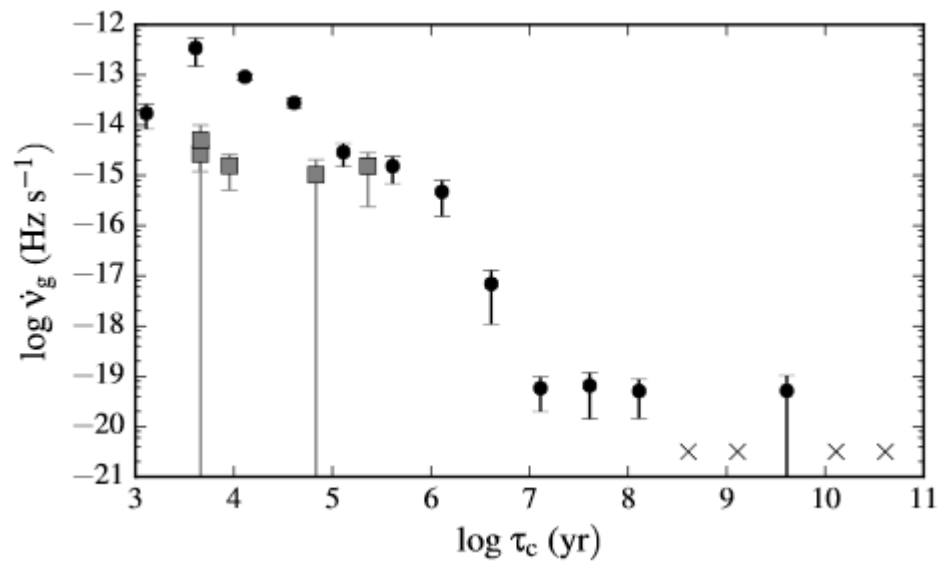
All PSRs in the survey are included, also those with no detected glitches.

$$\dot{\nu}_g = \frac{\sum_i \sum_j \Delta \nu_{ij}}{\sum_i T_i},$$

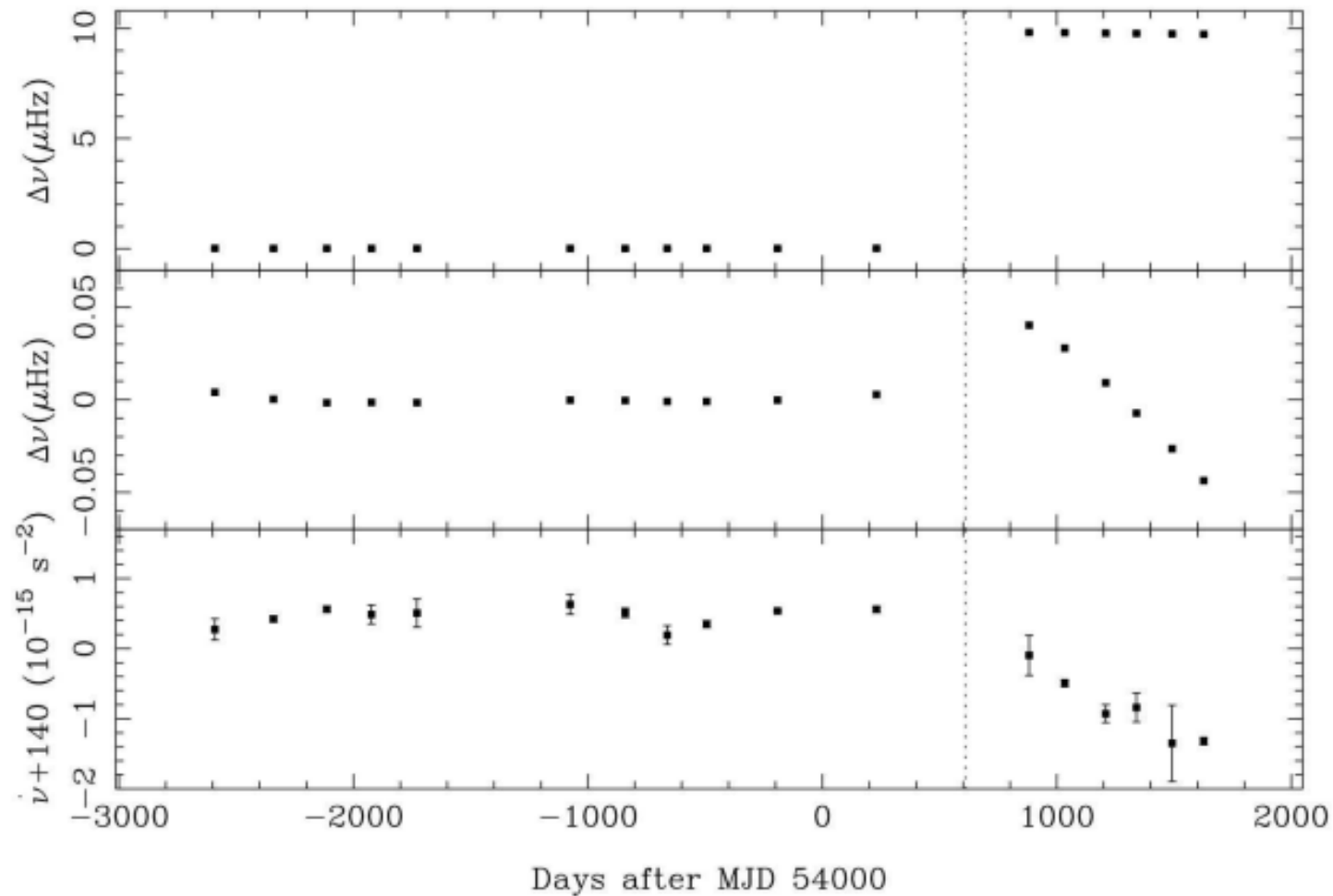
The double sum runs over every change in frequency  $\Delta \nu_{ij}$  due to the glitch  $j$  of the pulsar  $i$ , and  $T_i$  is the time over which pulsar  $i$  has been searched for glitches.



$$\dot{v}_g = \frac{\sum_i \sum_j \Delta v_{ij}}{\sum_i T_i},$$

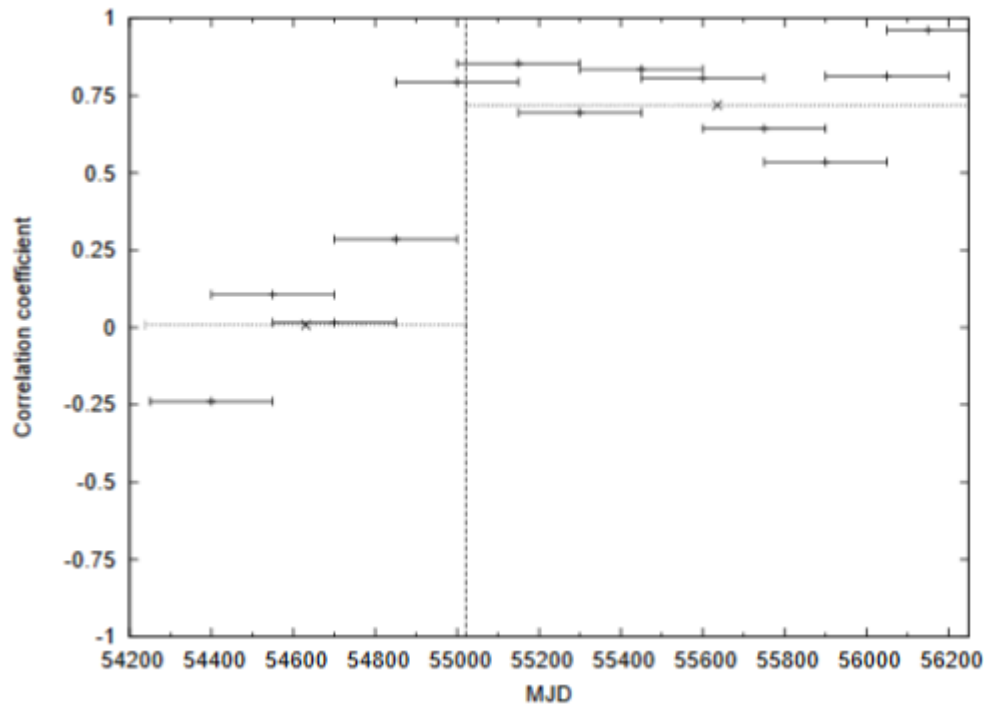


# The largest glitch



$33 \cdot 10^{-6}$

# Glitch and radio properties



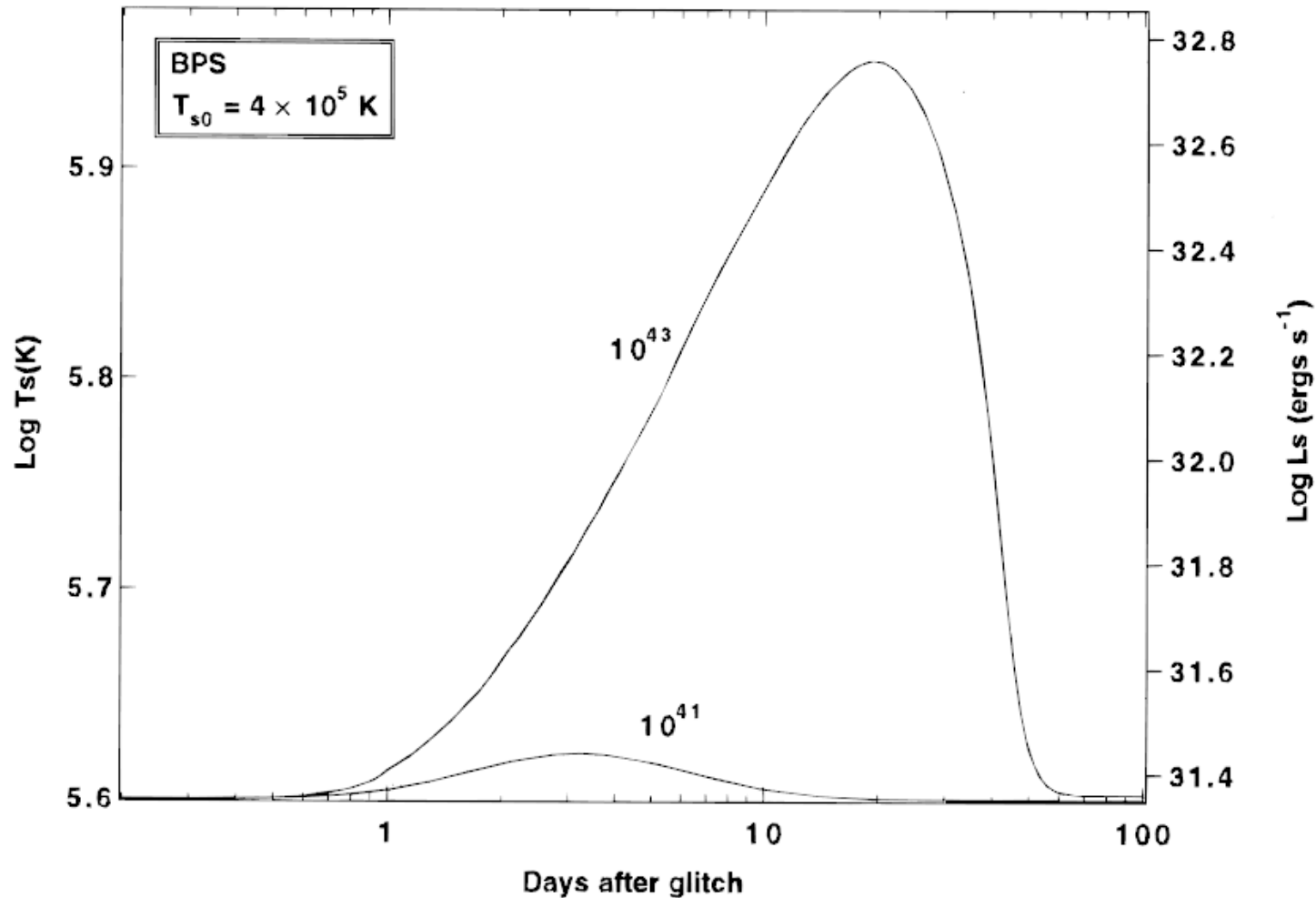
PSR J0742-2822

exhibits two distinct emission states that are identified by discrete changes in the observed pulse profile.

Correlation between frequency derivative and smoothed pulse shape parameter for overlapping 300-day intervals. The vertical dashed line at MJD 55022 indicates the epoch of a glitch.

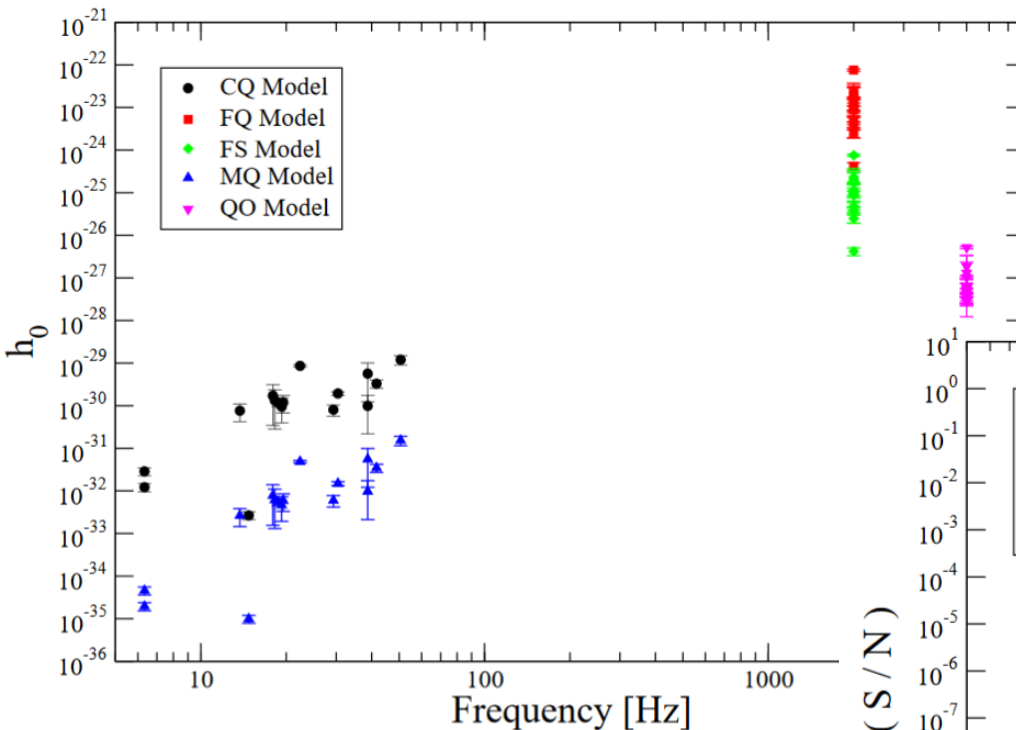
Also shown with dotted bars is the same correlation when computed for the entire pre and post-glitch epochs.

# Thermal effect of a glitch

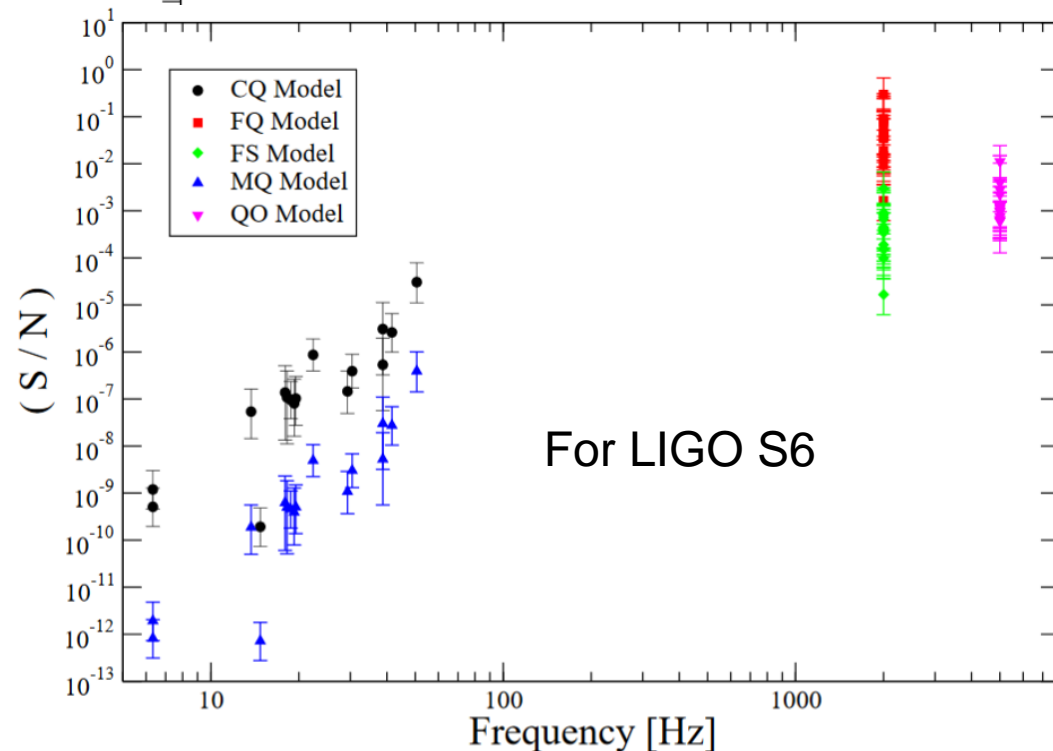




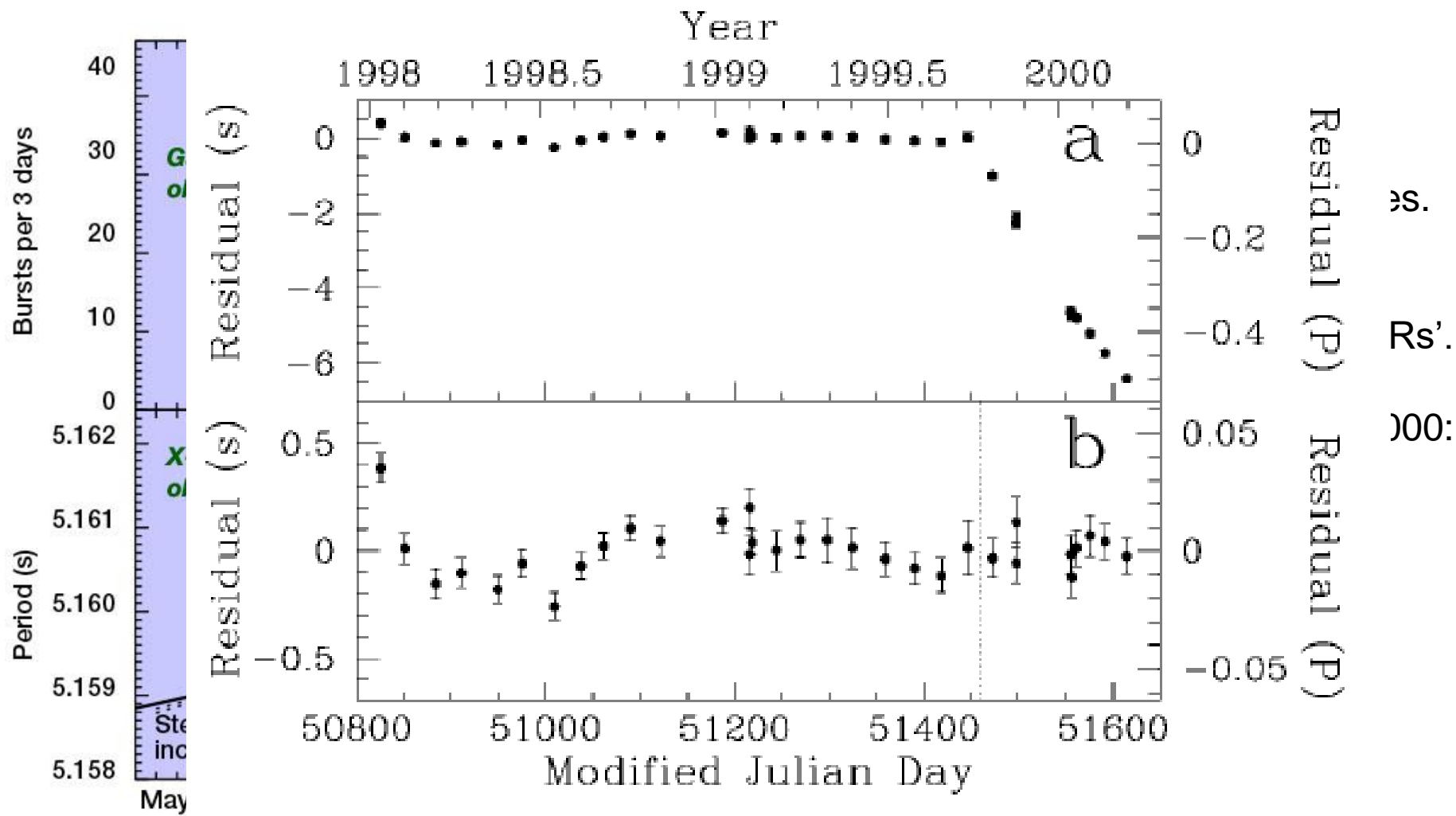
# Gravitational waves from glitches



In some very optimistic models GW signals from PSRs glitches can be detected already with existing detectors (aLIGO, adVIRGO).

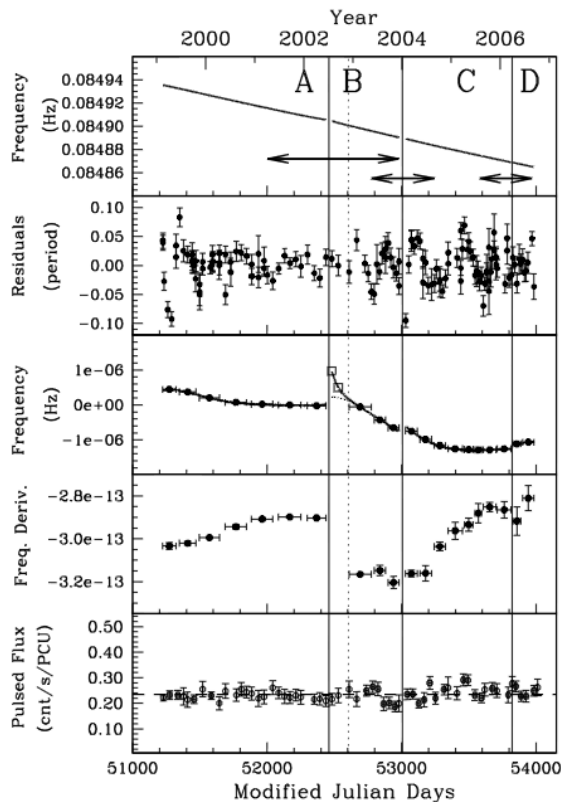


# Glitches of magnetars

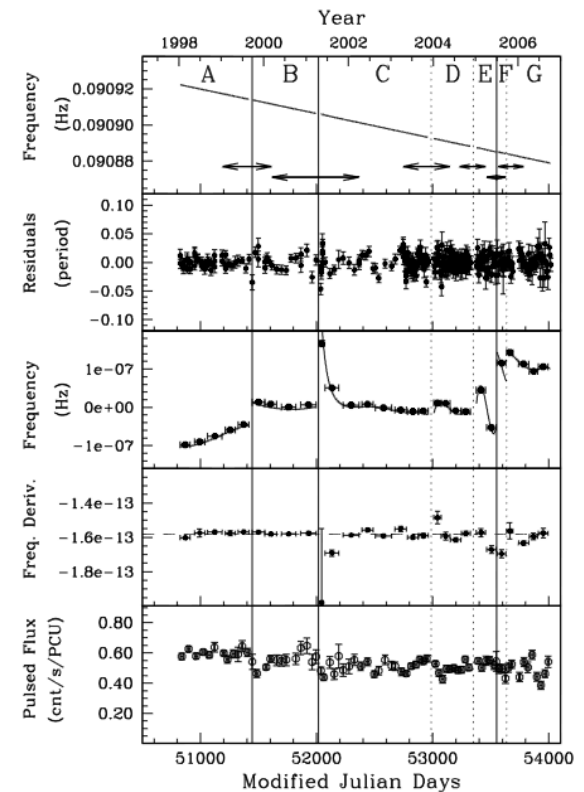


# Glitches and bursts

Sometime magnetar glitches are related to bursts, sometime – not.



The pulsed flux was nearly constant during glitches.

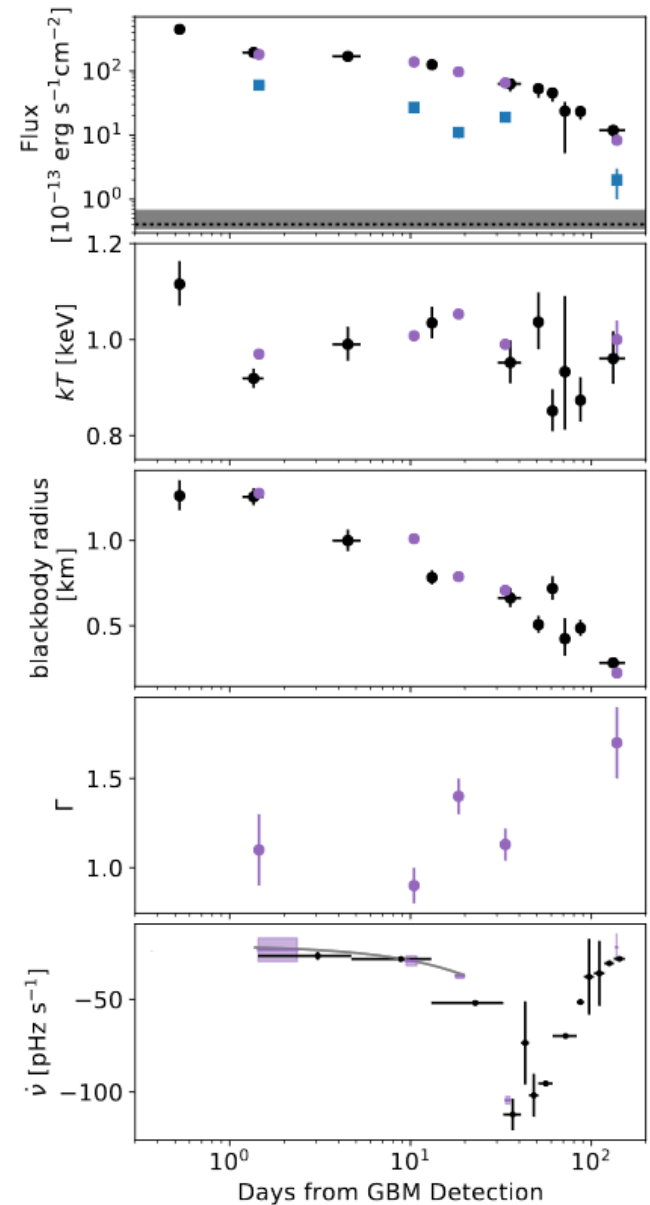
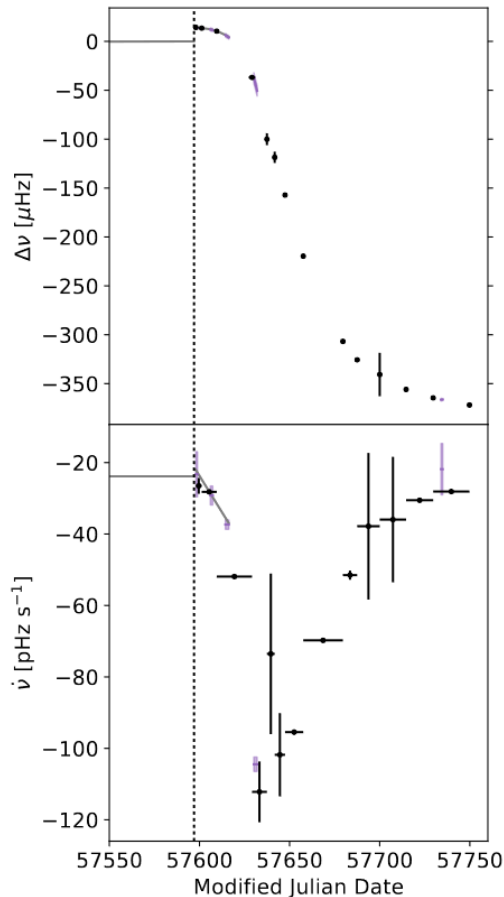


# Glitch and bursts from PSR J1119–6127

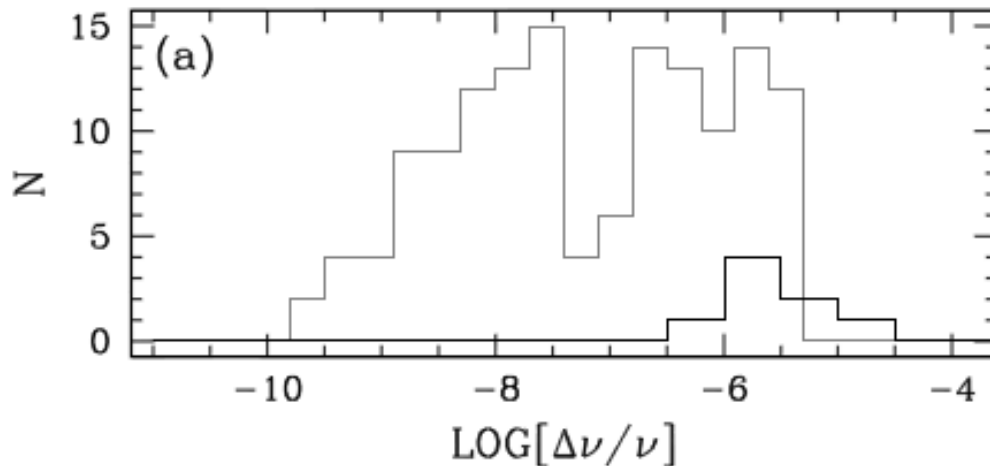
Young  
highly magnetizes  
radio pulsar.

Outburst with  
many flares.

Glitch properties  
confirm the model  
of magnetospheric  
perturbation and  
energy release.  
Spin behavior  
correlates with  
pulse profile  
and spectral changes.

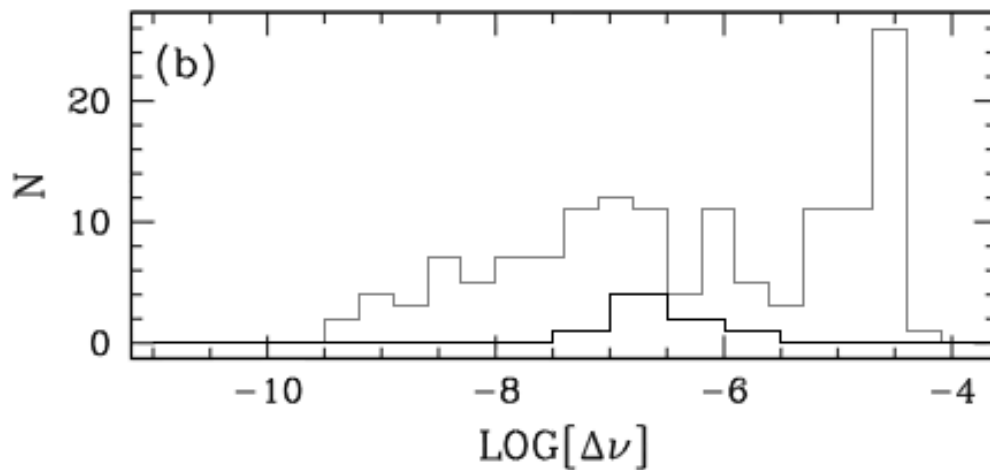


# PSRs vs. magnetars



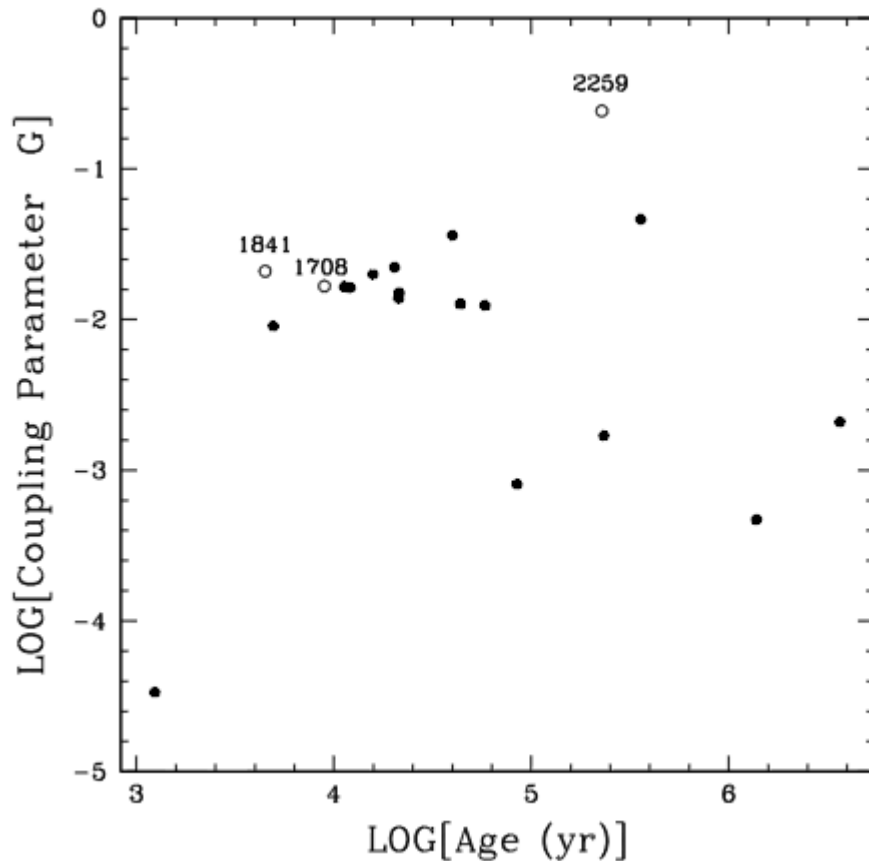
Nearly all known persistent AXPs now seem to glitch.

In terms of fractional frequency change, AXPs are among the most actively glitching neutron stars, with glitch amplitudes in general larger than in radio pulsars.



However, in terms of absolute glitch amplitude, AXP glitches are unremarkable.

# Are PSRs and magnetar glitches similar?



$$\dot{J}_{\text{res}} \leq I_{\text{res}} |\dot{\Omega}|, \quad \frac{I_{\text{res}}}{I_c} \geq \frac{\bar{\Omega}}{|\dot{\Omega}|} A \equiv G,$$

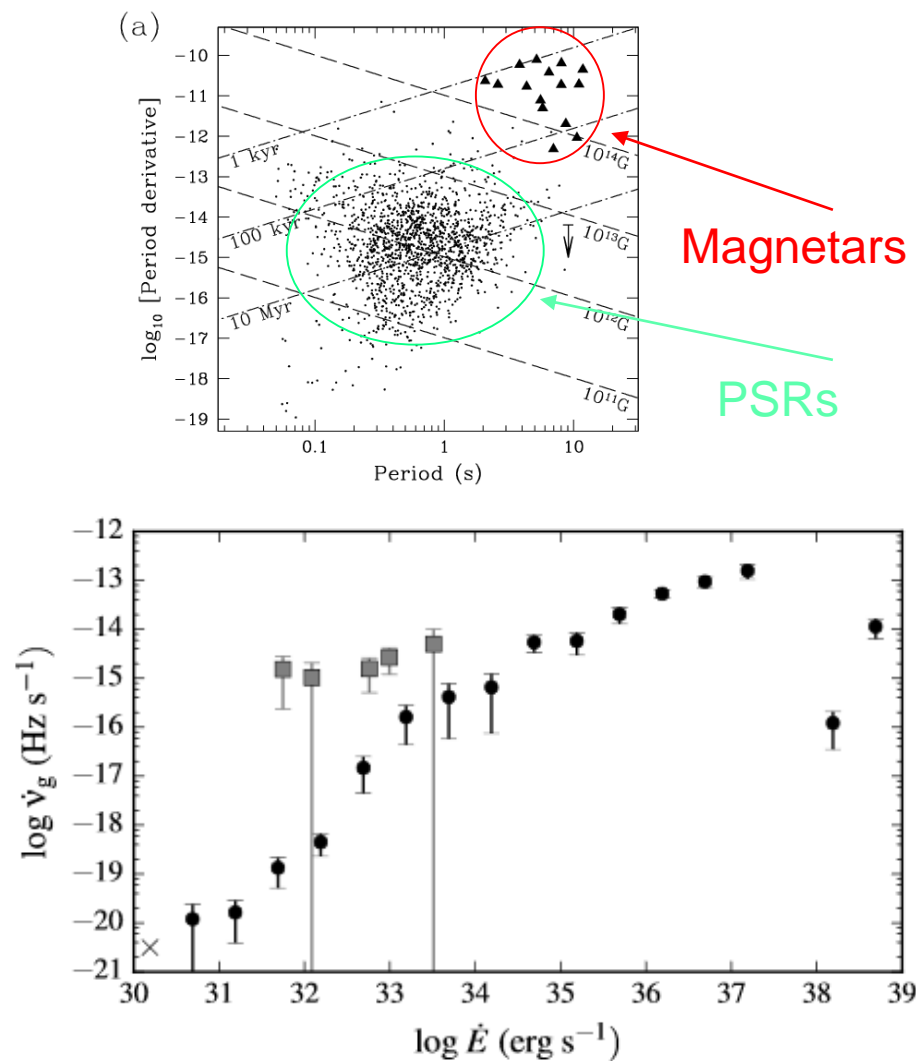
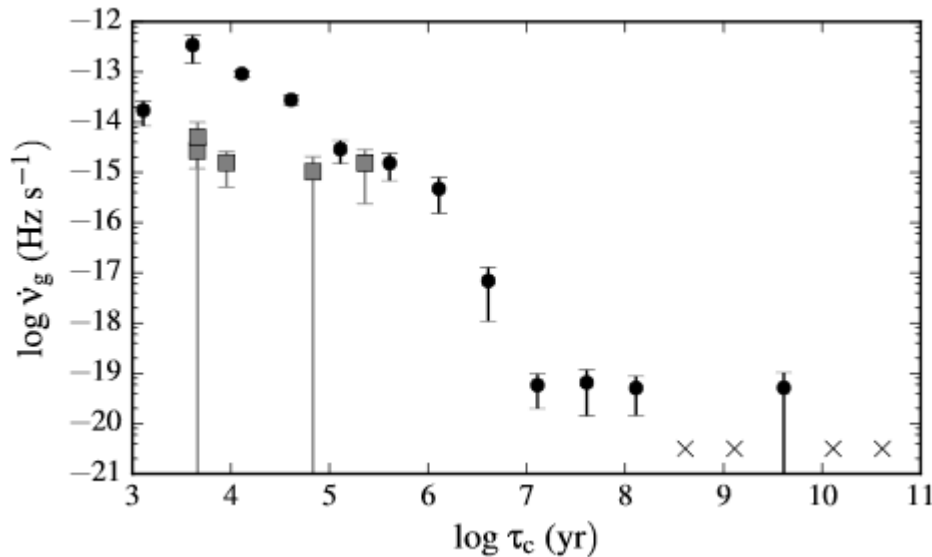
$$\frac{I_{\text{res}}}{I_c} \geq G_{\text{Vela}} = 1.4\%.$$

It seems that for some AXP glitches  $G$  is much larger than for PSRs. Dib et al. propose that it can be related to the role of core superfluid.

Many others proposed that glitches of magnetars can be related to magnetic field dissipation in the crust. As the field can be dynamically important there, its decay can result in crust cracking.

# PSRs vs. Magnetars

Glitch activity of the magnetars with the smallest characteristic ages is lower than that of the rotation-powered pulsars with similar characteristic ages. However, their activity is larger than that of pulsars of equal spin-down power.





# CCOs also glitch!

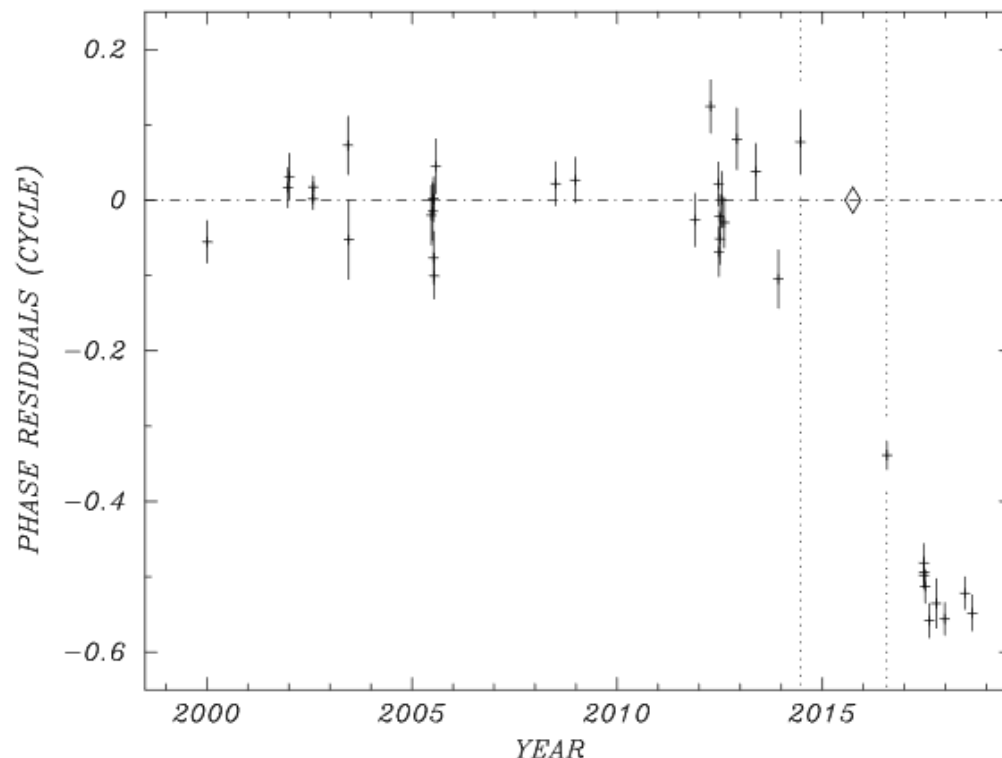
1E 1207.4–5209

0.424 s

$B_s \approx 9 \times 10^{10}$  G

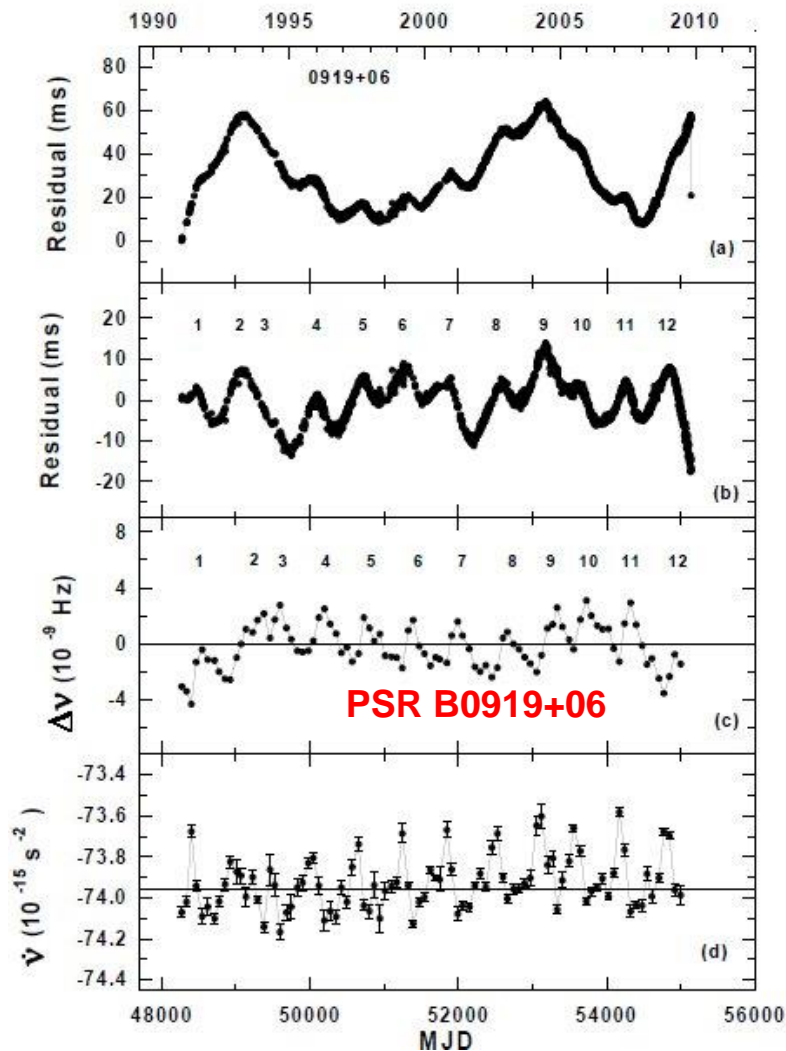
$\Delta f/f = (2.8 \pm 0.4) \times 10^{-9}$

Parameter	Pre-glitch <sup>a</sup>	Post-glitch <sup>b</sup>
Epoch	2012	2017
$N_H$ (cm <sup>-2</sup> ) (fixed)	$1.66 \times 10^{21}$	$1.66 \times 10^{21}$
$kT_1$ (keV)	$0.0801^{+0.0046}_{-0.0042}$	$0.0742^{+0.0042}_{-0.0037}$
$kT_2$ (keV)	$0.2513^{+0.0022}_{-0.0021}$	$0.2509^{+0.0021}_{-0.0021}$
$E_0$ (keV)	$0.712^{+0.012}_{-0.011}$	$0.710^{+0.017}_{-0.015}$
$\sigma_0$ (keV) (fixed)	0.08	0.08
$\tau_0$	0.26	0.22
$E_1$ (keV)	$1.4292^{+0.0087}_{-0.0088}$	$1.4216^{+0.0089}_{-0.0090}$
$\sigma_1$ (keV) (fixed)	0.08	0.08
$\tau_1$	0.098	0.10
$F_x$ (pn) <sup>c</sup>	$2.084^{+0.010}_{-0.011}$	$2.078^{+0.010}_{-0.012}$
$\chi^2_\nu$ (DoF)	1.39(283)	1.32(276)

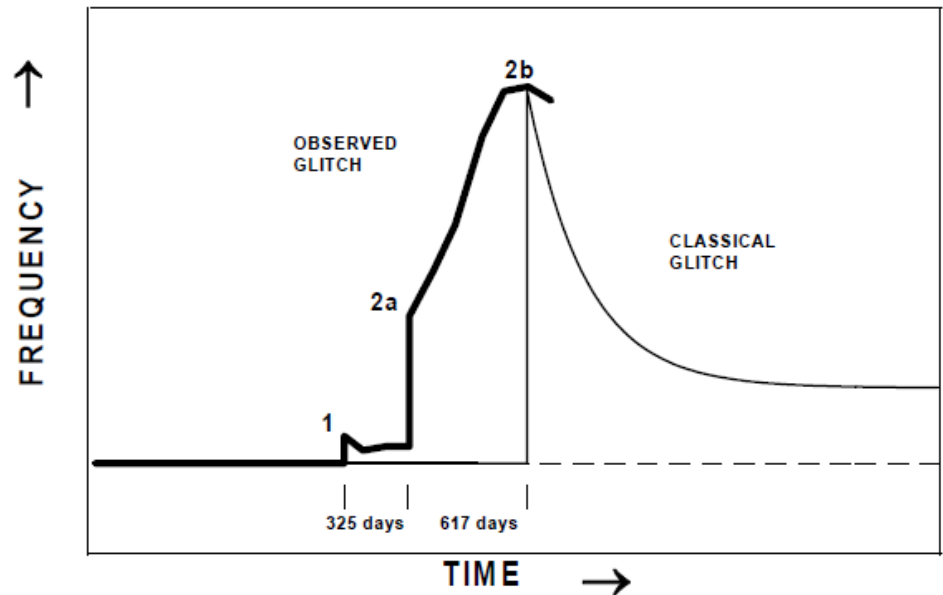


Glitch Parameters	
Epoch (MJD)	57295 <sup>b</sup>
$\Delta f$	$(1.23 \pm 0.19) \times 10^{-8} \text{ s}^{-1}$
$\Delta f/f_{\text{pred}}$	$(5.22 \pm 0.80) \times 10^{-9}$
$\Delta \dot{f}$	$(-1.58 \pm 0.31) \times 10^{-16} \text{ s}^{-2}$
$\Delta \dot{f}/\dot{f}_{\text{pred}}$	$1.27 \pm 0.25$

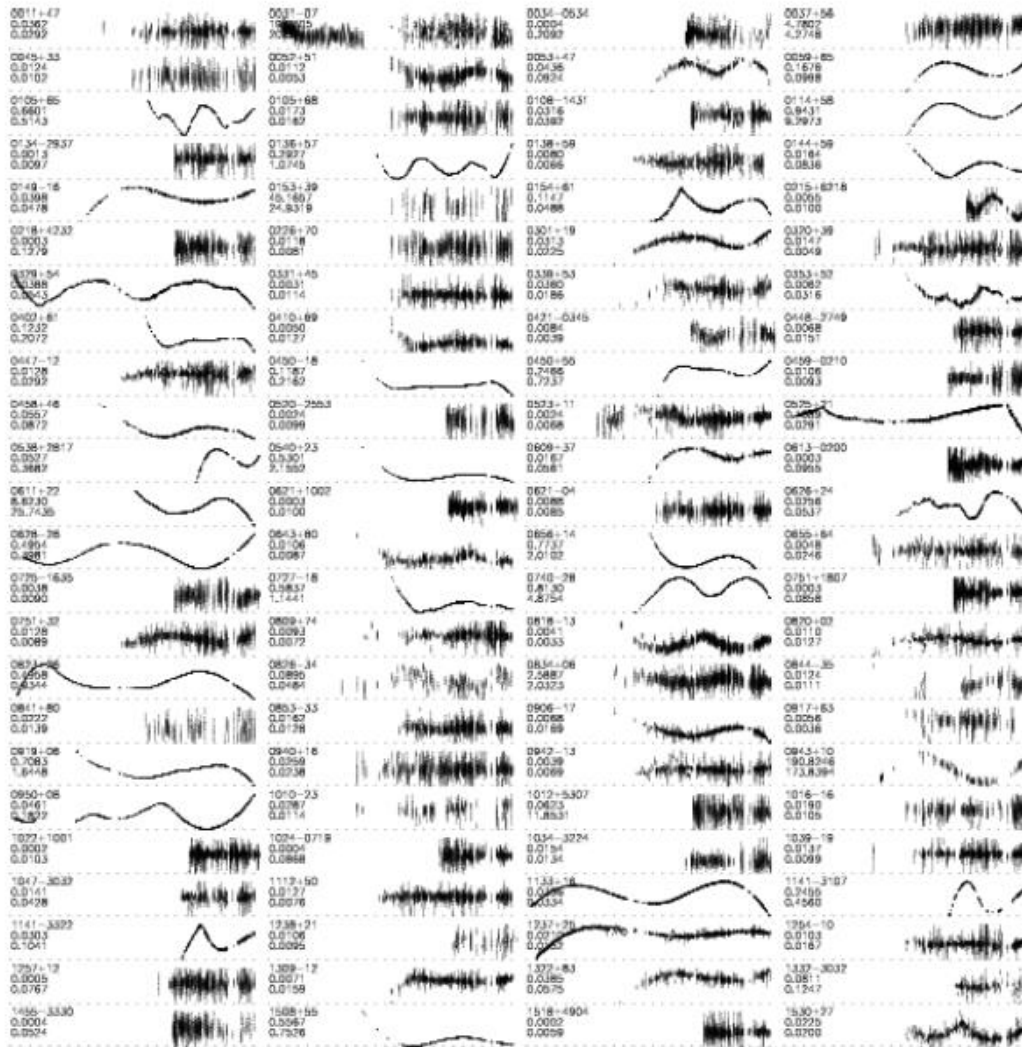
# Slow glitches



Below: a slow glitch by PSR B1822-09 (Shabanova 1998)

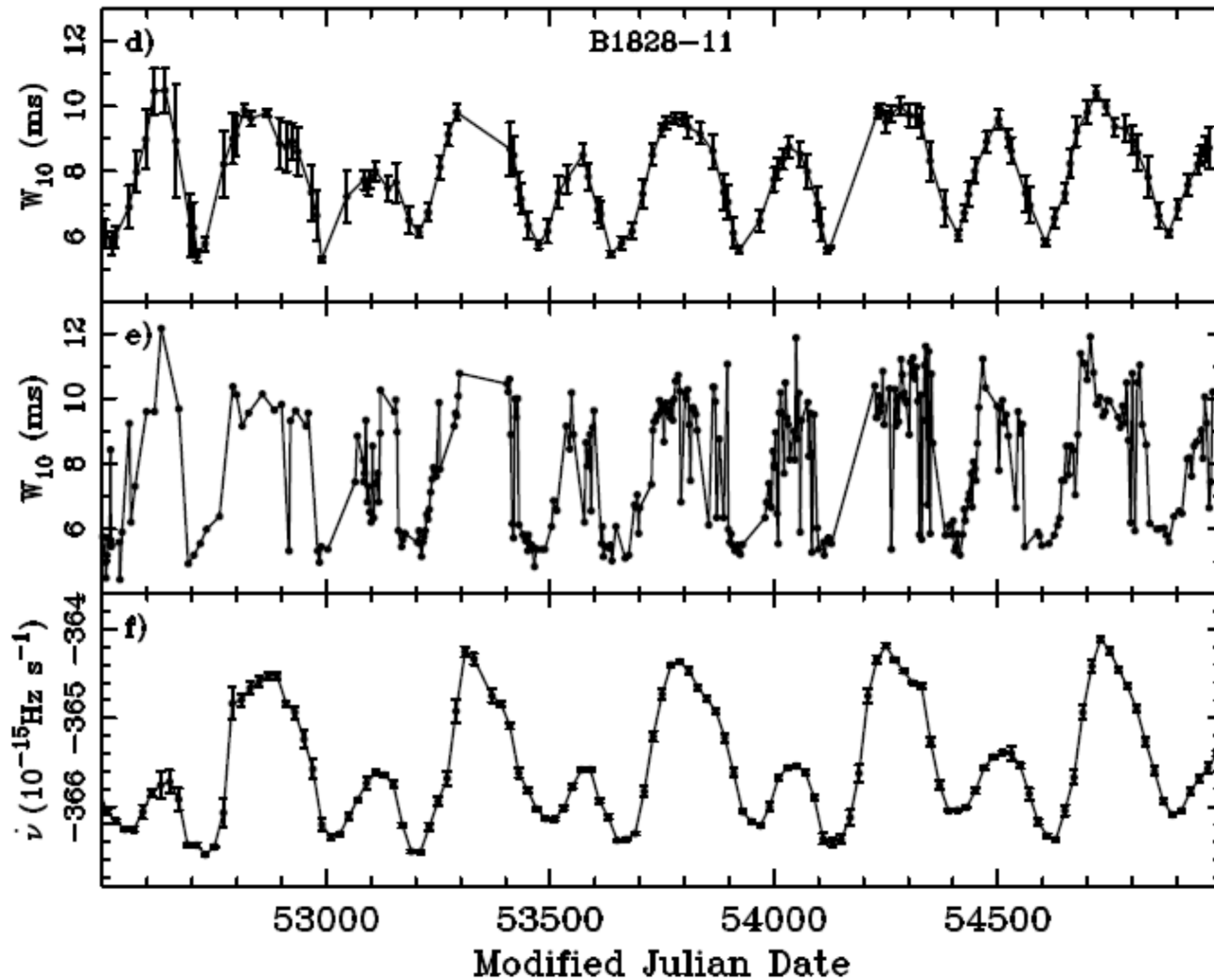


# Timing irregularities



Analysis demonstrates different type of irregularities including quasi-periodic.

# Possible explanation?

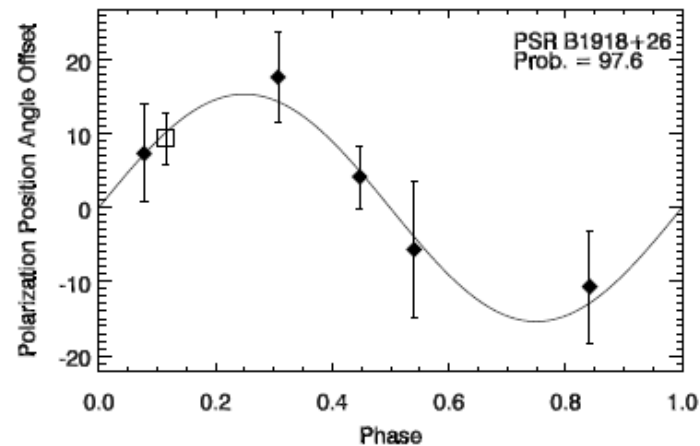
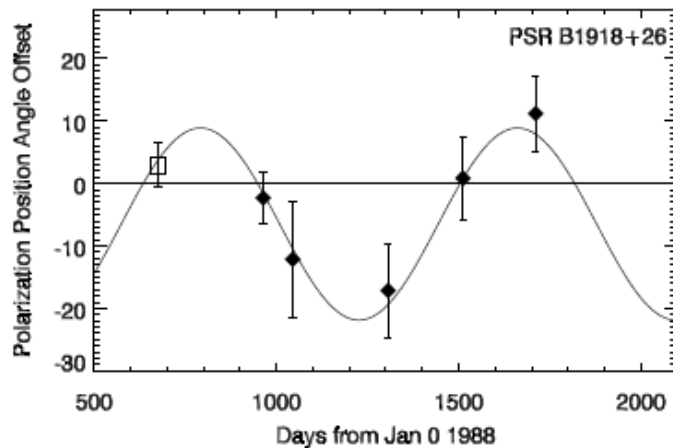
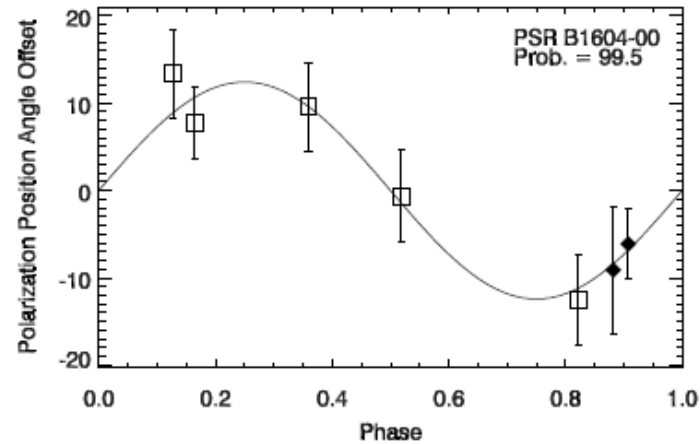
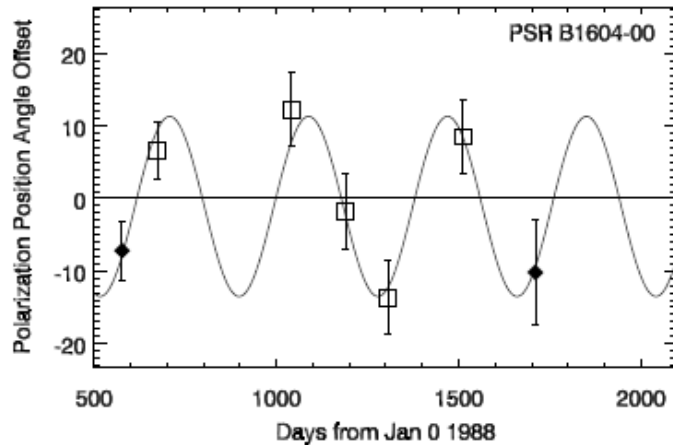


Magnetospheric effect?

Two stages characterized by particular pulse profile and spin-down rate.

Switching between these states happens rapidly.

# Polarization angle variations



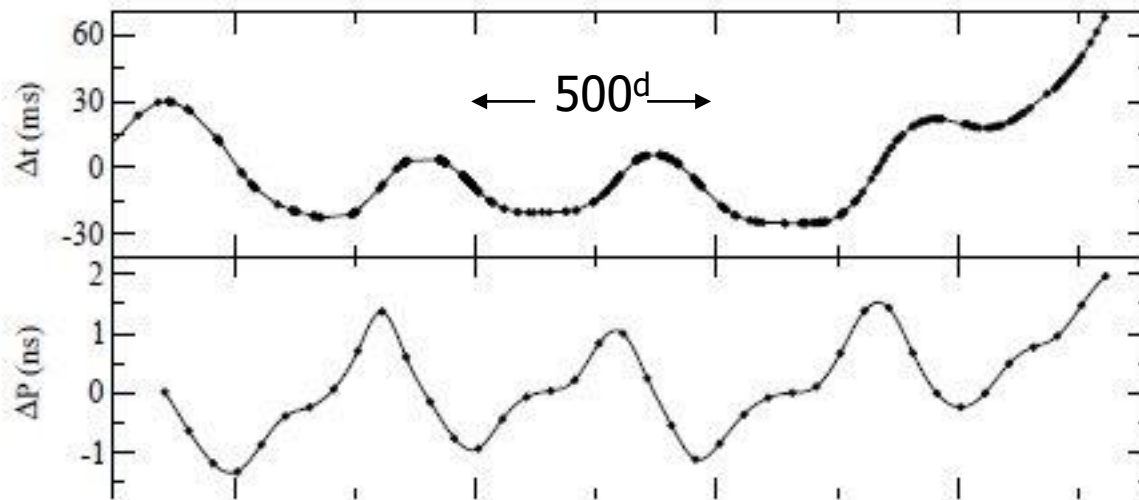
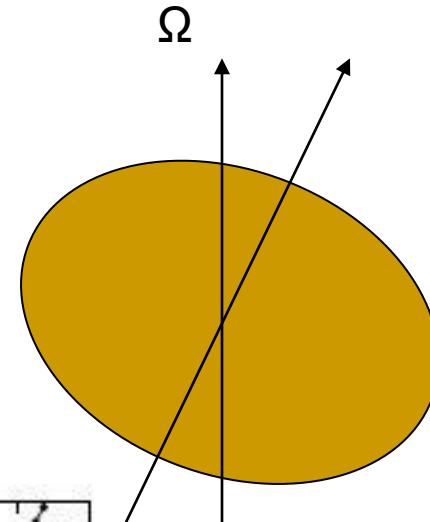
Such variations could be caused by precession

# Precession in NSs

$$P_{\text{prec}} = P/\epsilon,$$

$\epsilon$ -oblateness:  $\epsilon \sim 10^{-8}$   $\Rightarrow$   $P_{\text{prec}} \sim \text{year}$

(More complicated models are developed, too.  
See Akgun, Link, Wasserman, 2005)



Time of arrival  
and period residuals  
for PSR B1828-11.  
Wobbling angle is  $\sim 3-5^\circ$

But why among  $\sim 1500$   
there are just 1-2  
candidates... ?

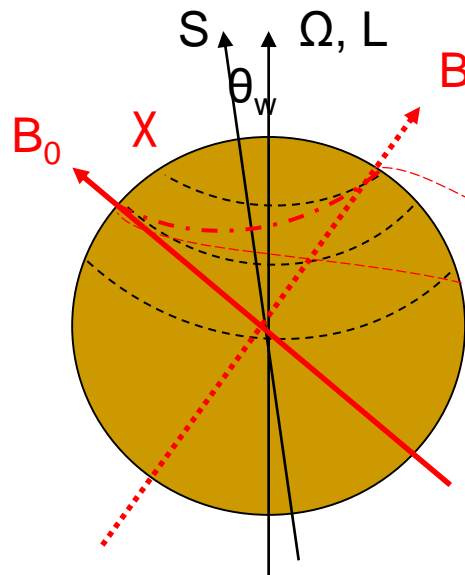
New analysis confirms that PSR 1826-11 can have precession (1510.03579).  
Still, it is difficult to bring it in correspondence with glitches from this PSR (1610.03509).

# Precession (nutation)

If we consider the free precession,  
then we have a superposition of two motions:

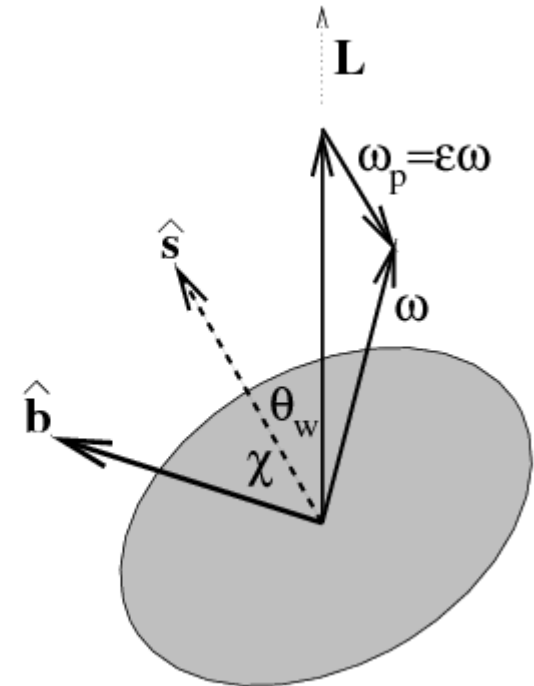
1. Rapid ( $\sim \Omega$ ) rotation around total angular momentum axis –  $L$
2. Slow ( $\Omega_p$ ) retrograde rotation around the symmetry axis ( $s$ )

$\Theta_w$  – is small  
 $\Omega$  and  $L$  are very close



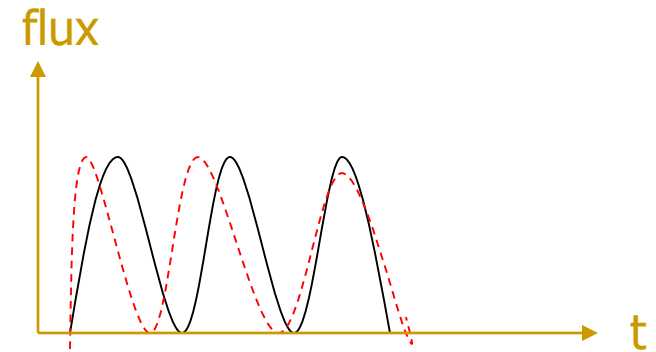
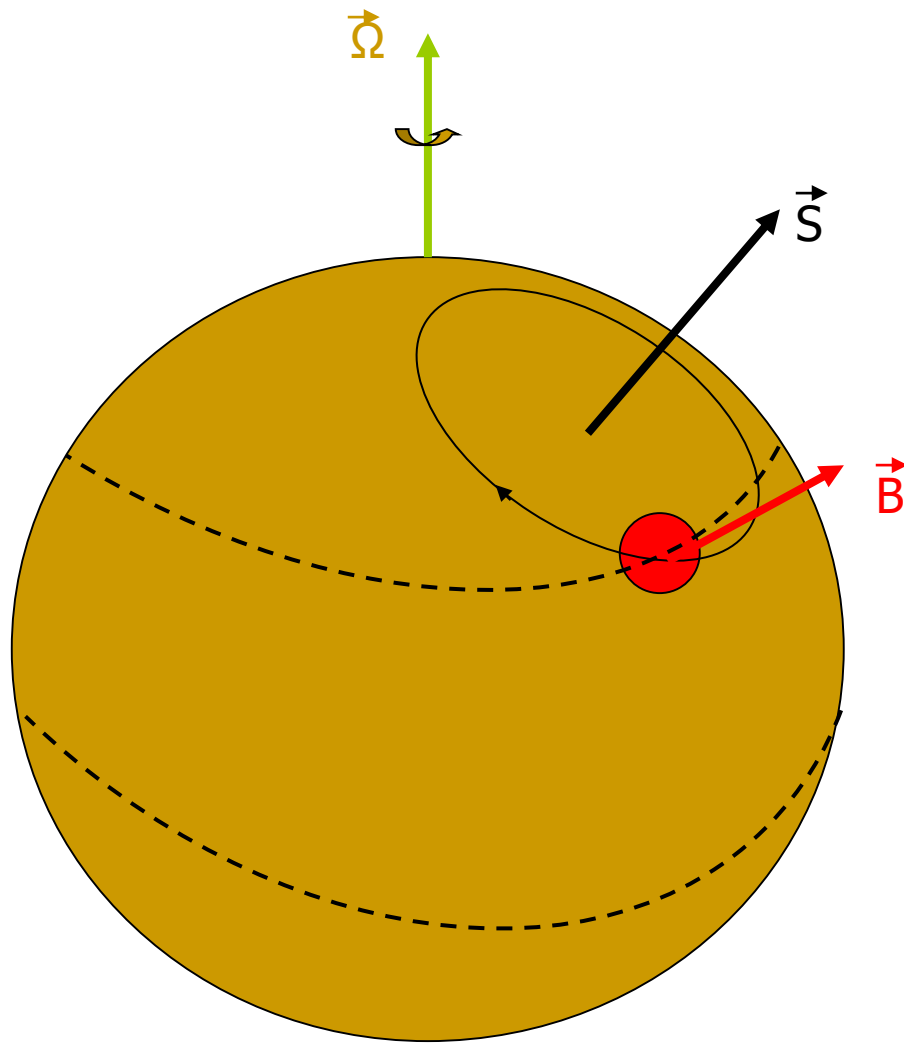
$$\Delta\varphi = \varphi_{\max} - \varphi_{\min} = (\chi + \theta_w) - (\chi - \theta_w) = 2\theta_w$$

Beam width variation



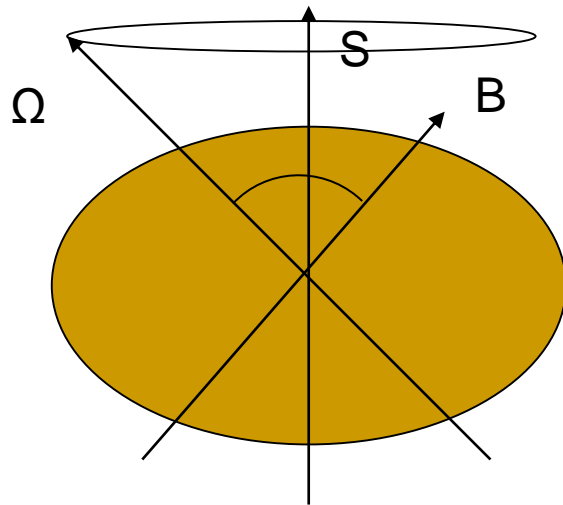


# A toy model



This is a picture seen  
by an external observer.

# In the coordinate frame of the body

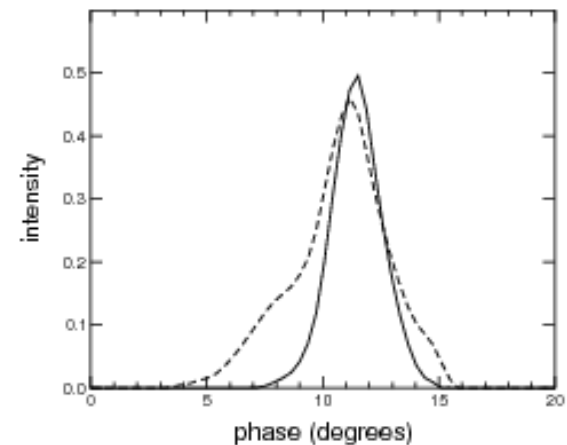
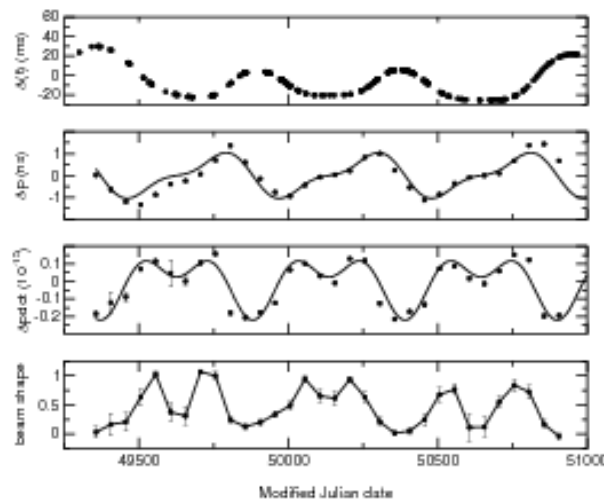


In this system the rotation axis is rotating around the symmetry axis.  
So, it is clear that the angle between spin axis and the magnetic axis changes.

This results in an additional effect in timing:  
Now the spin-down rate changes with the period of precession.

$$\frac{1}{2} I_1 \frac{d\omega^2}{dt} \simeq \boldsymbol{\omega} \cdot \mathbf{N},$$

$$\mathbf{N} = \frac{2\omega^2}{3c^3} (\boldsymbol{\omega} \times \mathbf{m}) \times \mathbf{m},$$

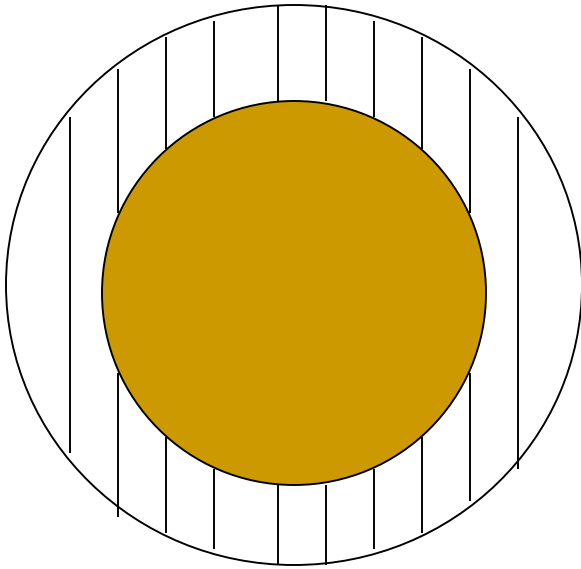


# Complications ...

A neutron star is not a solid body ...

At least crust contains superfluid neutron vortices.

They are responsible for  $I_p \sim 0.01$  of the total moment of inertia.



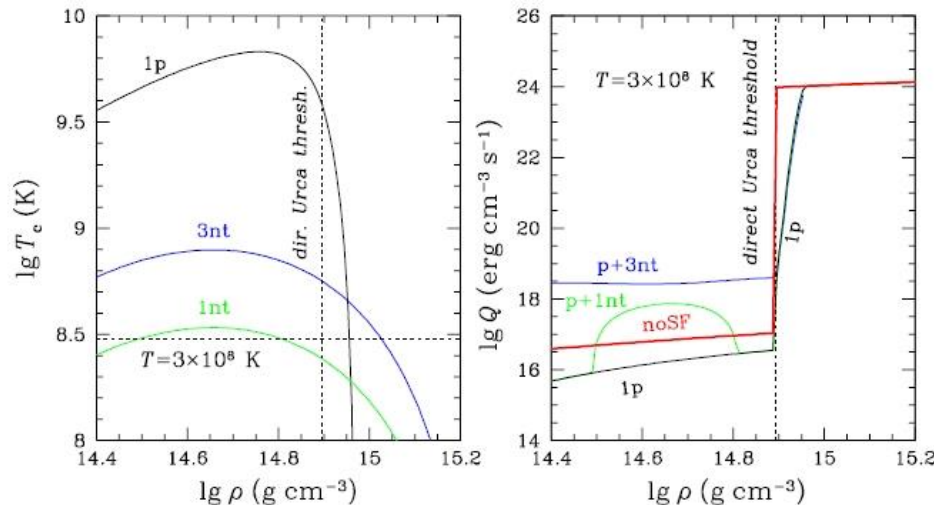
There are several effects related to vortices.

Neutron vortices can interact with the crust.  
So-called “pinning” can happen.

The vortex array works as a gyroscope.  
If vortices are absolutely pinned to the crust  
then  $\omega_{\text{prec}} = (I_p/I)\Omega \sim 10^{-2}\Omega$  (Shaham, 1977).  
But due to finite temperature the pinning is not  
that strong, and precession is possible  
(Alpar, Ogelman, 1987).

# Superfluidity in NSs

50 years ago it was proposed (Migdal, 1959) that neutrons in NS interiors can be *superfluid*.



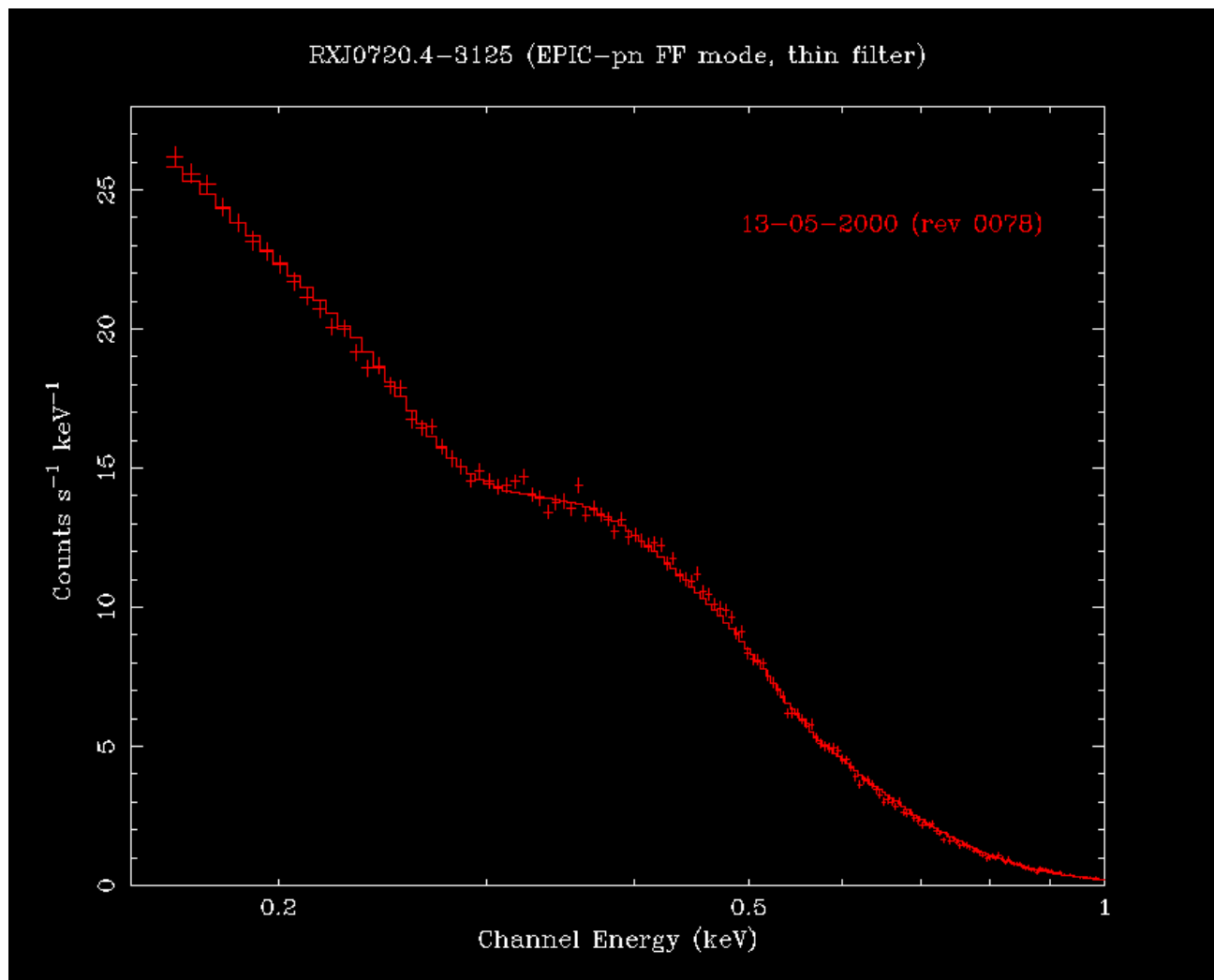
Various baryons in neutron star matter can be in *superfluid* state produced by Cooper pairing of baryons due to an attractive component of baryon-baryon interaction.

Now it is assumed that

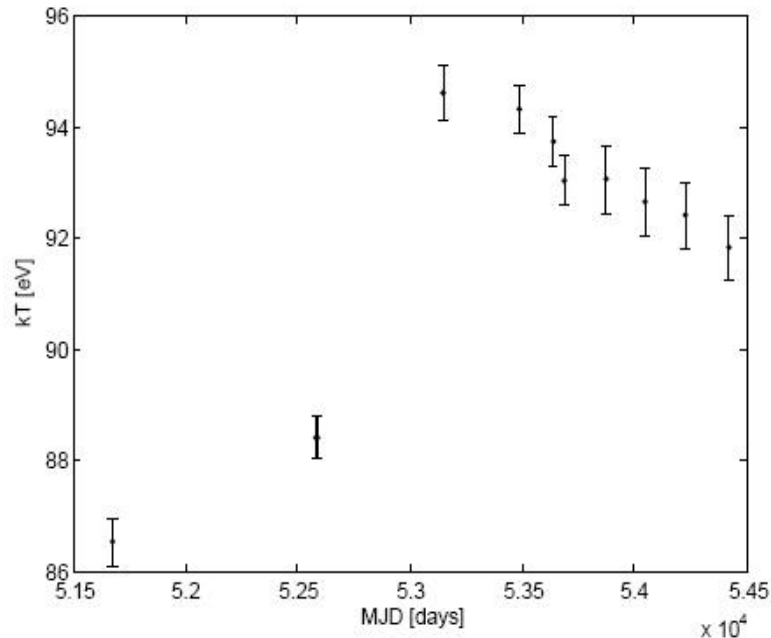
- neutrons are superfluid in the crust (singlet)
- protons are superfluid in the core (singlet)
- neutrons can also be superfluid in the core (triplet)

Onsager and Feynman revealed that rotating superfluids were threaded by an array of quantized vortex lines.

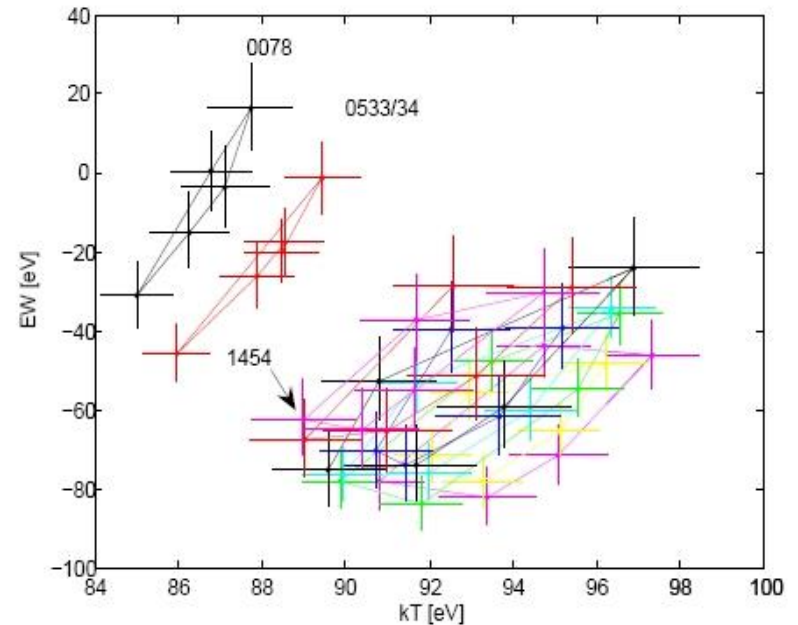
# Peculiar behavior of RX J0720



# RX J0720.4-3125 as a variable source

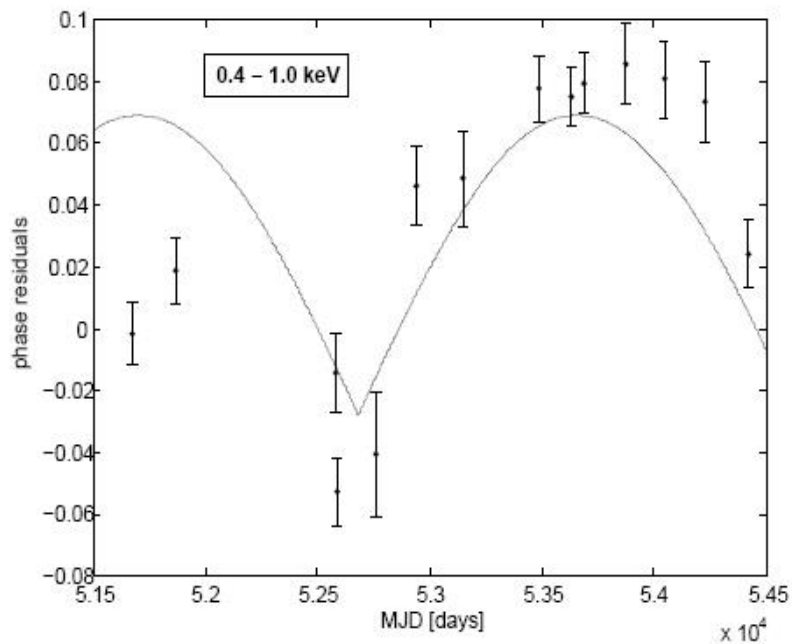


Long term phase averaged  
spectrum variations

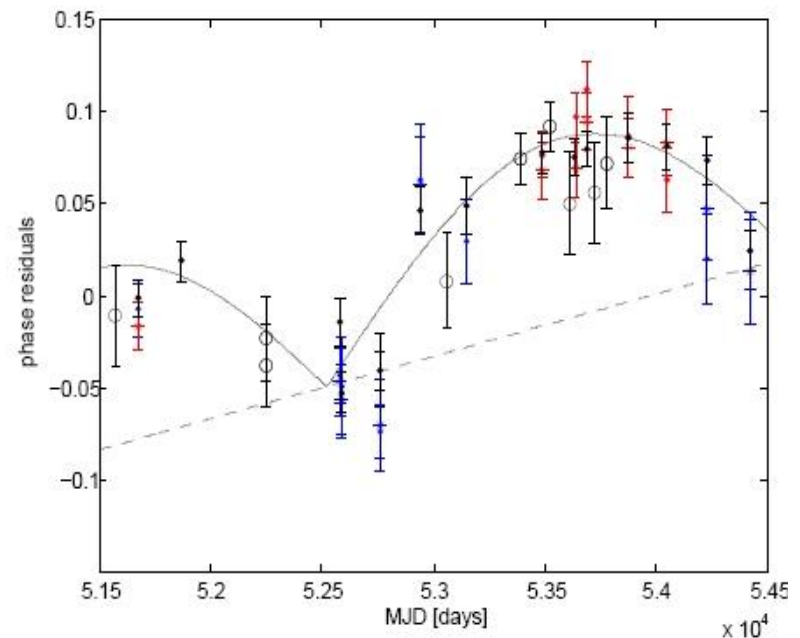


Phase dependent variations  
during different observations.

# $\sim 10$ years period: precession???



10.711  $\pm$  0.058 yrs



However, the situation is not clear.  
New results and a different timing solution.  
The estimate of the period of precession  
slightly changed down to  $\sim 7$  years.



# RX J0720.4-3125: timing residuals

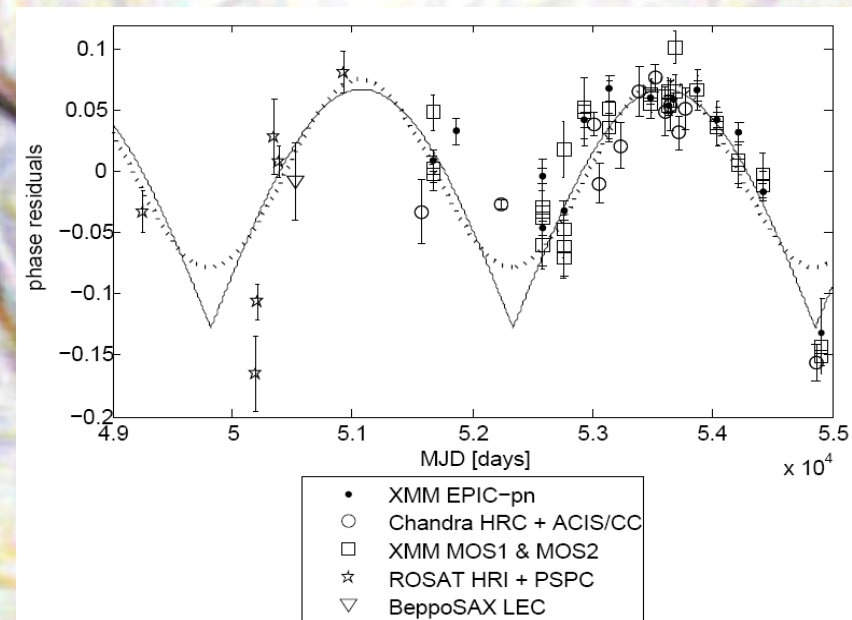
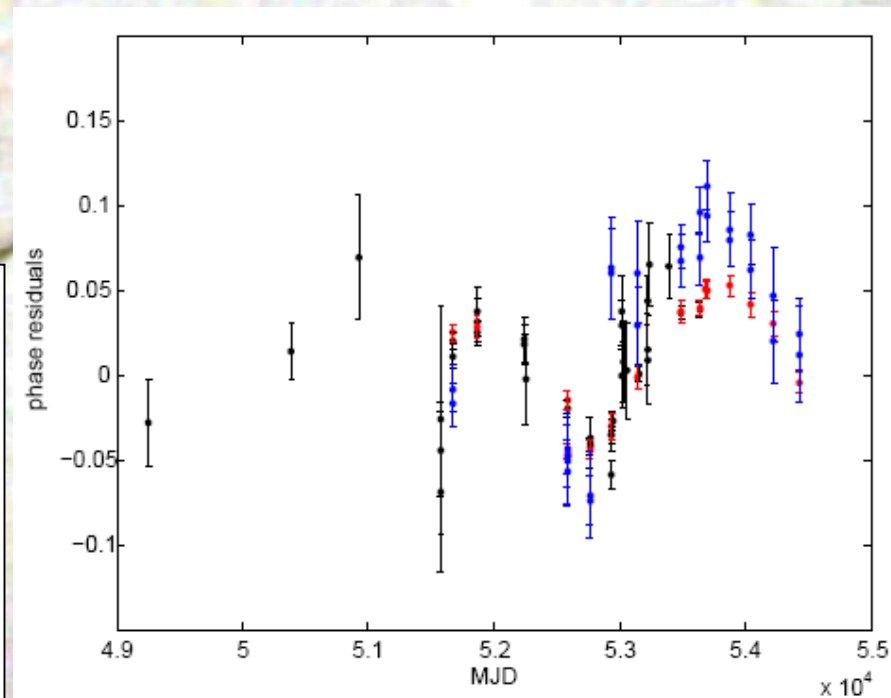
-for  $P(t_0)$  and  $dP/dt$ : phase coherent timing  
-in Kaplan & van Kerkwijk (2005) and  
van Kerkwijk 2007, without energy  
restriction

-now: restricting to the hard band  
(except for ROSAT and Chandra/HRC )  
+five new XMM-Newton  
+two new Chandra/HRC  
observations

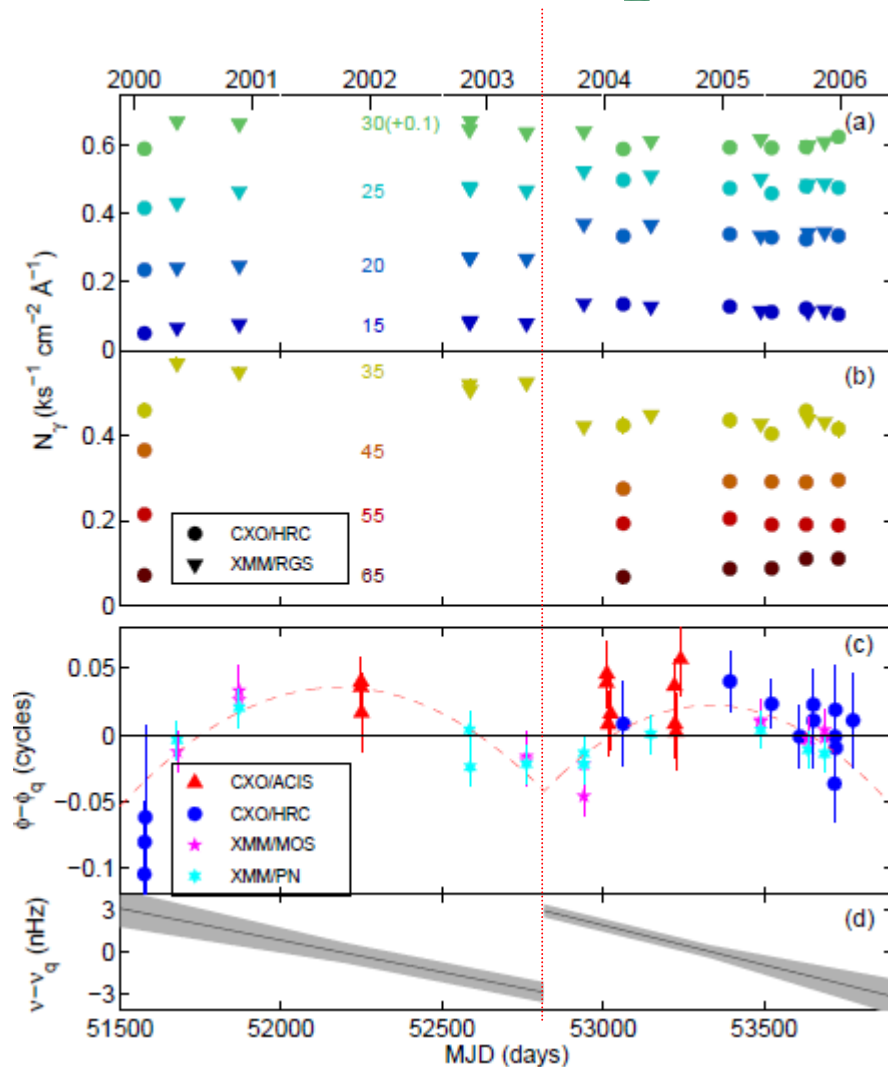
$P(t_0)=8.3911132650(91)\text{s}$   
 $dP/dt=6.9742(19) \cdot 10^{-14} \text{ s/s}$

-long term period:  $(6.91 \pm 0.17) \text{ yrs}$   
Haberl (2007):  $(7.70 \pm 0.60) \text{ yrs}$   
for two hot spots:  $\text{abs}(\text{sine})$   
with 13-15.5yrs period

The slide from a talk by  
Markus Hohle (Jena observatory).



# Another interpretation: glitch + ?

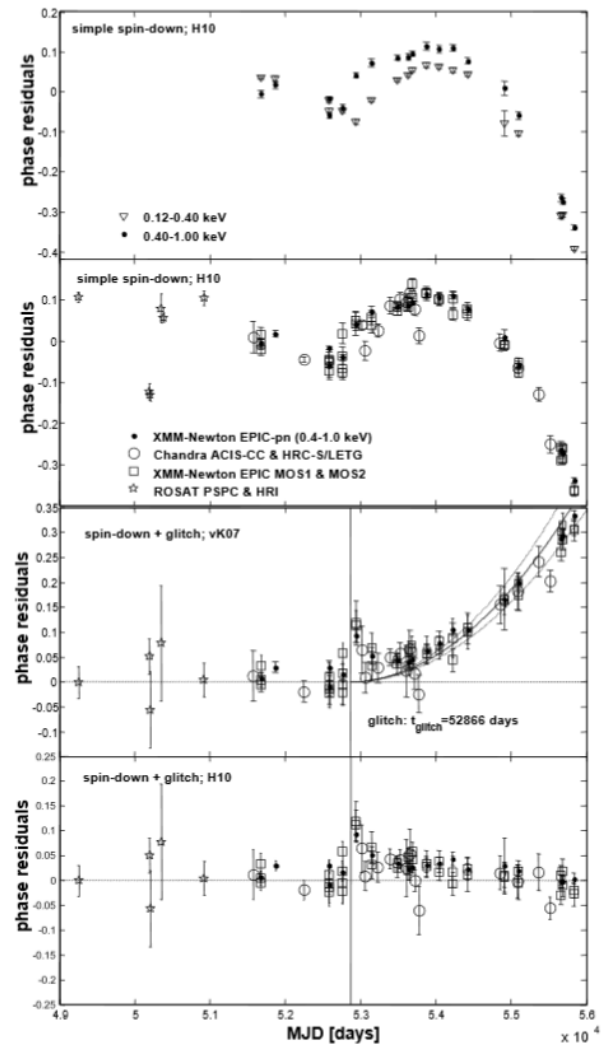
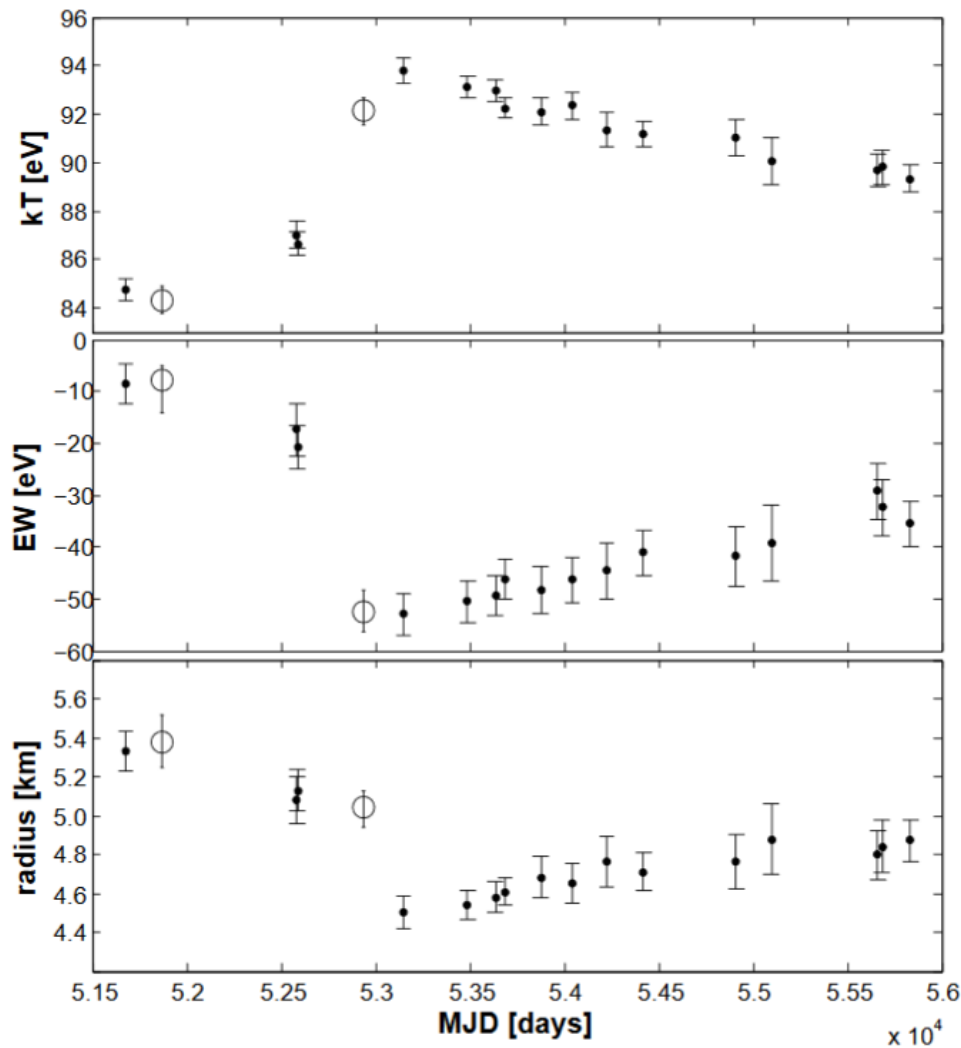


TIMING SOLUTIONS FOR RX J0720.4-3125

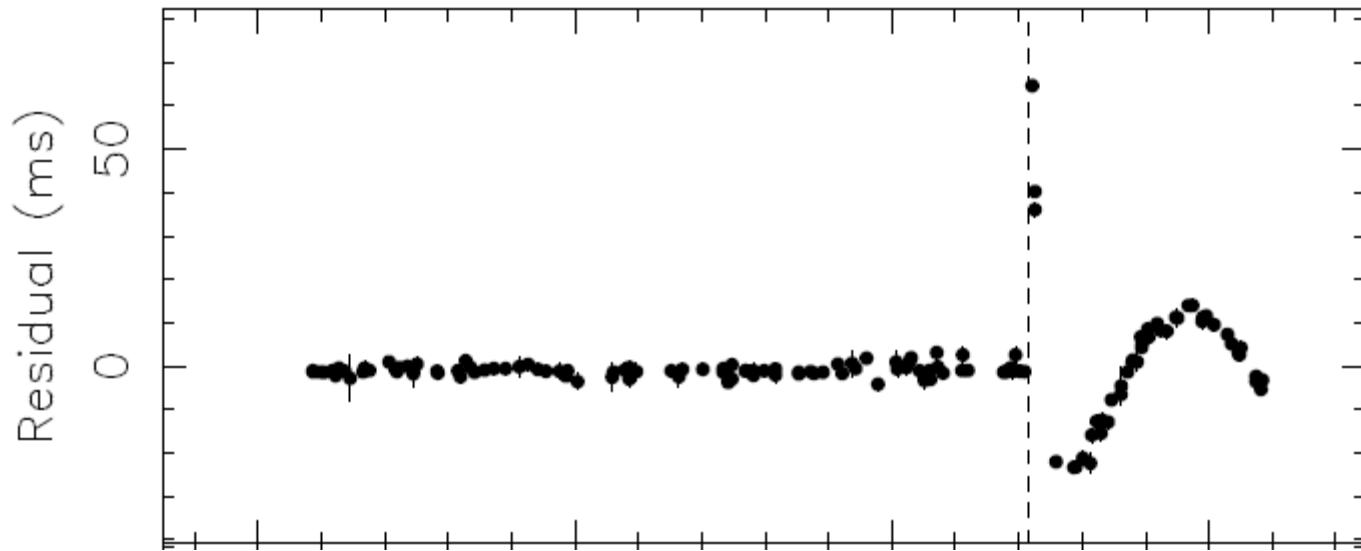
Quantity	Excl. <i>ROSAT</i>	All Data
Spindown only		
$t_0$ (MJD)	53010.2635646(7)	53010.2635626(6)
$\nu$ (Hz)	0.11917366979(12)	0.11917366954(11)
$\dot{\nu}$ (Hz s <sup>-1</sup> )	$-9.74(4) \times 10^{-16}$	$-9.88(13) \times 10^{-16}$
TOA rms (s)	0.26	0.29
$\chi^2/\text{dof}$	77.6/46=1.69	150.8/49=3.08
Spin-down + Glitch		
$t_0$ (MJD)	53010.2635686(10)	53010.2635667(10)
$\nu$ (Hz)	0.1191736716(9)	0.1191736716(9)
$\dot{\nu}$ (10 <sup>-15</sup> Hz s <sup>-1</sup> )	-1.04(3)	-1.04(3)
$t_g$ (MJD)	52817(61)	52866(73)
$\Delta\nu$ (nHz)	5.7(17)	4.1(12)
$\Delta\dot{\nu}$ (10 <sup>-17</sup> Hz s <sup>-1</sup> )	-1(4)	-4(3)
TOA rms (s)	0.15	0.24
$\chi^2/\text{dof}$	37.0/43=0.86	45.1/46 = 0.98

NOTE. — The parameters determine the cycle count plus phase via  $\phi(t) = \nu(t - t_0) + \frac{1}{2}\dot{\nu}(t - t_0)^2 + \Delta\phi_g(t)$ , where  $\Delta\phi_g(t) = -\Delta\nu(t - t_g) - \frac{1}{2}\Delta\dot{\nu}(t - t_g)^2$  for  $t < t_g$  in the glitch model and zero otherwise. For all fits, a 0.11 s systematic uncertainty has been added in quadrature to the times of arrival (TOAs), and the uncertainties quoted are twice the formal 1 $\sigma$  values.

# RX J0720.4-3125: a glitch



# Glitch+? in a PSR



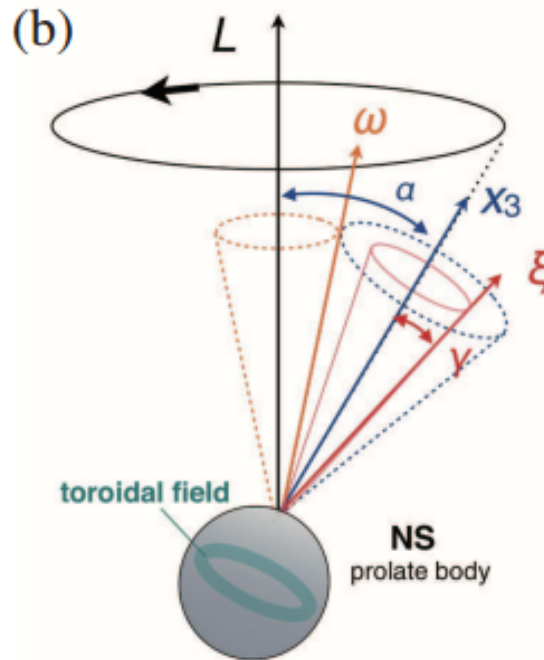
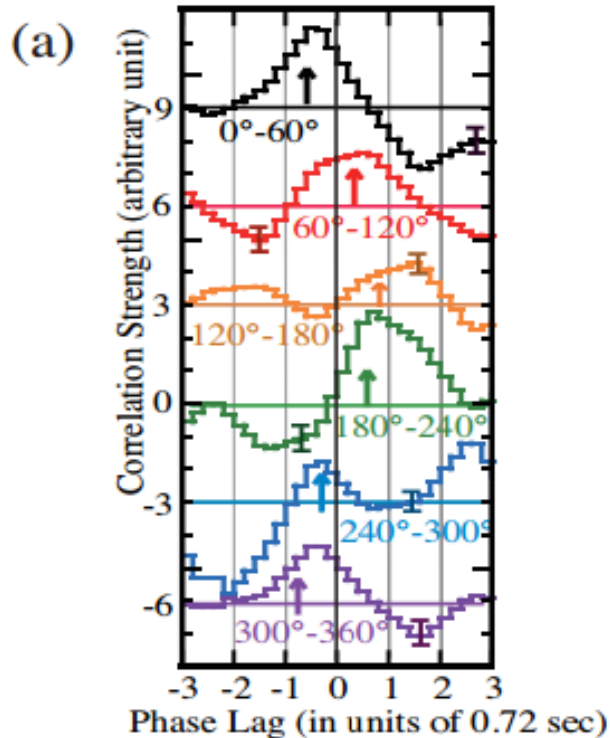
PSR B2334+61

arXiv: 1007.1143

Precession after a glitch was proposed as possible feature due to Tkachenko waves excitation (arXiv: [0808.3040](#) ).

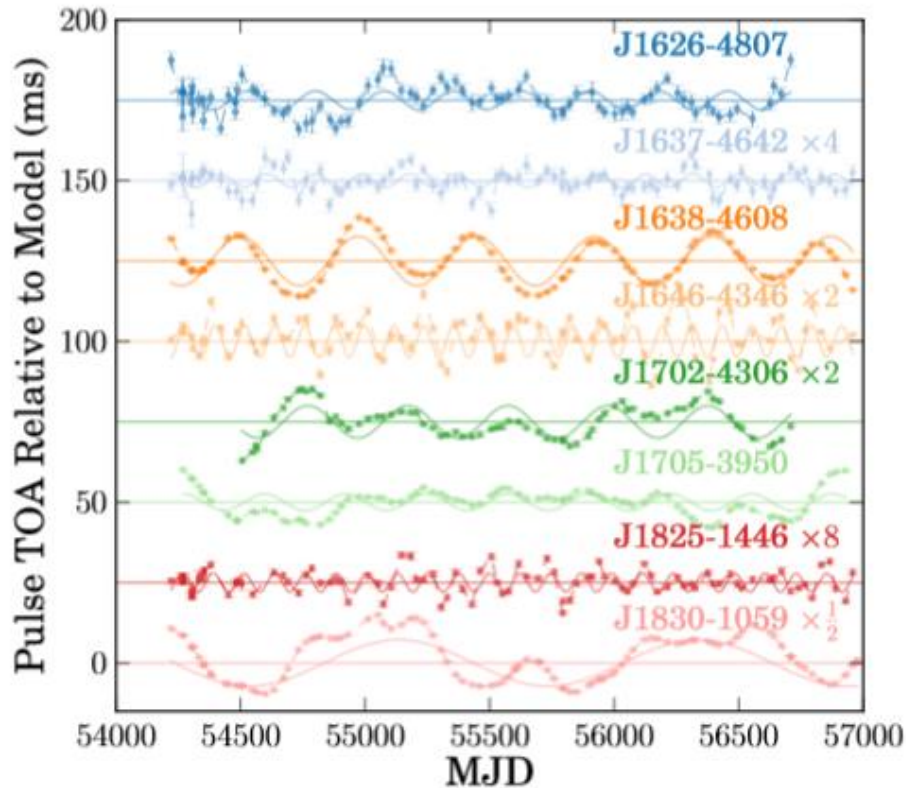
Precession as a viable mechanism for long-term modulation was recently discussed in details in 1107.3503.

# Free precession of a magnetar?



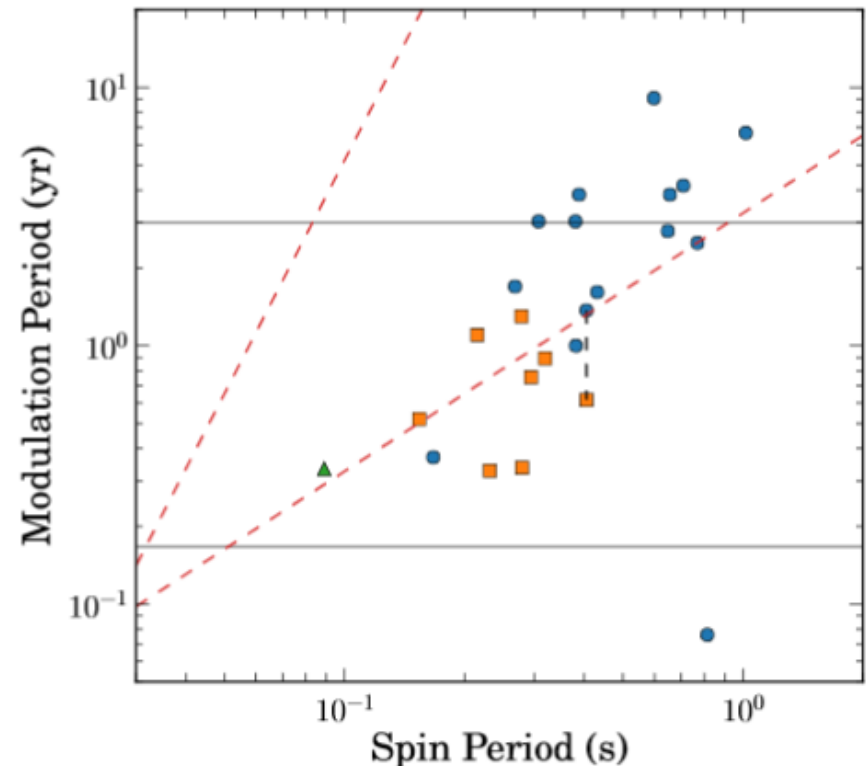
The authors observe modulation of the pulse profile with a period  $\sim 15$  hours. If it is interpreted by a free precession, then the NS is significantly deformed which can be due to strong toroidal field. This field might be  $\sim 10^{16}$  G.

# New precession candidates among PSRs



Correlations of the modulation period with spin period, characteristic age and spin-down power.

Periodic modulations which can be interpreted as free precession.



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# Conclusion

Many observed phenomena are related to internal dynamics of NSs.

- Glitches
- Precession

Glitches are related to the existence of some reservoir for angular momentum. Most probably, it is a layer of superfluid neutrons in the inner crust.

Some glitches of magnetars can be related to a different process.

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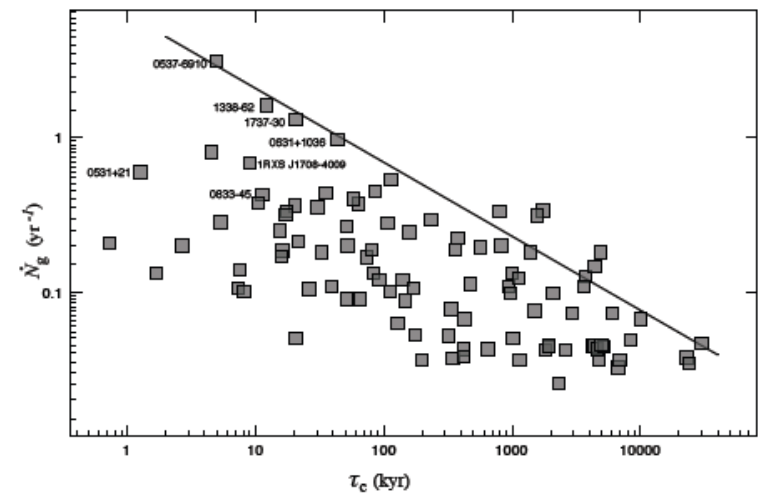
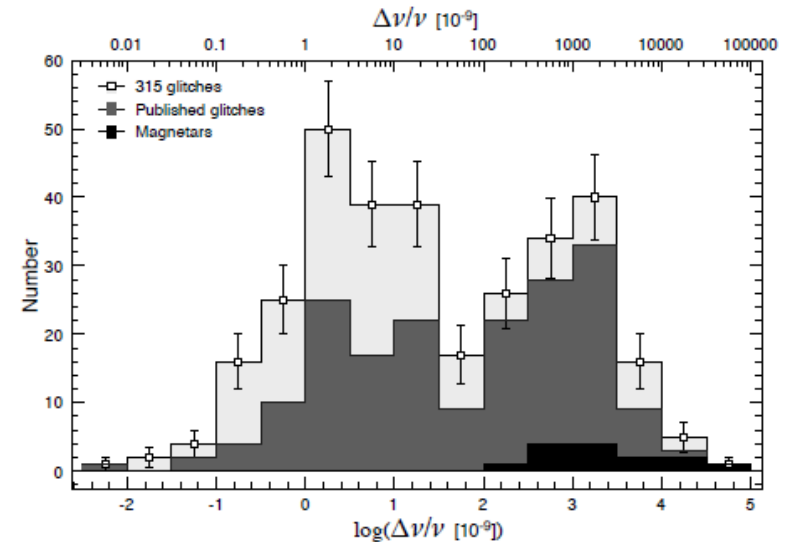
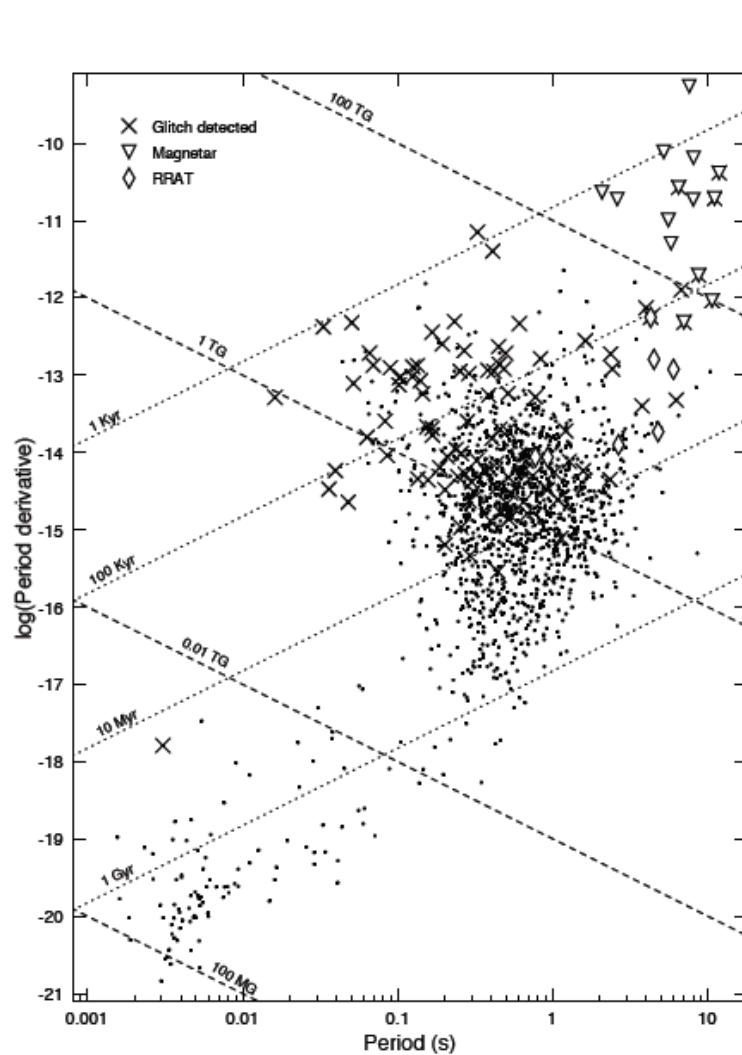


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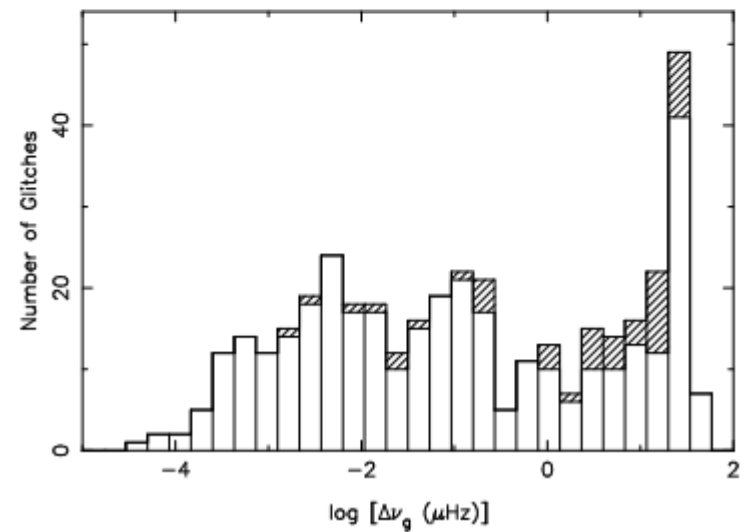
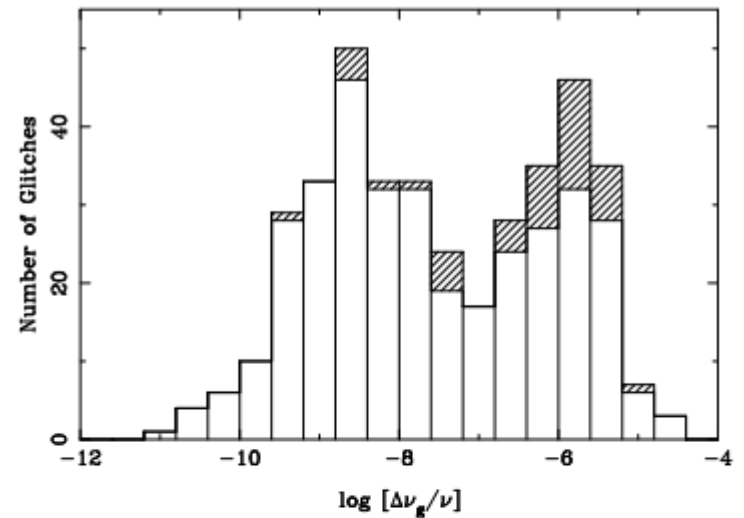
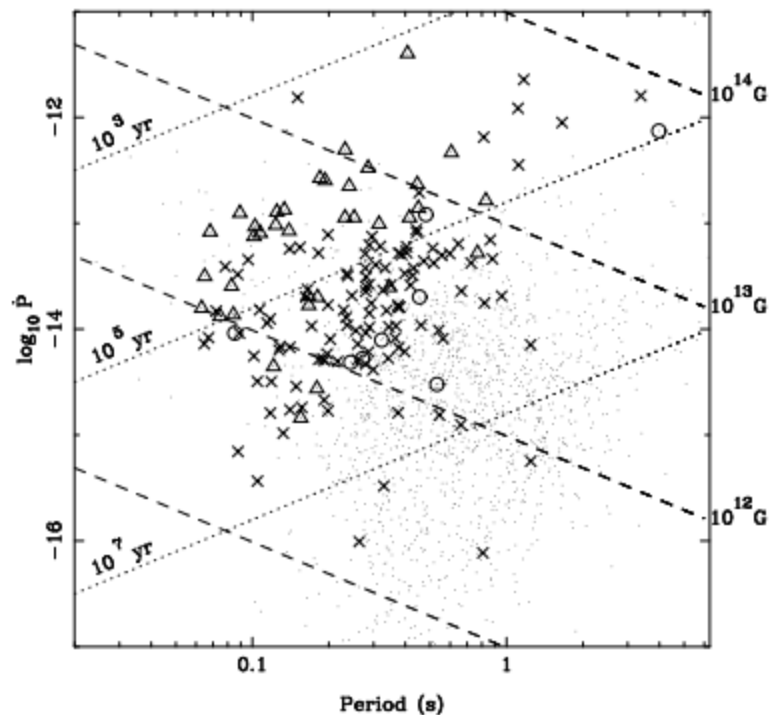
# Main papers

- Link et al. [astro-ph/0001245](#) Glitches
  - Link [astro-ph/0211182](#) Precession
  - Jones, Andersson [astro-ph/0011063](#) Precession
  - Dib et al. arXiv: 0706.4156 AXP glitches
  - Haskell, Melatos arXiv: 1502.07062 Big review
  - Haskell, Sedrakian arXiv: 1709.10340 Big review on superfluidity
  - Fuentes et al. arXiv: 1710.00952 Glitch statistics
  - Manchester arXiv: [1801.04332](#) Brief review on glitches
-

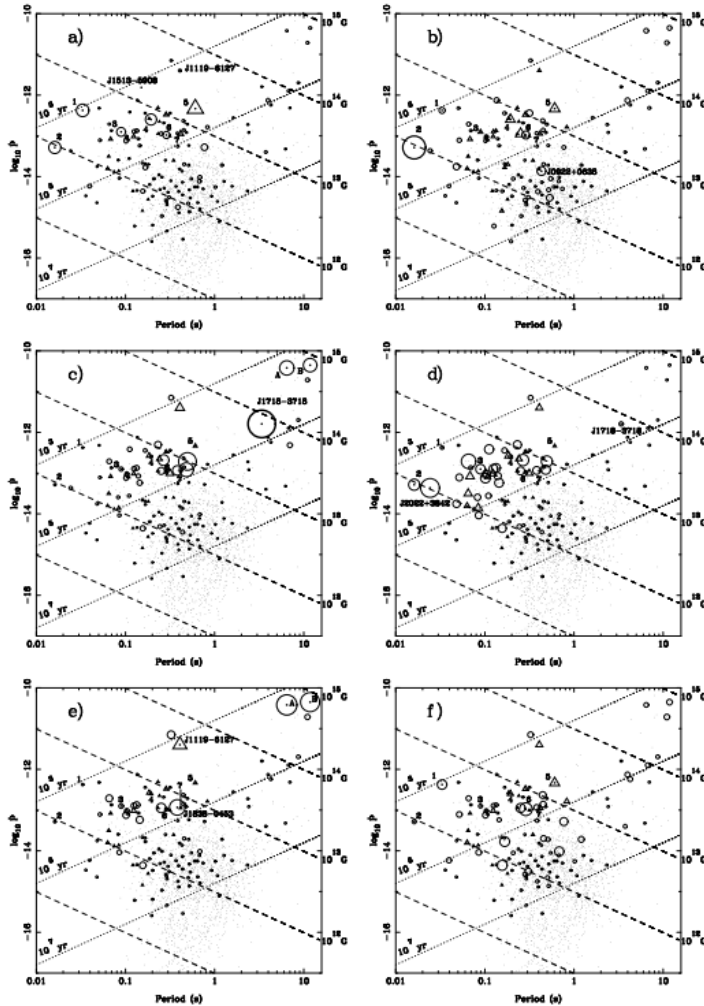
# Many-many glitches ...



# 107 new glitches in 36 pulsars



# P–Pdot diagrams for glitch-related quantities



a) number of detected glitches; b) average number of glitches per year; c) maximum fractional glitch size; d) maximum glitch size; e) rms fractional glitch size; and f) rms fractional size normalised by the mean. A circle indicates the parameter was obtained from the ATNF Pulsar Catalogue glitch table, whereas a triangle symbol indicates a parameter from this work. In the various plots, the seven pulsars exhibiting ten or more glitches are marked: 1 – PSR B0531+21 (Crab pulsar); 2 – PSR J0537–6910; 3 – PSR B0833–45 (Vela pulsar); 4 – PSR J1341–6220; 5 – PSR J1740–3015; 6 – PSR J0631+1036; 7 – PSR J1801–2304; and two magnetars: A – PSR J1048–5937 (1E 1048.1–5937) and B – PSR J1841–0456 (1E 1841–045).

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# Modeling glitches

Mean field approach to describe vortex dynamics

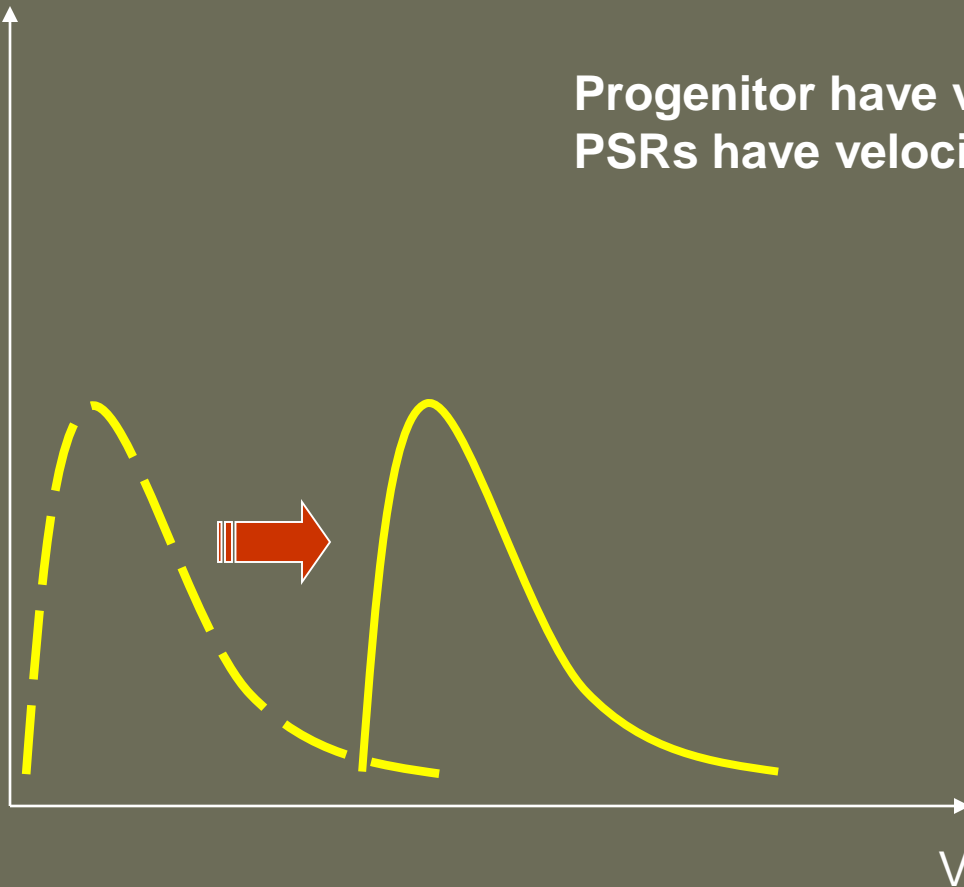
# Kick velocity



Why do neutron stars move so rapidly?

# Stars vs. Neutron Stars

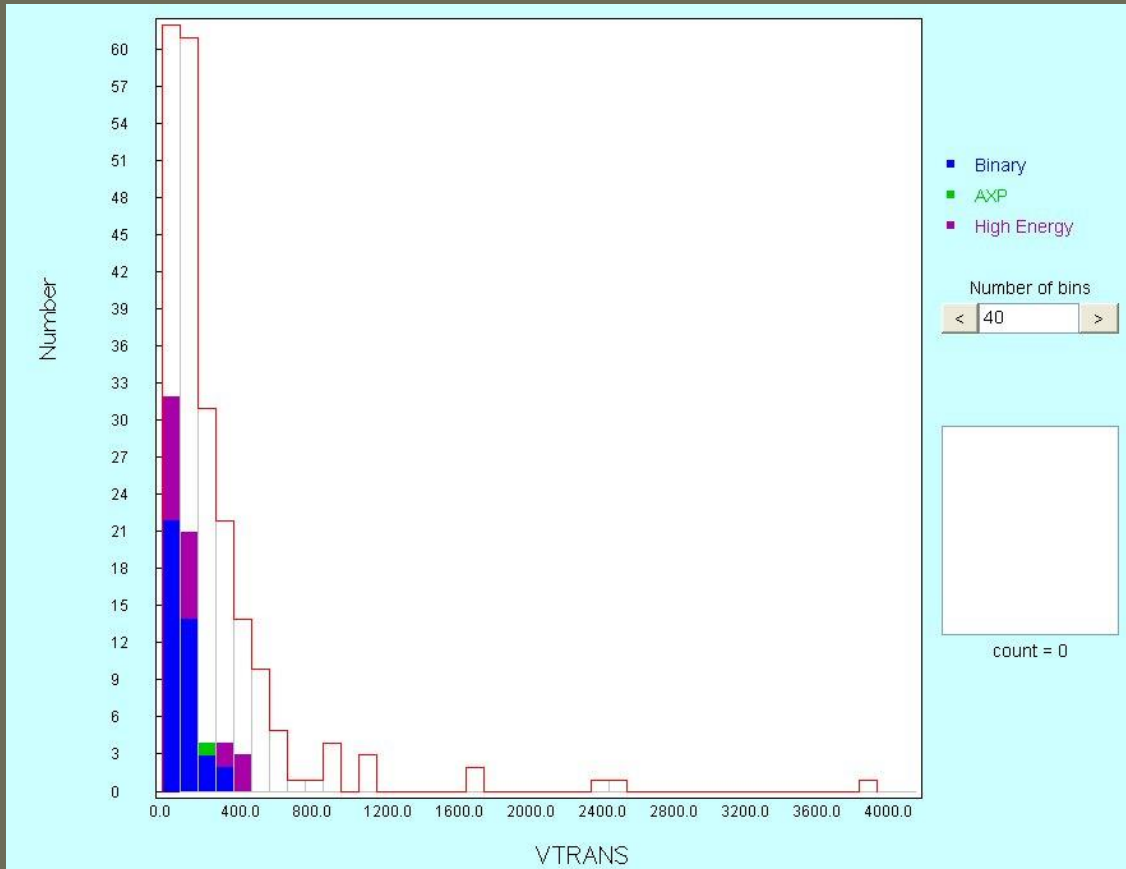
Progenitor have velocities about  $\sim 10\text{-}30$  km/s  
PSRs have velocities  $\sim 100\text{-}500$  km/s





# Pulsar velocity distribution

Normal stars have velocities  $\sim 10\text{-}30$  km/s.



Already in 70s it became clear that PSRs have high spatial velocities ( $\gg 10$  km/s).

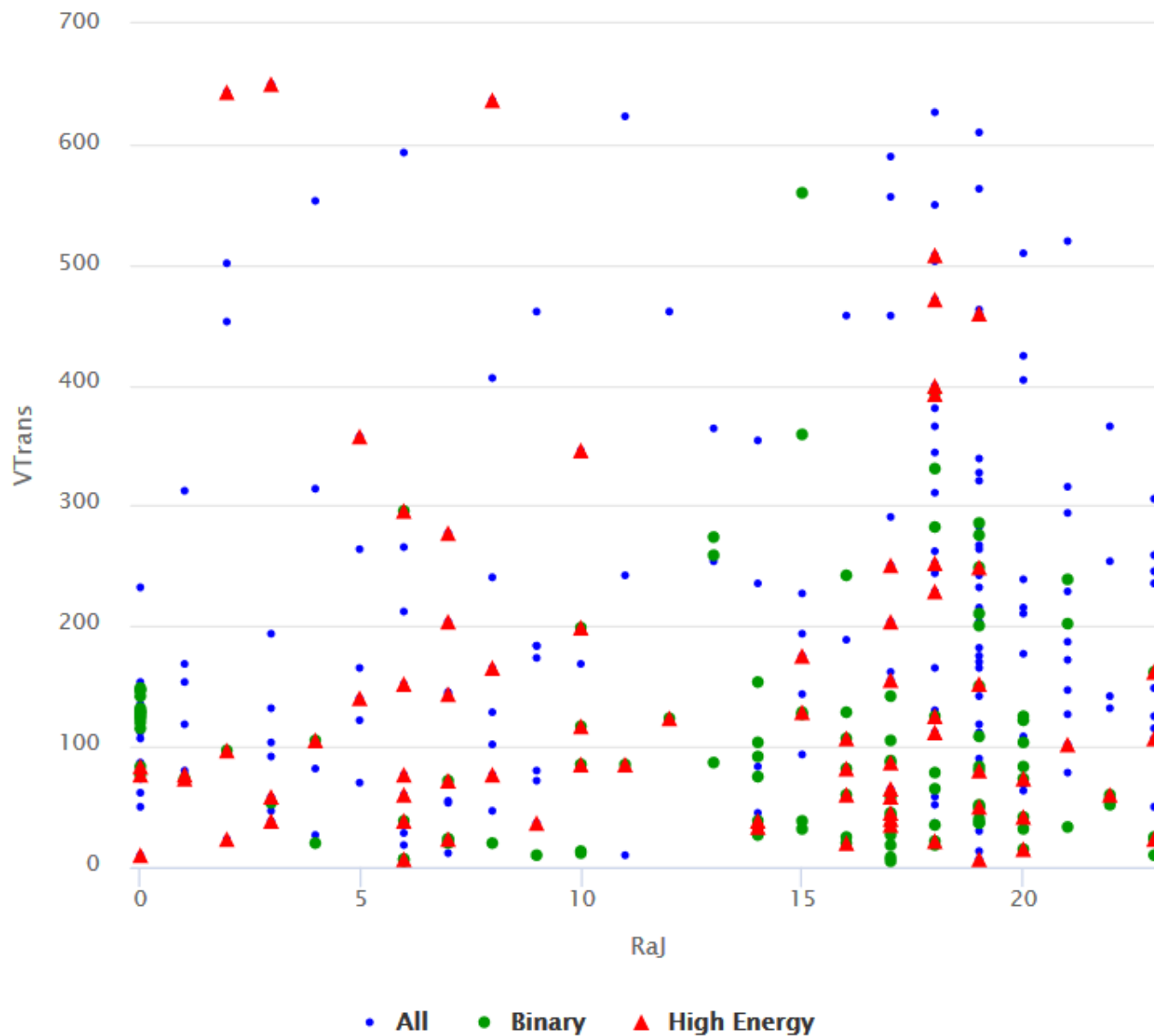
A breakthrough happened in 1994 when Lyne and Lorimer in a seminal paper in *Nature* showed that velocities are even higher than it was thought before – hundreds km/s.

Note, that the observed distribution is much different from the initial one. To derive the later it is necessary to calculate a model.

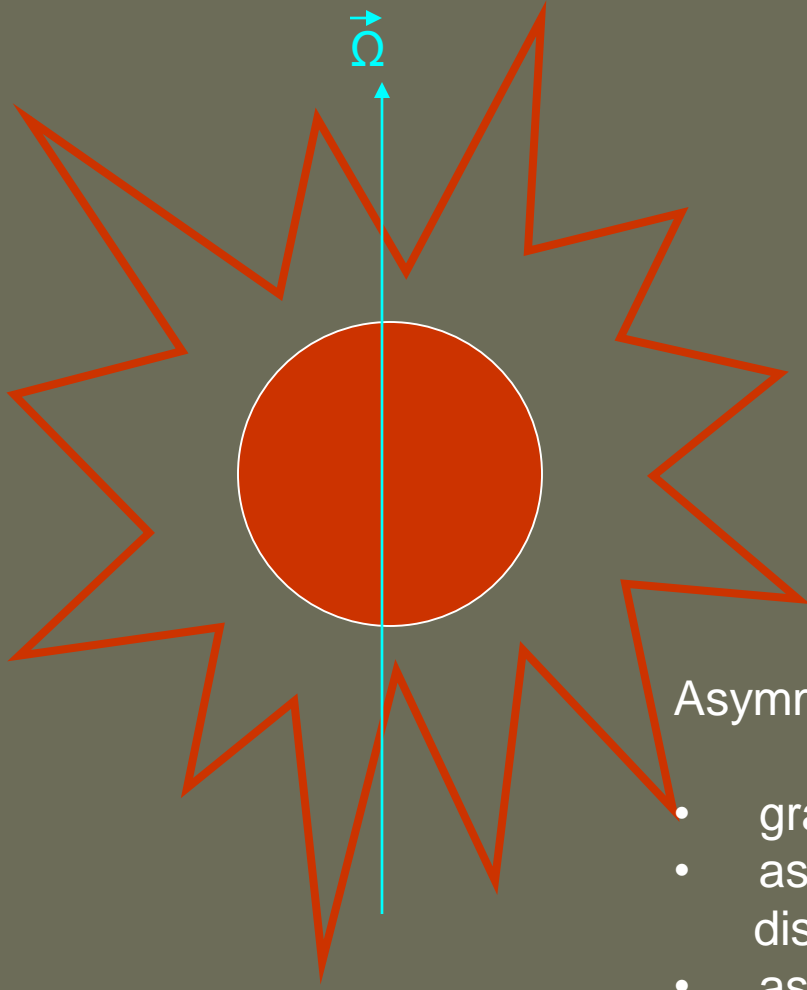
## PSRCAT plot (Catalogue v1.58)



Source: <http://www.atnf.csiro.au/research/pulsar/psrcat>



# SN explosions should not be symmetric!



$E_{\text{total}} \sim 3 \cdot 10^{53} \text{ erg}$

Most of energy is carried away by neutrinos.

~Few % asymmetry in energy release  
can produce a strong kick up to 1000 km/s.

## Main kick mechanisms

- Asymmetric mass ejection (Shklovsky 1970)
- Asymmetric neutrino emission (Chugai 1984)

Asymmetric mass ejection includes three mechanisms:

- gravitational pull due to asymmetric matter
- asymmetric neutrino emission due to matter distribution
- asymmetric matter jets (Khokhlov et al. 1999)

*Leonid Ozernoy in 1965 discussed asymmetry  
of SN explosions in the context of GW radiation.*

# SN and kick explosion mechanisms

Mechanism	Time scale	$V_{\max},$ $\text{km s}^{-1}$	Alignment (spin and $V$ )	Main recent refs.
Hydrodynamical	0.1 s	$\sim (100 - 200)$	random	<a href="#">Lai et al. (2001)</a>
$\nu$ -driven	$\sim \text{few s}$	$\sim 50 B_{15}$	parallel	<a href="#">Lai et al. (2001)</a>
Electromagnetic rocket	long	$1400 R_{10}^2 P_{\text{ms}}^{-2}$	parallel	<a href="#">Lai et al. (2001)</a> , <a href="#">Huang et al. (2003)</a>
Binary disruption (without add. kick)	$\ll P_{\text{orb}}$	$\sim 1000$	perpendicular	<a href="#">Iben &amp; Tutukov (1996)</a>
NS instability	few ms	$\sim 1000$	perpendicular	<a href="#">Colpi &amp; Wasserman (2002)</a> , <a href="#">Imshennik &amp; Ryazhskaya (2004)</a>
Magnetorotational	0.2 s – minutes	$\sim 300$ (up to 1000)	quasirandom	<a href="#">Moiseenko et al. (2003)</a> , <a href="#">Ardeljan et al. (2004)</a>

For neutrino emission:  $V_{\text{kick}} = \varepsilon E_{\text{tot}} / Mc \sim 1000 \text{ km/s } (\varepsilon/0.1) (E_{\text{tot}}/10^{53} \text{ erg})$ .  
Also it depends on the magnetic field.

# Kick modeling

Recently, new results on the origin of NS and BH kicks have been obtained:

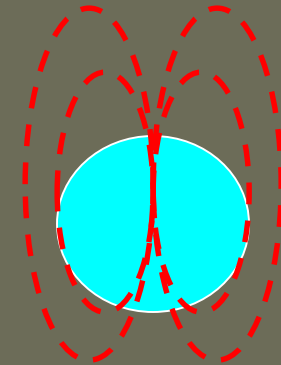
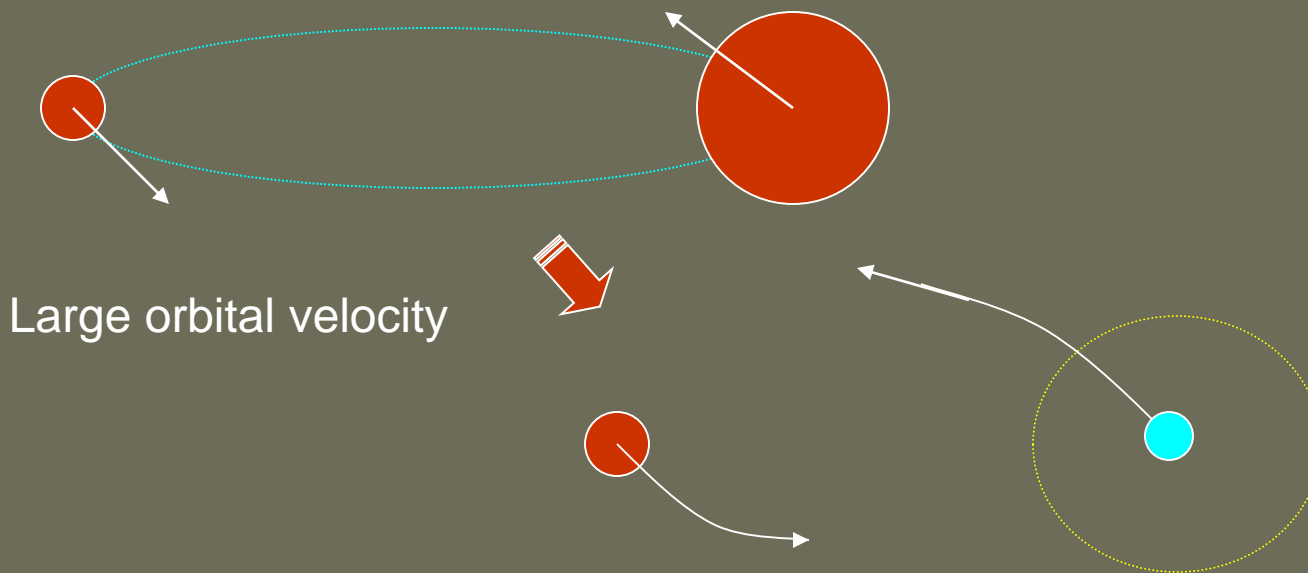
- Neutrino-triggered asymmetric magnetorotational mechanism [arXiv:1110.1041](#)
- Hydrodynamic Origin of Neutron Star Kicks [arXiv: 1112.3342](#)
- Three-dimensional neutrino-driven supernovae [arXiv:1210.8148](#)
- BH kicks [arXiv:1203.3077](#)

A review on SNe properties and explosion mechanisms: [arXiv:1210.4921](#)

# To kick or not to kick?

Up to mid-90s it was not clear if kicks are absolutely necessary.

- Tademaru (rocket) mechanism
- Binary disruption (Blaauw mechanism)
- Core fragmentation (Berezinski et al., Imshennik)

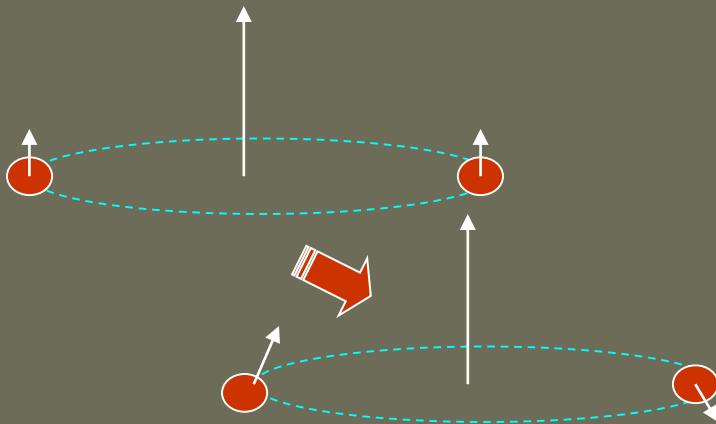


Asymmetric dipole

However, some discoveries directly point to necessity of natal kicks.

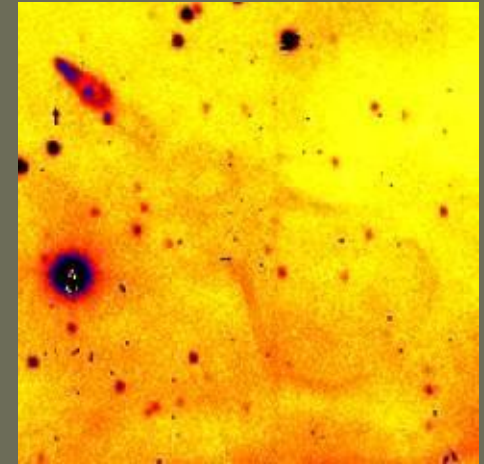
# Direct evidence

1. High-velocity NSs and binaries
2. Spin inclination in binaries and geodetic precession



Orbit inclination relative to a normal star equator can be measure due to:

- orbital precession due to spin-orbit interaction (Kaspi et al. 1996)
- circumstellar disc inclination (Prokhorov, Postnov 1997)



Guitar nebula, B2224+65

The most spectacular 3D velocity measurements for NSs are related to nebulae around these objects.

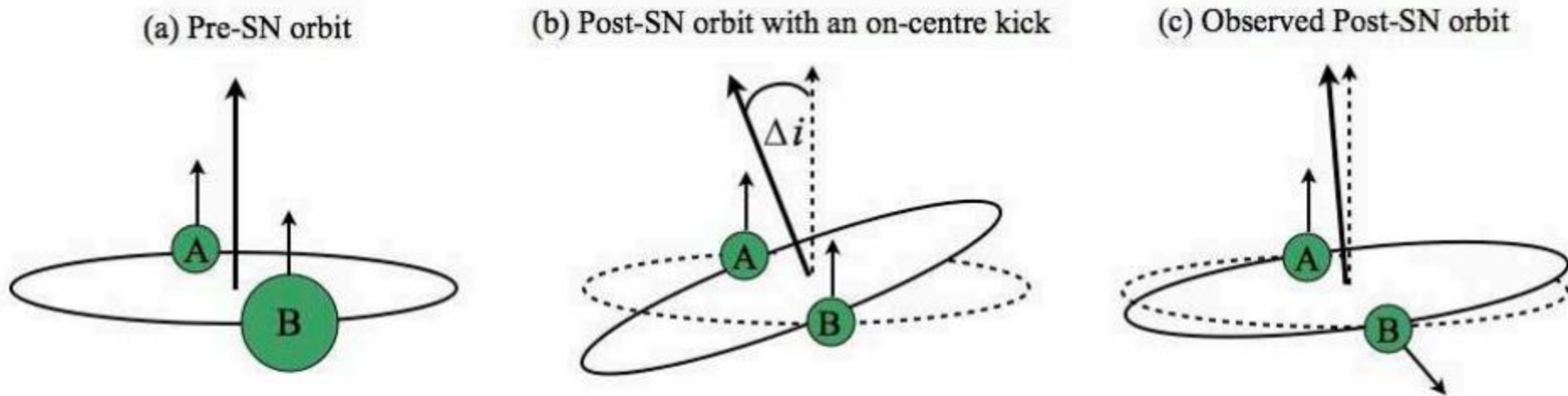
The transversal velocity can be measured by proper motion observations of radio pulsars and other neutron stars

For binaries large velocities are measured (Cir X-1: Johnston et al. 1999).



# Double pulsar PSR J0737-3039

Pulsar A's spin is tilted from the orbital angular momentum by no more than 14 degrees at 95% confidence;  
pulsar B's -- by  $130 \pm 1$  degrees at 99.7% confidence.

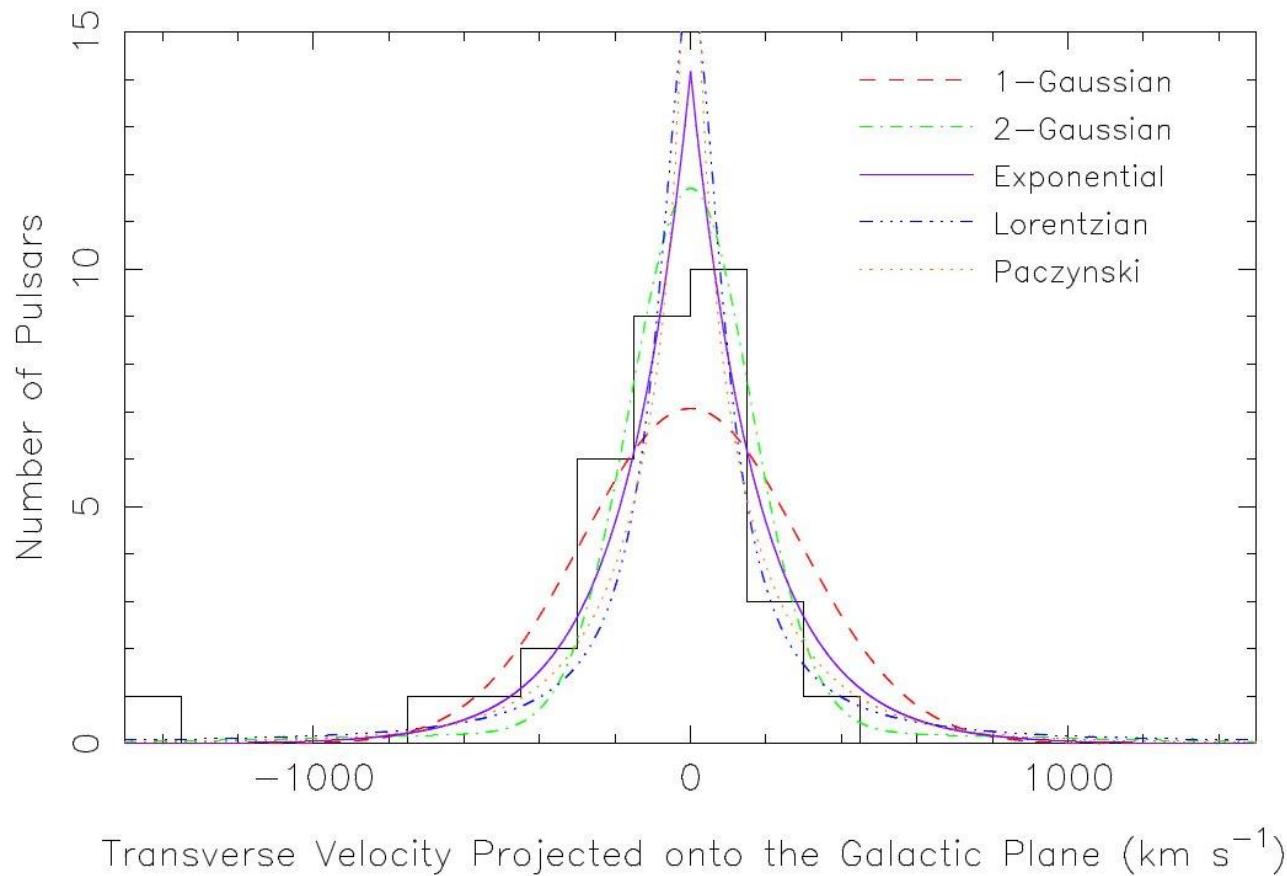


This spin-spin misalignment requires that the origin of most of B's present-day spin is connected to the supernova that formed pulsar B.  
The spin could be thought of as originating from the off-center nature of the kick.

1104.5001

See also 1302.2914 about probably near-zero kick for the pulsar A.

# Many kick velocity distributions are proposed



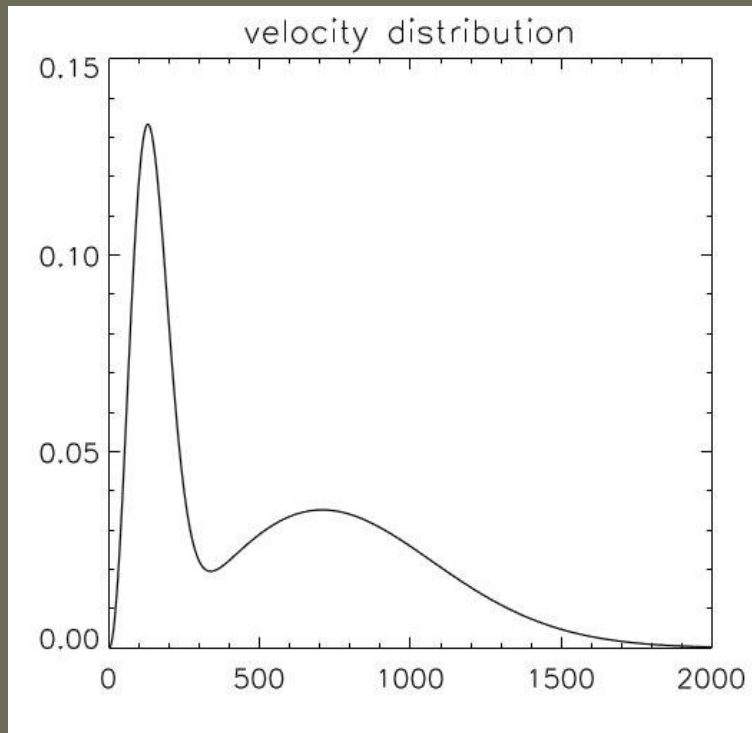
Three popular models:

- Arzoumanian, Chernoff, Cordes (2002)
- Hobbs et al. (2005)
- Faucher-Giguere and Kaspi (2006)

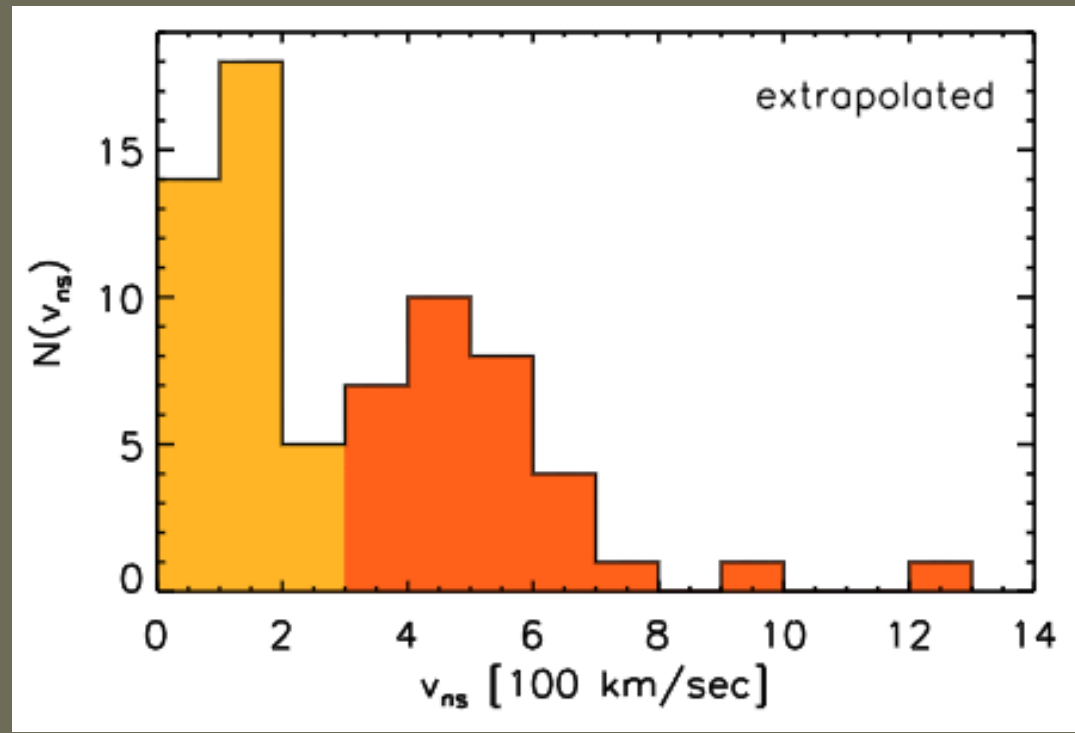
Note the difference:  
We observe present day velocities with selection and evolutionary effects, but we are interested in the velocity at birth!

(Faucher-Giguere, Kaspi 2006)

# Bimodal distribution

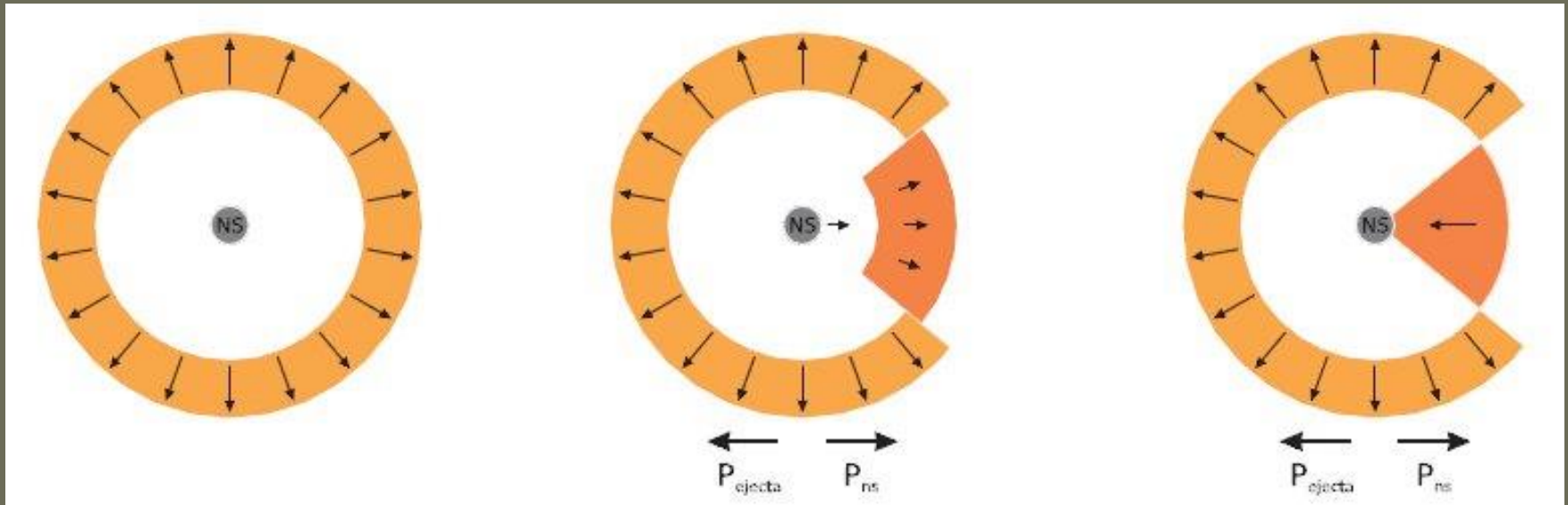


Arzoumanian et al. 2002



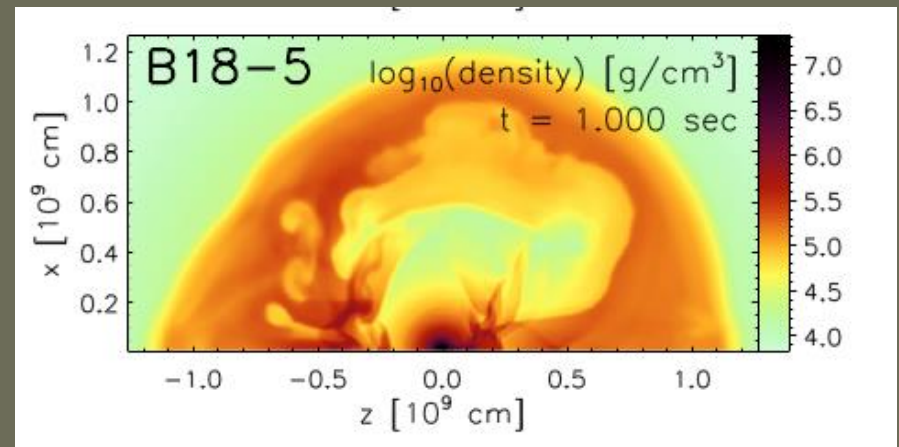
Scheck et al. 2006

# Hydrodynamical models

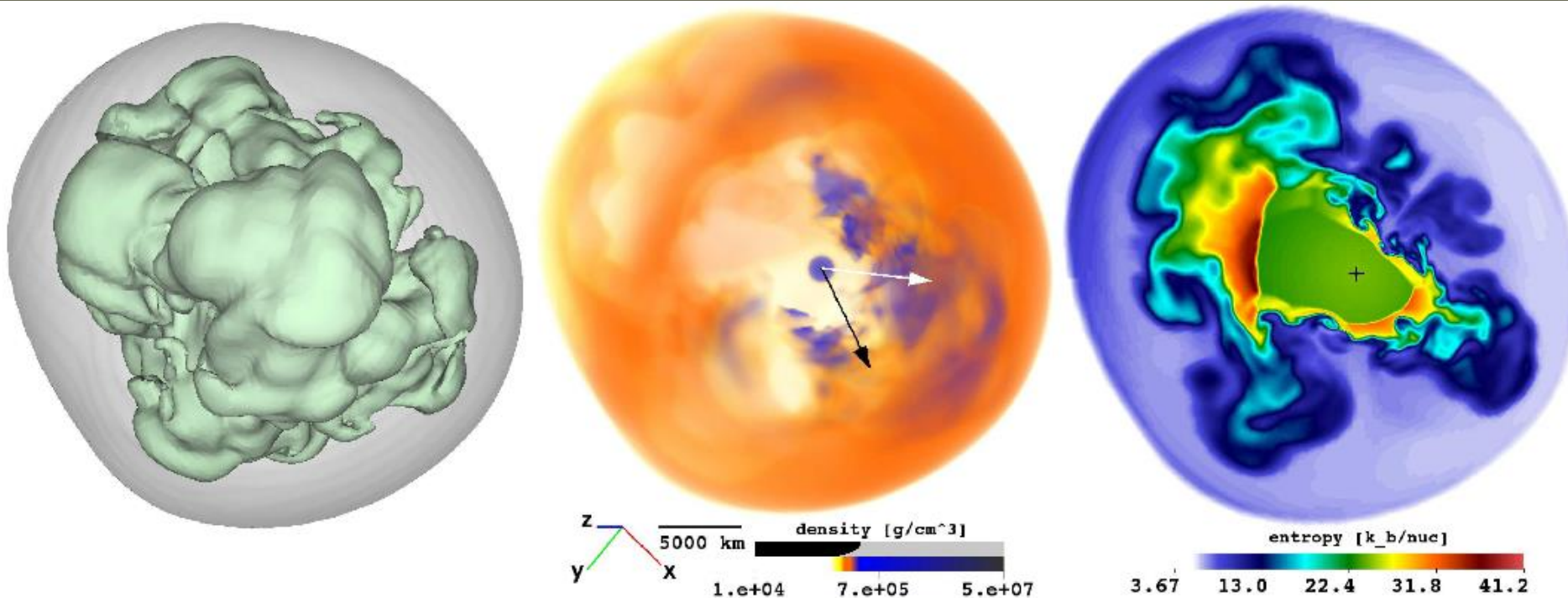


## 2D simulations

Acceleration of a NS is mainly due to gravitational pull of the anisotropic ejecta



# 3D hydrodynamics kicks



$$\mathbf{v}_{\text{ns}}(t) = -\mathbf{P}_{\text{gas}}(t)/M_{\text{ns}}(t)$$

$$\mathbf{P}_{\text{gas}} = \int_{R_{\text{ns}}}^{R_{\text{ob}}} dV \rho \mathbf{v}.$$

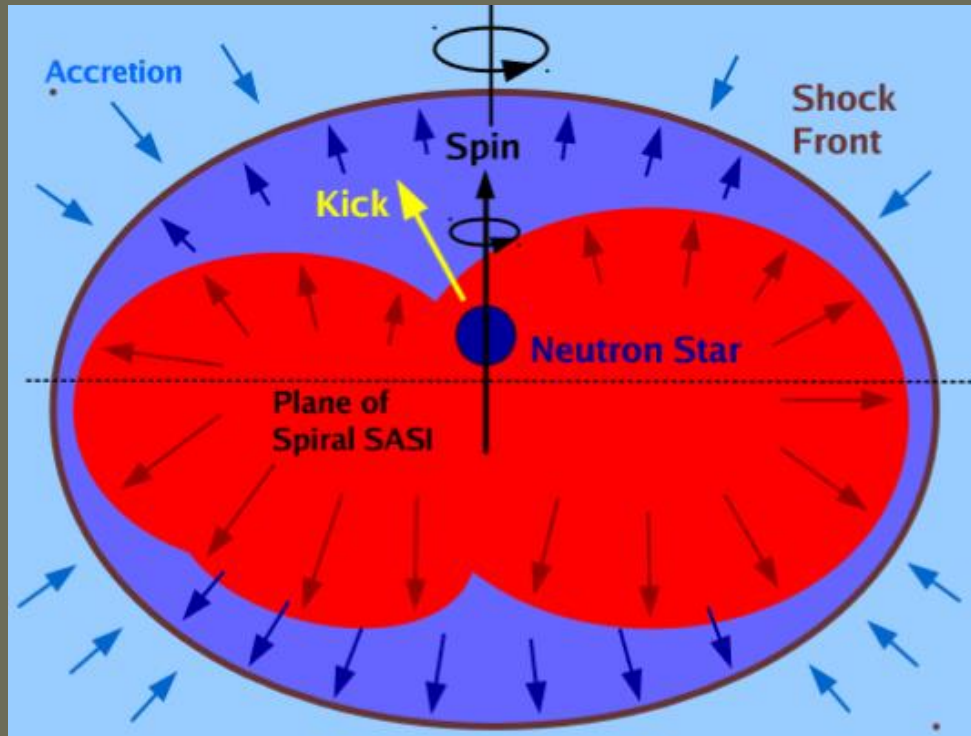
$$v_{\text{ns}} \approx 2G\Delta m/(r_i v_s) \approx 2700 \text{ km s}^{-1}$$

$$r_i = 100 \text{ km} \quad \Delta m = \pm 10^{-3} M_{\odot}$$

$$v_s = 1000 \text{ km s}^{-1}$$



# NS kick models



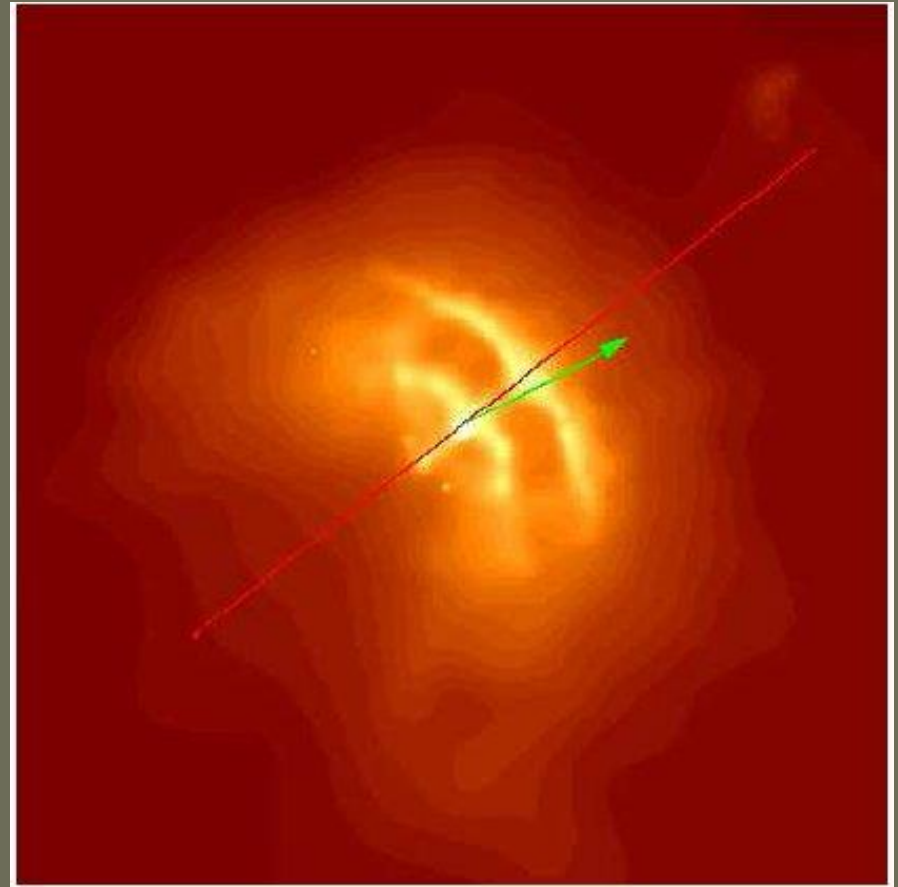
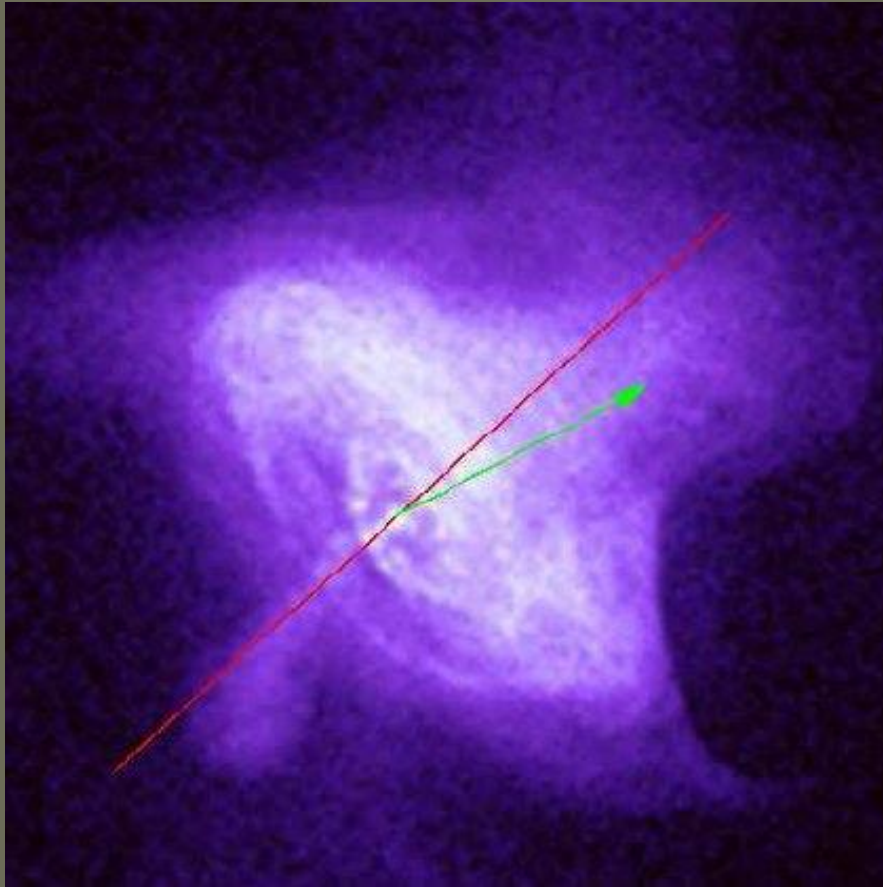
Spin-kick alignment resulting from a neutrino-driven explosion launched from a phase of strong spiral-SASI activity.

While the explosion starts by equatorial expansion, the final NS kick is determined by the slower mass ejection in the polar directions.

The NS is accelerated by the gravitational attraction of the mass in these more slowly expanding, dense regions.

In the cartoon the NS is pulled more strongly towards the northern direction and therefore opposite to the (southern) hemisphere where the explosion is more powerful.

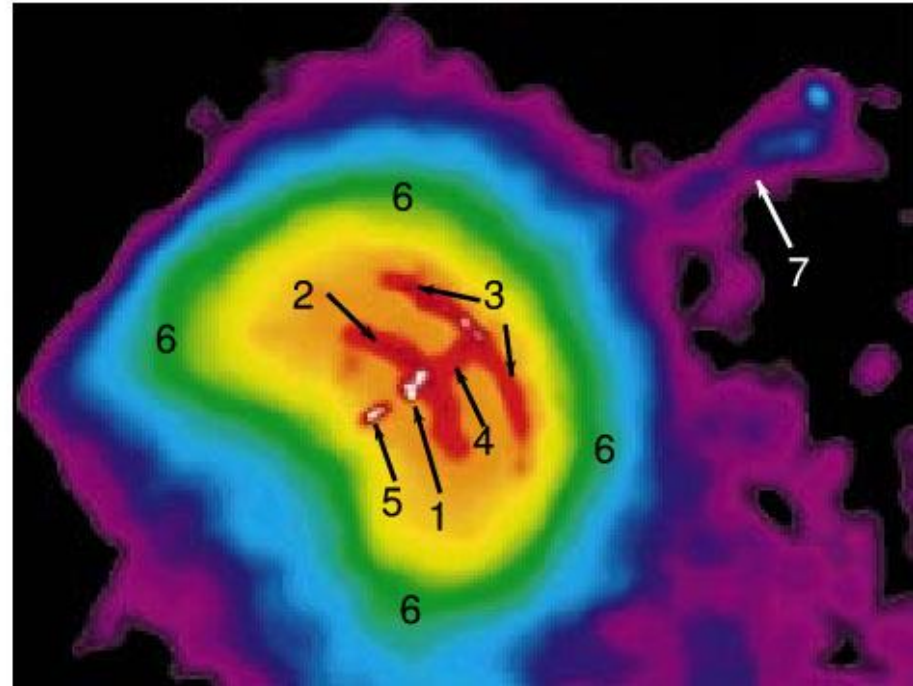
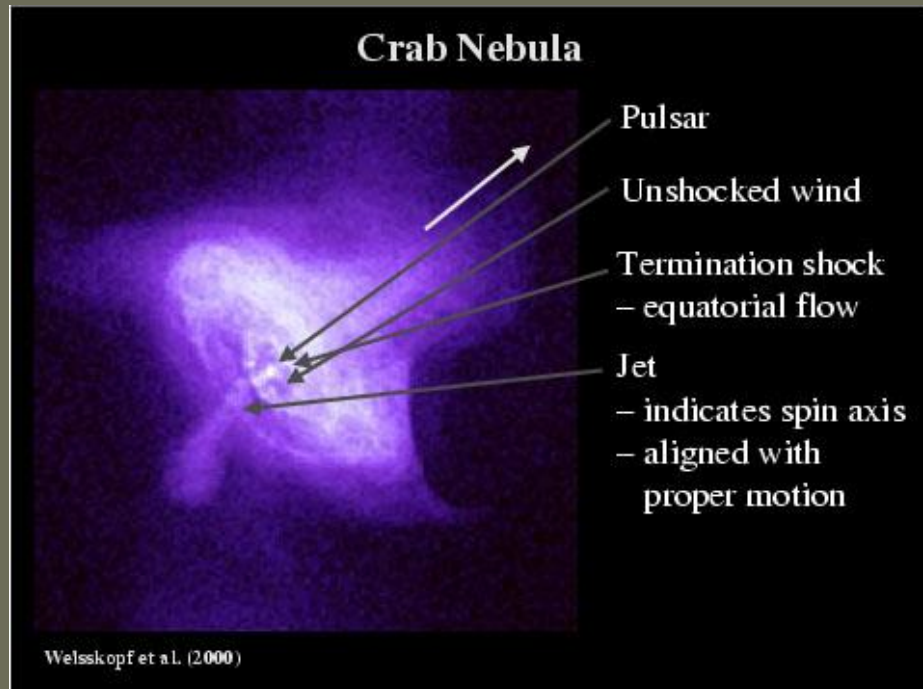
# Spin-velocity alignment



Spatial velocity and spin axis are nearly coincident.  
*Nearly* is important: there is some misalignment.



# The best studied cases: Crab and Vela

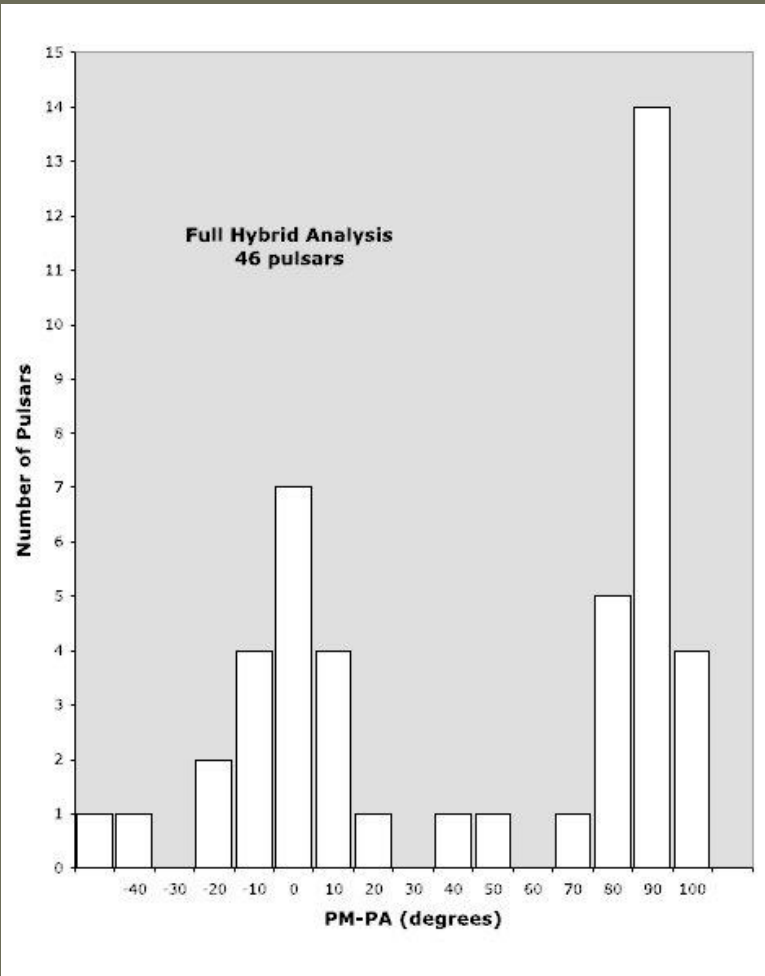


Crab and Vela are not the only cases, but are the best studied ones. Spin-velocity correlation (in direction) is reported for many radio pulsar. For some of them pulsar wind nebula observations are used, for some only direction of proper motion and polarization properties can be used.

# Some set of PSRs with known spin-velocity orientation

2D Pulsars			
B0628–28.....	318	+61/–64	5 ± 4
B0740–28.....	259	+190/–149	7 ± 5
B0823+26.....	189	+55/–34	21 ± 7
B0835–41.....	170	±30	13 ± 11
B0919+06.....	506	±80	32 ± 17
B1133+16.....	639	+38/–35	22 ± 2
B1325–43.....	597	±254	31 ± 22
B1426–66.....	150	+40/–24	5 ± 9
B1449–64.....	219	+55/–18	1 ± 3
B1508+55.....	1082	+103/–90	23 ± 7
B1642–03.....	160	+34/–32	26 ± 5
B1800–21.....	347	+48/–57	7 ± 8
B1842+14.....	512	+51/–50	5 ± 15
B1929+10.....	173	+4/–5	16 ± 2
B2045–16.....	304	+39/–38	3 ± 6
IC 443 .....	250	±50	45 ± 10
3D Pulsars			
J0205+6449.....	838	±251	21 ± 10
B0531+21.....	140	±8	26 ± 3
J0537–6910.....	634	±50	3 ± 5
J0538+2817.....	407	+116/–74	12 ± 4
B0540–69.....	1300	±612	34 ± 33
B0833–45.....	61	±2	10 ± 2
B1706–44.....	645	±194	35 ± 10
J1833–1034.....	125	±30	16 ± 15
B1951+32.....	273	±11	18 ± 5

# Recent data on radio pulsars



Rankin (2007)

J name	B name	log[age] (yr)	$V_T$ $\text{km s}^{-1}$	$PA_v$ ( $^\circ$ )	$PA_0$ ( $^\circ$ )	$\Psi$ ( $^\circ$ )
J0452-1759	B0450-18	6.2	185	72(23)	47(3)	25(23)
J0659+1414	B0656+14	5.0	65	93.1(4)	-86(2)	-1(5)
J0738-4042	B0736-40	6.6	180	313(5)	-21(2)	-26(5)
J0837+0610	B0834+06	6.5	170	2(5)	18(5)	-16(7)
J0837-4135	B0835-41	6.5	360	187(6)	-84(5)	-89(8)
J1604-4909	B1600-49	6.7	510	268(6)	-17(3)	-75(7)
J1735-0724	B1732-07	6.7	570	355(3)	55(5)	-60(6)
J1801-2451	B1757-24	4.2	300	270	-55(5)	-35(5)
J1820-0427	B1818-04	6.2	190	338(17)	42(3)	-64(17)
J1850+1335	B1848+13	6.6	300	237(16)	-45(3)	-78(16)
J1915+1009	B1913+10	5.6	280	174(15)	85(3)	89(15)
J1937+2544	B1935+25	6.7	210	220(9)	-9(5)	49(10)
J2048-1616	B2045-16	6.5	330	92(2)	-13(5)	-75(6)
J2330-2005	B2327-20	6.7	180	86(2)	21(10)	65(10)

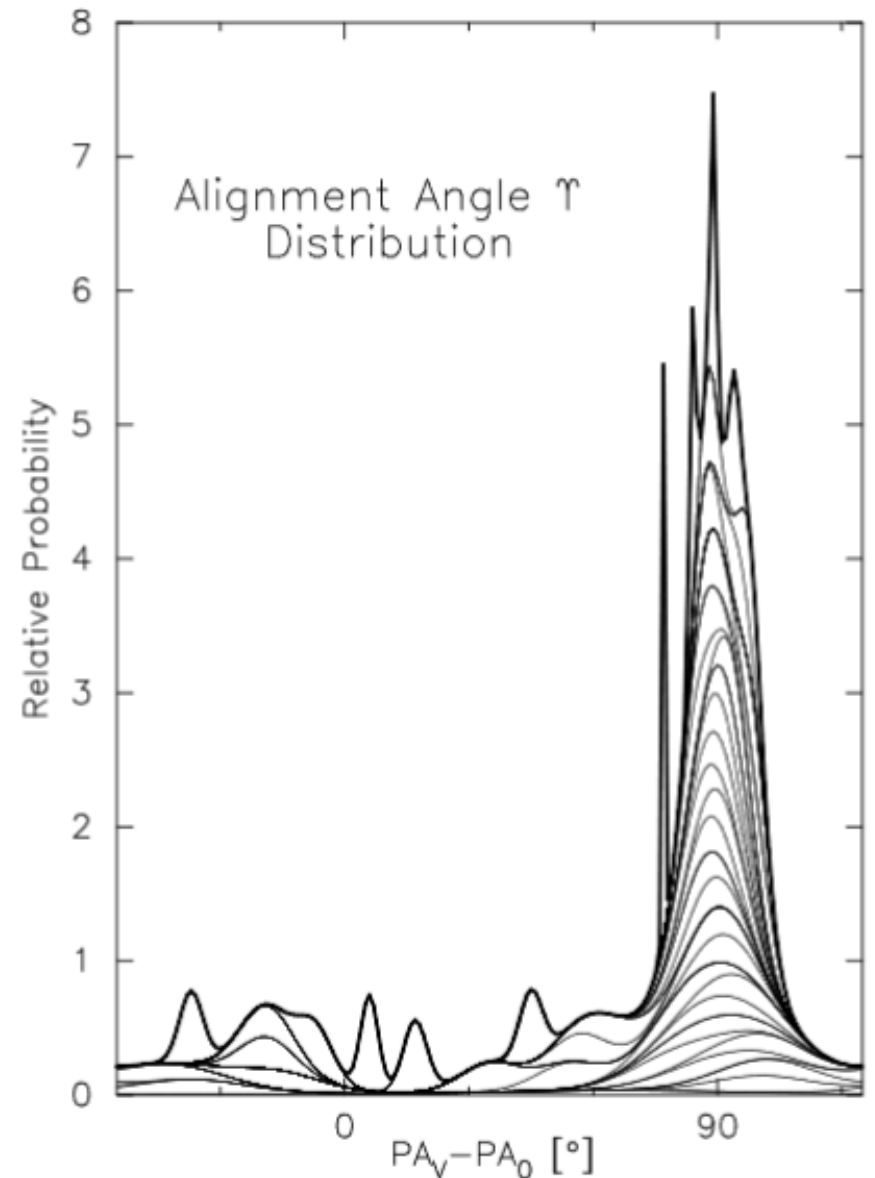
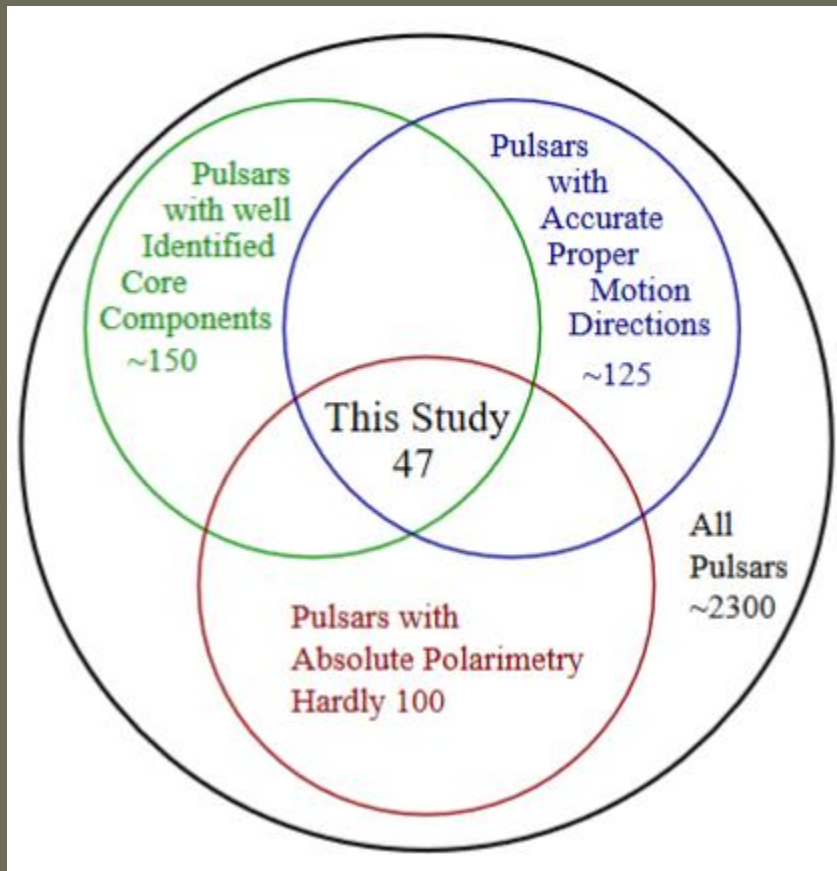
Johnston et al. (2007)

The tendency is clear,  
but it is only a tendency.

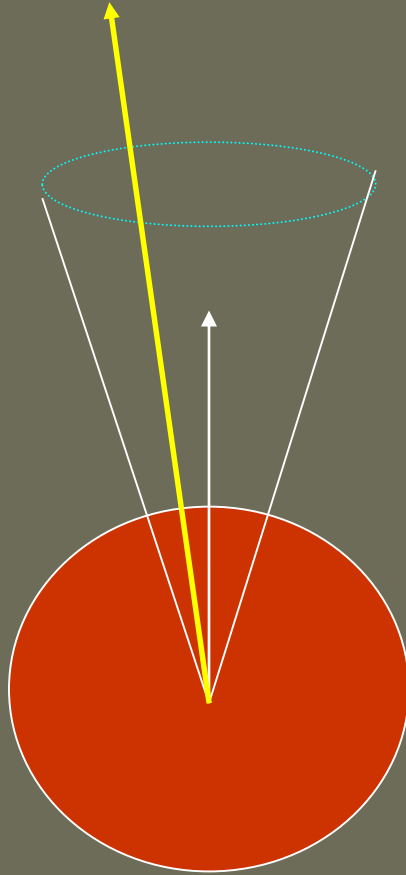
New data and discussion in 1502.05270

# Alignment

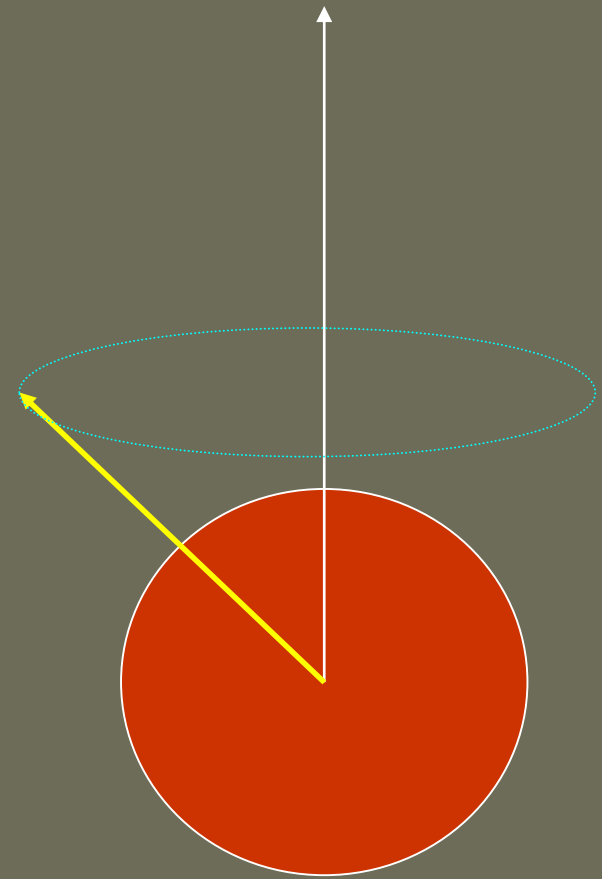
47 pulsars with well-determined parameters.



# Why?

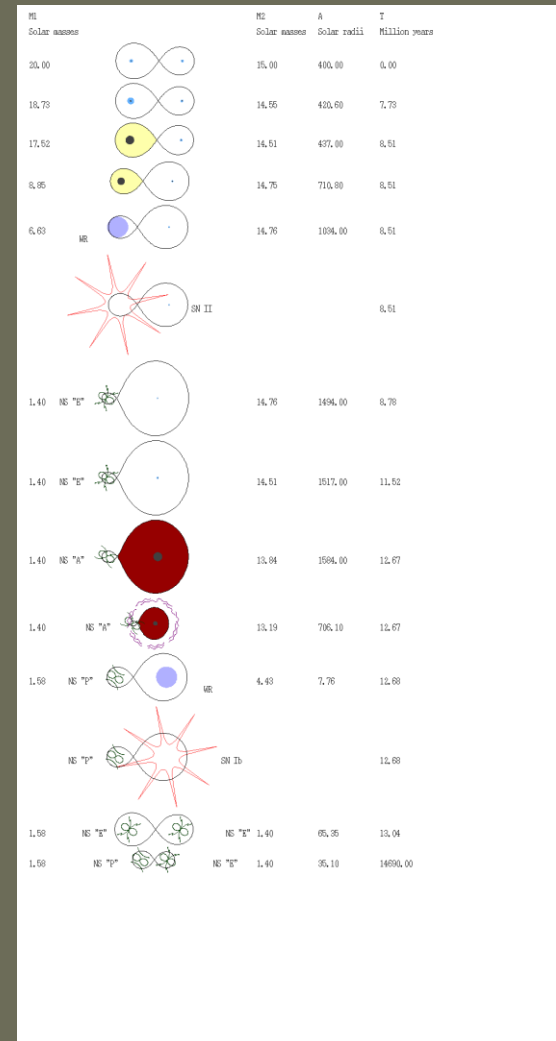
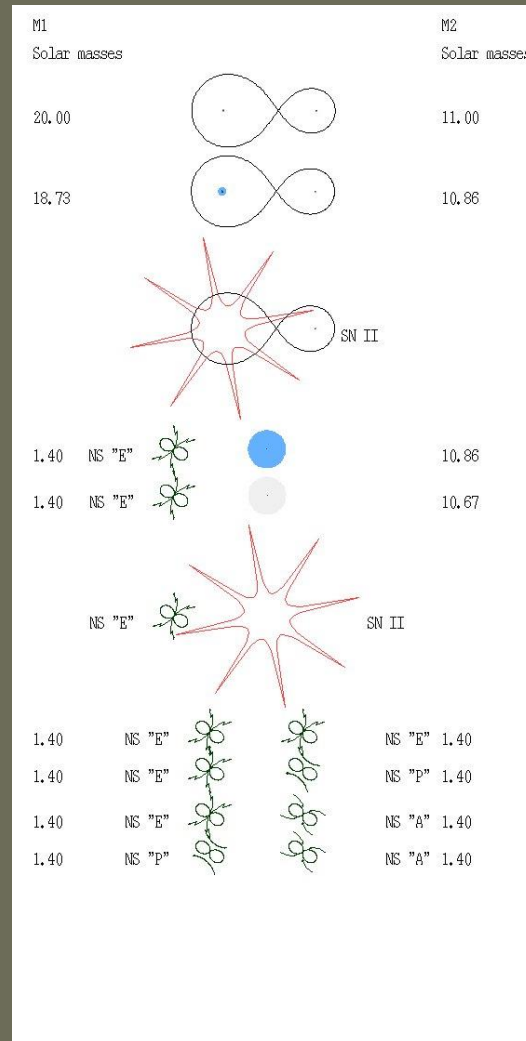
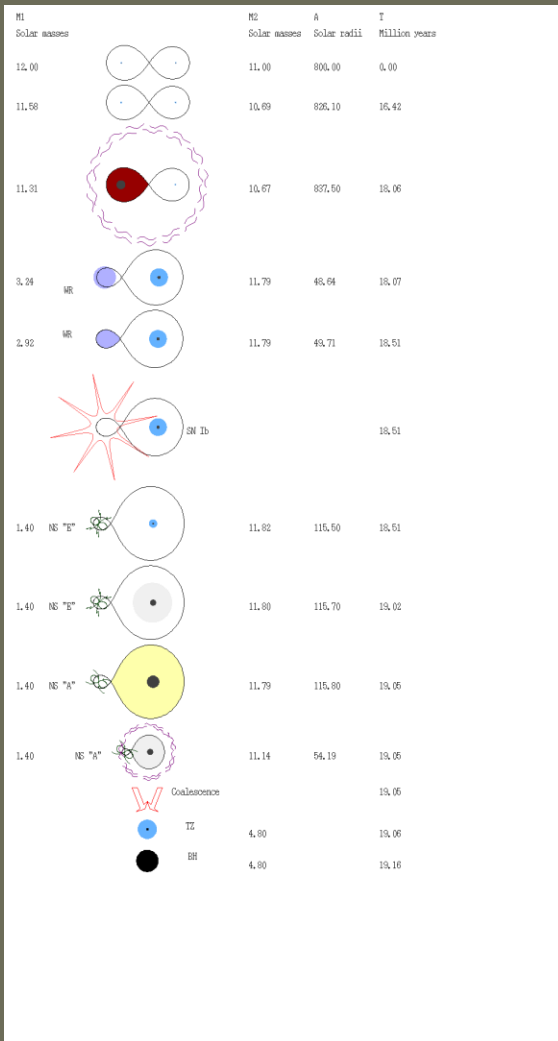


Kick can be confined in some angle around the spin axis. Typical cones must be  $< \sim 10^\circ$  (see, for example, Kuranov et al. 2009).



Kick mechanism can be operative for a long time (many spin periods), so that its influence is average. Typical duration must be 1-10 sec.

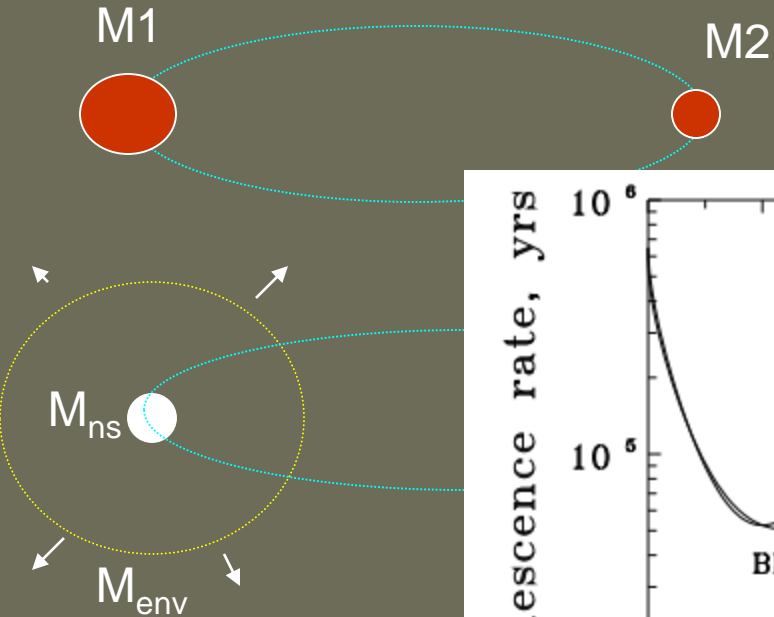
# Kicks in binary evolution





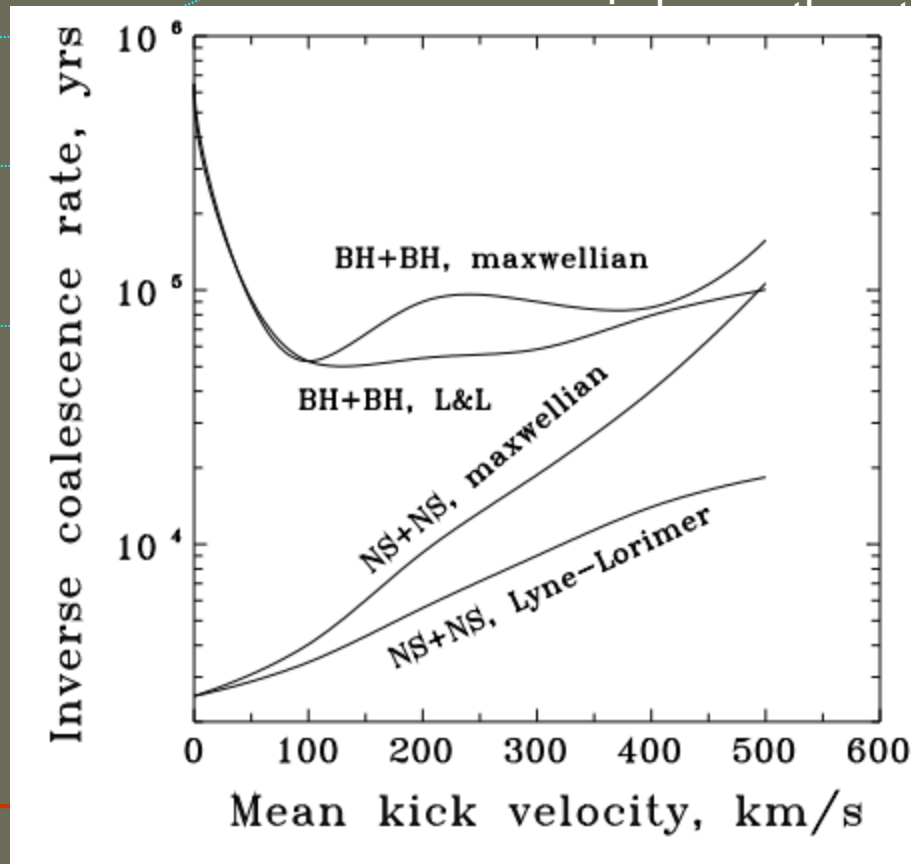
# Influence of kicks on binaries

Kicks can both – destroy and **save** – binaries!



If a more massive star is about to explode, and the lost mass

is less than the sum of the  
and the secondary mass,  
should be destroyed.  
kick can save it.



+BH, BH+BH binaries.  
the more massive star  
ns to produce GW.



# Parameters of binaries after kicks

Kicks significantly influence binary parameters (for example, eccentricity distribution). This is specially important for systems which survived the second explosion (NS+NS).

There are examples, when a NS rotates “in a wrong direction”, i.e. its orbital motion is in the direction opposite to the spin of the second companion.

For detailed description see Postnov, Yungelson (astro-ph/0701059) pp. 18-22.

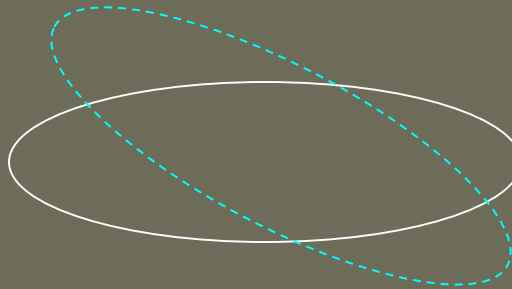
$$\frac{a_f}{a_i} = \left[ 2 - \chi \left( \frac{w_x^2 + w_z^2 + (V_i + w_y)^2}{V_i^2} \right) \right]^{-1}$$

$$1 - e^2 = \chi \frac{a_i}{a_f} \left( \frac{w_z^2 + (V_i + w_y)^2}{V_i^2} \right)$$

$$\chi \equiv (M_1 + M_2)/(M_c + M_2) \geq 1.$$

$$\cos \theta = \frac{\vec{J}_f \cdot \vec{J}_i}{|\vec{J}_f| |\vec{J}_i|},$$

$$\cos \theta = \frac{V_i + w_y}{\sqrt{w_z^2 + (V_i + w_y)^2}}.$$



$$\frac{V_f}{V_i} \geq \sqrt{\frac{2}{\chi}}.$$



Disruption  
condition

# $e^-$ -supernovae with low kicks

In 80s it was proposed by Nomoto, Miyaji et al. that in some cases a SN explosion can happen due to electron capture by  $^{24}\text{Mg}$  and  $^{20}\text{Ne}$  (no iron core is formed).

It was noticed (Pfahl et al. 2002, Podsiadlowski et al. 2004; van den Heuvel 2004, 2007) that among Be/X-ray binaries there is a group of systems with small eccentricities.

But they suffered one SN explosion and there was no Roche-lobe overflow.

This means that kicks in these systems were low.

The same is true for some of NS+NS binaries.

The proposed mechanism is related to  $e^-$ -capture SN.

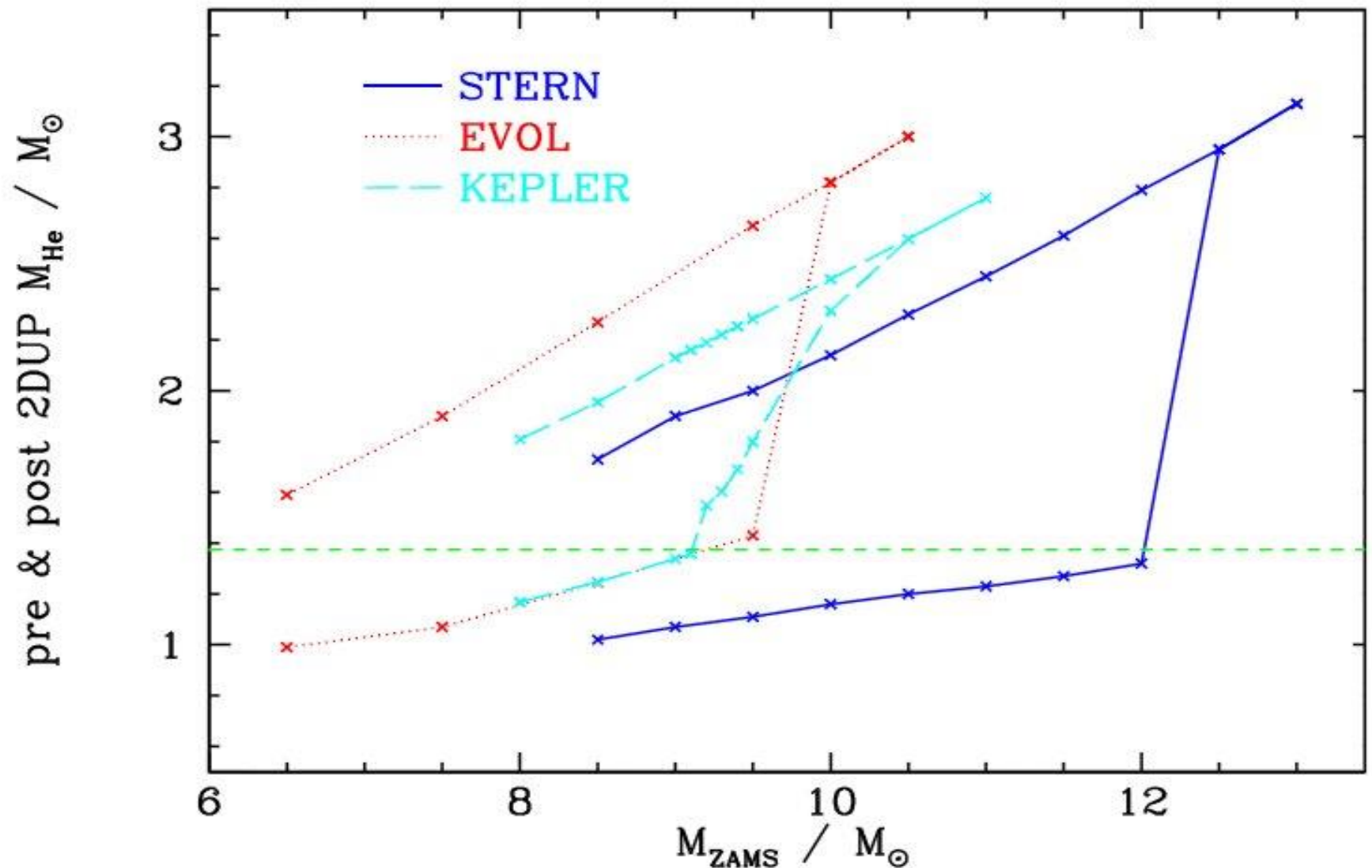
Such explosions can appear not only in binary systems, but in binaries they can be more frequent.

Among isolated stars about 4% (up to ~20%!) of SN can be of this type (Poelarends et al. 2008). [It is not clear if they appear among normal PSRs.]

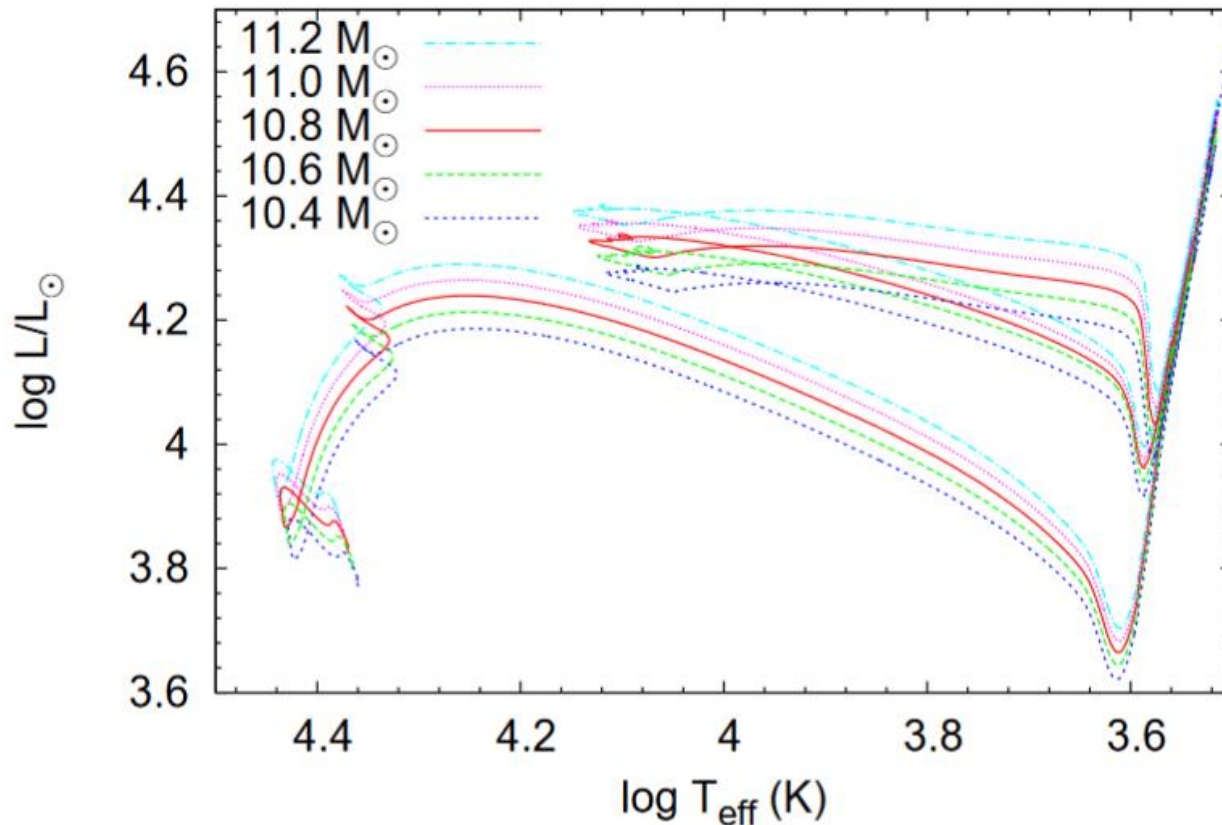
Why kick is low? Uncertain. Low core mass, rapid explosion, low mass ejection...

# $e^-$ -capture SN in binaries

Poelarends et al. 2008. 0705.4643



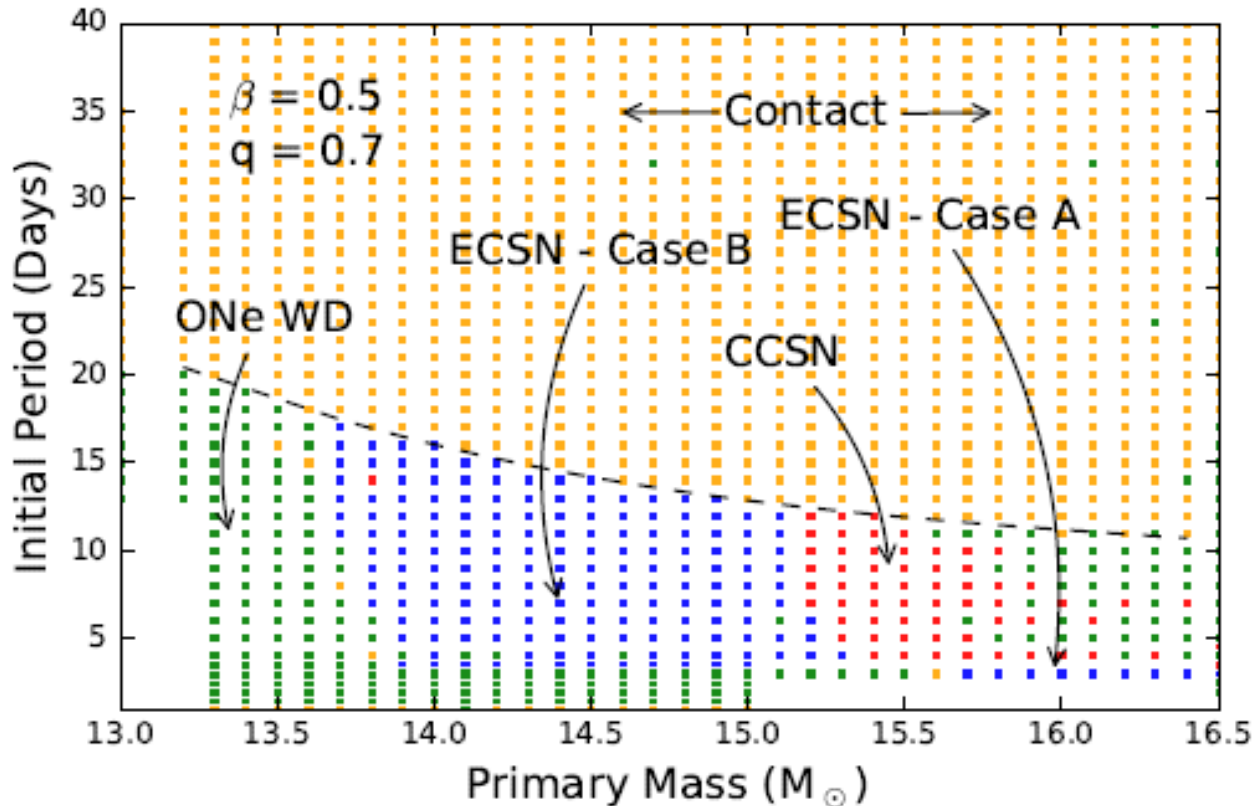
# Evolution of $e^-$ -capture SN progenitors



Critical core mass 1.367 solar masses.

For initial stellar masses  $>11$  solar neon is ignited, and later on a Fe-core is formed.

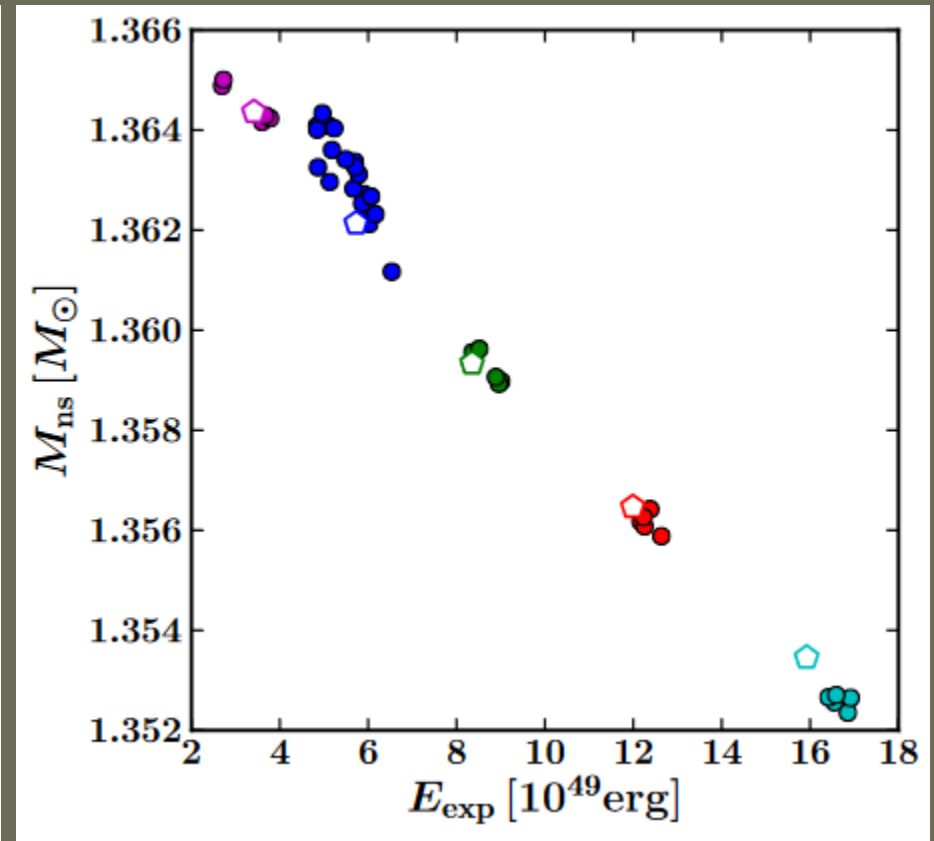
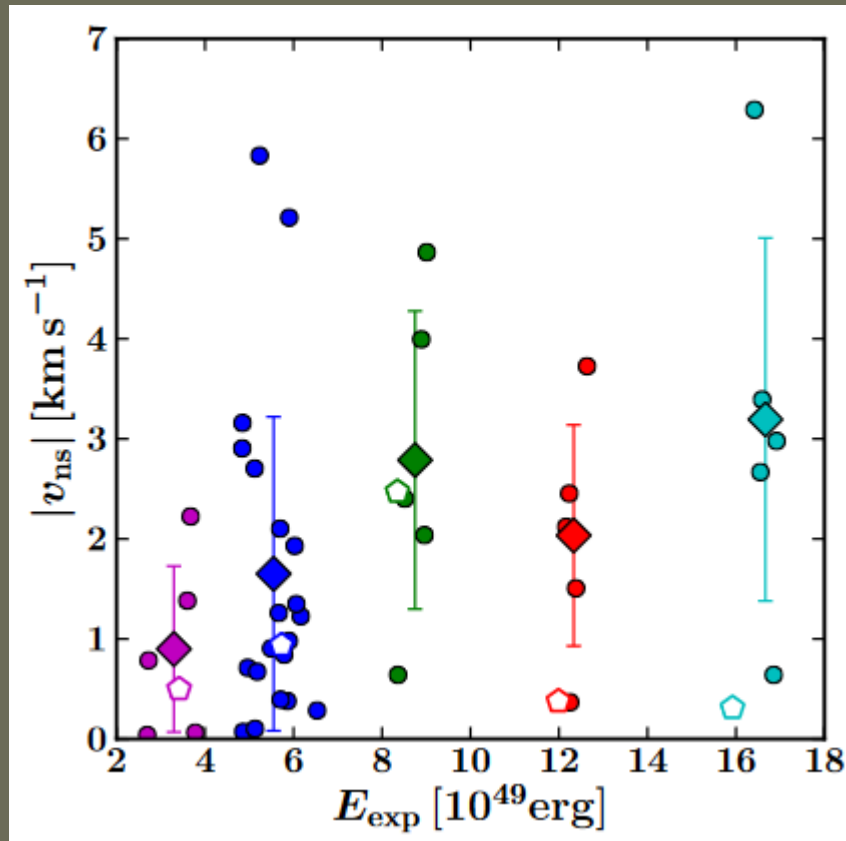
# $e^-$ -capture SN in close binaries



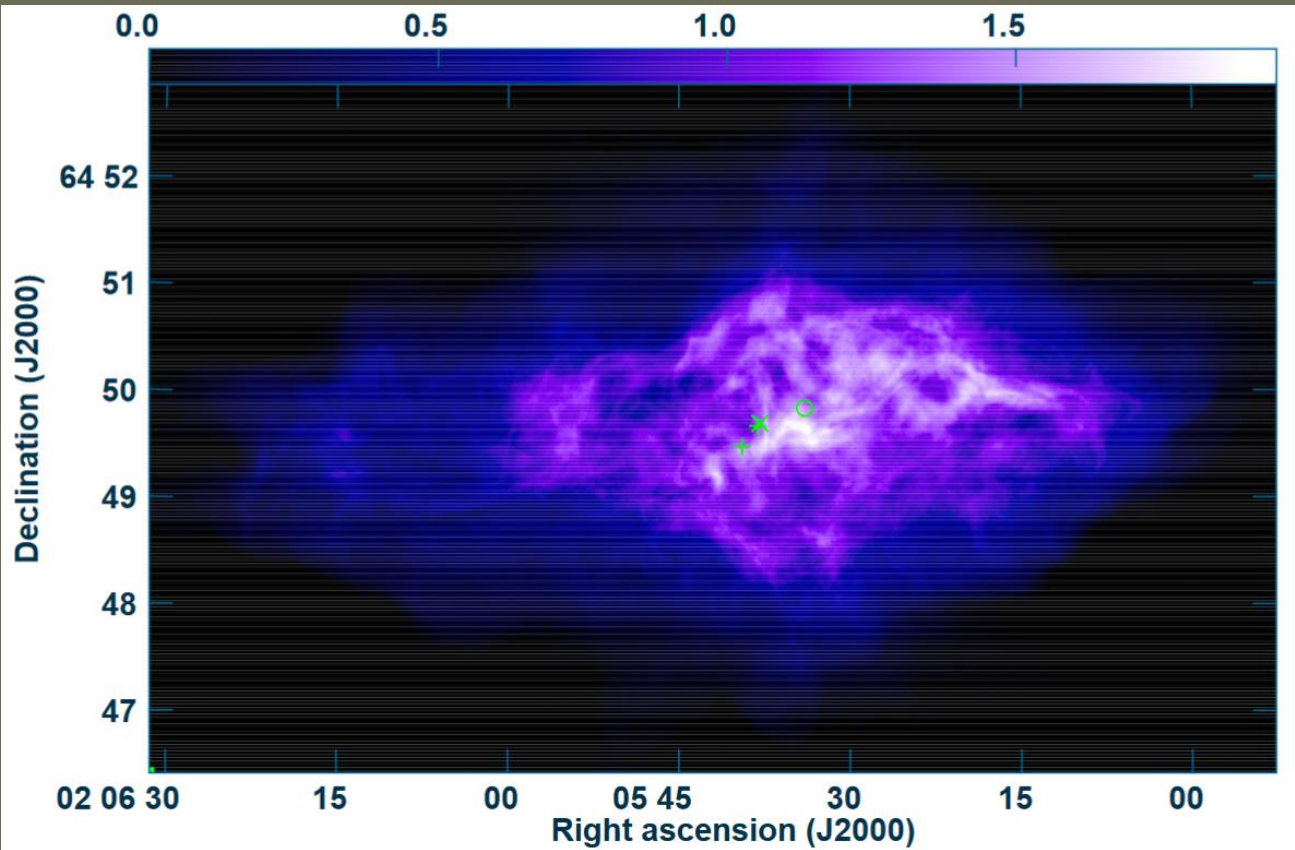
The initial primary mass and the mass transfer evolution are important factors in the final fate of stars in this mass range

# $e^-$ -capture SN and Crab

Calculations confirm that in  $e^-$ -capture SN kicks are low (tag-boat, i.e. gravitational pull mechanism, is not effective). Thus, Crab pulsar was not born in an  $e^-$ -capture explosion.



# Pulsars with low velocities



Some NSs demonstrate low spatial velocities. Obviously, this is due to low kicks.

3C58.

Low kick velocity.

Projected velocity 30-40 km/s



# Kicks as fingerprints

Think about young highly magnetized NSs of different types:

- SGR
- AXP
- RRATs
- Magnificent Seven

Are they relatives?

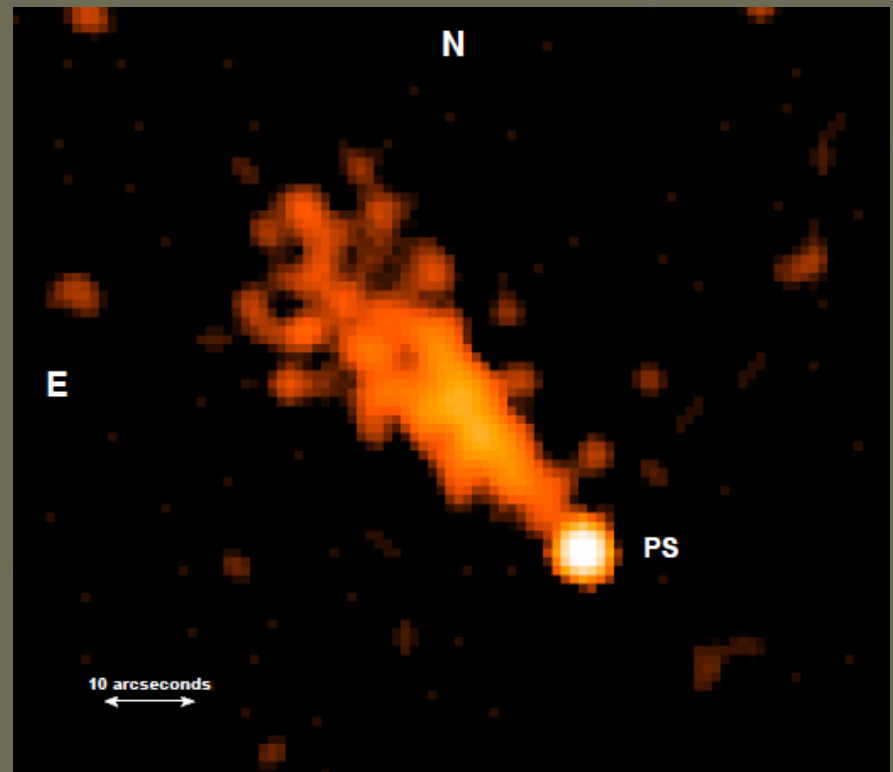
It is a difficult question, but velocity measurements can give you a hint.  
Even if fields are decayed, rotation is slowed down, thermal energy is emitted ...  
if they are relatives – velocity distributions must be identical.  
Unfortunately, now we do not know the answer.

# Magnetar velocity measurements

SGR 1806-20	350 +/- 100 km/s	arXiv:1210.8151
SGR 1900+14	130 +/- 30 km/s	arXiv:1210.8151
PSR J1550-5418	280 +/- 130 km/s	arXiv:1201.4684
XTE J1810-197	200 km/s	Helfand et al. (2007)

# Record velocities

1. PSR J1357-6429 1600-2000 km/s [arXiv:1206.5149](#) - shown to be wrong
2. IGR J11014-6103 2400-2900 km/s [arXiv: 1204.2836](#)

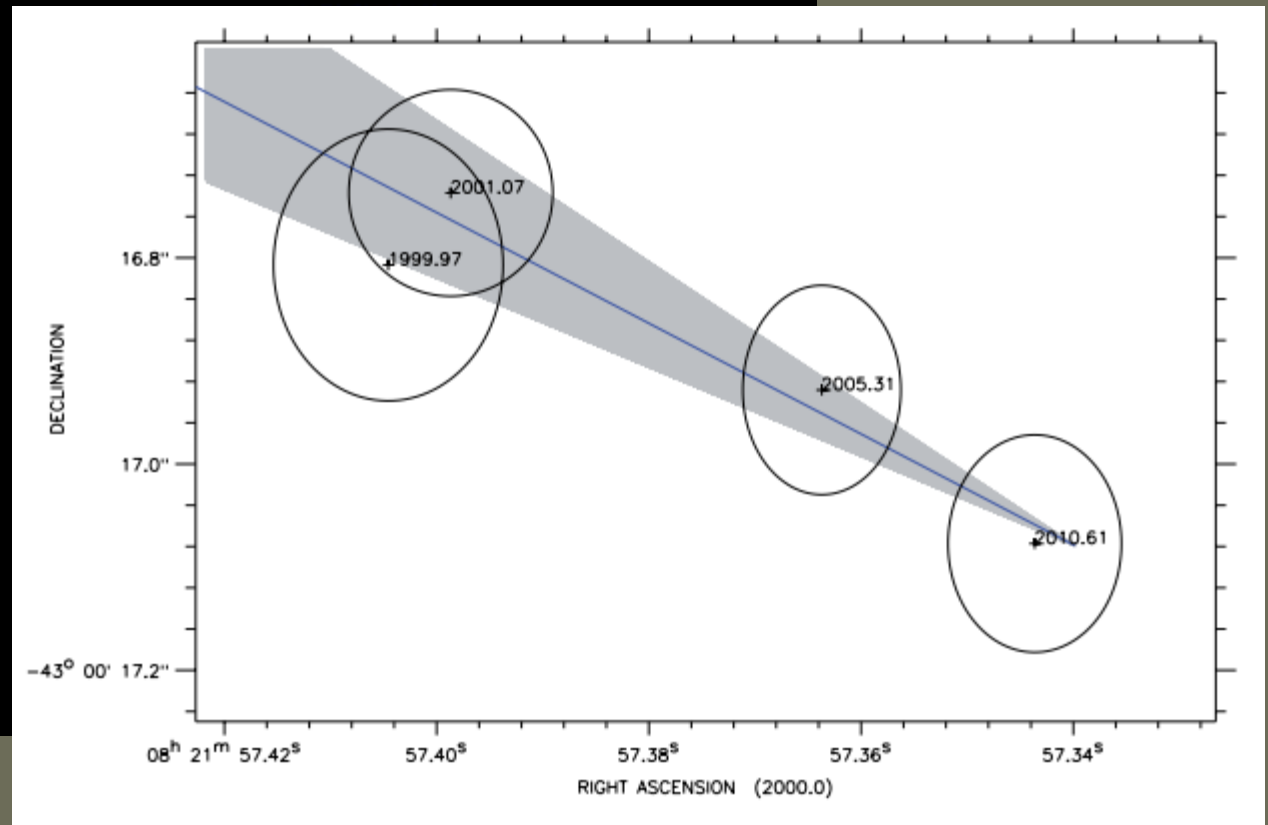


# CCO velocities

RX J0822-4300 in the Supernova Remnant Puppis A

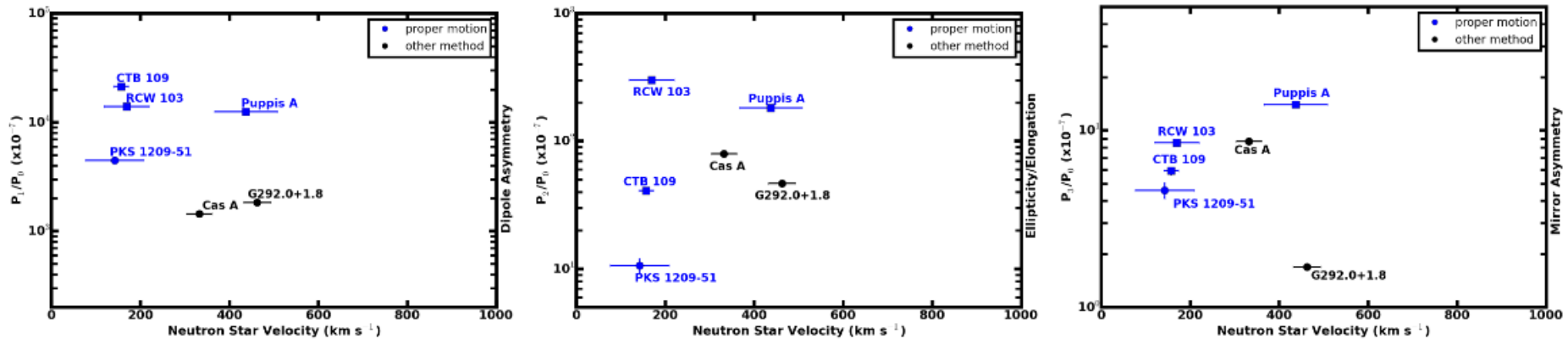
672 +/- 115 km/s

arXiv: 1204.3510

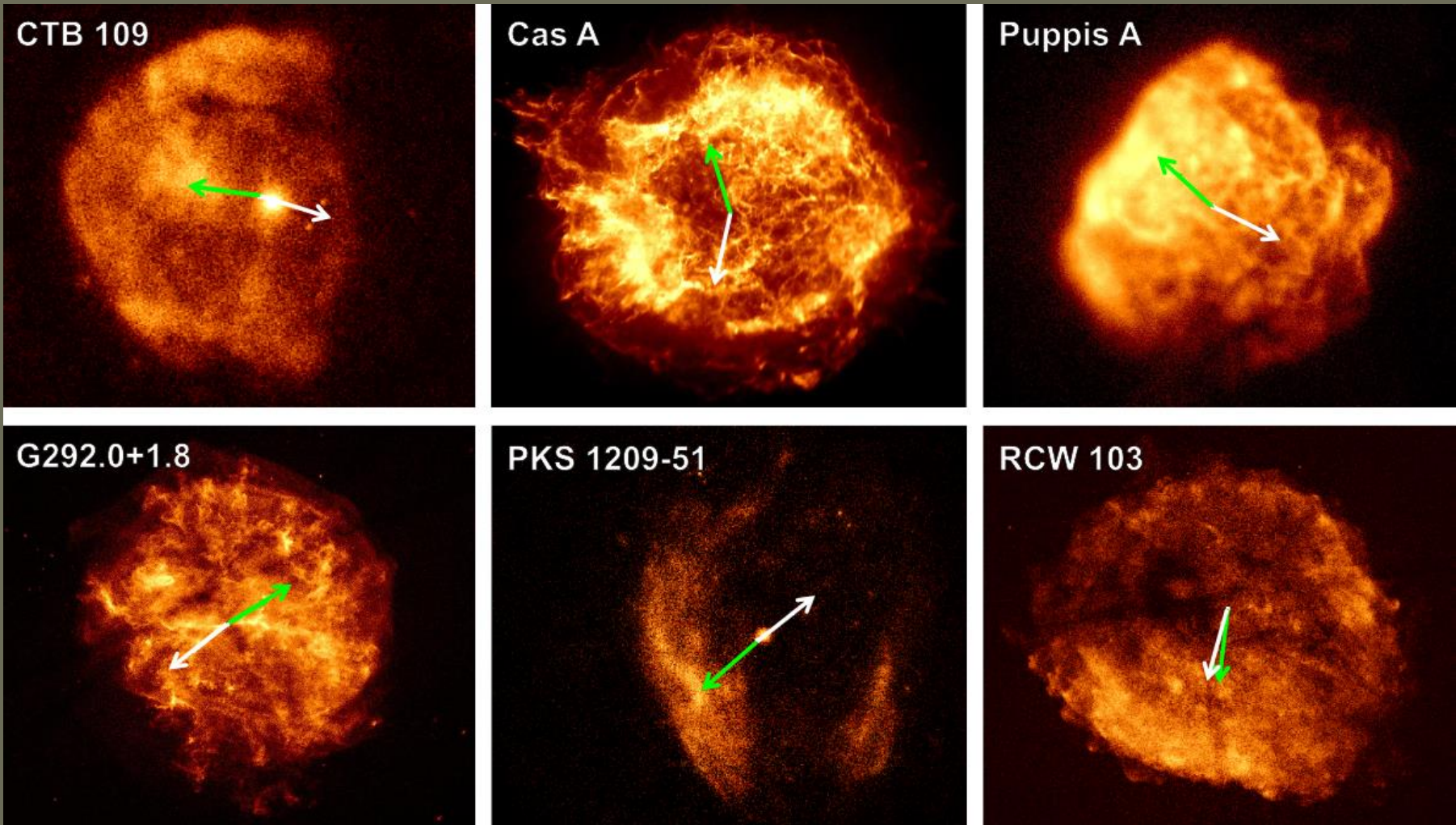


# Kick velocity and SNR morphology

18 young (<20 kyr) SNR with NSs (with velocity) fully imaged by Chandra or ROSAT. Thermal X-ray emission distribution is studied.



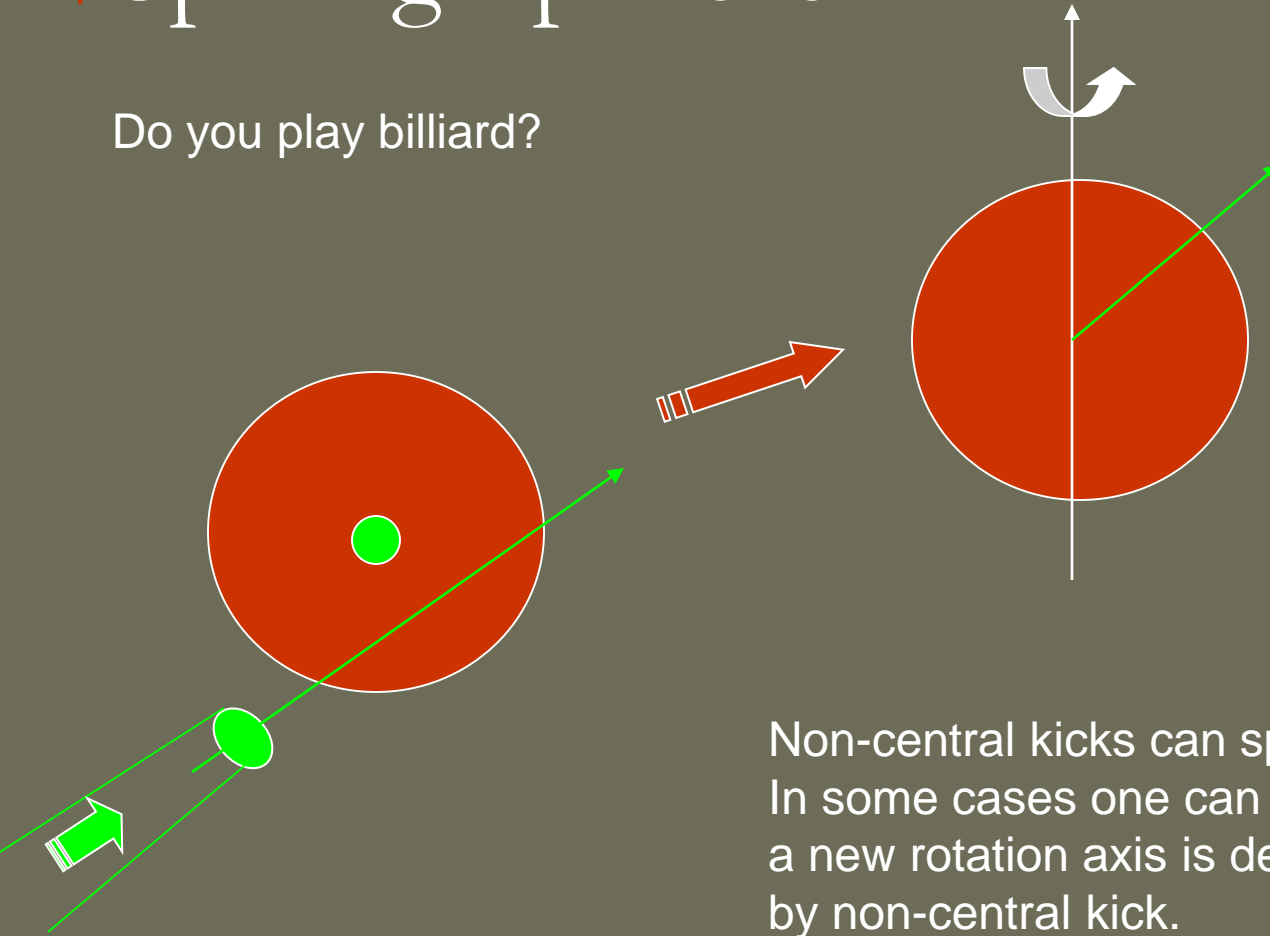
# Dipole anisotropy and velocity



Green – dipole anisotropy of X-ray thermal emission distribution

# Spinnig-up kicks

Do you play billiard?

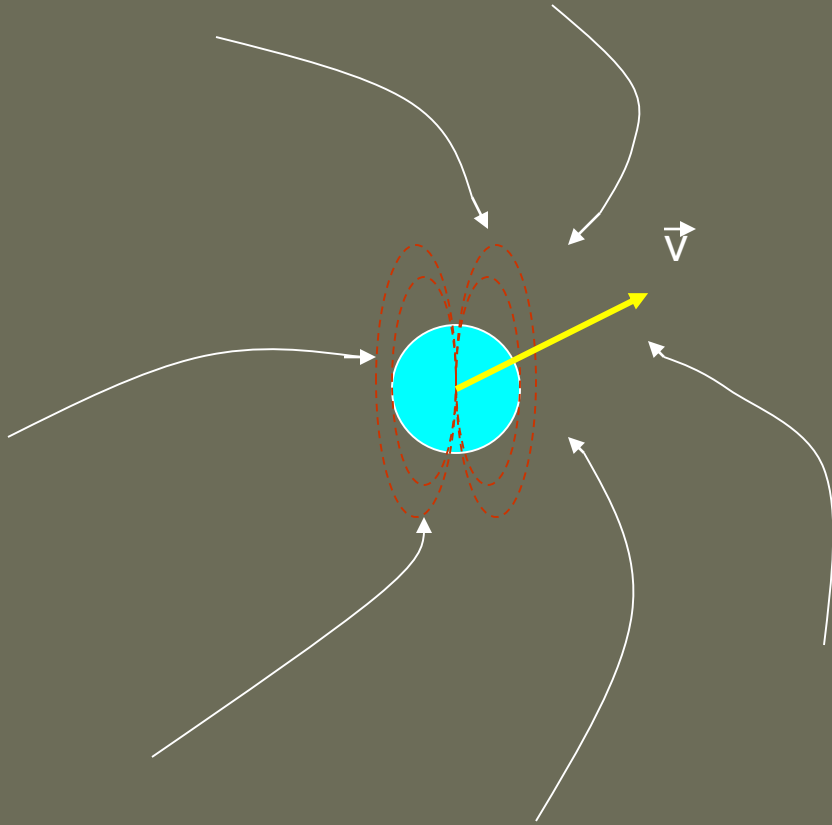


Non-central kicks can spin-up a NS.  
In some cases one can speculate that  
a new rotation axis is determined mainly  
by non-central kick.  
But then velocity - spin period correlation is expected.



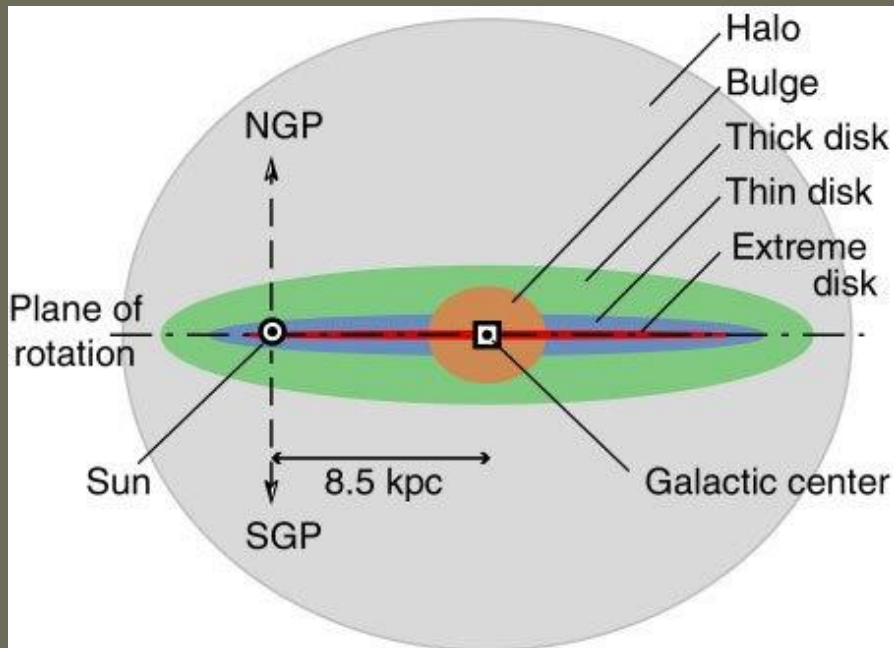
# Evolution of isolated NSs and kicks

Evolution of an isolated NS depends on the intensity of its interaction with the ISM. This intensity depends on the relative velocity of a NS and the ISM.



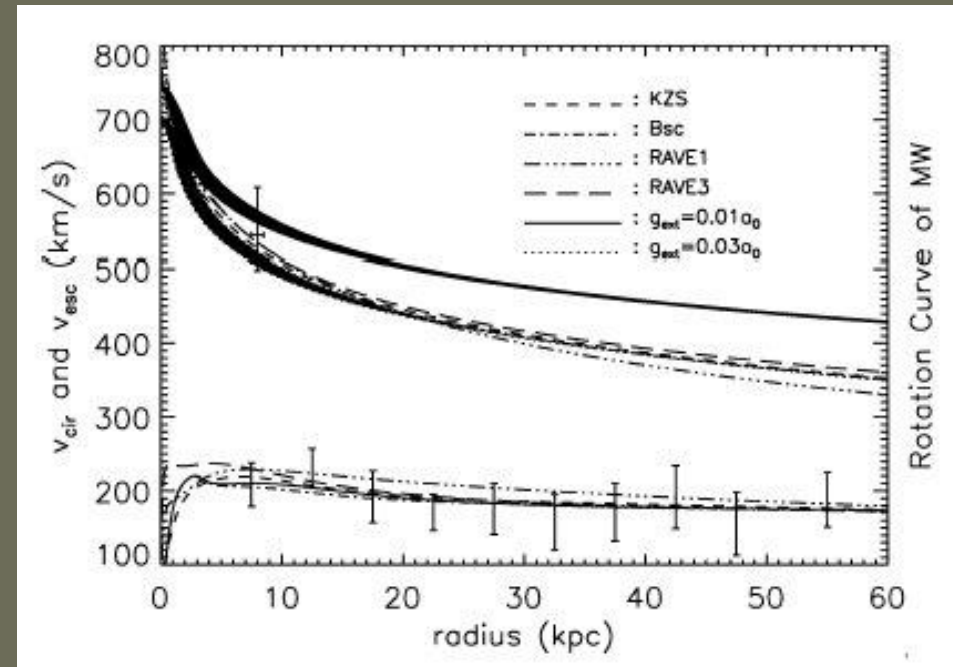
Will a NS start to accrete from the ISM, or will it stay as Ejector, or Propeller, or will it enter another regime strongly depends on the relative velocity of a NS and the ISM.

# Galactic potential

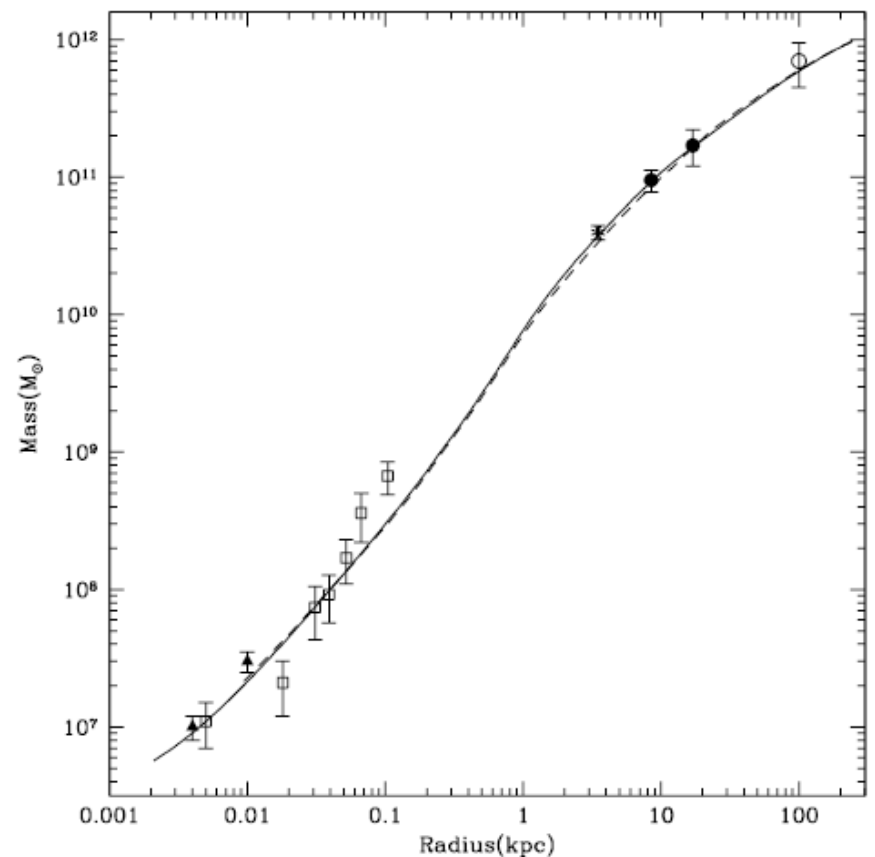
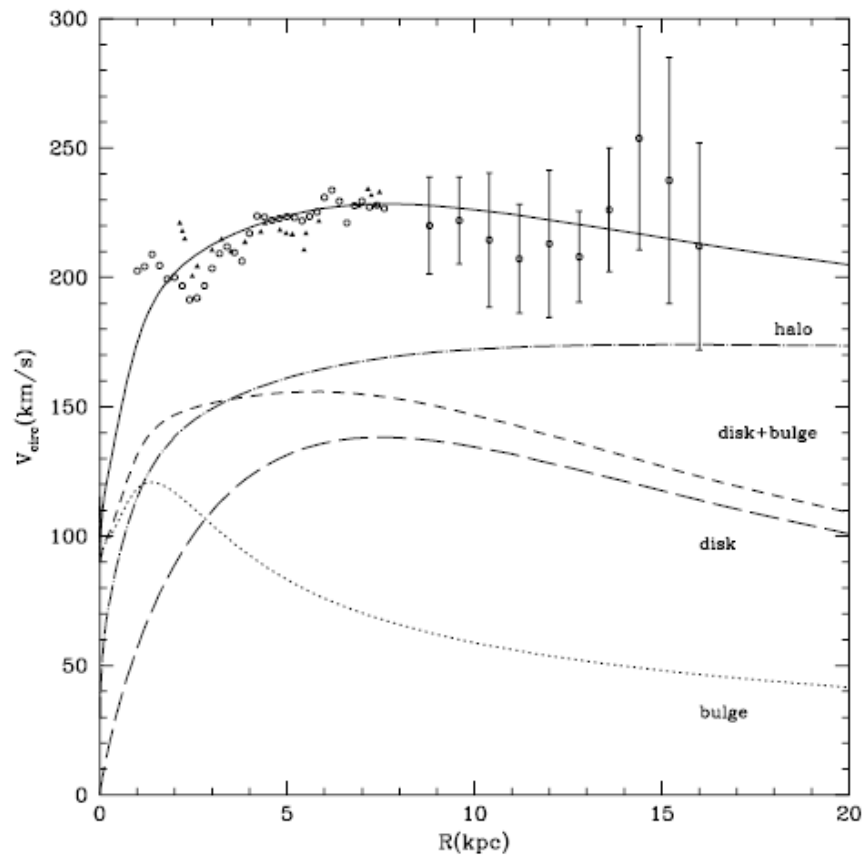


Clearly, some NSs are rapid enough to leave the Galaxy.

Z-distribution of PSRs is much wider than the progenitors' one.



# Mass distribution in the Galaxy



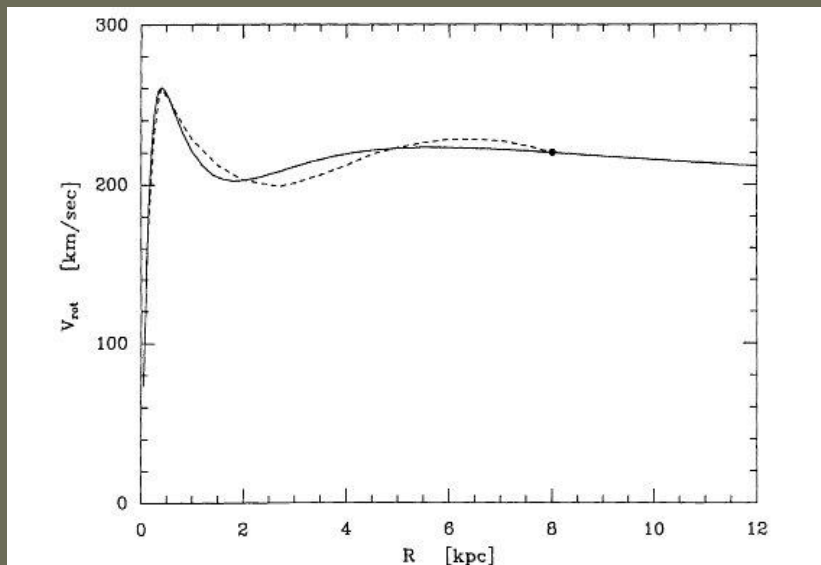
Klypin et al. (2002)

# “Paczynski” model

Disc+Bulge+Halo

Actually, it is Miyamoto, Nagai (1975) model.

It is simple and popular in NS motion calculations.



$$\Phi_i(R, z) = \frac{GM_i}{\{R^2 + [a_i + (z^2 + b_i^2)^{1/2}]^2\}^{1/2}}, \quad R^2 = x^2 + y^2$$

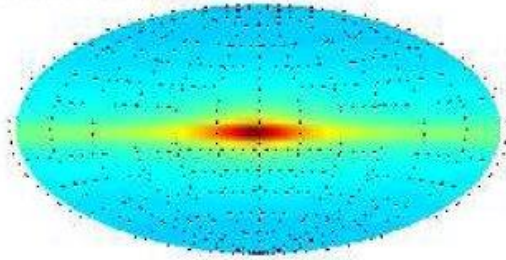
$$\Phi_h = -\frac{GM_c}{r_c} \left[ \frac{1}{2} \ln \left( 1 + \frac{r^2}{r_c^2} \right) + \frac{r_c}{r} \operatorname{atan} \left( \frac{r}{r_c} \right) \right],$$
$$M_c \equiv 4\pi\rho_c r_c^3$$

$$a_1 = 0, \quad b_1 = 0.277 \text{ kpc}, \quad M_1 = 1.12 \times 10^{10} M_\odot,$$
$$a_2 = 3.7 \text{ kpc}, \quad b_2 = 0.20 \text{ kpc}, \quad M_2 = 8.07 \times 10^{10} M_\odot,$$
$$r_c = 6.0 \text{ kpc}, \quad M_c = 5.0 \times 10^{10} M_\odot,$$

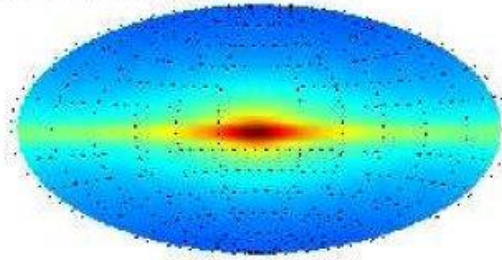
At the very center one has to add the central BH potential

# Examples of old NS distribution

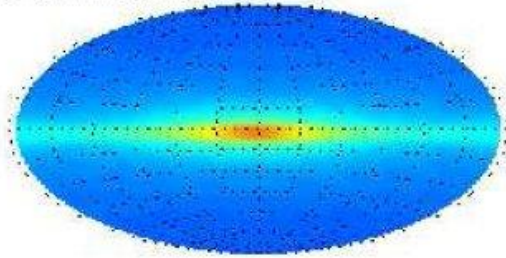
a: Unimodal all



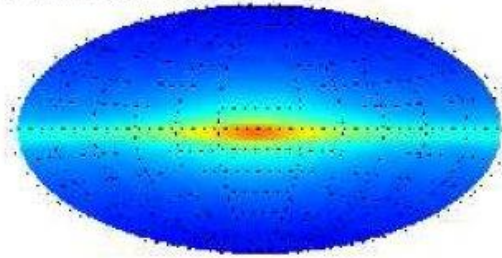
d: Bimodal all



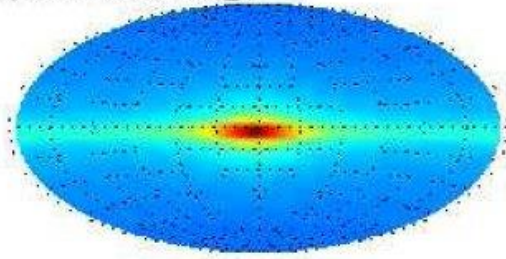
b: Unimodal disk



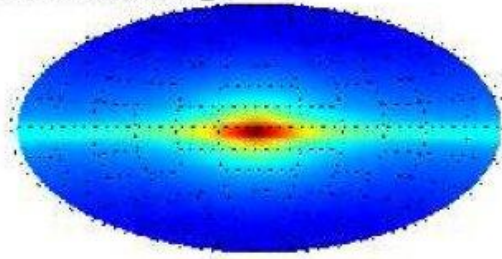
e: Bimodal disk



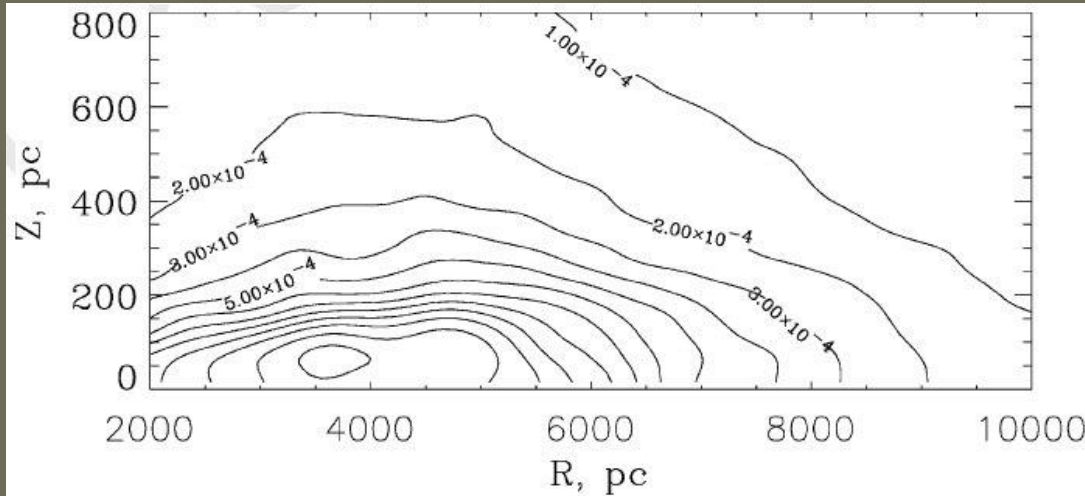
c: Unimodal bulge



f: Bimodal bulge

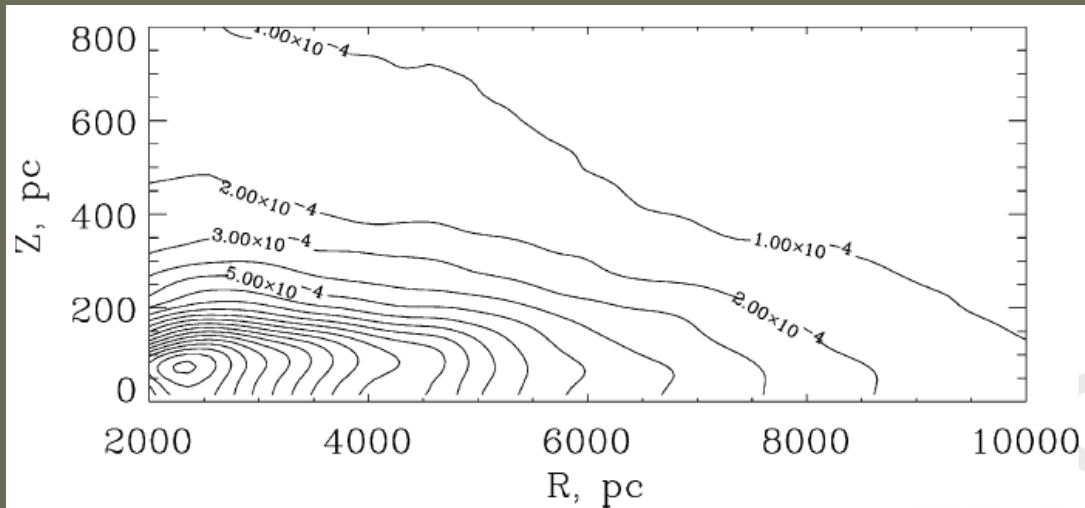


# Spatial density of NSs



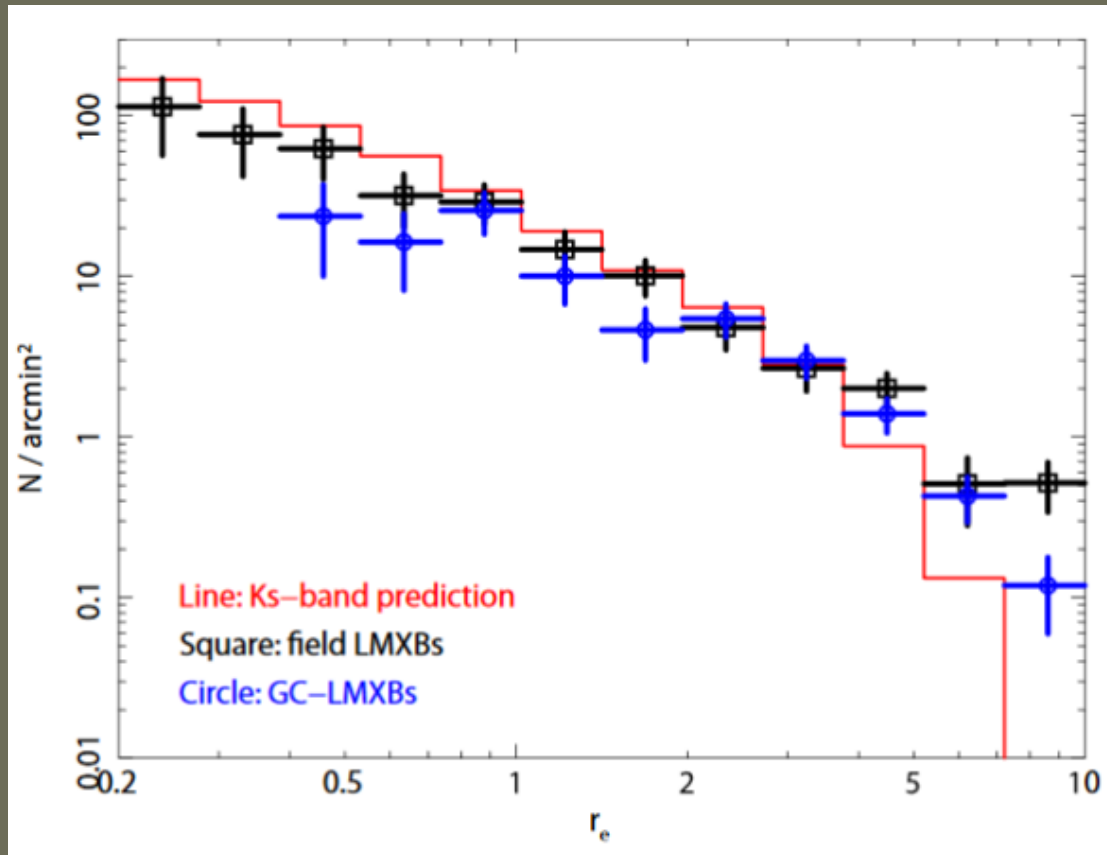
In both models  $N=5 \cdot 10^8$ .  
Kick: ACC02.  
Potential: Paczynski 1990

NS formation rate is assumed to be proportional to the square of the ISM density at the birthplace.



Formation rate is proportional to  $[\exp(-z/75 \text{ pc}) \exp(-R/4 \text{ kpc})]$ .

# X-ray sources in other galaxies



X-ray sources are shifted from the stellar light distribution.  
This might be due to kicks, especially in the case of NS binaries.

The effect cannot be explained by sources in globular clusters.



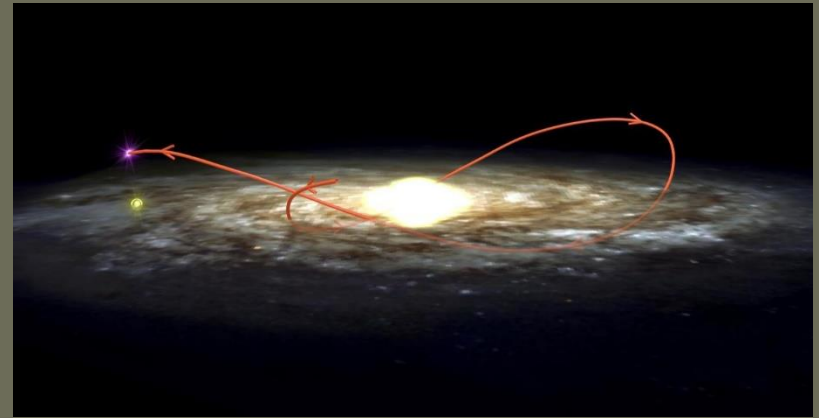
# Black hole kicks

Do BHs obtain kicks?

- they are more massive
- horizon is formed
- SN mechanism can be different

If before the horizon formation a “protoNS-like” object is formed, then there should be a kick, but smaller (in km/s) due to larger mass.

We do not know isolated BHs, but we know binaries. It is possible to measure velocity.

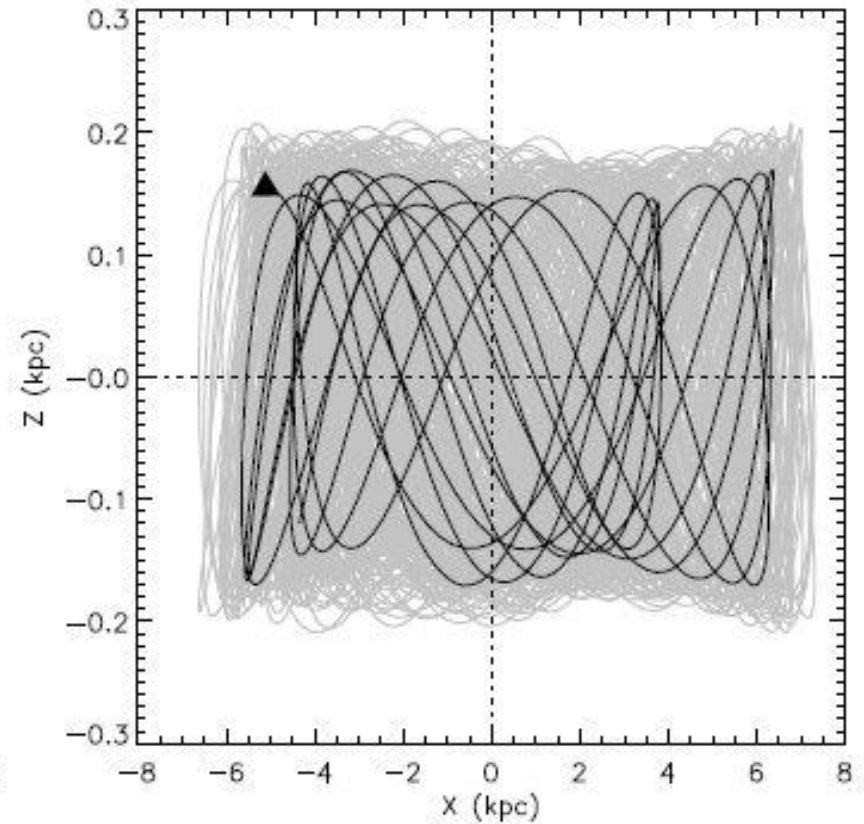
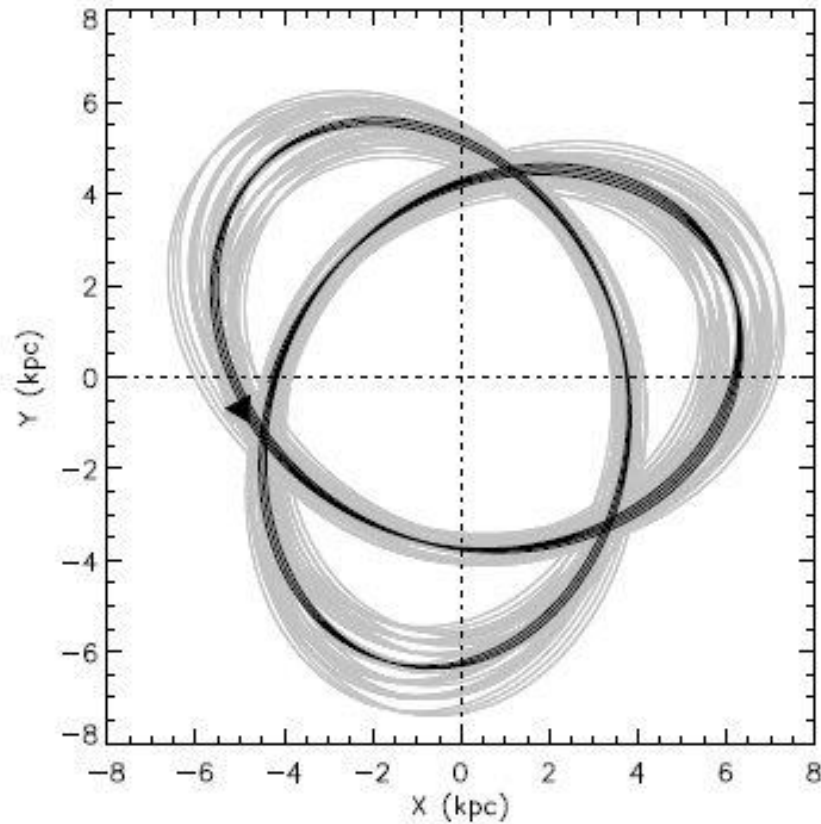


XTE J1118+480

Knowing just a velocity it is difficult to distinguish kick from dynamical interaction or initially large velocity (for example, a system can be from a globular cluster).

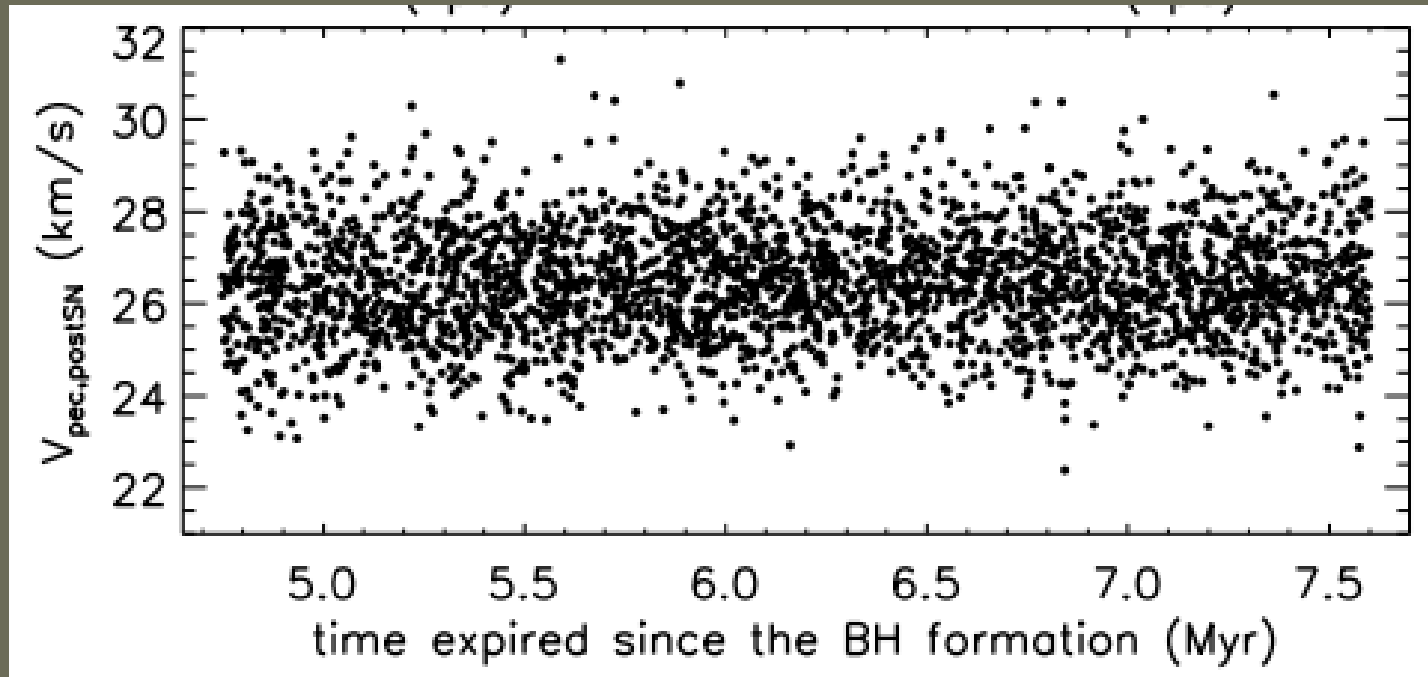
# GROJ1655-40

Kick 45-115 km/s



Willems et al. (2005)

# Cyg X-1

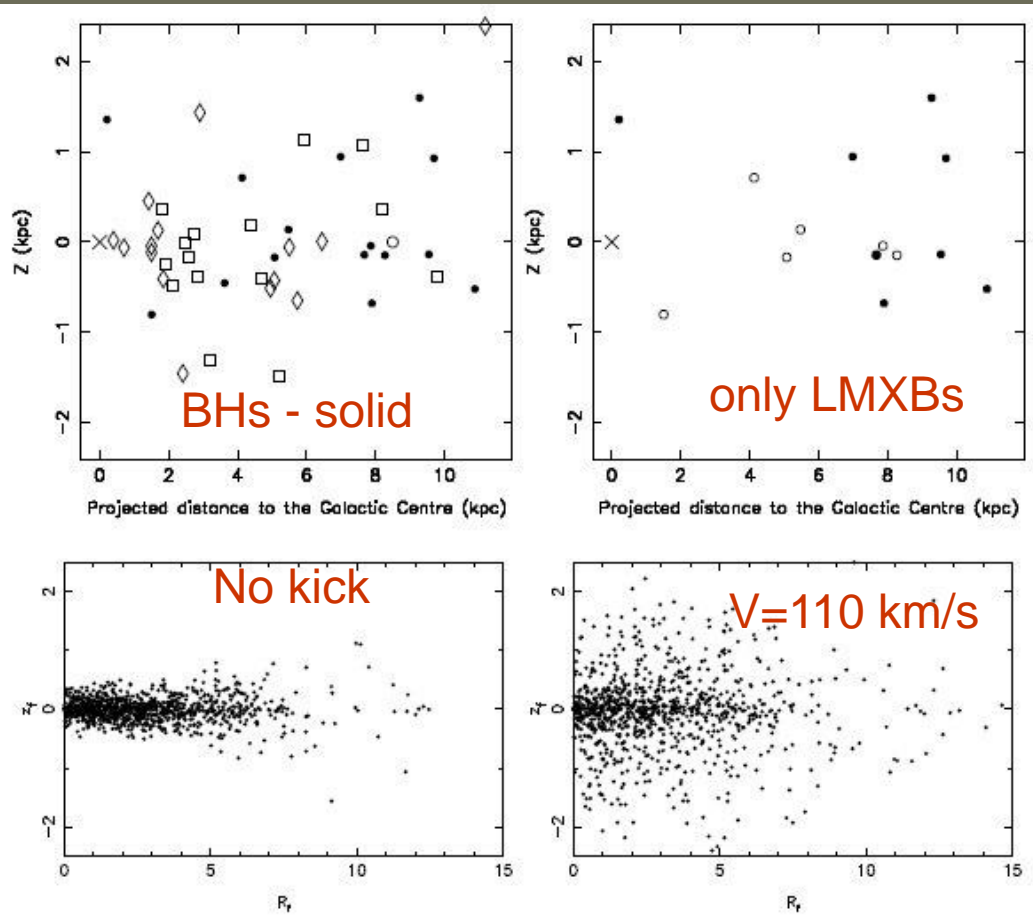


For this system the distance is known very precisely.  
This allows to trace the trajectory back  
and derive the value of post-SN peculiar velocity.

It is equal to 22-32 km/s.

Probably, the BH obtained a moderate kick  $< 77$  km/s.

# BH binaries in the Galaxy

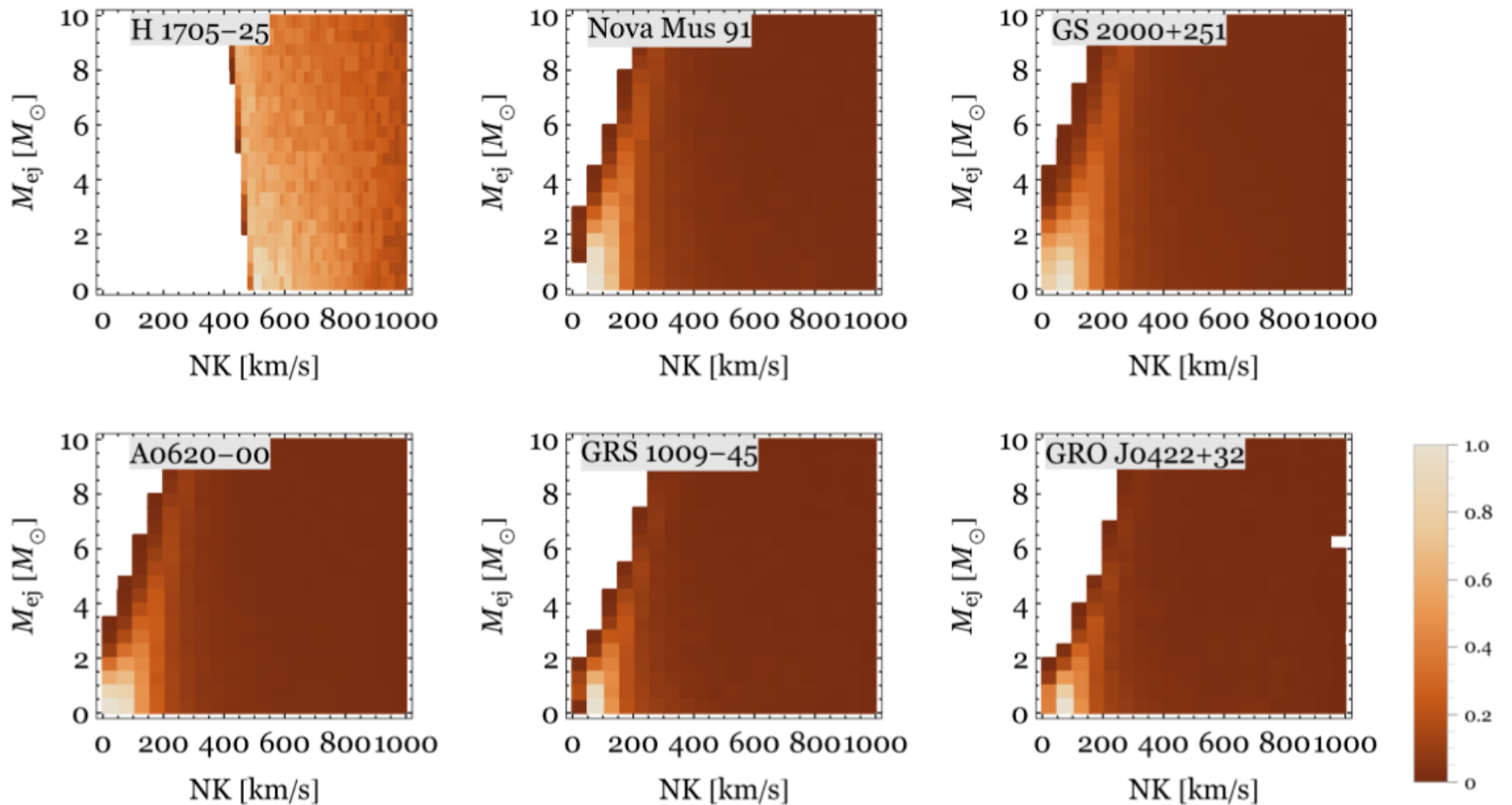


The situation is not clear when we look at the whole population:

- Distribution for BHs is similar to the one for NS (for kick)
- Modeled distribution for zero kick can explain, roughly, the spatial distribution (against large kick)

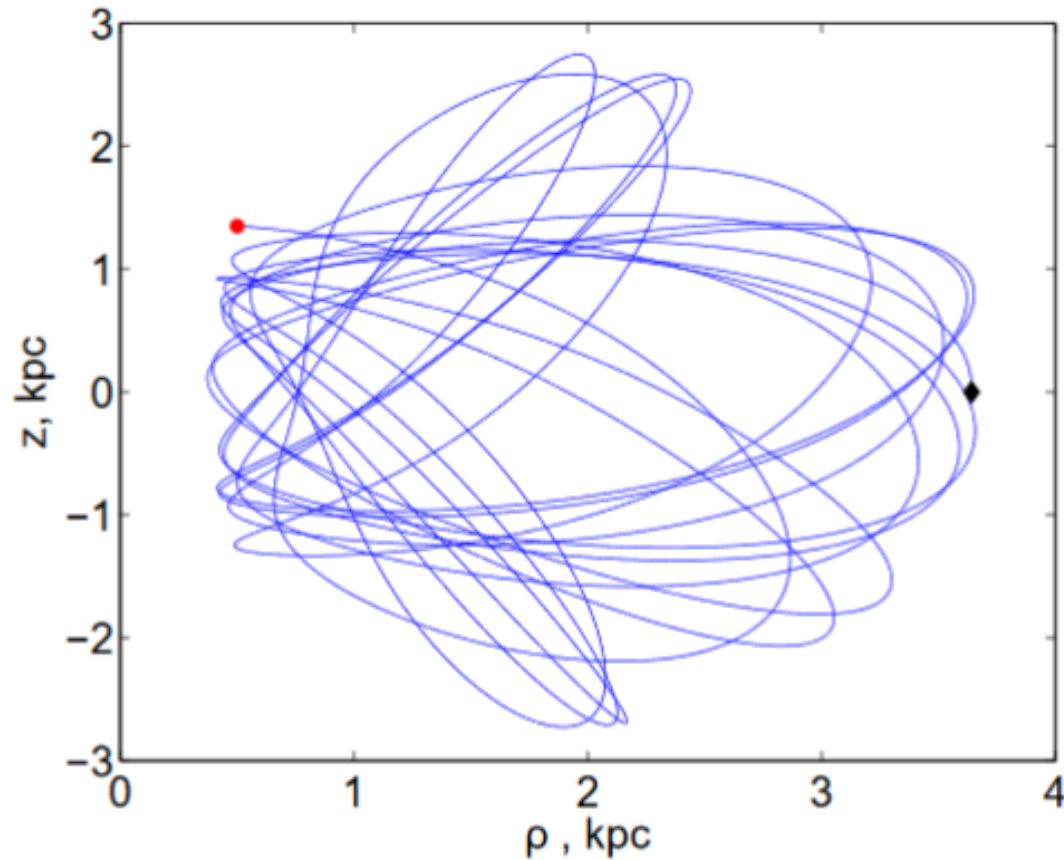
Also line-of-sight velocities are not high

# Black hole kick velocities



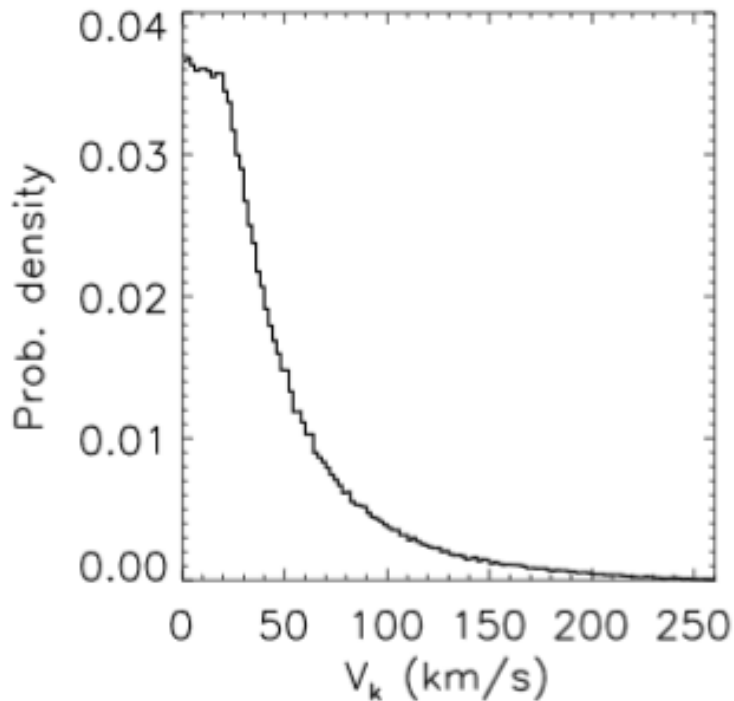
Some BHs receive large kicks at birth. Difficult to explain by scaling from NSs.

# H 1705-250

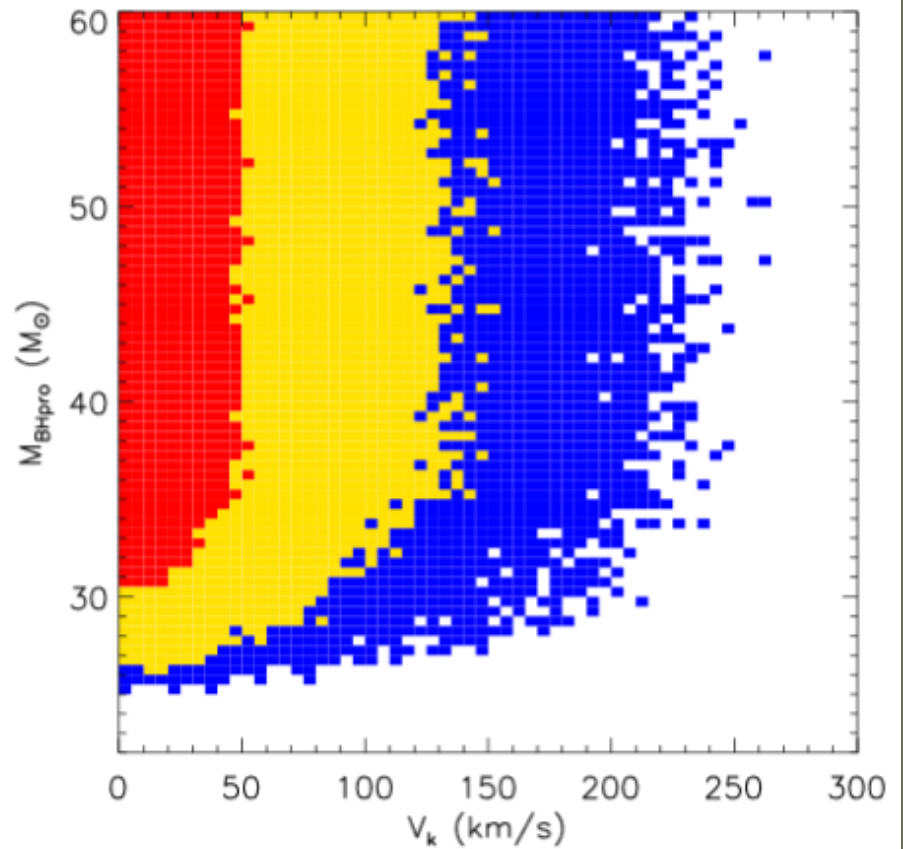


Large kick is not necessary.  
~100 km/s is enough.

# IC 10 X-1



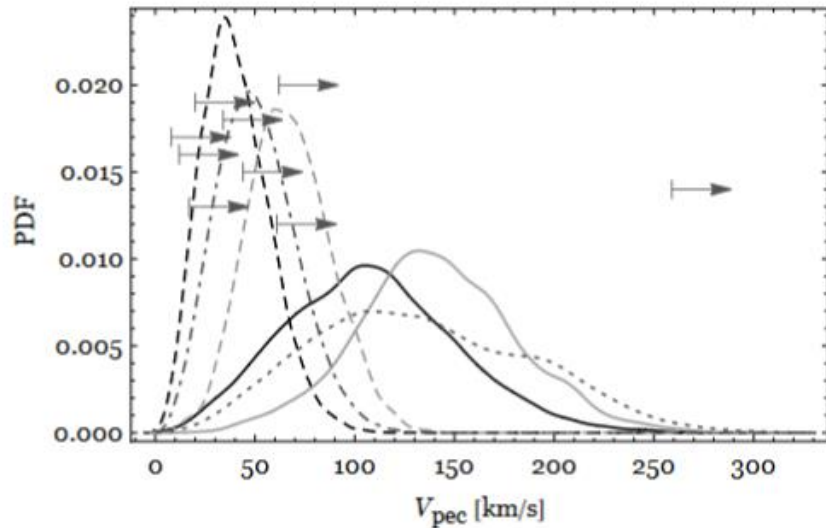
Low kick  $<130$  km/s.



$M_{\text{BH Pro}}$  – progenitor mass  
before BH formation

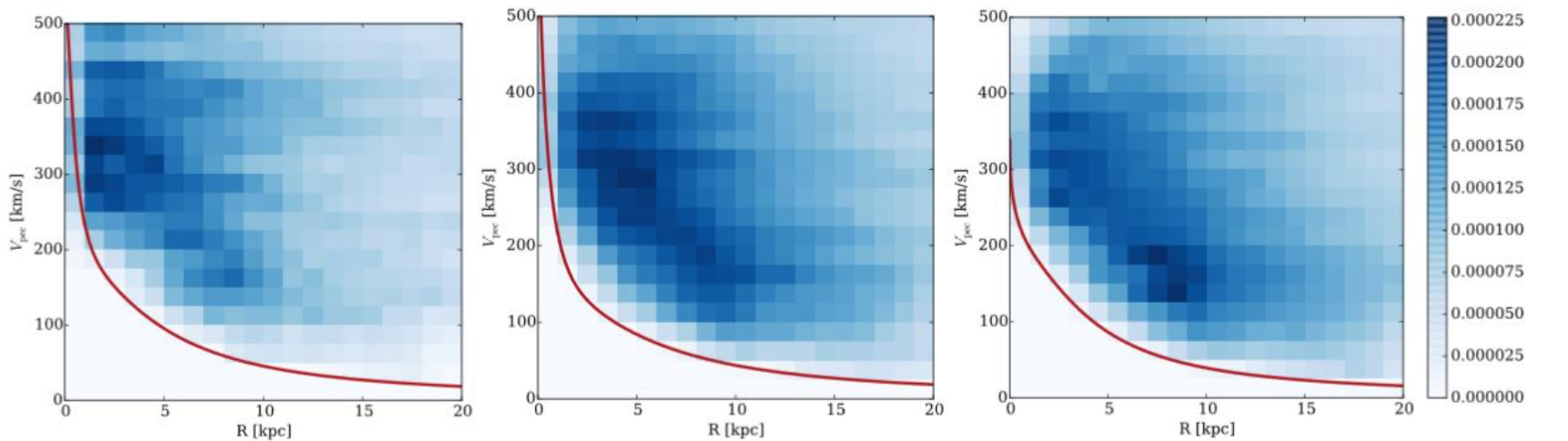


# Velocity of BH and NS X-ray binaries



Some BHs might obtain significant kick.

NS binaries kick distribution is compatible with the one derived from PSRs.



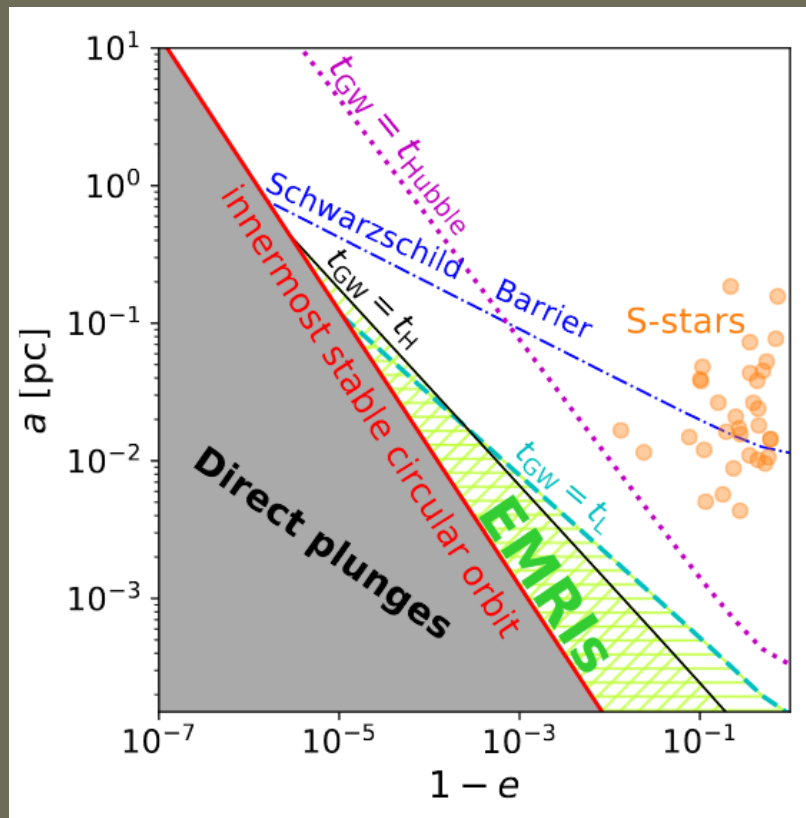
$$V_{\text{pec,min}} = \sqrt{2[\Phi(R_0, z) - \Phi(R_0, 0)]},$$

# EMRI and compact objects kicks

Kicks received by NSs and BHs in the nuclear cluster around a SMBH can result in extreme mass ratio inspirals (EMRI).

The rate is  $>\sim 10^{-8}$  per year per galaxy.

eLISA can detect up to tens of event per year of observations.

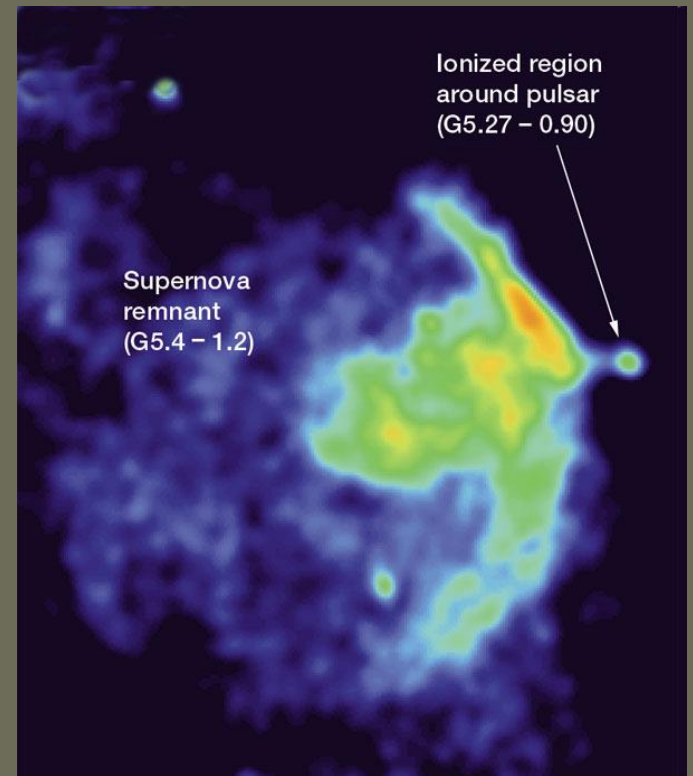


Populations with higher kick produce more SN-EMRI.

SN-EMRI contribute  $\sim 10\%$  of all EMRI (in the case of the Milky Way).

# Conclusions

- NSs and (most probably) BHs obtain natal kicks
- For NSs kick velocity can be as large as  $>1000$  km/s
- The direction of the kick and rotation are correlated
- Kicks depend on the SN mechanism
- Kicks influence parameters of binaries
- Kicks influence evolution of isolated NSs



# Important papers

- Lai astro-ph/0212140– different kick mechanisms
- ATNF catalogue – database including PSR transversal velocities
- Ng & Romani, ApJ 660, 1357 (2007) – spin-velocity alignment in PSRs with nebulae
- Johnston et al. MNRAS 381, 1625 (2007) and Rankin ApJ 664, 443 (2007) – spin-velocity alignment in dozens of radio pulsars (polarization)
- Postnov, Yungelson astro-ph/0701059 – kicks in binaries (pp.18-23)
- Ofek et al. NS spatial distribution. arXiv: 0910.3684

# Spin evolution of NSs

# Evolution of neutron stars



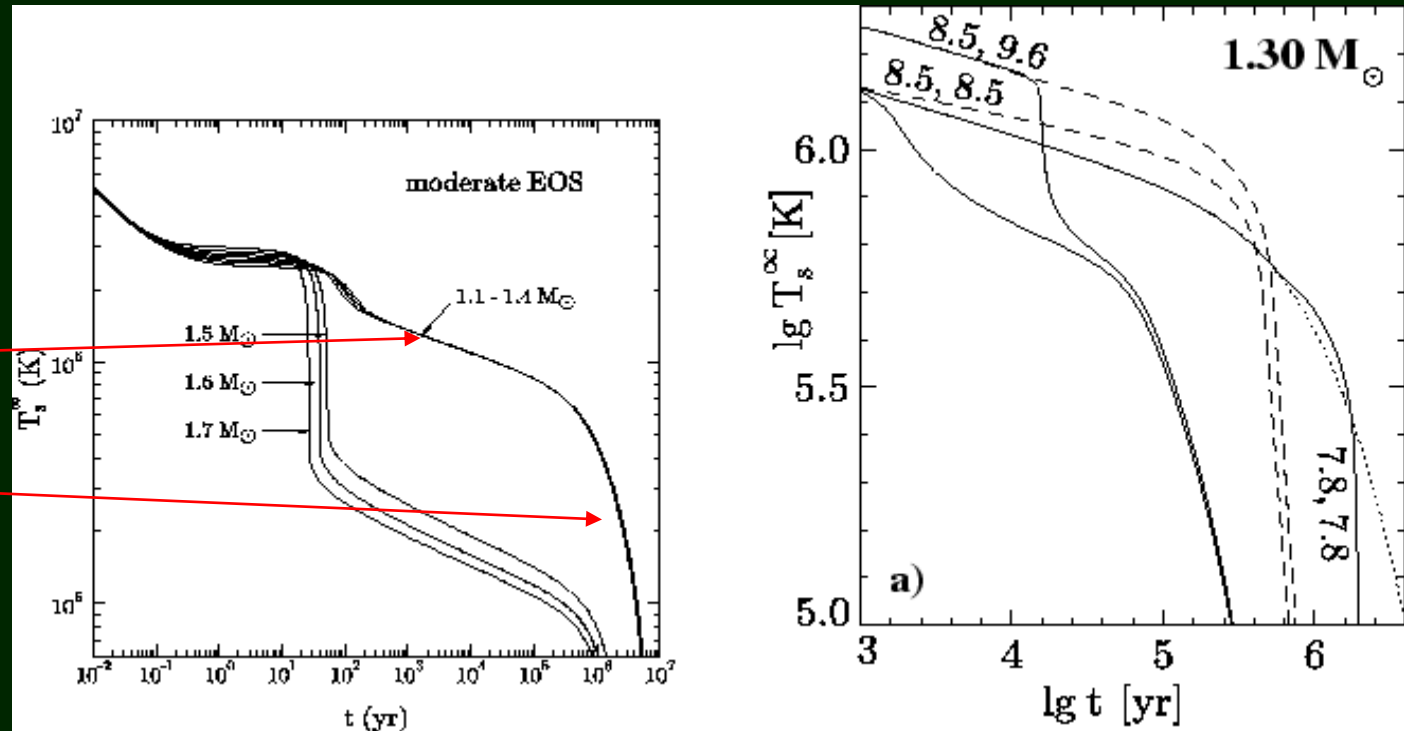
Observational appearance of a NS can depend on:

- Temperature
- Period
- Magnetic field
- Velocity

# Evolution of NSs: temperature

Neutrino  
cooling stage

Photon  
cooling stage



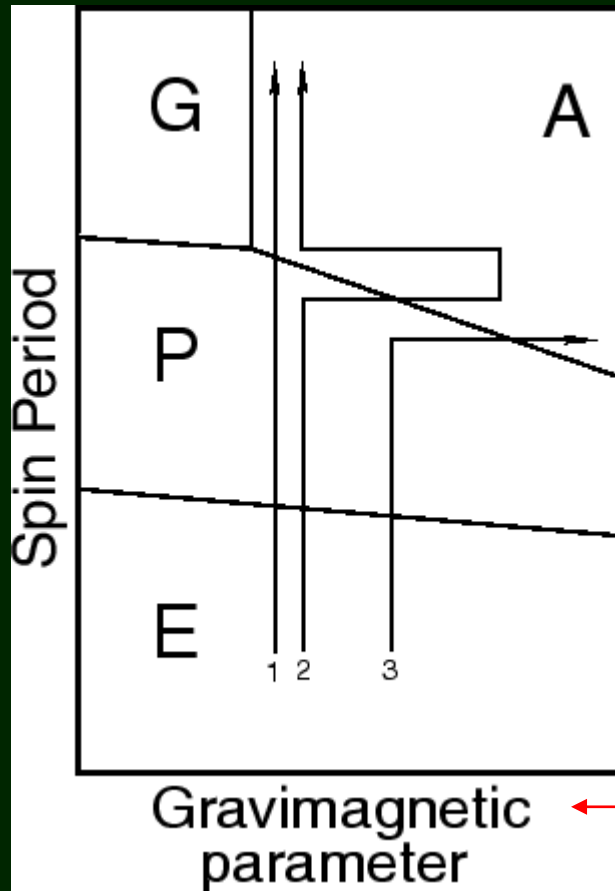
First papers on the thermal evolution appeared already in early 60s, i.e. before the discovery of radio pulsars.

[Yakovlev et al. (1999) Physics Uspekhi]

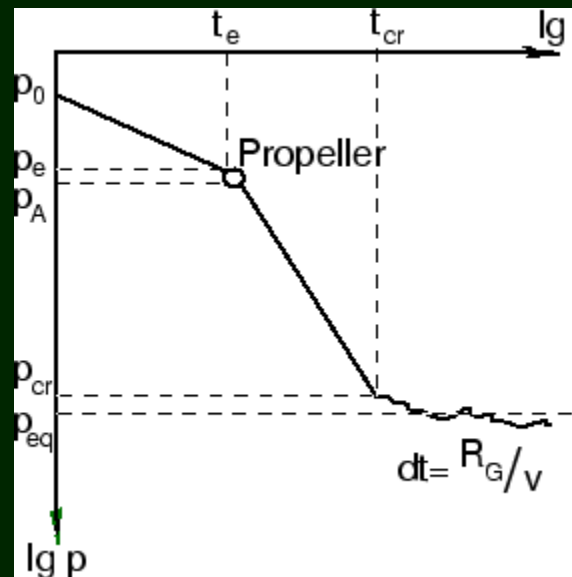


# Evolution of neutron stars: rotation + magnetic field

Ejector → Propeller → Accretor → Georotator



- 1 – spin down
- 2 – passage through a molecular cloud
- 3 – magnetic field decay

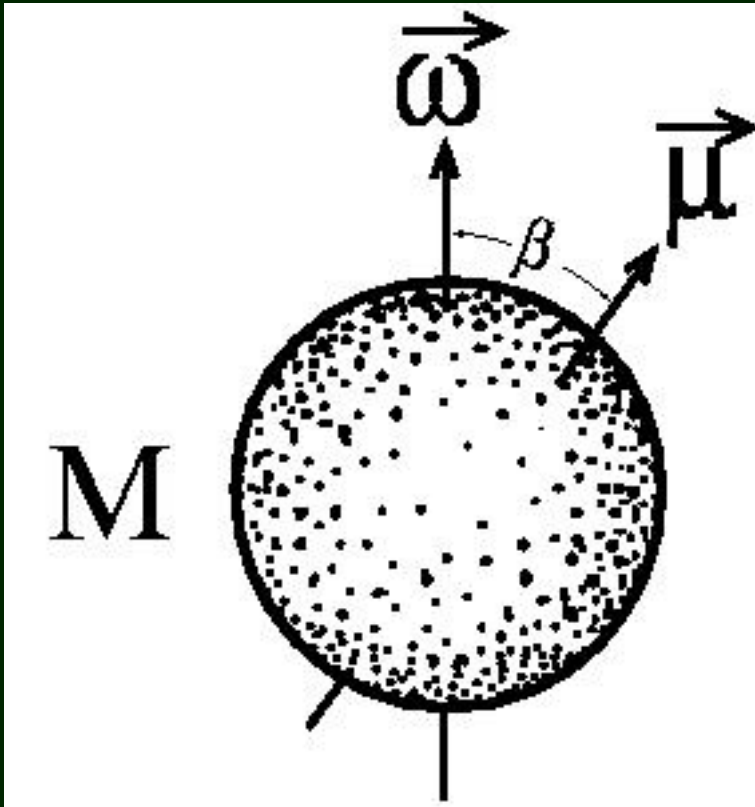


astro-ph/0101031

$\dot{M}/\mu^2$

See the book by Lipunov (1987, 1992)

# Magnetic rotator



Observational appearances of NSs (if we are not speaking about cooling) are mainly determined by  $P$ ,  $\dot{P}$ ,  $V$ ,  $B$ , (and, probably, by the inclination angle  $\chi$ ), and properties of the surrounding medium.  $B$  is not evolving significantly in most cases, so it is important to discuss spin evolution.

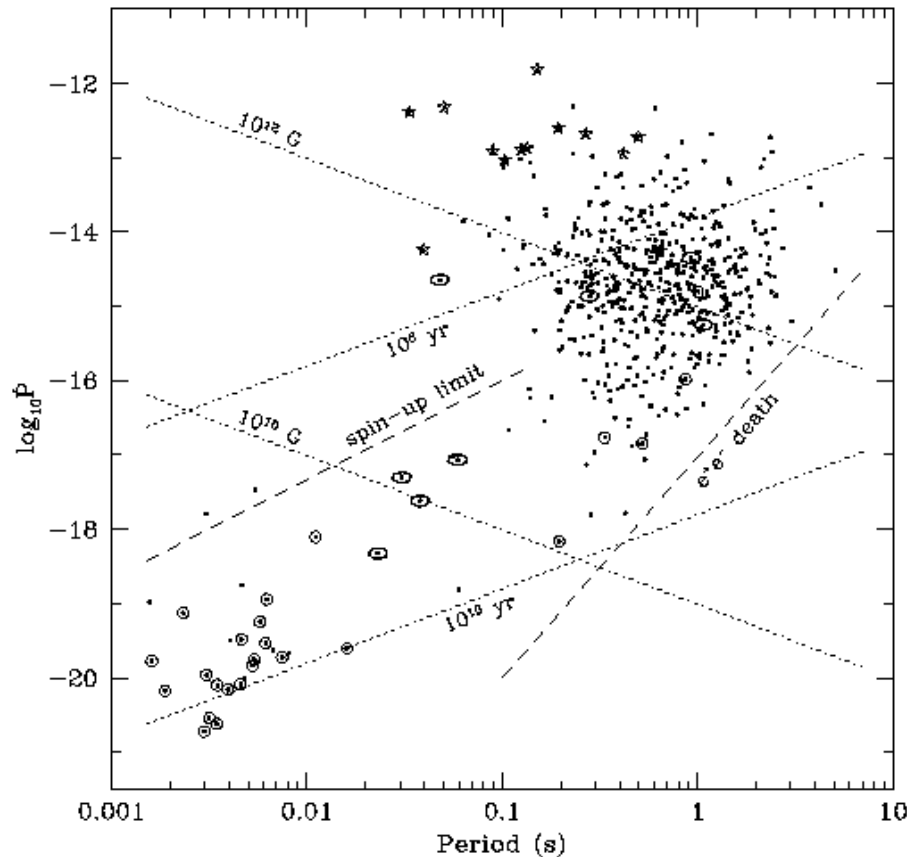
Together with changes in  $B$  (and  $\chi$ )  
one can speak about

## **magneto-rotational evolution**

We are going to discuss the main stages of this evolution, namely:

*Ejector, Propeller, Accretor, and Georotator* following the classification by Lipunov

# Magneto-rotational evolution of radio pulsars



For radio pulsar magneto-rotational evolution is usually illustrated in the P-Pdot diagram.

However, we are interested also in the evolution after this stage.

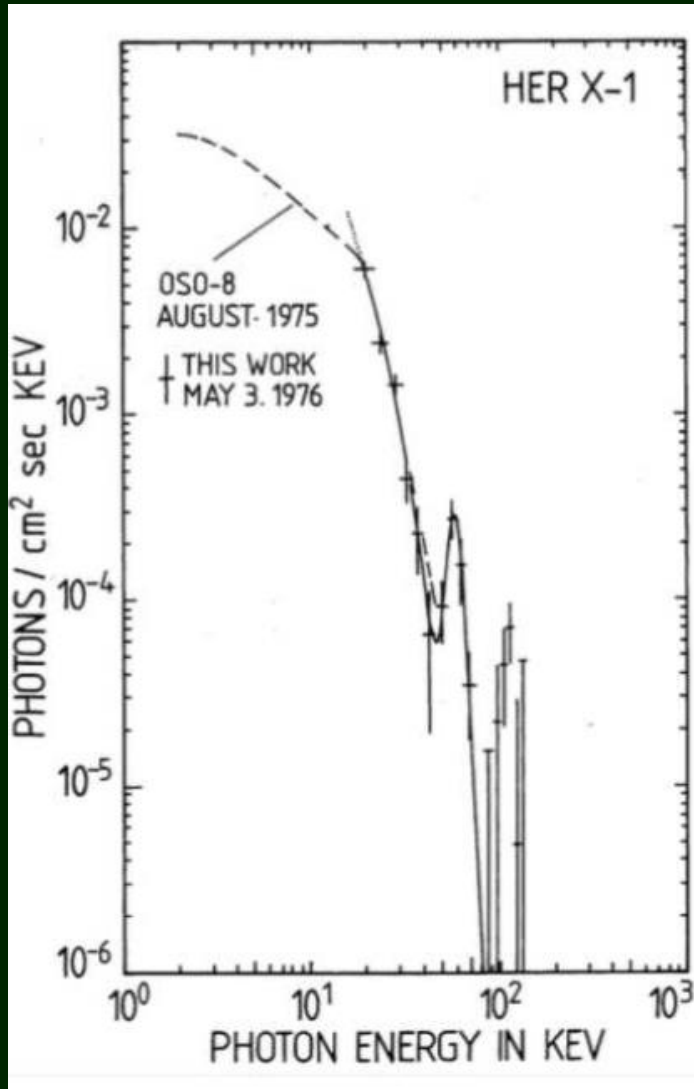
$$L_m = \frac{2}{3} \frac{\mu^2 \omega^4}{c^3} \sin^2 \beta = \kappa_t \frac{\mu^2}{R_l^3} \omega,$$

$$B \sim 3.2 \times 10^{19} (PdP/dt)^{1/2} \text{ G}.$$

Spin-down.

Rotational energy is released.  
The exact mechanism is still unknown.

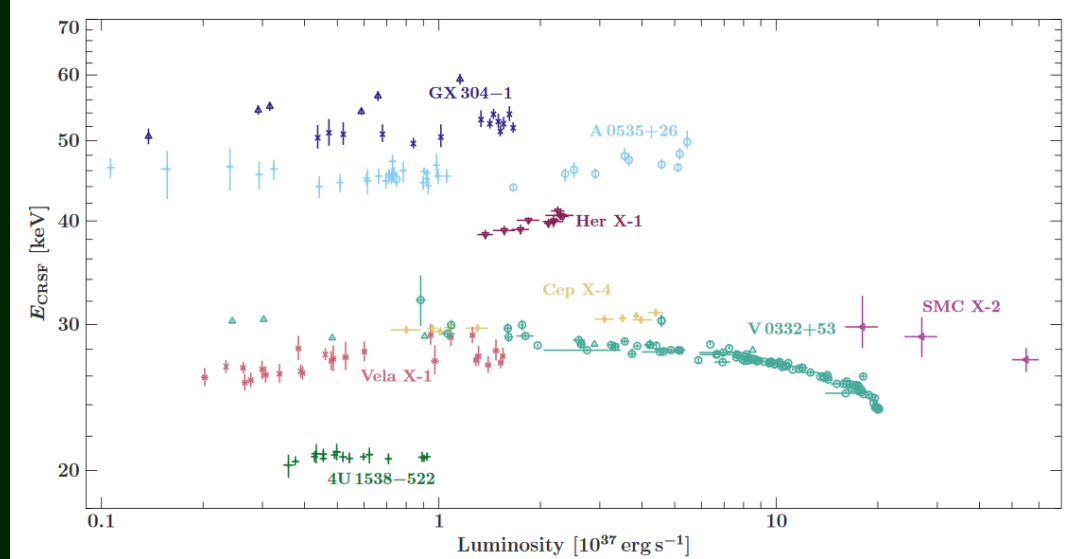
# Fields in binaries: cyclotron line



Now 36 X-ray pulsars in binaries.

$$E_{cyc} = \frac{n}{(1+z)} \frac{\hbar e B}{m_e c} \approx \frac{n}{(1+z)} 11.6 [\text{keV}] \times B_{12},$$

$$E_{cyc}(\dot{M}) = E_0 \left( \frac{R_{NS}}{H_l(\dot{M}) + R_{NS}} \right)^3$$



# Radio pulsar braking: current losses

The model of pulsar emission is not known, and also the model for spin-down is not known, too. Well-known magneto-dipole formula is just a kind of approximation.

One of models is the ***longitudinal current losses*** model (Beskin et al. see astro-ph/0701261)

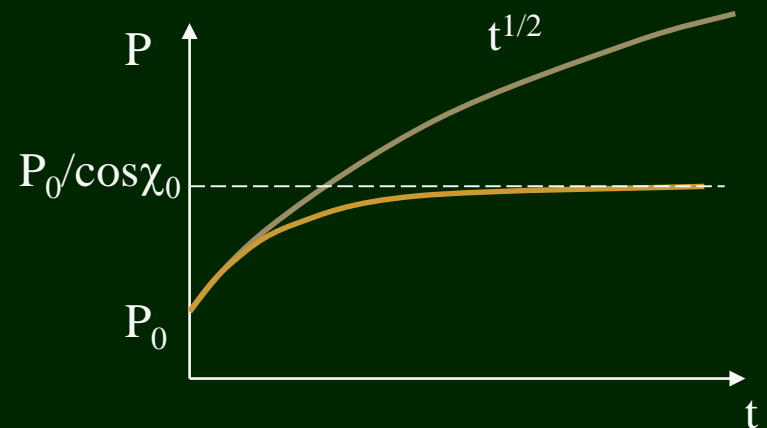
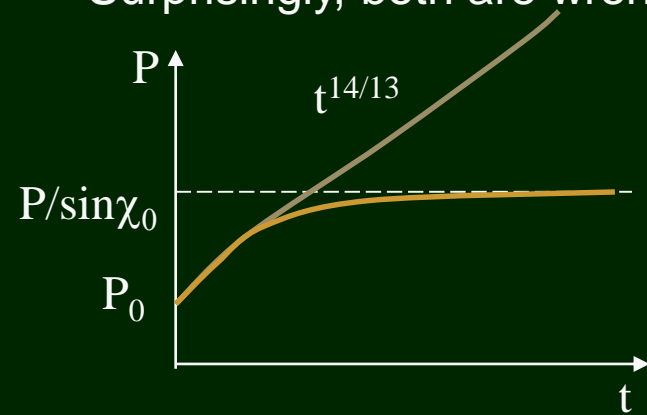
$$\dot{P} = 10^{-15} B_{12}^{10/7} P^{1/14} \cos^{3/2} \chi.$$

Longitudinal current losses

$$\dot{P} = 0.24 \times 10^{-15} B_{12}^2 P^{-1} \sin^2 \chi.$$

Magneto-dipole

Both models predict evolution of the angle between spin and magnetic axis. Surprisingly, both are wrong!



Models of spin-down are not certain up to now, see 1903.01528

# Radio pulsar braking: braking index

$$n_{\text{br}} = \frac{\Omega \ddot{\Omega}}{\dot{\Omega}^2}.$$

Braking index (definition)

$$n_{\text{br}} = 3 + 2 \cot^2 \chi.$$

Magneto-dipole formula

$$n_{\text{br}} = 1.93 + 1.5 \tan^2 \chi.$$

Longitudinal current losses

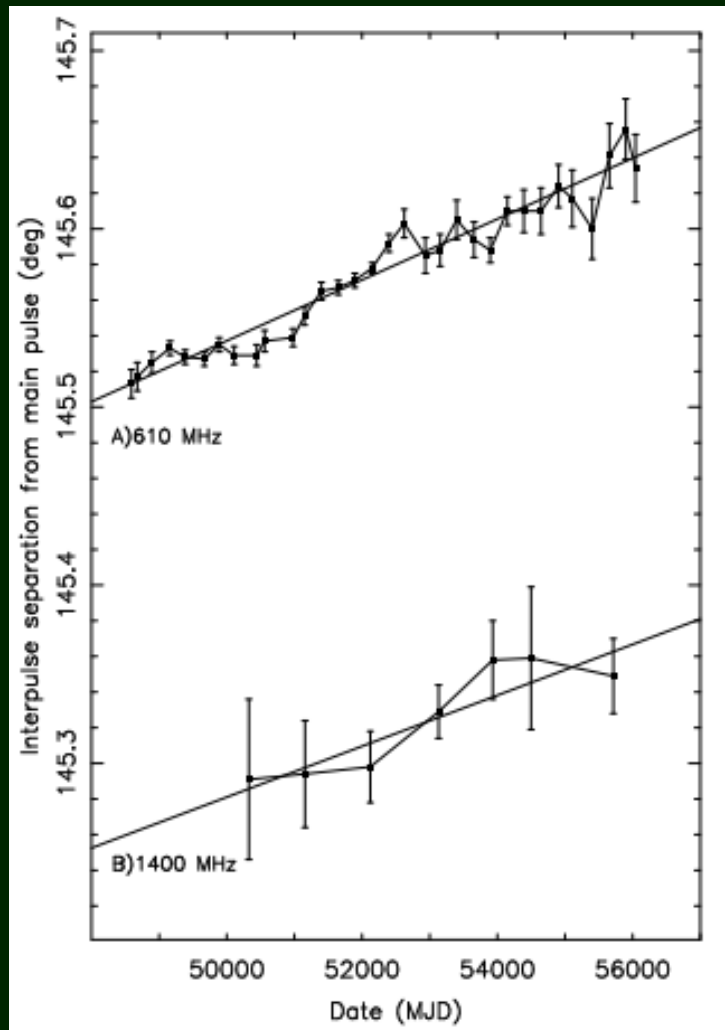
For well-measured braking indices  $n < 3$ .

However, for many pulsars they are very large.

This can be simply an observational effect (microglitches, noise, etc.), but it can also be something real.

For example, related to the magnetic field evolution.

# Crab pulsar and angle evolution



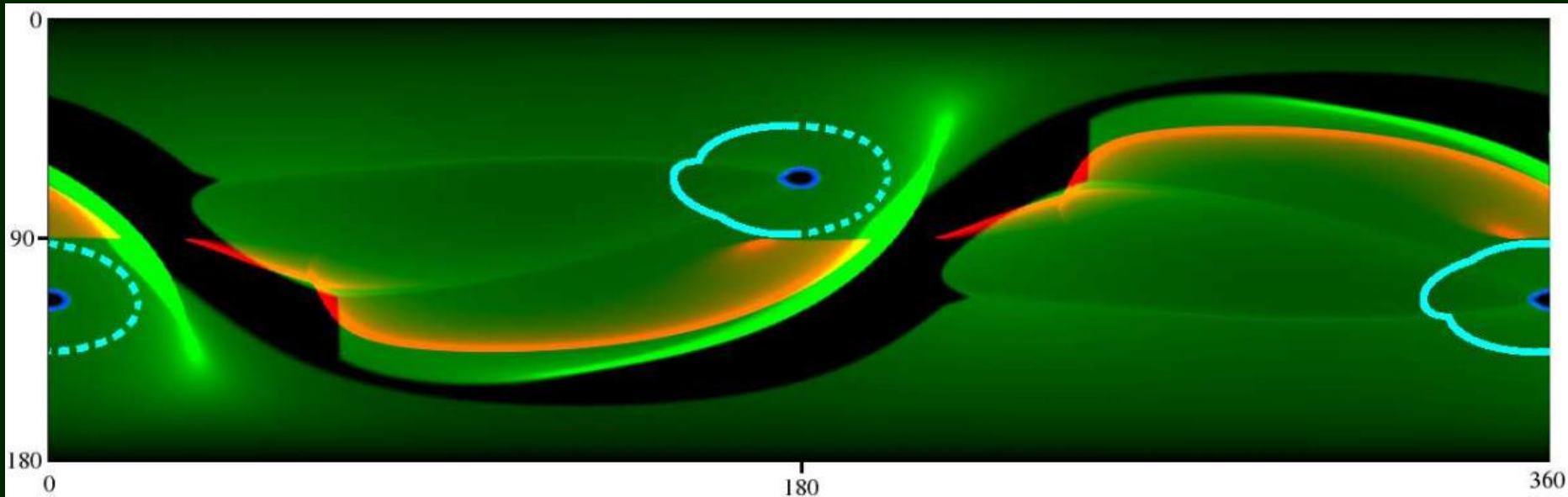
It seems that the angle is changing with the rate 0.6 degrees per century. It is visible as the separation between the main pulse and interpulse is changing.

The axis of the dipolar magnetic field is moving towards the equator.

$$n = 3 + 2\nu/\dot{\nu} \times \dot{\alpha}/\tan(\alpha).$$



# Pulsar emission

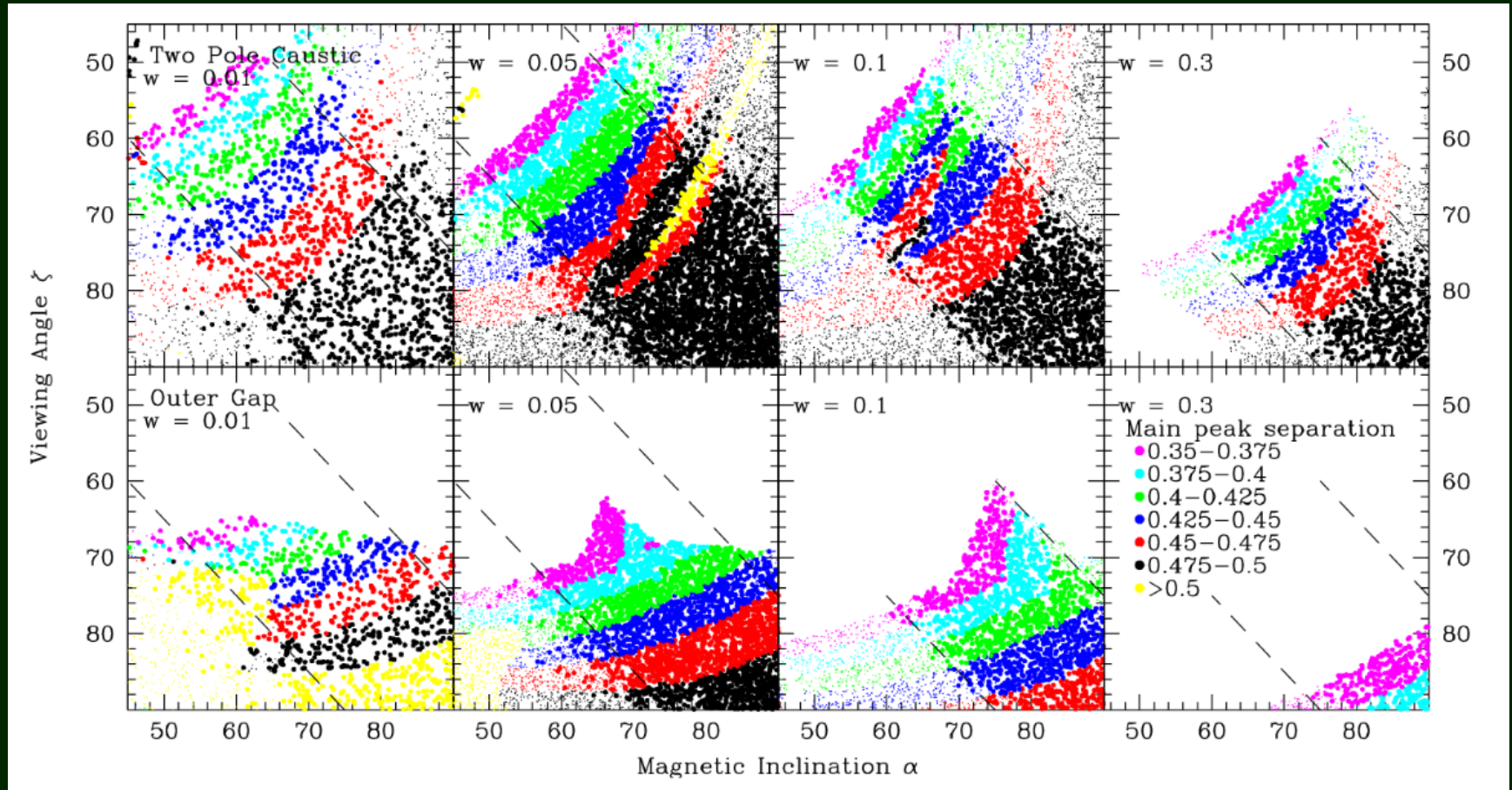


The TPC model for  $w=0.05$  is shown in green, the OG model for  $w=0.1$  is shown in red and the PC emission site is shown in blue.

The cyan lines show the locus of the possible high altitude ( $r=500$  km, here for  $P=0.2$ s) radio emission, with the radiating front half shown solid and the back half dashed

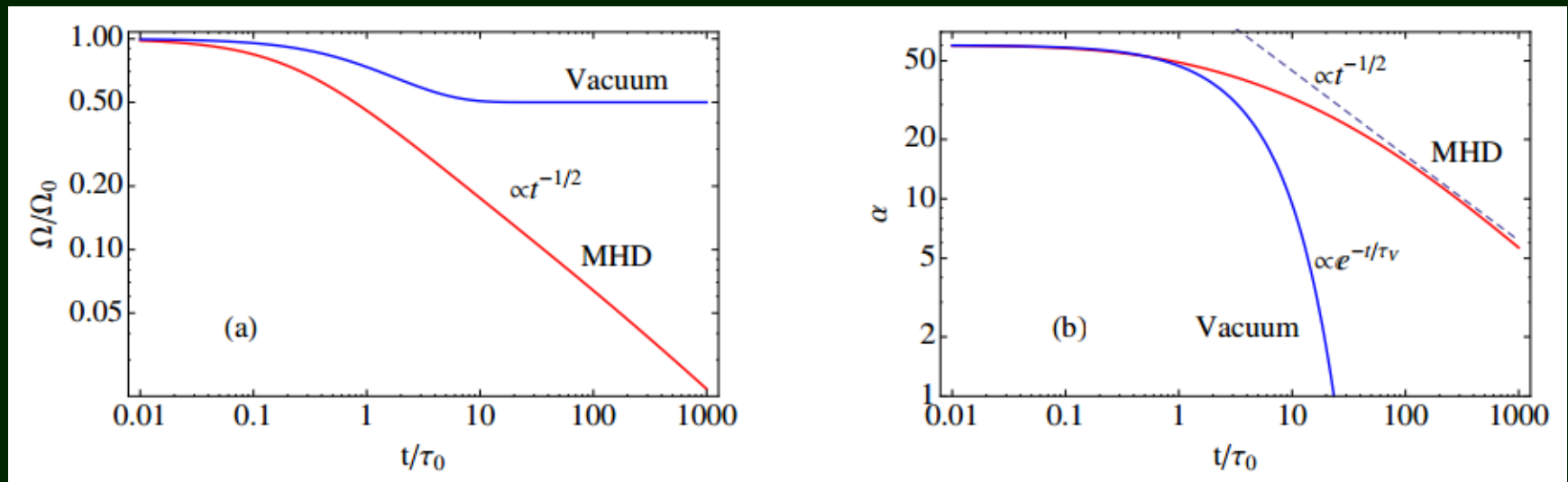
# Peaks separation for different parameters

Increasing of magnetic inclination results in growth of the separation up to  $180^\circ$



# Theoretical studies of the angle evolution

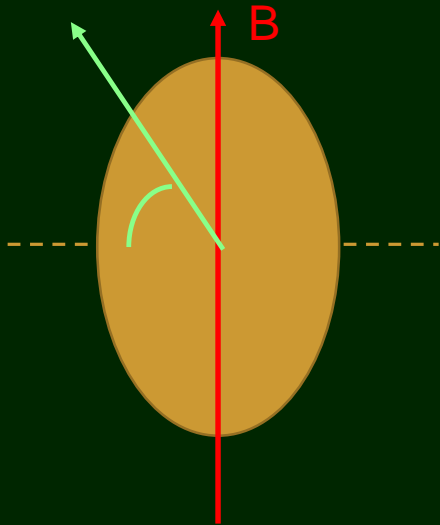
The authors studied the case of plasma filled magnetosphere.  
The angle should evolve towards zero.



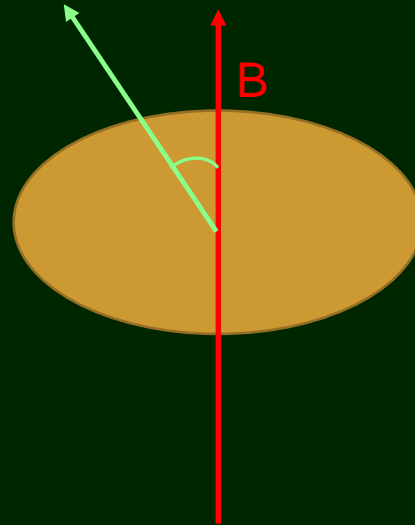
# Initial tilt angle distribution

The distribution of pulsar tilt angles is not consistent with a random distribution at birth. Deficit of PSRs with intermediate angles. Bimodality of initial angles?

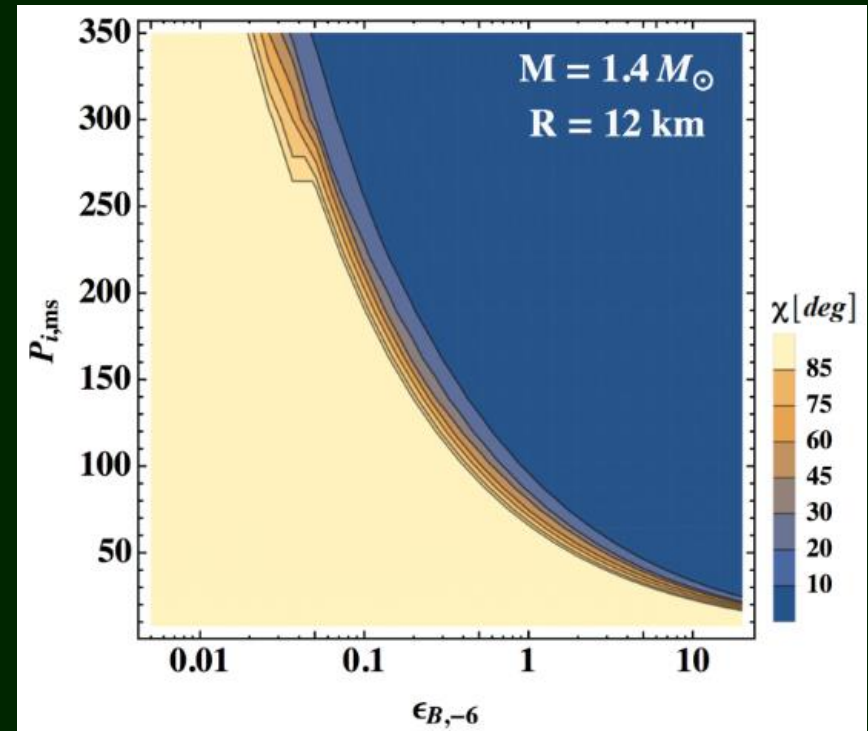
Viscous damping of precession results in the angle evolution for an oblique rorator.



Toroidal field.  
Prolate. The angle evolves towards 90 degrees.



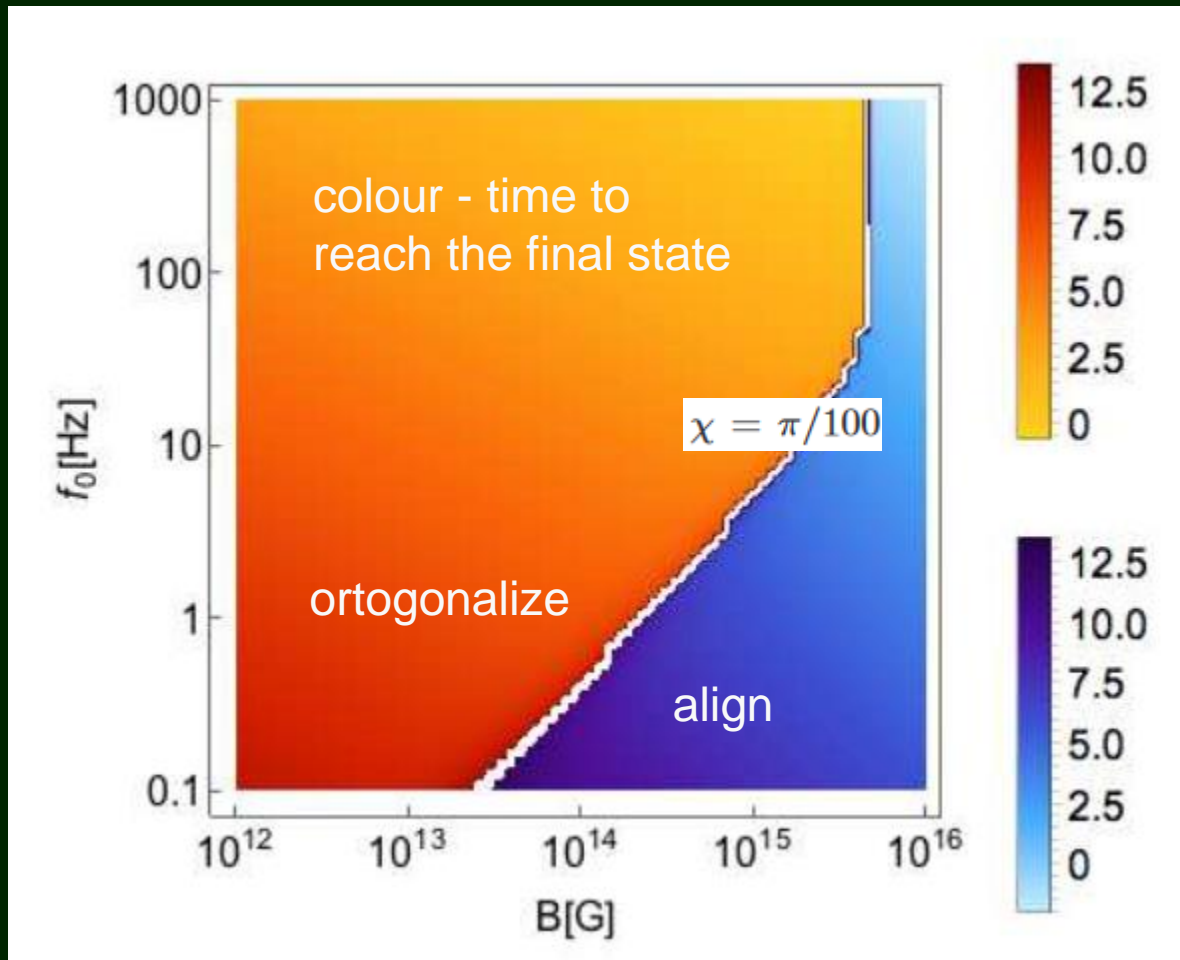
Poloidal field.  
Oblate. The angle evolves towards alignment



- (i) the NS has a non-spherical shape which is mostly determined by magnetic stresses,
- (ii) the internal magnetic field is dominated by a toroidal component, thus - prolate deformation,
- (iii) at birth, the magnetic axis has a small tilt angle.

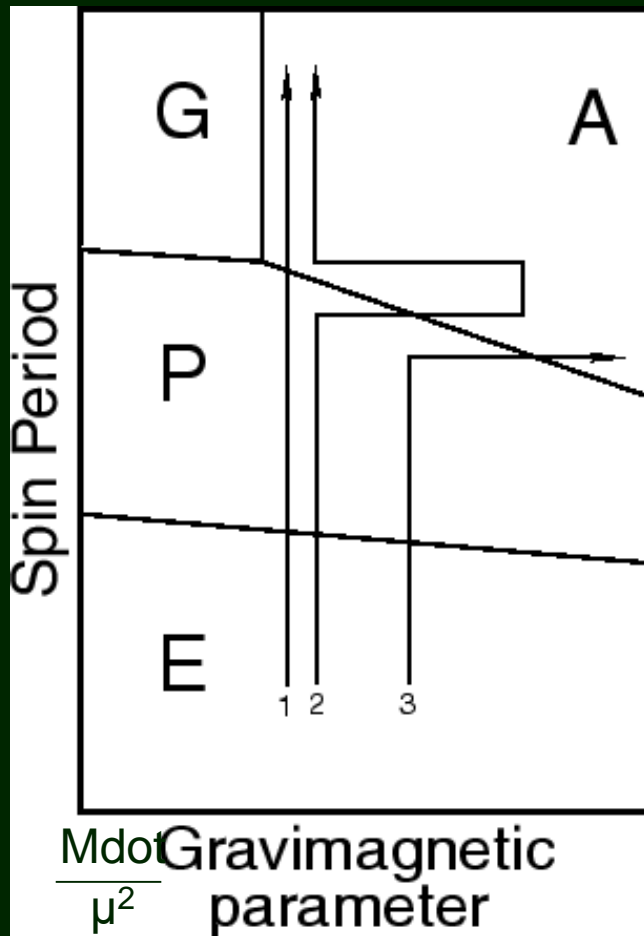
# Angle evolution

Precession, viscous dissipation, electromagnetic radiation reaction.

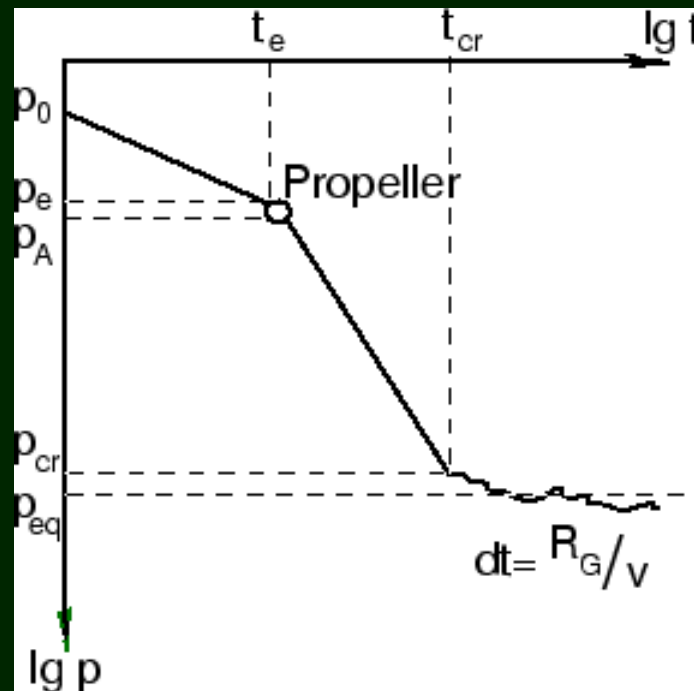


# Magneto-rotational evolution of NSs

Ejector → Propeller → Accretor → Georotator



- 1 – spin down
- 2 – passage through a molecular cloud
- 3 – magnetic field decay



astro-ph/0101031

See the book by Lipunov (1987, 1992)

# Critical radii -I

Transitions between different evolutionary stages can be treated in terms of critical radii

- Ejector stage. Radius of the light cylinder.  $R_l=c/\omega$ .  
Shvartsman radius.  $R_{sh}$ .
- Propeller stage. Corotation radius.  $R_{co}$
- Accretor stage. Magnetospheric (Alfven) radius.  $R_A$
- Georotator stage. Magnetospheric (Alfven) radius.  $R_A$

As observational appearance is related to interaction with the surrounding medium the radius of *gravitational capture* is always important.  $R_G=2GM/V^2$ .



# Critical radii-II

## 1. Shvartsman radius

It is determined by  
relativistic particles wind

$$R_{\text{sh}} = \left( \frac{8\kappa_t \mu^2 G^2 M^2 \omega^4}{\dot{M}_c v_\infty^5 c^4} \right)^{1/2}, \quad R_{\text{sh}} > R_G$$

## 2. Corotation radius

$$\omega R_{\text{st}} < \sqrt{GM_x / R_{\text{st}}}$$

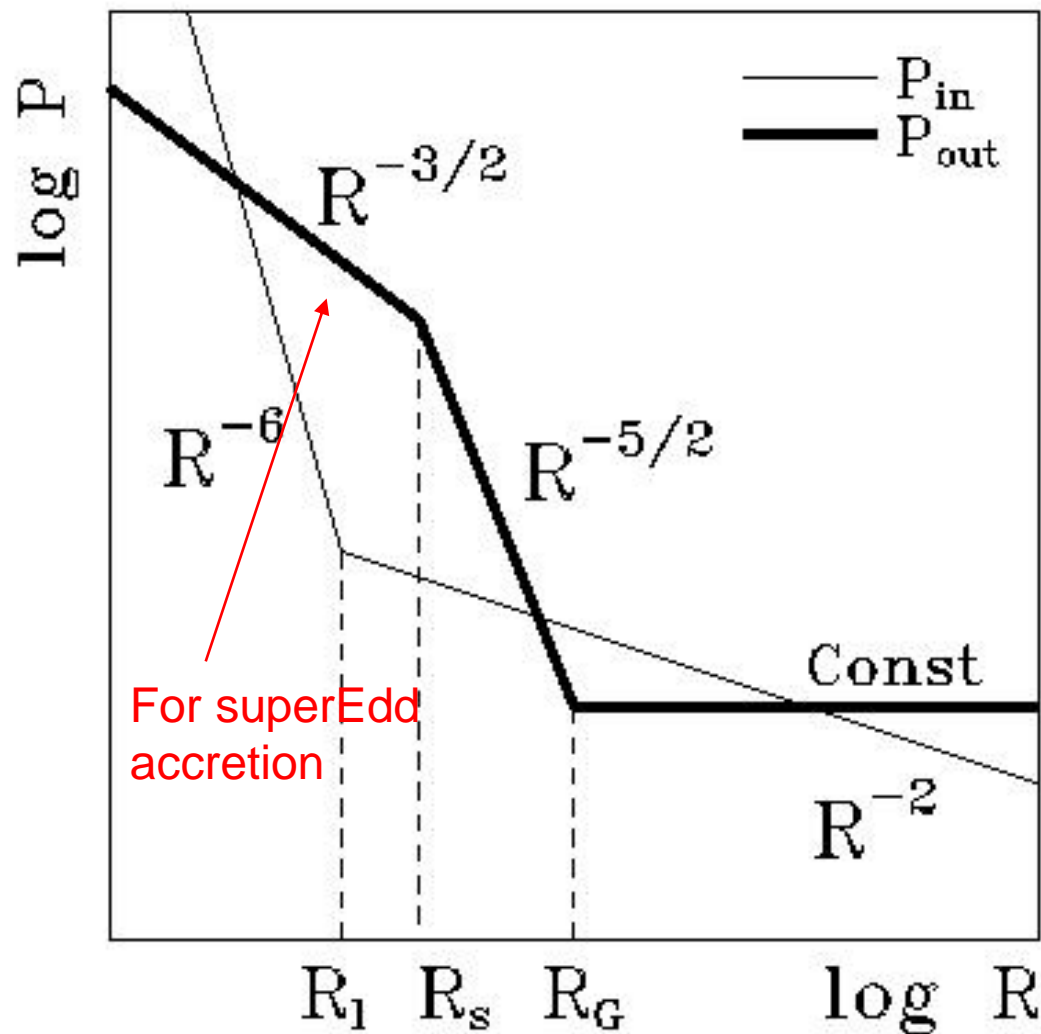
$$R_c = (GM_x / \omega^2)^{1/3} \sim 2.8 \times 10^8 m_x^{1/3} (P/1 \text{ s})^{2/3} \text{ cm}$$

## 3. Alfven radius

$$P_m(R_{\text{st}}) = P_a(R_{\text{st}})$$

$$R_A = \begin{cases} \left( \frac{2\mu^2 G^2 M^2}{\dot{M}_c v_\infty^5} \right)^{1/6}, & R_A > R_G \\ \left( \frac{\mu^2}{2\dot{M}_c \sqrt{2GM}} \right)^{2/7}, & R_A \leq R_G \end{cases}$$

# Pressure



$$P_m = \begin{cases} \frac{\mu^2}{8\pi R^6}, & R \leq R_t \\ \frac{L_m}{4\pi R^2 c}, & R > R_t \end{cases}$$

$$L_m = \kappa_t \frac{\mu^2}{R_t^3} \omega$$

We can define a stopping radius  $R_{st}$ , at which external and internal pressures are equal.

The stage is determined by relation of this radius to other critical radii.

# Classification

Abbreviation	Type	Characteristic radii relation	Accretion rate	Observational appearances
E	Ejector	$R_{\text{st}} > R_G$ $R_{\text{st}} > R_l$	$\dot{M}_c \leq \dot{M}_{\text{cr}}$	Radiopulsars, Soft $\gamma$ -ray repeaters, Cyg X-3? LSI+61 303?
P	Propeller	$R_c < R_{\text{st}}$ $R_{\text{st}} \leq R_G$ $R_{\text{st}} \leq R_l$	$\dot{M}_c \leq \dot{M}_{\text{cr}}$	X-ray transients? Rapid burster? $\gamma$ -bursters?? Magnetic Ap-stars
A	Accretor	$R_{\text{st}} \leq R_G$ $R_{\text{st}} \leq R_l$	$\dot{M}_c \leq \dot{M}_{\text{cr}}$	X-ray pulsars, bursters, cataclysmic variables, intermediate polars
G	Georotator	$R_G < R_{\text{st}}$ $R_{\text{st}} \leq R_c$	$\dot{M}_c \leq \dot{M}_{\text{cr}}$	Earth, Jupiter
M	Magnetor	$R_{\text{st}} > a$ $R_c > a$ ???	$\dot{M}_c \leq \dot{M}_{\text{cr}}$	AM Her, polars

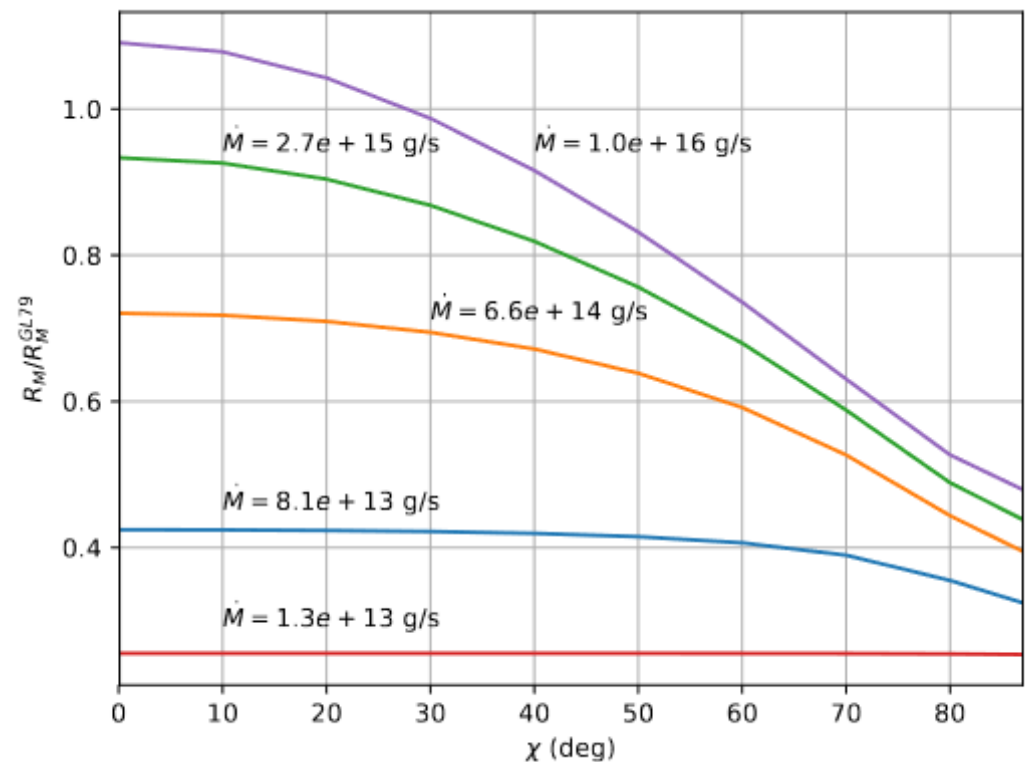
# Alfven radius in different situations

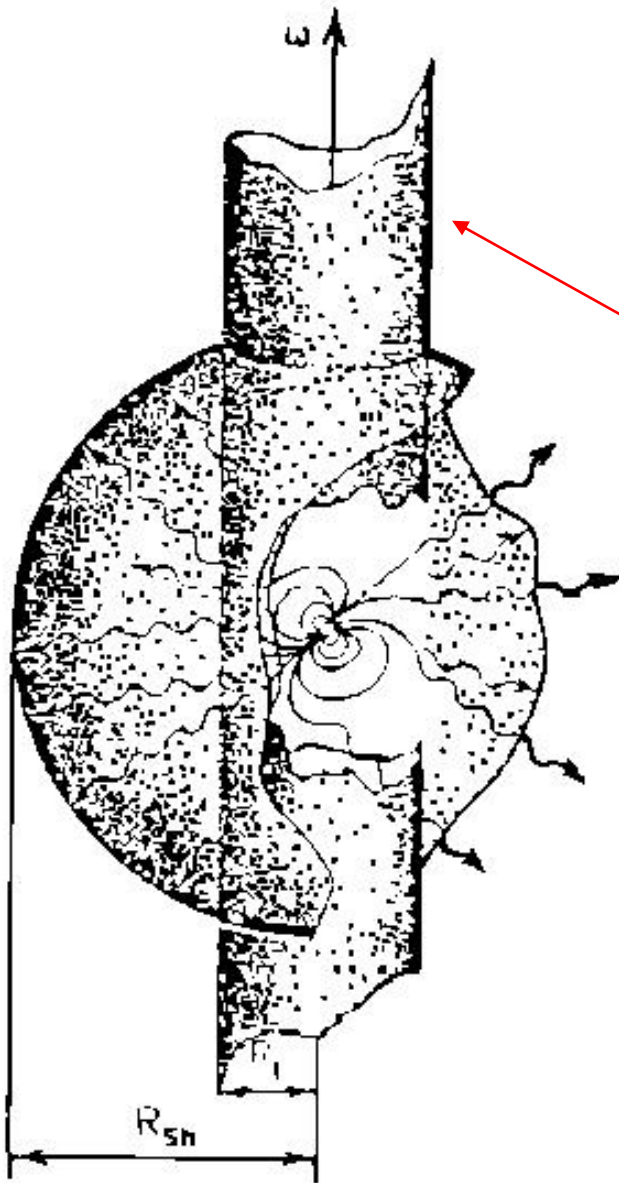
Simple estimate of  $R_A$  presented before is just a zero approximation. Many different variants for different accretion regimes were obtained. In particular,  $R_A$  is modified in the case of disc accretion, and for low accretion rates.

1806.11516

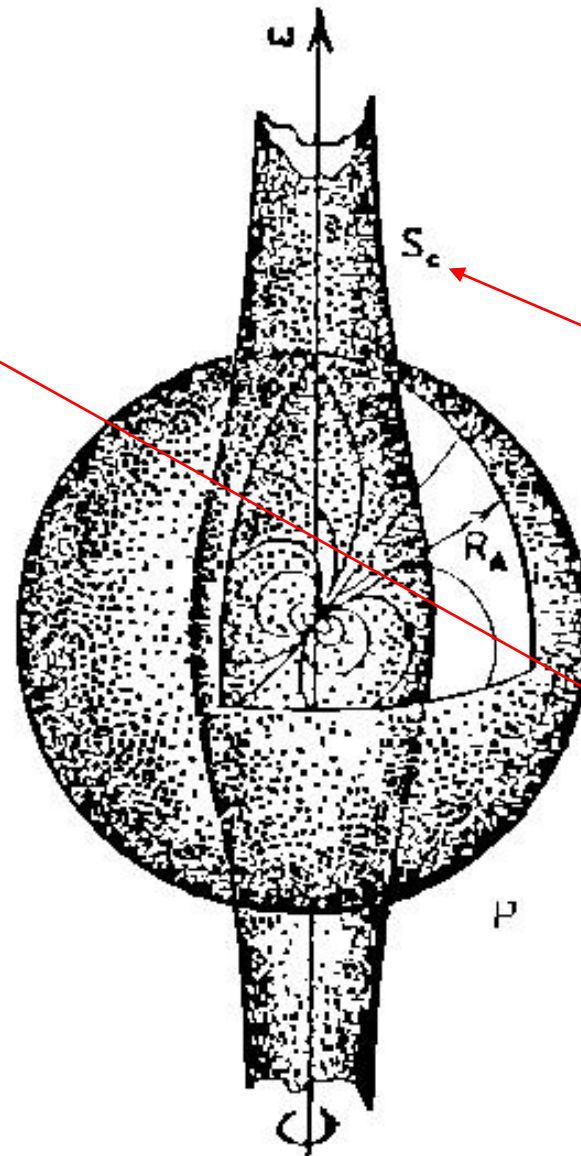
$$R_M^{\text{GL79}} \simeq 0.52 R_A = 0.52 \mu^{4/7} (2GM)^{-1/7} \dot{M}^{-2/7} \\ \simeq 1.6 \times 10^6 \mu_{26}^{4/7} M_{1.4}^{-1/7} \dot{M}_{16}^{-2/7} \text{ cm},$$

In the plot the radius in the case of disc accretion according to GL79 is compared for different accretion rate and inclination with the model originally developed by Wang (1997).





**Ejector**

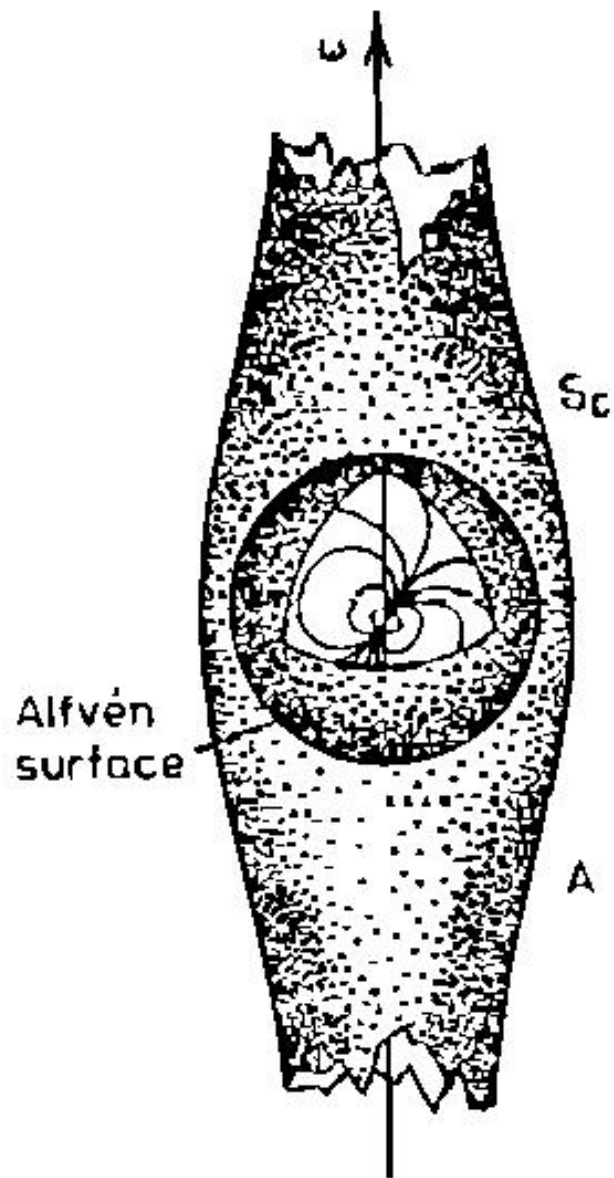


**Propeller**

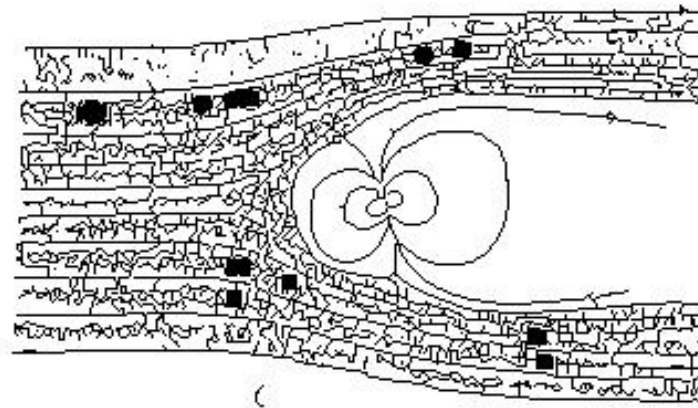
$$R = R_{co} \cos^{-2/3} \theta$$

$$R_{co} = (GM/\omega^2)^{1/3}$$

Light cylinder  
 $R_l = \omega/c$



**Accretor**



**Georotator**

# Critical periods for isolated NSs

$$P_E(E \rightarrow P) \simeq 10 \mu_{30}^{1/2} n^{-1/4} v_{10}^{1/2} \text{ s}$$

Transition from Ejector to Propeller (supersonic)

$$t_E \simeq 10^9 \mu_{30}^{-1} n^{-1/2} v_{10} \text{ yr}$$

Duration of the ejector stage

$$P_A(P \rightarrow A) \simeq 420 \mu_{30}^{6/7} n^{-3/7} v_{10}^{9/7} \text{ s}$$

Transition from supersonic Propeller to subsonic Propeller or Accretor

$$P_{eq} = 2.6 \times 10^3 v_{(t)10}^{-2/3} \mu_{30}^{2/3} n^{-2/3} v_{10}^{13/3} \text{ s}$$

A kind of equilibrium period for the case of accretion from turbulent medium

$$v < 410 n^{1/10} \mu_{30}^{-1/5} \text{ km s}^{-1}$$

Condition for the Georotator formation (instead of Propeller or Accretor)

(see, for example, astro-ph/9910114)



# Spin-up/down at the stage of accretion

$$\frac{dI\omega}{dt} = \dot{M}k_{\text{su}} - \kappa_t \frac{\mu^2}{R_t^3},$$

$$k_{\text{su}} = \begin{cases} (GM_x R_d)^{1/2}, & \text{Keplerian disk accretion,} \\ \eta_t \Omega R_G^2, & \text{wind accretion in a binary,} \\ \sim 0, & \text{a single magnetic rotator.} \end{cases}$$

For a single rotator (i.e. an isolated NS) spin-up can be possible due to turbulence in the interstellar medium.

In the case of  
isolated accreting NS  
one can estimate  
the accretion rate as:

$$\dot{M}_c = 4\pi R_G^2 \rho_\infty v_\infty$$

# Unified approach to spin-down

One can find it comfortable to represent the spin-down moment by such a formula

$$- \kappa_t \frac{\mu^2}{R_t^3}$$

$k_t$  and  $R_t$  are different for different stages.  
 $k_t$  can be also frequency dependent.

Parameter	Regime					
	E, SE	P, SP	A	SA	G	M
$\dot{M}$	0	0	$\dot{M}_c$	$\dot{M}_c(R_A/R_s)$	0	$\dot{M}_c$
$\kappa_t$	$\sim 2/3$	$\lesssim 1/3$	$\sim 1/3$	$\sim 1/3$	$\sim 1/3$	$\sim 1/3$
$R_t$	$R_t$	$R_m$	$R_c$	$R_c$	$R_A$	$a$

# Equilibrium period

$$\dot{M} k_{su} = -\kappa_t \frac{\mu^2}{R_t^3}$$

The hypothesis of equilibrium can be used to determine properties of a NS.

The corotation radius is decreasing as a NS is spinning up.

So, before equilibrium is reached the transition to the propeller stage can happen.

---

Looking at this formula (and remembering that for Accretors  $R_t = R_{co}$ ) it is easy to understand why millisecond PSRs have small magnetic field.

Spin-up can not be very large (Eddington rate).

So, to have small spin periods (and so small corotation radii), it is necessary to have small magnetic fields.

High magnetic field NS can not be spun-up to millisecond periods.

---

# Accreting isolated neutron stars

## Why are they so important?

- Can show us how old NSs look like
  1. Magnetic field decay
  2. Spin evolution
- Physics of accretion at low rates
- NS velocity distribution
- New probe of NS surface and interiors
- ISM probe

# Expected properties

## 1. Accretion rate

An upper limit can be given by the Bondi formula:

$$\dot{M} = \pi R_G^2 \rho v, \quad R_G \sim v^{-2}$$

$$\dot{M} = 10^{11} \text{ g/s} \quad (v/10 \text{ km/s})^{-3} n$$

$$L = 0.1 \dot{M} c^2 \sim 10^{31} \text{ erg/s}$$

However, accretion can be smaller due to the influence of a magnetosphere of a NS

## 2. Periods

Periods of old accreting NSs are uncertain, because we do not know evolution well enough.

$$a) \quad p_A = 2^{5/14} \pi (GM)^{-5/7} (\mu^2 / \dot{M})^{3/7} \simeq$$

$$R_A = R_{\text{co}}$$

$$300 \mu_{30}^{6/7} (v/10 \text{ km s}^{-1})^{9/7} n^{-3/7} \text{ s.}$$

# Subsonic propeller

Even after  $R_{\text{co}} > R_A$  accretion can be inhibited.

This have been noted already in the pioneer papers by Davies et al.

Due to rapid (however, subsonic) rotation a hot envelope is formed around the magnetosphere. So, a new critical period appear.

$$P_{\text{br}} \simeq 450 \mu_{30}^{16/21} \dot{M}_{15}^{-5/7} m^{-4/21} \text{ s.}$$

(Ikhsanov astro-ph/0310076)

If this stage is realized (inefficient cooling) then

- accretion starts later
- accretors have longer periods

# Initial spin periods

Determination of initial spin periods is closely linked with models of magneto-rotational evolution of neutron stars.

Among thousands of known NSs just for a few tens there are estimates of initial spin periods. Just for a few such estimates a robust enough.

Typically, it is necessary to have a independent estimate of a NS age. Then, using some model of magneto-rotational evolution the initial spin period is reconstructed.

Independent ages:

- SNR
- Kinematic
- Cooling



# Sample of NSs+SNRs

Table 1. Sample of PSRs associated with SNRs

PSR	SNR	$\tau_{SNR}/10^3$ yrs	$\tau_{sd}/10^3$ yrs	Ref.
J0537-6910	N157B	as the PSR	4.9	Wang and Gotthelf (1998)
J1119-6127	G292.2-0.5	as the PSR	1.6	Pivovarov et al. (2001)
J1747-2809	G0.9+0.1	as the PSR	5.3	Aharonian and et al. (2005)
				Porquet et al. (2003)
J1747-2958	G359.23-0.82	as the PSR	25.5	Camilo et al. (2002b)
J1846-0258	Kes75	as the PSR	0.73	Leahy and Tian (2008)
J1930+1852	G54.1+0.3	as the PSR	2.9	Camilo et al. (2002a)
J0007+7303	CTA 1	10.2-15.8	13.9	Slane et al. (2004)
J0205+6449	3C58	4.3-7	5.4	Slane et al. (2008)
J0538+2817	S147	40-200	618.1	Anderson et al. (1996)
				Ng et al. (2007)
B0540-69	0540-693	0.66-1.1	1.67	Williams et al. (2008)
B0656+14	Monogem Ring	86-170	110.9	Thorsett et al. (2003)
J0821-4300	Puppis A	3.3-4.1	1489.	Gotthelf and Halpern (2009)
B0833-45	Vela	11-27	11.3	Aschenbach et al. (1995)
J1124-5916	G292.0+1.8	2.4-2.85	2.85	Gonzalez and Safi-Harb (2003)
B1509-58	G320.4-1.2	6-20	1.6	Yatsu et al. (2005)
J1809-2332	G7.5-1.7	10-100	67.6	Roberts and Brogan (2008)
J1813-1749	G12.8-0.0	0.285-2.5	4.7	Brogan et al. (2005)
J1833-1034	G21.5-0.9	0.8-40.	4.9	Safi-Harb et al. (2001)

Table 1—Continued

PSR	SNR	$\tau_{SNR}/10^3$ yrs	$\tau_{sd}/10^3$ yrs	Ref.
B1853+01	W44	6.5-20	20.3	Harrus et al. (1997)
J1957+2831	G65.1+0.6	40-140	1568.	Tian and Leahy (2006)
B1951+32	CTB80	> 18	107.	Castelletti et al. (2003)
B1338-62	G308.8-0.1	< 32.5	12.1	Caswell et al. (1992)
J2229+6114	G106.6+2.9	> 3.9	10.5	Kothes et al. (2006)
B0531+21	Crab	0.957	1.24	Stephenson and Green (2002)
J1210-5226	G296.5+10.0	10-20	101817.	Vasisht et al. (1997)
J1437-5959	G315.9-0.0	22	114.	Camilo et al. (2009)
J1811-1925	G11.2-0.3	1.6	23.2	Torii et al. (1999)
J1852+0040	Kes79	6	191502.	Sun et al. (2004)
J2021+4026	G78.2+2.1	6.6	76.9	Uchiyama et al. (2002)
B2334+61	G114.3+0.3	7.7	40.6	Yar-Uyaniker et al. (2004)

30 pairs: PSR+SNR  
Popov, Turolla arXiv: 1204.0632

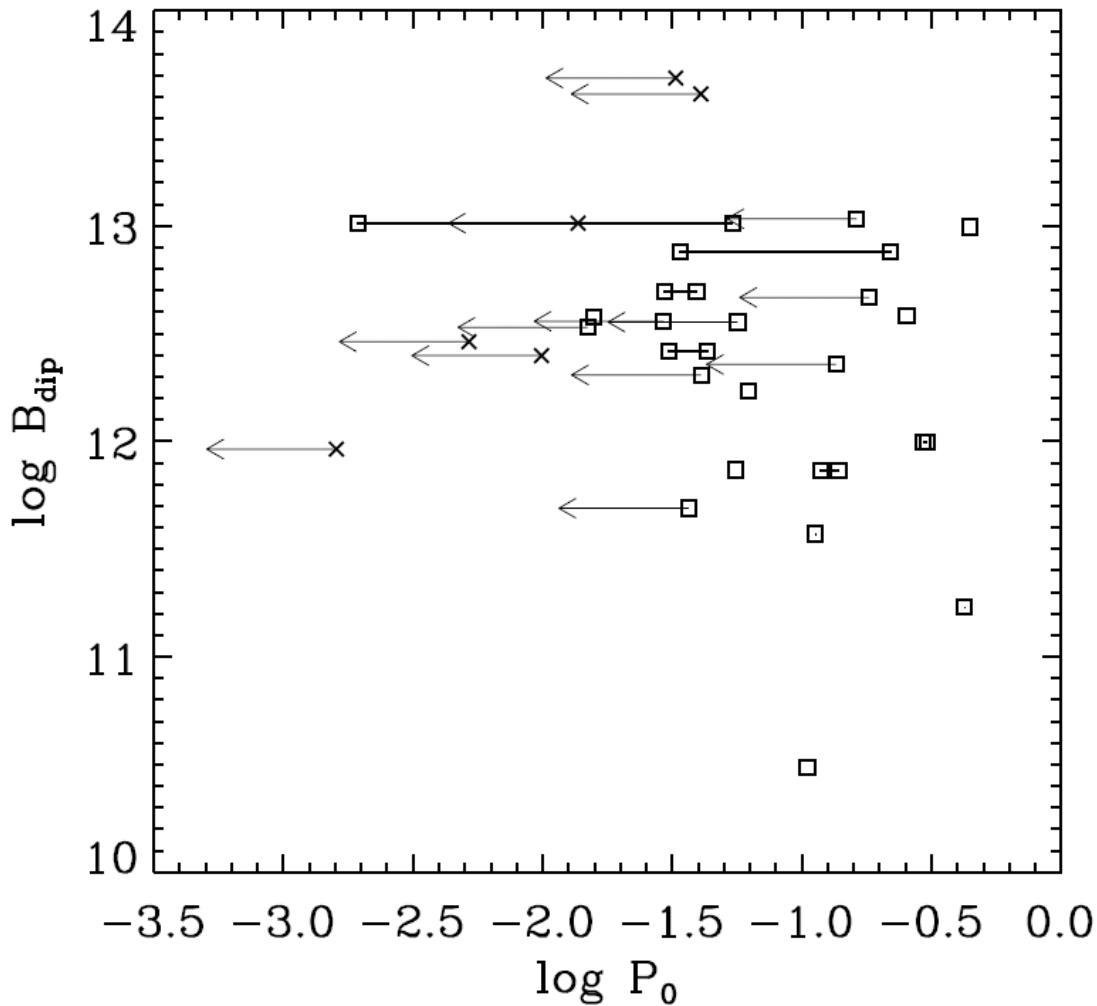
Table 2. Spin parameters of PSRs in the sample

PSR	$P$ s	$\dot{P}$	$B/10^{12}$ G	$P_0$ s	$P_0/P$
J0537-6910	0.016	5.18E-14	0.92	$\ll P$	$\sim 0$
J1119-6127	0.408	4.02E-12	41.	$\ll P$	$\sim 0$
J1747-2809	0.052	1.56E-13	2.9	$\ll P$	$\sim 0$
J1747-2958	0.099	6.13E-14	2.5	$\ll P$	$\sim 0$
J1846-0258	0.326	7.08E-12	48.6	$\ll P$	$\sim 0$
J1930+1852	0.137	7.51E-13	10.3	$\ll P$	$\sim 0$
J0007+7303	0.316	3.6E-13	10.8	$< 0.163$	$< 0.52$
J0205+6449	0.066	1.94E-13	3.6	$< 0.029$	$< 0.45$
J0538+2817	0.143	3.67E-15	0.73	$< 0.134$	$< 0.93$
	0.143	3.67E-15	0.73	$> 0.118$	$> 0.82$
B0540-69	0.05	4.79E-13	5.0	$< 0.039$	$< 0.78$
	0.05	4.79E-13	5.0	$> 0.03$	$> 0.59$
B0656+14	0.385	5.5E-14	4.7	$< 0.183$	$< 0.48$
J0821-4300	0.113	1.2E-15	0.37	$< 0.113$	$\sim 1$
	0.113	1.2E-15	0.37	$> 0.113$	$\sim 1$
B0833-45	0.089	1.25E-13	3.4	$< 0.016$	$< 0.2$
J1124-5916	0.135	7.53E-13	10.2	$< 0.054$	$< 0.40$
	0.135	7.53E-13	10.2	$> 0.004$	$> 0.03$
J1210-5226	0.424	6.6E-17	0.17	0.424	$\sim 1$
B1509-58	0.151	1.54E-12	15.4	—	—

Table 2—Continued

PSR	$P$ s	$\dot{P}$	$B/10^{12}$ G	$P_0$ s	$P_0/P$
J1809-2332	0.147	3.44E-14	2.3	$< 0.136$	$< 0.92$
J1813-1749	0.045	1.5E-13	2.6	$< 0.043$	$< 0.97$
	0.045	1.5E-13	2.6	$> 0.031$	$> 0.69$
J1833-1034	0.062	2.02E-13	3.6	$< 0.057$	$< 0.91$
B1853+01	0.267	2.08E-13	7.5	$< 0.221$	$< 0.83$
	0.267	2.08E-13	7.5	$> 0.036$	$> 0.14$
J1957+2831	0.308	3.11E-15	0.99	$< 0.3$	$< 0.99$
	0.308	3.11E-15	0.99	$> 0.29$	$> 0.95$
B1951+32	0.04	5.84E-15	0.49	$< 0.036$	$< 0.91$
B1338-62	0.193	2.53E-13	7.1	—	—
J2229+6114	0.052	7.83E-14	2.0	$< 0.041$	$< 0.79$
B0531+21	0.033	4.23E-13	3.8	0.016	0.48
J1437-5959	0.062	8.59E-15	0.74	0.055	0.9
J1811-1925	0.065	4.40E-14	1.7	0.062	0.97
J1852+0040	0.105	8.68E-18	0.03	0.105	$\sim 1$
J2021+4026	0.265	5.47E-14	3.9	0.254	0.96
B2334+61	0.495	1.93E-13	9.9	0.45	0.91

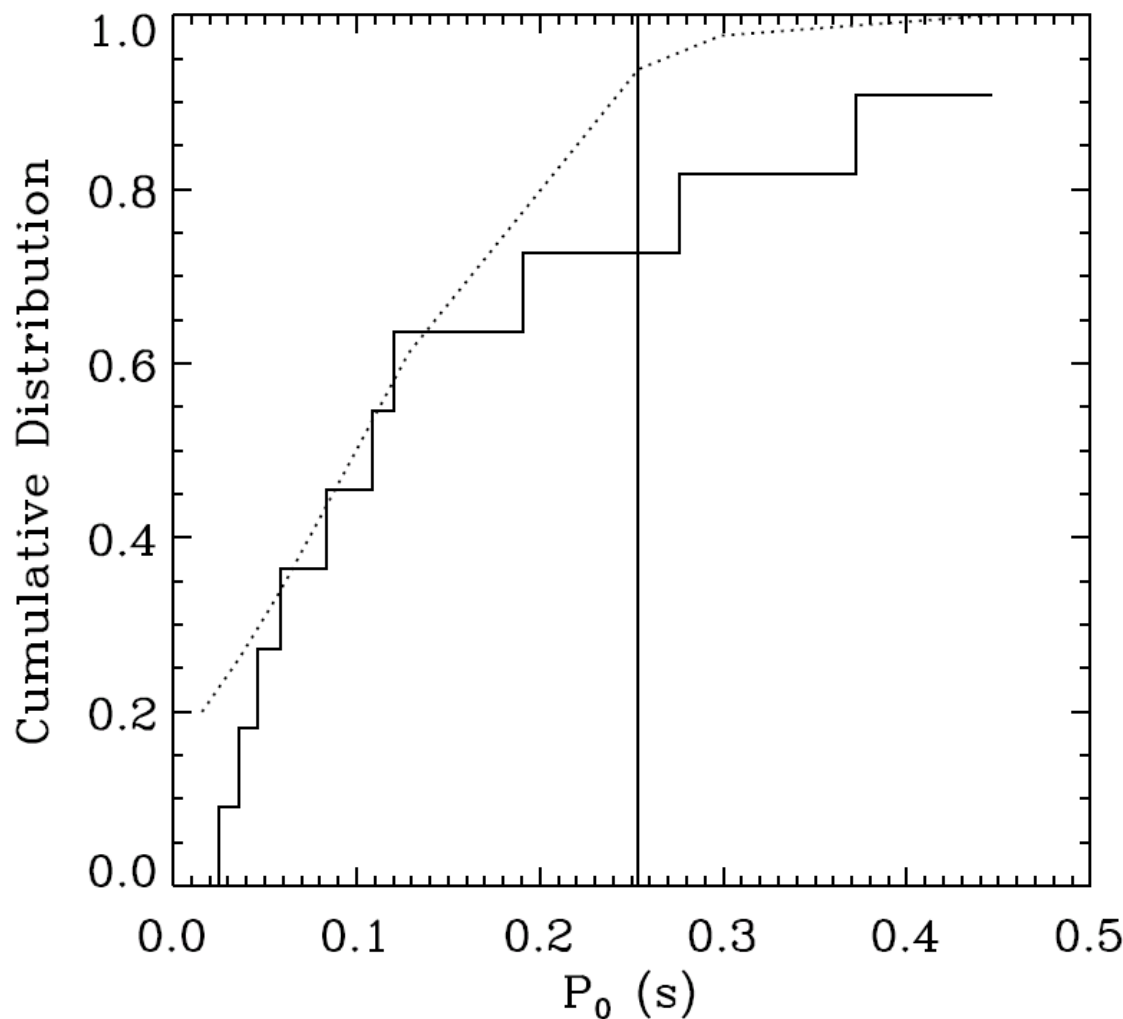
# B vs. $P_0$



All presented estimates  
are made for standard  
assumptions:  
 $n = \text{const} = 3$ .  
So, field is assumed to be  
constant, as well as the angle  
between spin and magnetic axis.

Crosses – PSRs in SNRs (or PWN) with ages just consistent with spin-down ages. We assume that  $P_0 < 0.1 P$

# Checking gaussian



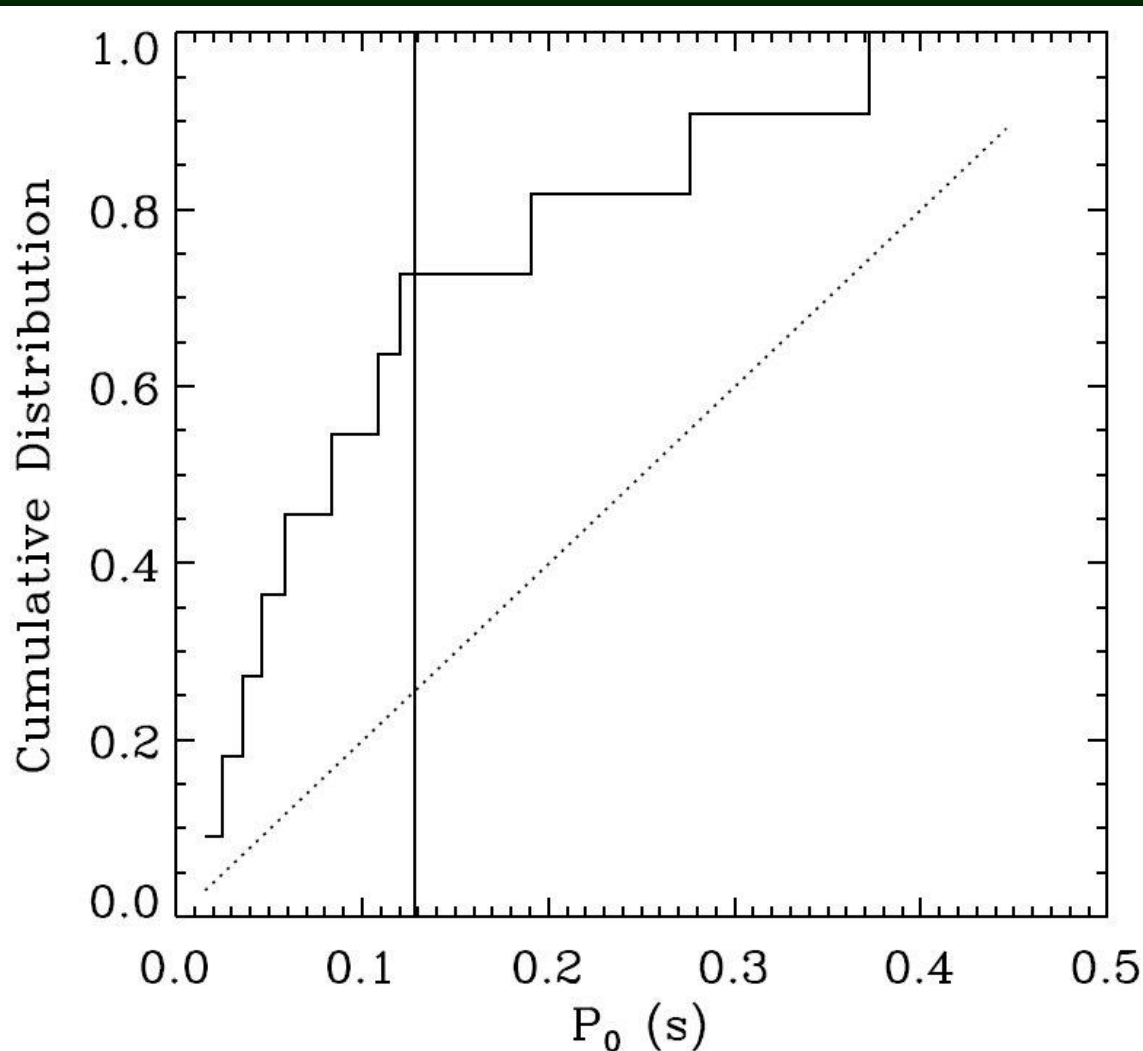
The data we have is not enough to derive the shape of the  $P_0$  distribution. However, we can exclude very wide and very narrow distributions, and also we can check if some specific distributions are compatible with our results.

Here we present a test for a gaussian distribution, which fits the data.

Still, we believe that the fine tuning is premature with such data.

$$P_0 = 0.1 \text{ s}; \sigma = 0.1 \text{ s}$$

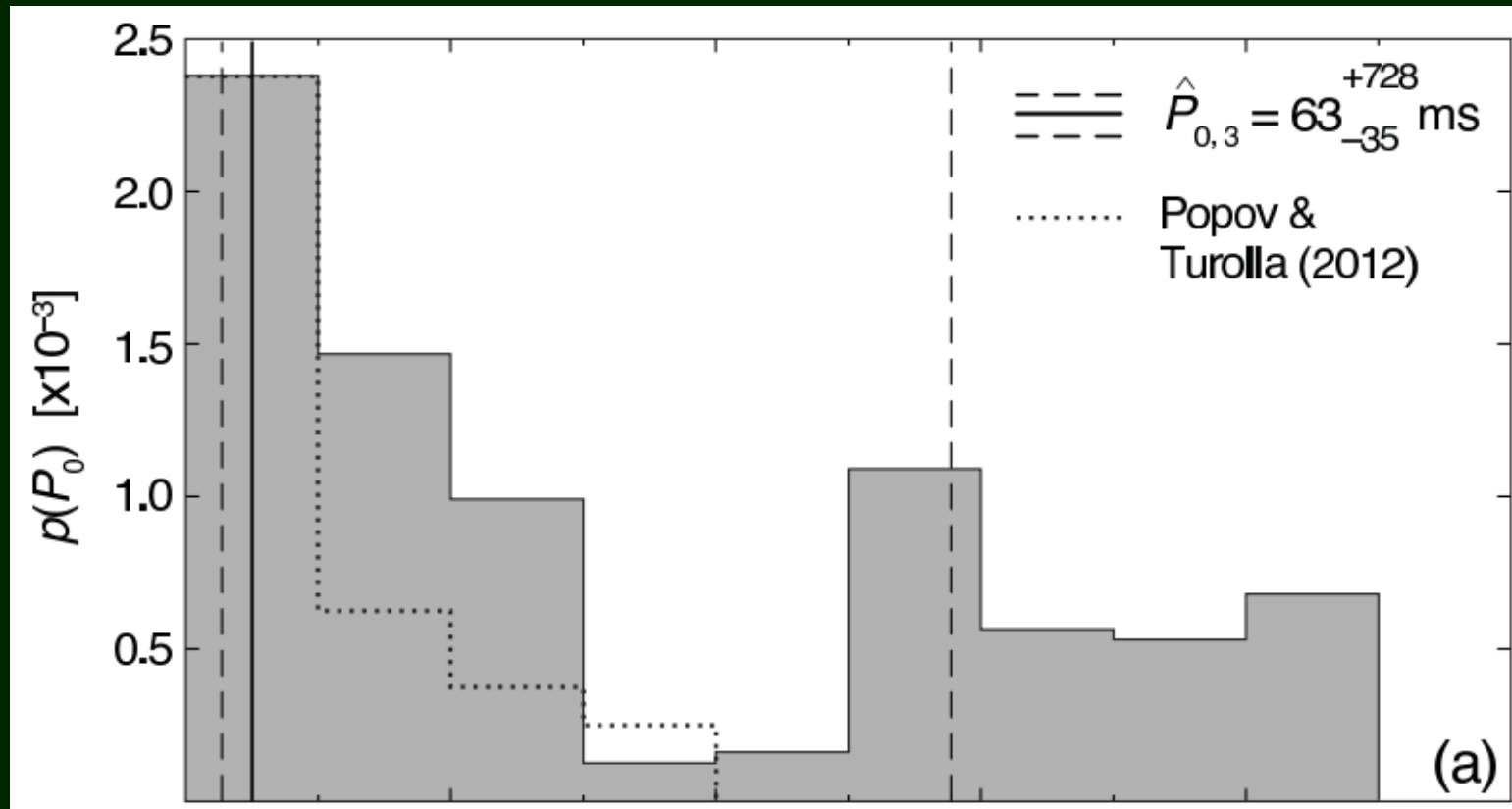
# Checking flat distribution



Flat between 0.001 and 0.5 s.

Very wide distributions  
in general do not fit  
the data we have.

# Wide initial spin period distribution



Based on kinematic ages. Mean age – few million years.  
Note, that in Popov & Turolla (2012) only NSs in SNRs  
were used, i.e. the sample is much younger!  
Can it explain the difference?

1301.1265

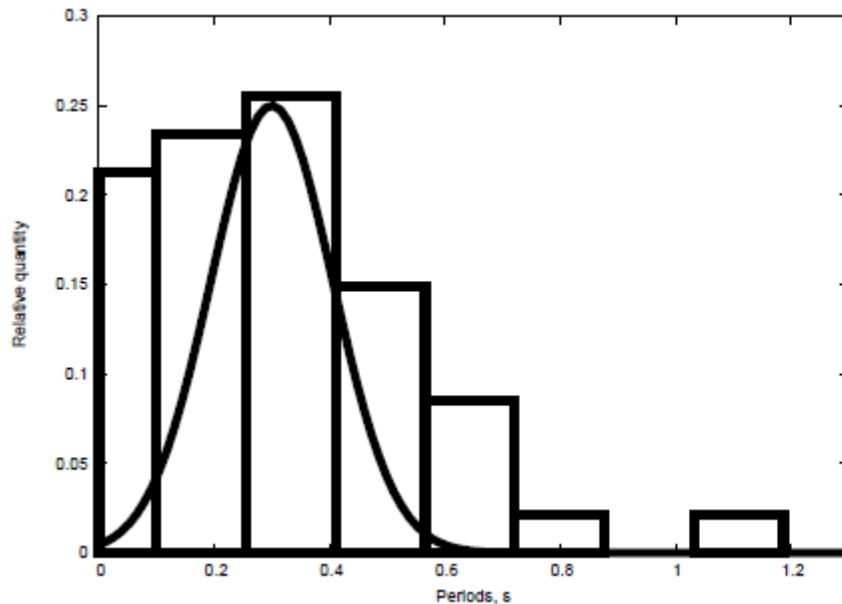
# Magnetic field decay and $P_0$

One can suspect that magnetic field decay can influence the reconstruction of the initial spin period distribution.

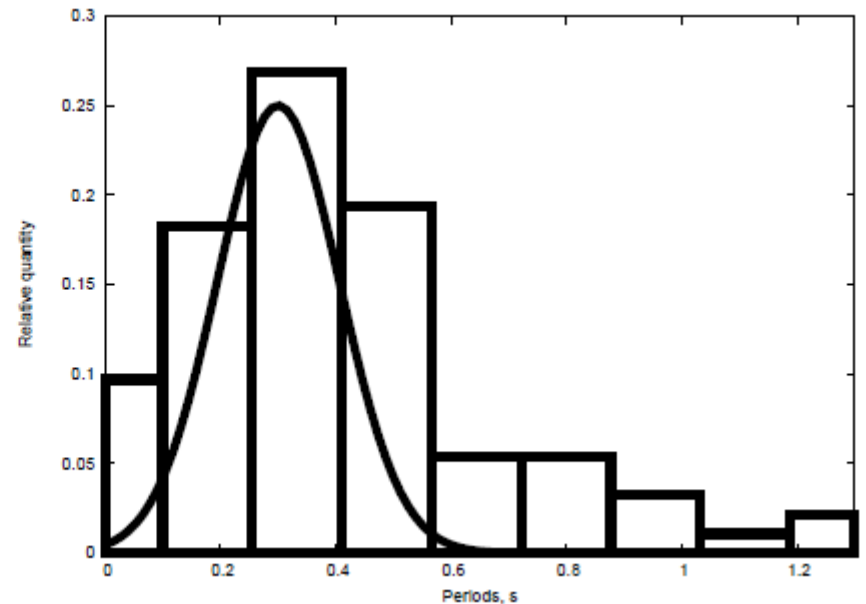
Exponential field decay with  $\tau=5$  Myrs.

$\langle P_0 \rangle = 0.3$  s,  $\sigma_P = 0.15$  s;  $\langle \log B_0/[G] \rangle = 12.65$ ,  $\sigma_B = 0.55$

$$P_0 = P \sqrt{1 - \frac{t}{\tau}}$$



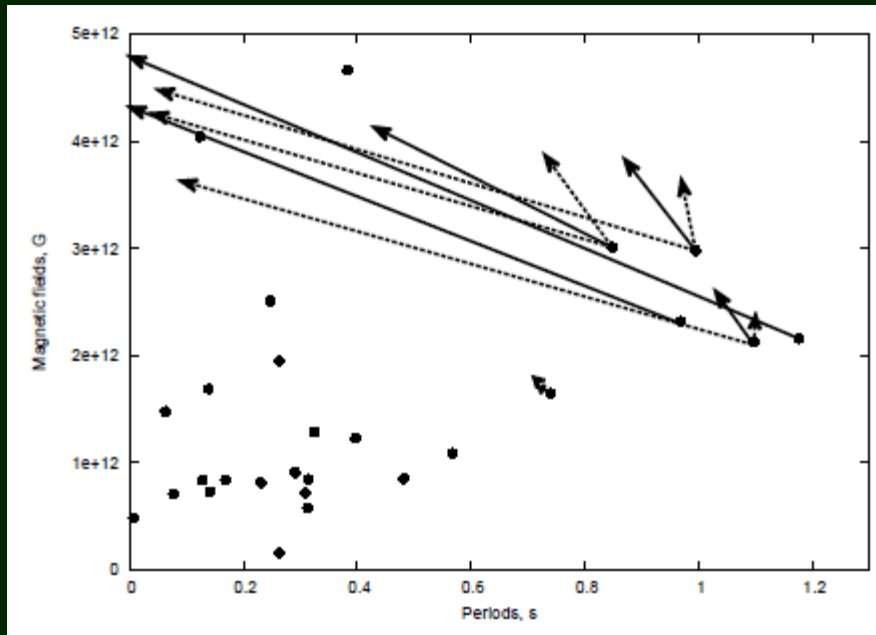
$\tau < 10^7$  yrs,  $10^5 < t$



$10^5 < t < 10^7$  yrs



# Real vs. reconstructed $P_0$



How much the reconstructed initial periods are changed due to not taking into account the exponential field decay?

Igoshev, Popov 2013

# Complications for magneto-rotational evolution

1. Internal structure can be important.

For example, neutron vortices can pin magnetic flux tubes (1106.5997).

Estimates indicate that this can be important for magnetars.

2. In young NSs a core can rotate faster than the crust (1210.5872).

3. Non-trivial topology of the magnetosphere can be important.

In magnetars a twisted magnetosphere can result in a different spin-down rate (1201.3635, and see the lecture on magnetars)

4. Magnetic field can have a very non-trivial evolution (see the next lecture)

5. Initial spin-periods can depend on additional phenomena.

Gravitational wave emission (1302.2649 ).

Neutrino emission (1301.7495).

Different instabilities (1110.3937).

# Conclusions

- We have some framework for spin evolution of NSs.  
They are expected to pass several well-defined stages:  
Ejector (including radio pulsar),  
Propeller (probably, with subsonic substage), Accretor.  
NSs with large velocities (or fields) after the Ejector stage  
can appear as Georotators.
- In binaries we observe Ejectors, Propellers and Accretor.  
For isolated NSs – only Ejectors (even, mostly radiopulsars).
- There are still many uncertainties related to the spin evolution:
  1. Spin-down rate and angle evolution for radio pulsars
  2. Subsonic propeller stage for isolated NSs
  3. Inhibition of accretion at low rates
  4. The role of the field decay

# Conclusions-2

- Observations of isolated accreting NSs can help a lot to understand all unknown questions of NS spin evolution and low-rate accretion.
- Magnetic field decay can be important also for young NSs, especially for highly magnetized ones, as a source of energy.

So, we have some coherent picture ..... But .....

**A lot of funny thing a still waitng for us!**



# Papers and books to read

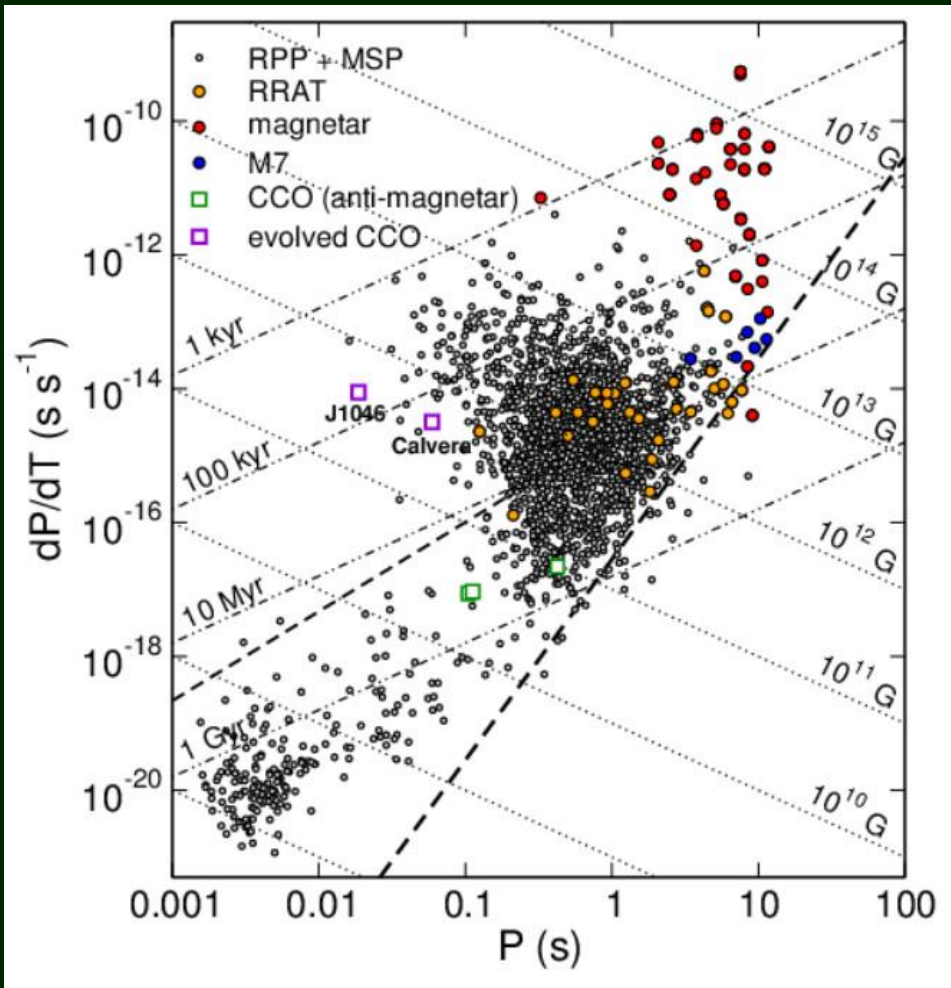
- Lipunov V.M. **“Astrophysics of neutron stars”** (1992)
- Lipunov, Postnov, Prokhorov **“The Scenario Machine: Binary Star Population Synthesis”**  
**Astrophysics and Space Science Reviews** (1996)  
<http://xray.sai.msu.ru/~mystery/articles/review/>
- Boldin, Popov ``Evolution of isolated neutron stars till accretion” 1004.4805

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Evolution with decaying and  
re-emerging magnetic field

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# Diversity of young neutron stars



Young isolated neutron stars can appear in many flavors:

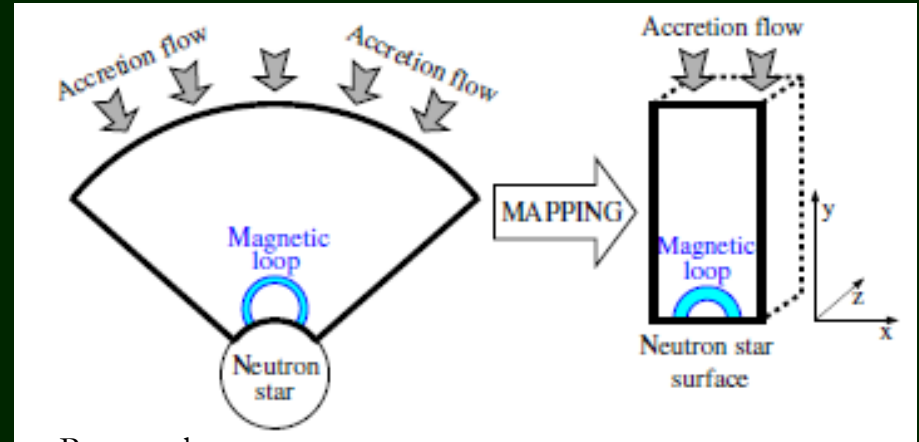
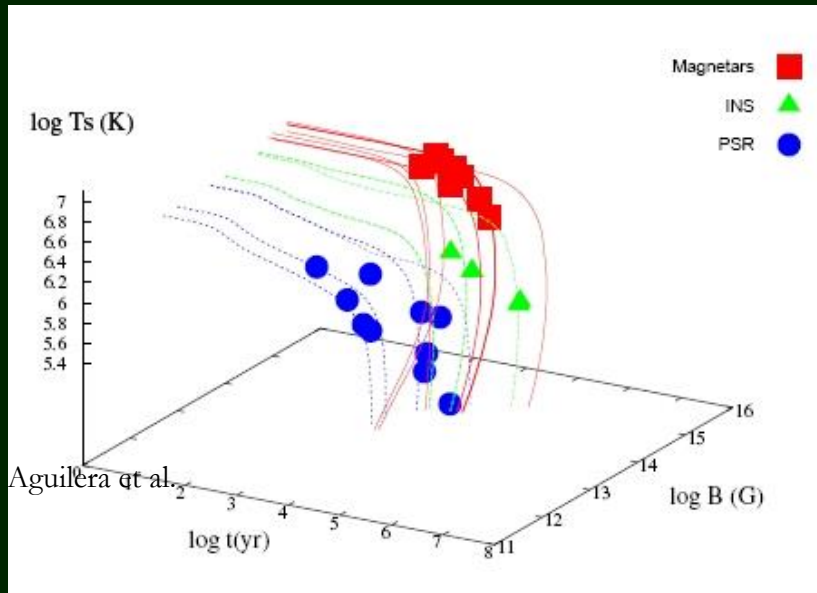
- o Radio pulsars
- o Compact central X-ray sources in supernova remnants.
- o Anomalous X-ray pulsars
- o Soft gamma repeaters
- o The Magnificent Seven & Co.
- o Transient radio sources (RRATs)

The term  
“GRAND UNIFICATION  
FOR NEUTRON STARS”  
was coined by Kaspi (2010)

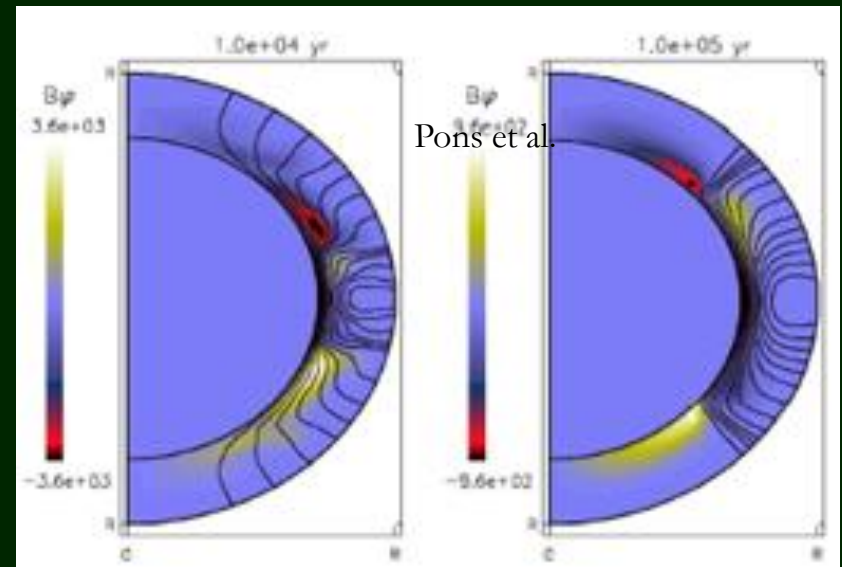
PSRs, magnetars and M7  
unified in the model by  
Popov et al. (2010).



# Three main ingredients of a unified model

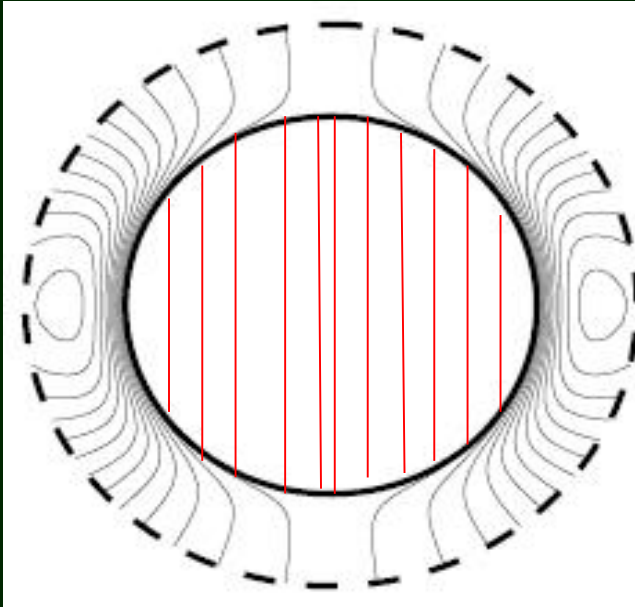


Page et al.



# Magnetic field decay

**Magnetic fields of NSs are expected to decay due to decay of currents which support them.**



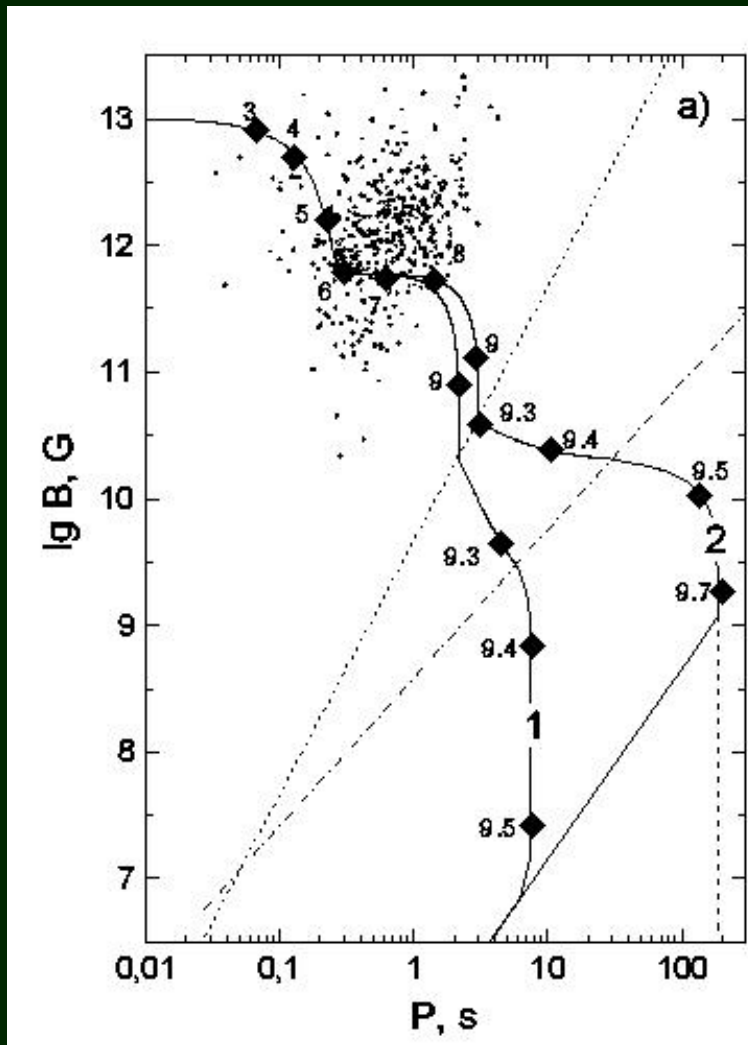
Crustal field of core field?

It is easy to decay in the crust.

In the core the field is in the form of superconducting vortices. They can decay only when they are moved into the crust (during spin-down).

Still, in most of models strong fields decay.

# Period evolution with field decay



An evolutionary track of a NS is very different in the case of decaying magnetic field.

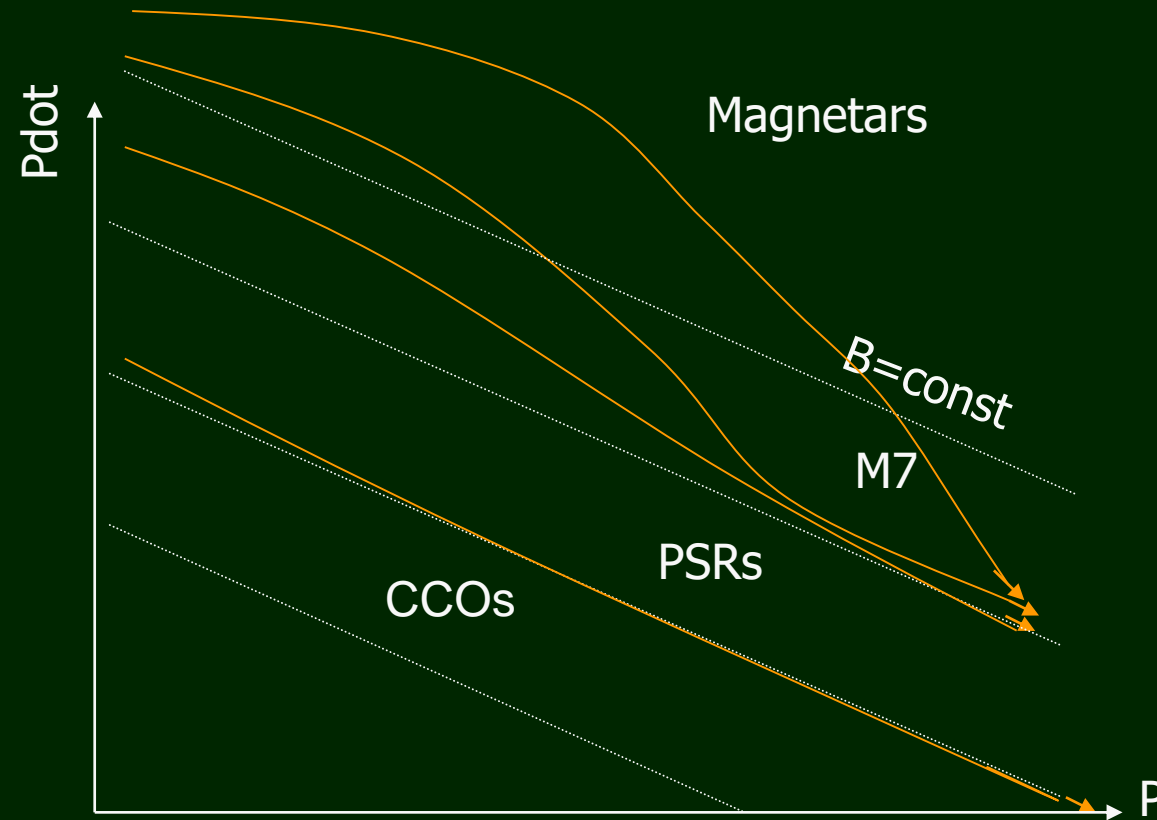
The most important feature is slow-down of spin-down. Finally, a NS can nearly freeze at some value of spin period.

Several episodes of relatively rapid field decay can happen.

Number of isolated accretors can be both decreased or increased in different models of field decay. But in any case their average periods become shorter and temperatures lower.

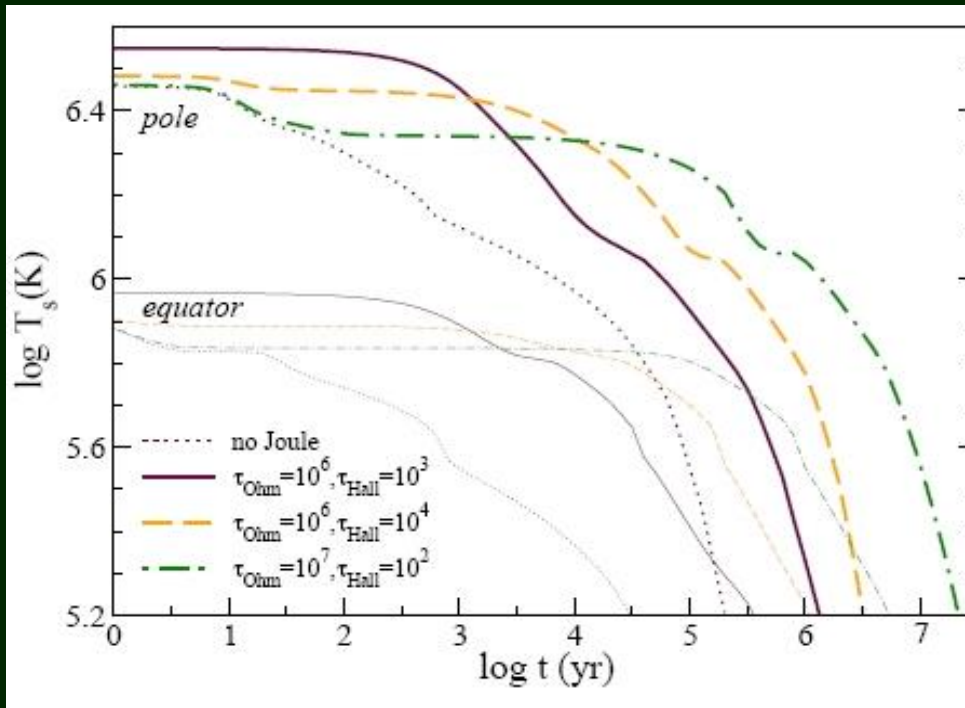
# Magnetars, field decay, heating

A model based on field-dependent decay of the magnetic moment of NSs can provide an evolutionary link between different populations (Pons et al.).



# Magnetic field decay vs. thermal evolution

**Magnetic field decay can be an important source of NS heating.**



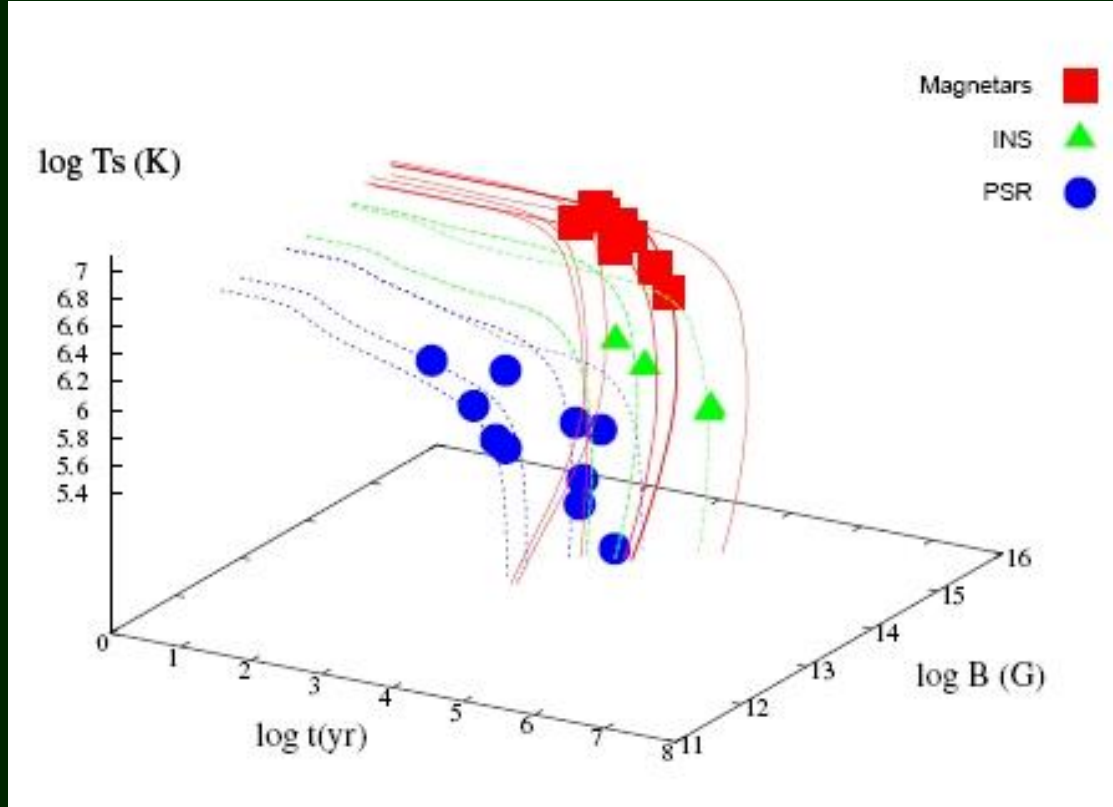
Heat is carried by electrons. It is easier to transport heat along field lines. So, poles are hotter. (for light elements envelope the situation can be different).

Ohm and Hall decay

arxiv:0710.0854 (Aguilera et al.)

$$B = B_0 \frac{\exp(-t/\tau_{\text{Ohm}})}{1 + \frac{\tau_{\text{Ohm}}}{\tau_{\text{Hall}}}(1 - \exp(-t/\tau_{\text{Ohm}}))}$$

# Joule heating for everybody?



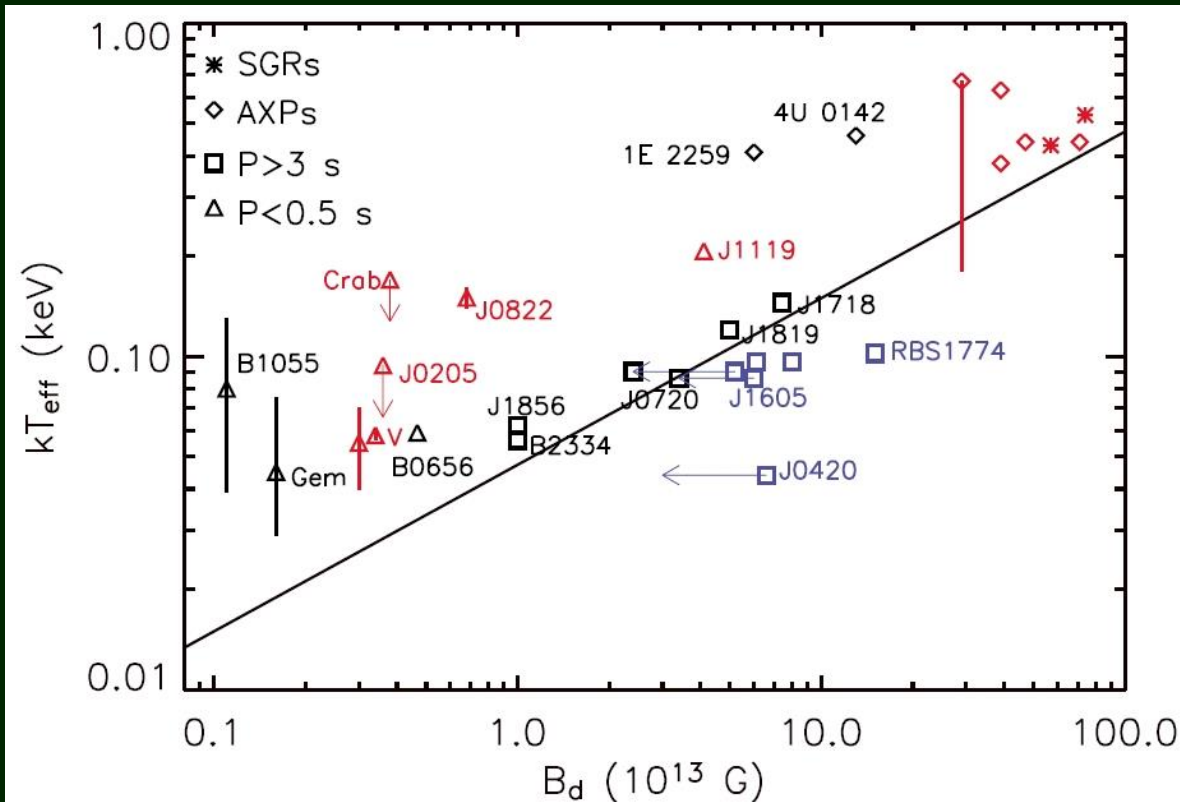
It is important to understand the role of heating by the field decay for different types of INS.

In the model by Pons et al. the effect is more important for NSs with larger initial  $B$ .

Note, that the characteristic age estimates ( $P/2 \dot{P}$ ) are different in the case of decaying field!

arXiv: 0710.4914 (Aguilera et al.)

# Magnetic field vs. temperature



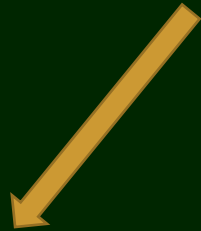
The line marks balance between heating due to the field decay and cooling. It is expected that a NS evolves downwards till it reaches the line, then the evolution proceeds along the line:  $T_{\text{eff}} \sim B_d^{1/2}$

Selection effects are not well studied here. A kind of population synthesis modeling is welcomed.

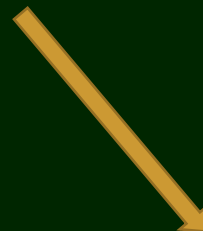
(astro-ph/0607583)



# What kind of decay do we see?



Ohmic decay due to phonons



Hall cascade

Both time scales fit, and in both cases we can switch of decay at  $\sim 10$  either due to cooling, or due to the Hall attractor.

$$B = B_0 \frac{\exp(-t/\tau_{\text{Ohm}})}{1 + \frac{\tau_{\text{Ohm}}}{\tau_{\text{Hall}}}(1 - \exp(-t/\tau_{\text{Ohm}}))}$$

# Hall cascade and field evolution

$$\frac{\partial B}{\partial t} = -c \nabla \times E,$$

advection

Ohm

Hall

$$\tau_{\text{Ohm}} = \frac{4\pi\sigma L^2}{c^2}.$$

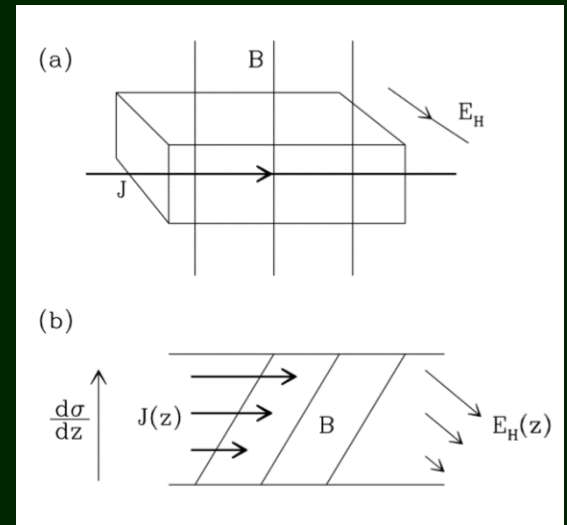
$$E = -\frac{1}{c} v \times B + \frac{J}{\sigma} + \frac{J \times B}{n_e e c},$$

$$J = (c/4\pi)(\nabla \times B)$$

With only Hall term we have:

$$\frac{\partial B}{\partial t} = -\nabla \times \left( \frac{J \times B}{n_e e} \right),$$

$$t_{\text{Hall}} = \frac{n_e e L}{J} = \frac{4\pi n_e e L^2}{c B},$$



# Characteristic timescales

$$\tau_{\text{Hall}} = \frac{4\pi en_e L^2}{cB(t)},$$

Hall time scale strongly depends on the current value of the field.

$$\tau_{\text{Hall}} = \tau_{\text{Hall},0} \frac{B_0}{B(t)}.$$

$$\tau_{\text{Ohm}} = \frac{4\pi\sigma L^2}{c^2},$$

Ohmic decay depends on the conductivity

$$\sigma = \frac{\sigma_Q \sigma_{\text{ph}}}{\sigma_Q + \sigma_{\text{ph}}}.$$

$$\tau_{\text{Ohm}}^{-1} = \tau_{\text{Ohm,ph}}^{-1} + \tau_{\text{Ohm,Q}}^{-1}.$$

Resistivity can be due to

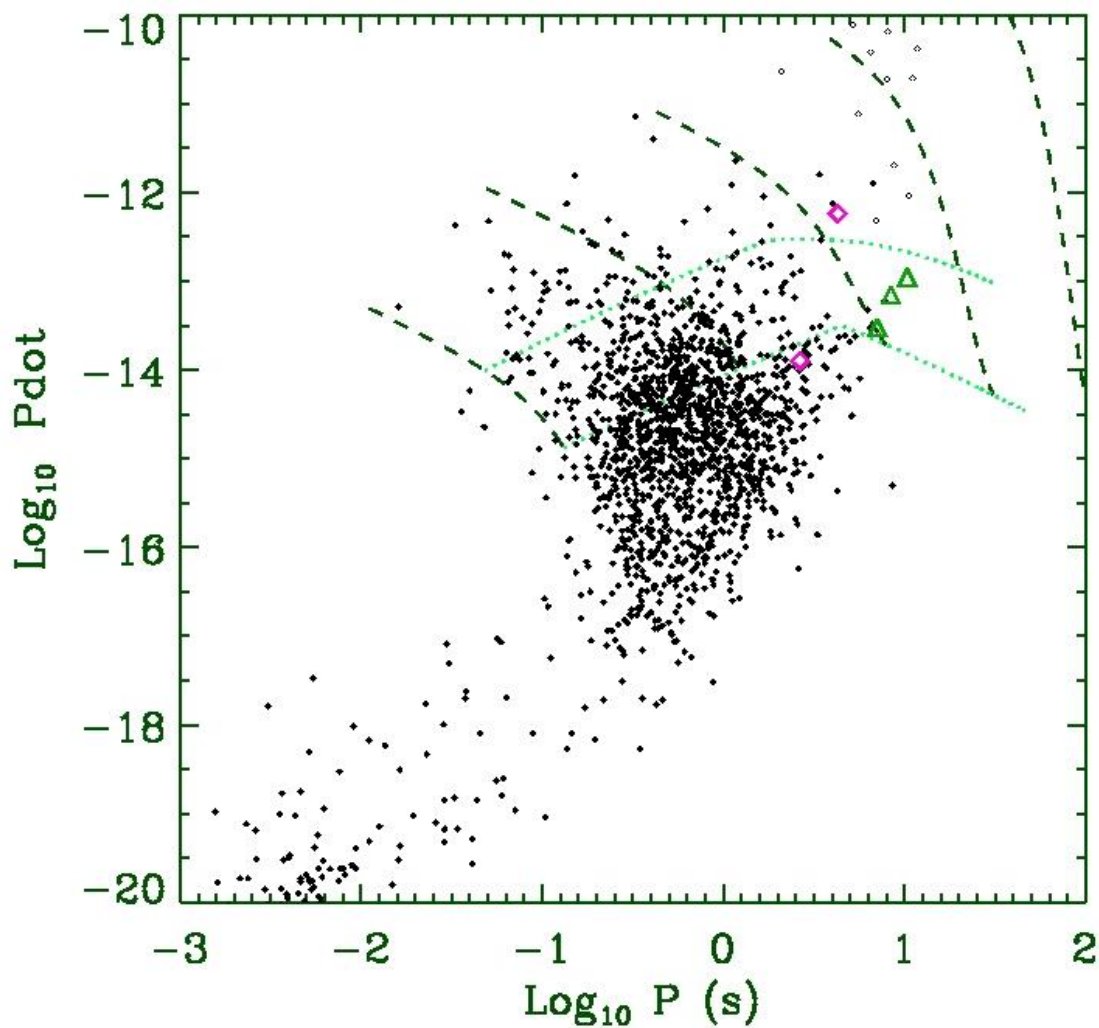
- Phonons
- Impurities

$$\sigma_Q = 4.4 \times 10^{25} \text{s}^{-1} \left( \frac{\rho_{14}^{1/3}}{Q} \right) \left( \frac{Y_e}{0.05} \right)^{1/3} \left( \frac{Z}{30} \right),$$

$$Q = n_{\text{ion}}^{-1} \sum_i n_i \times (Z^2 - \langle Z \rangle^2).$$

$$\sigma_{\text{ph}} = 1.8 \times 10^{25} \text{s}^{-1} \left( \frac{\rho_{14}^{7/6}}{T_8^2} \right) \left( \frac{Y_e}{0.05} \right)^{5/3},$$

# P-Pdot diagram and field decay



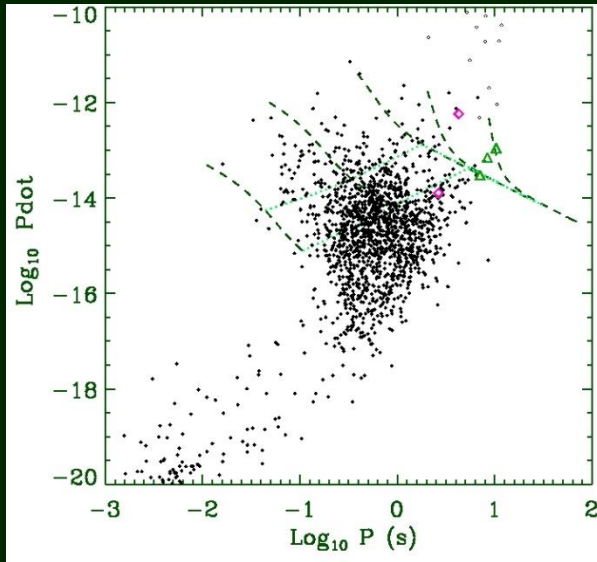
$$B = B_0 \frac{\exp(-t/\tau_{\text{Ohm}})}{1 + \frac{\tau_{\text{Ohm}}}{\tau_{\text{Hall}}} (1 - \exp(-t/\tau_{\text{Ohm}}))}$$

$$\tau_{\text{Ohm}} = 10^6 \text{ yrs}$$

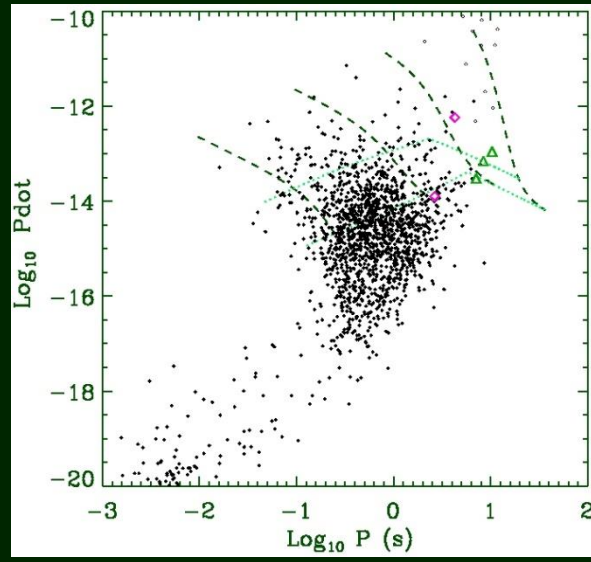
$$\tau_{\text{Hall}} = 10^4 / (B_0 / 10^{15} \text{ G}) \text{ yrs}$$

(Popov et al. MNRAS 2010. arXiv: 0910.2190)

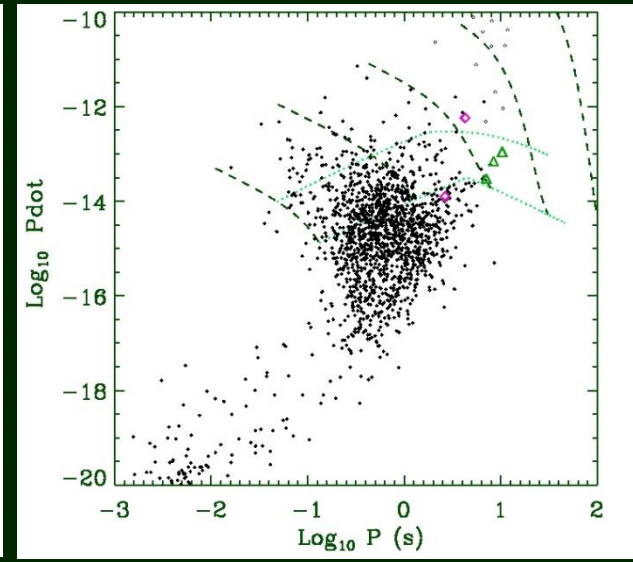
# Decay parameters and P-Pdot



$$\begin{aligned} \tau_{\text{Ohm}} &= 10^7 \text{ yrs} \\ \tau_{\text{Hall}} &= 10^2 / (B_0 / 10^{15} \text{ G}) \end{aligned}$$



$$\begin{aligned} \tau_{\text{Ohm}} &= 10^6 \text{ yrs} \\ \tau_{\text{Hall}} &= 10^3 / (B_0 / 10^{15} \text{ G}) \end{aligned}$$

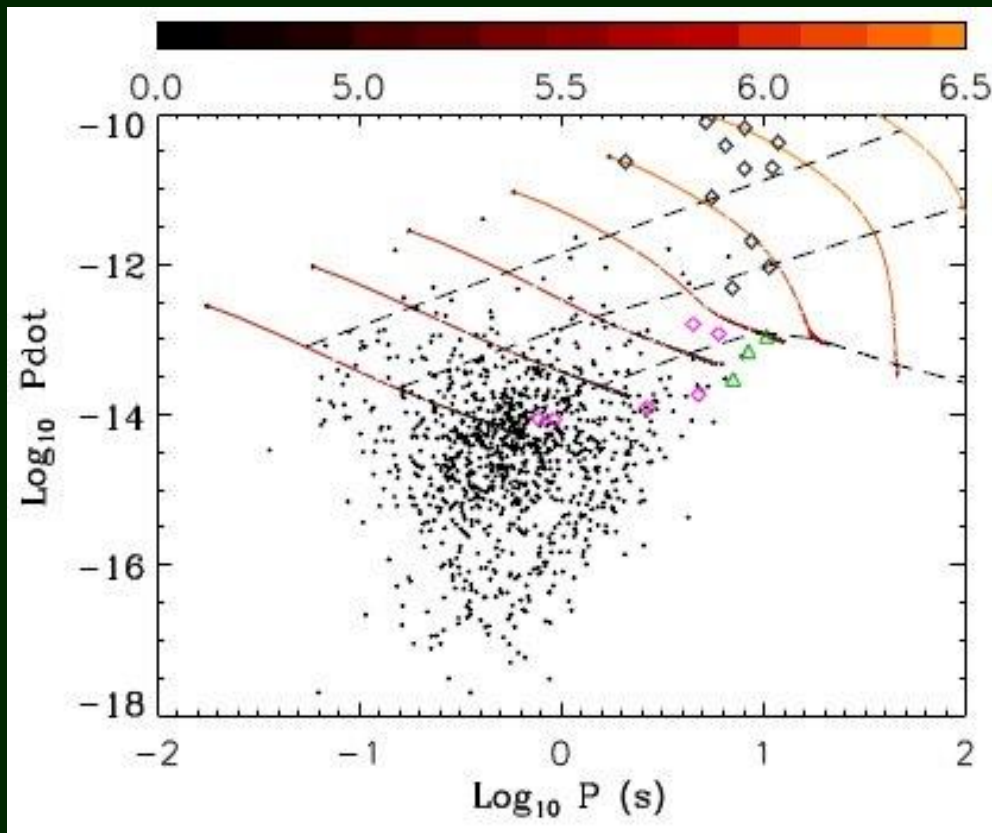


$$\begin{aligned} \tau_{\text{Ohm}} &= 10^6 \text{ yrs} \\ \tau_{\text{Hall}} &= 10^4 / (B_0 / 10^{15} \text{ G}) \end{aligned}$$

**Longer time scale for the Hall field decay is favoured.**

**It is interesting to look at HMXBs to see if it is possible to derive the effect of field decay and convergence.**

# Realistic tracks



Using the model by Pons et al. (arXiv: 0812.3018) we plot realistic tracks for NS with masses 1.4 Msolar.

Initial fields are:  
 $3 \cdot 10^{12}$ ,  $10^{13}$ ,  $3 \cdot 10^{13}$ ,  $10^{14}$ ,  
 $3 \cdot 10^{14}$ ,  $10^{15}$

Color on the track encodes surface temperature.

Tracks start at  $10^3$  years, and end at  $2 \cdot 10^6$  years.

(Popov et al. MNRAS 2010)

# Joint description of NS evolution with decaying magnetic field

The idea to describe all types of NSs with a unique model using one initial distribution (fields, periods, velocities) and to compare with observational data, i.e. to confront vs. all available observed distributions:

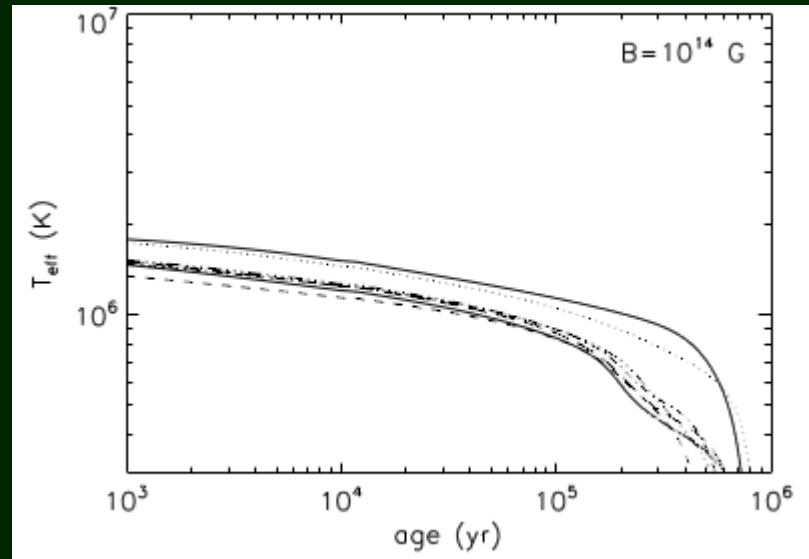
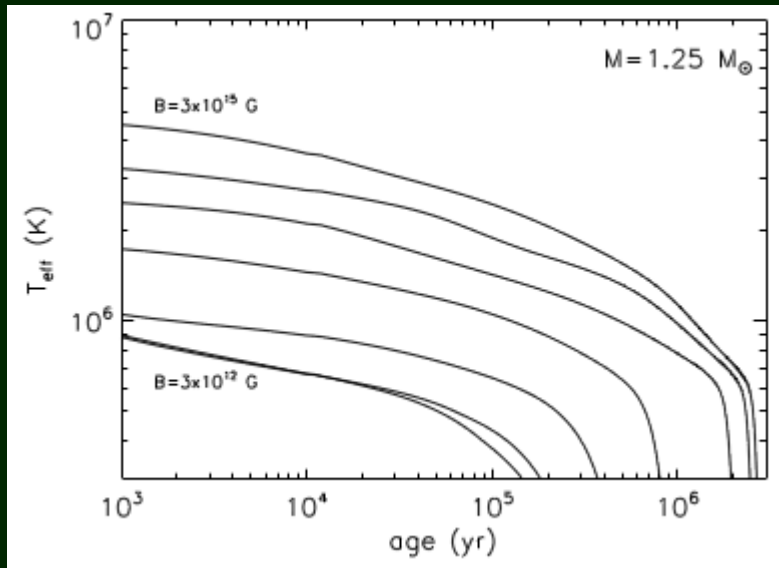
- $P$ - $\dot{P}$  for PSRs and other isolated NSs
- $\log N - \log S$  for cooling close-by NSs
- Luminosity distribution of magnetars (AXPs, SGRs)
- .....

The first step is done in Popov et al. (2010)

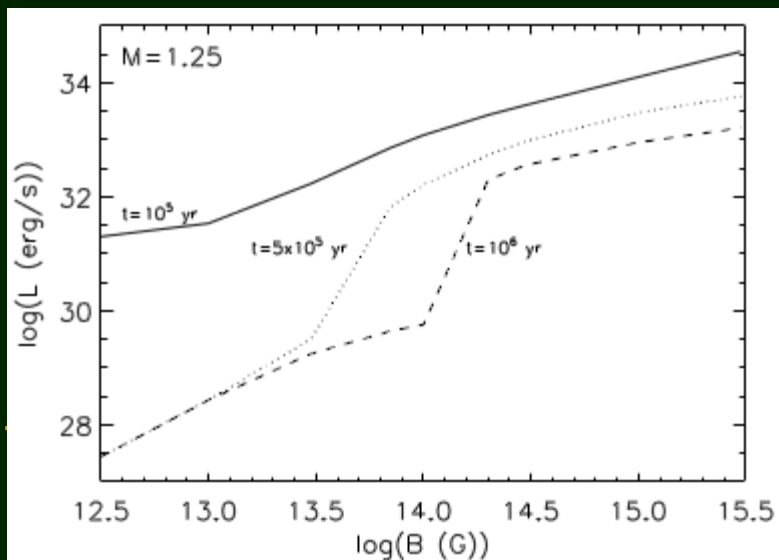
The initial magnetic field distribution with  $\langle \log B_0 \rangle \sim 13.25$  and  $\sigma \sim 0.6$  gives a good fit.  $\sim 10\%$  of magnetars.



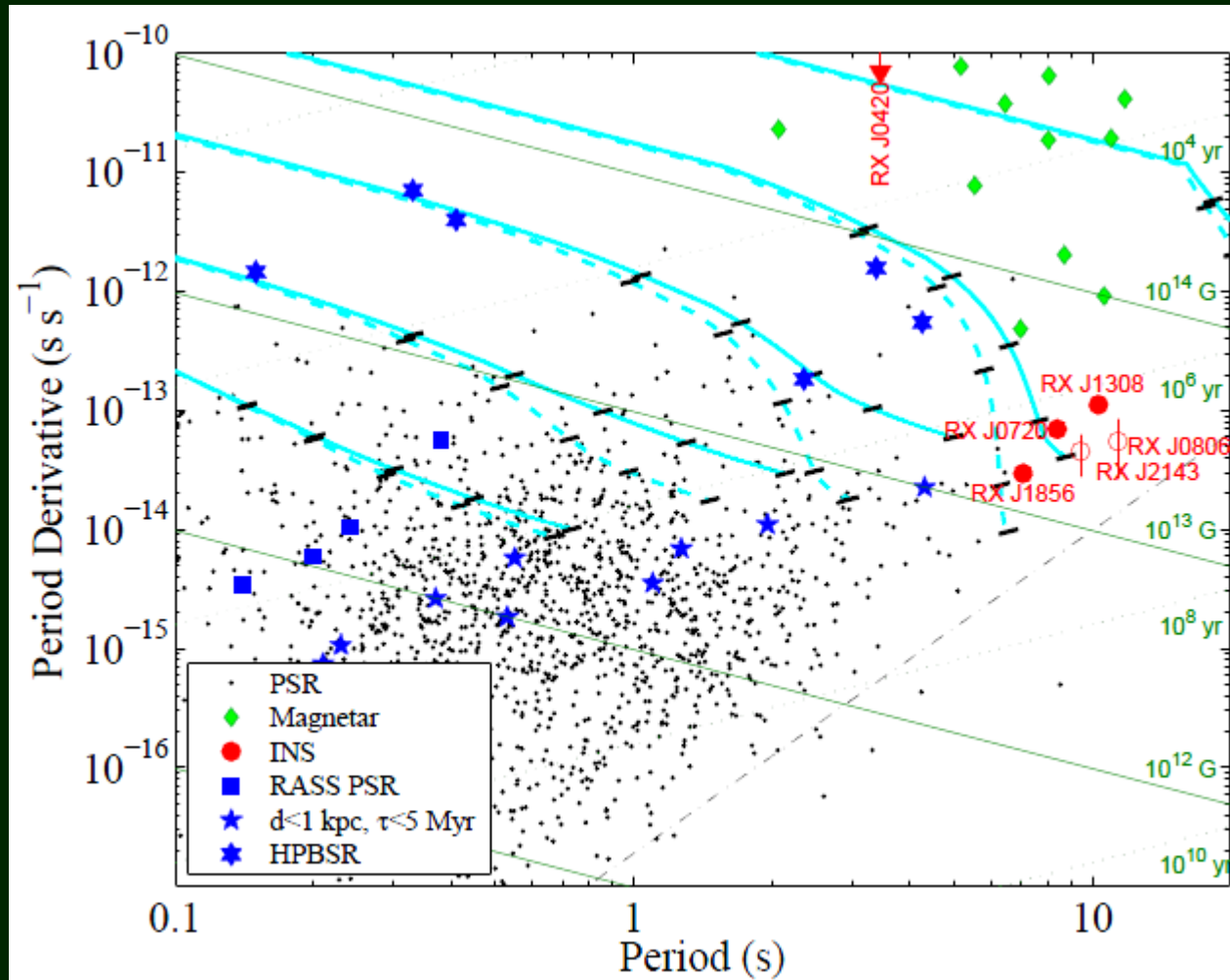
# Cooling curves with decay



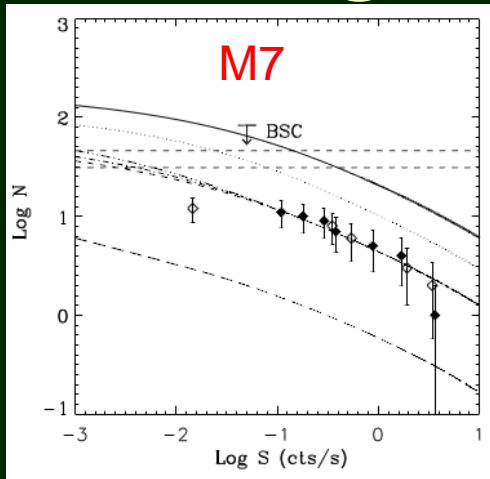
Magnetic field distribution is more important than the mass distribution.



# Observational evidence?



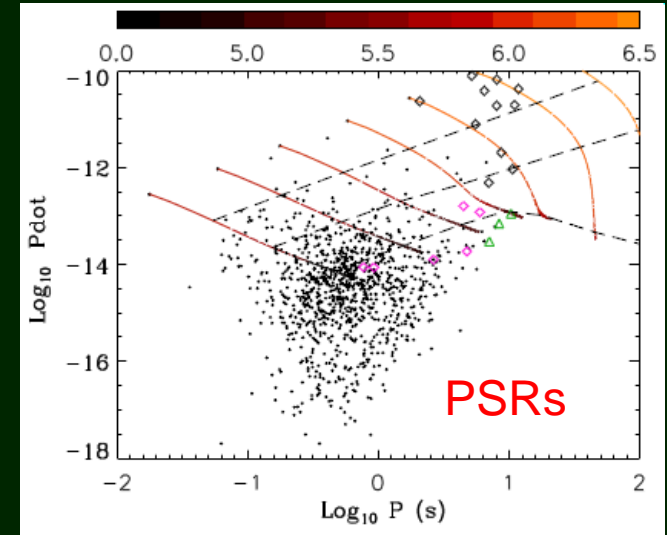
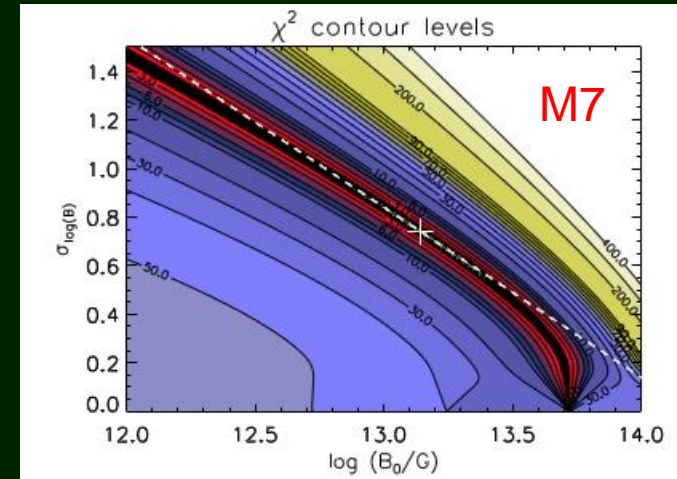
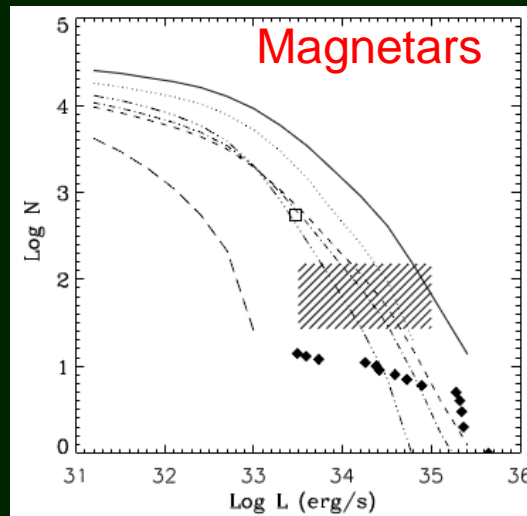
# Extensive population synthesis: M7, magnetars, PSRs



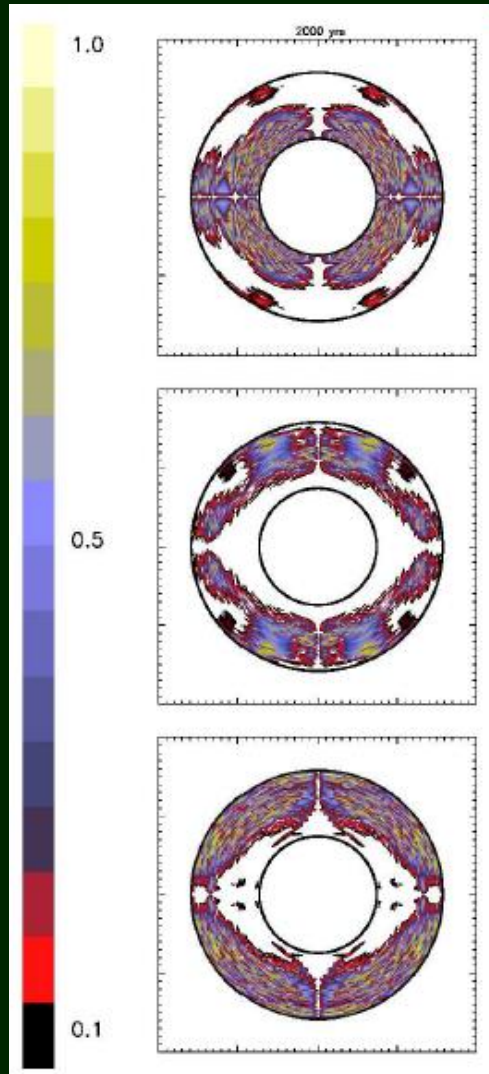
Using one population  
it is difficult or impossible  
to find unique initial  
distribution for the  
magnetic field

All three populations are  
compatible with a  
unique distribution.

Of course, the result  
is model dependent.

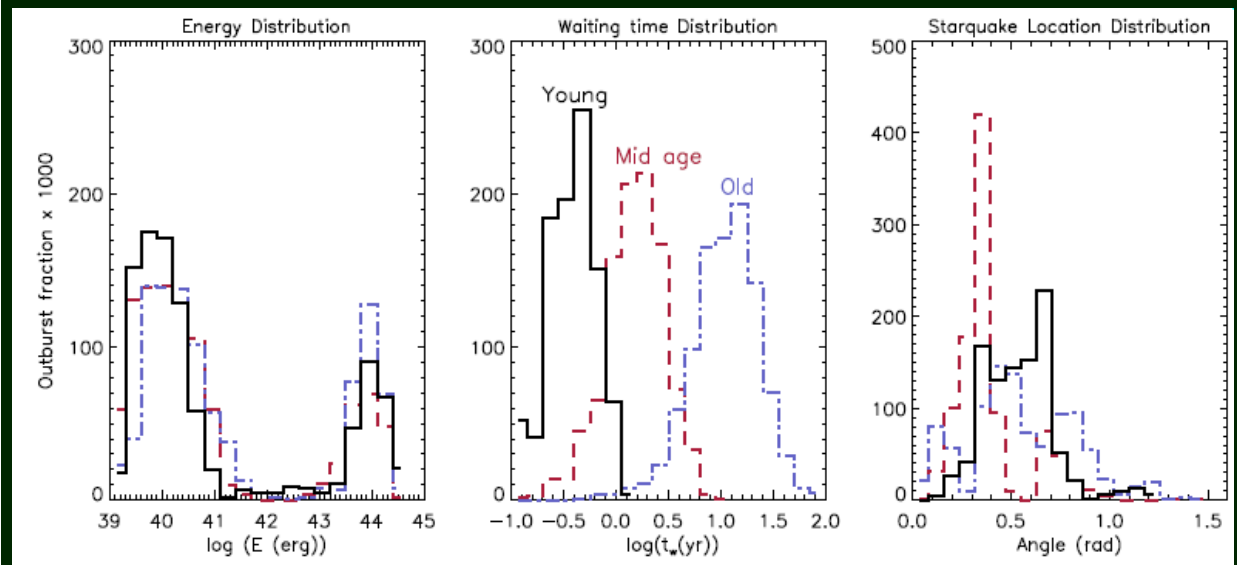


# Magnetars bursting activity due to decay



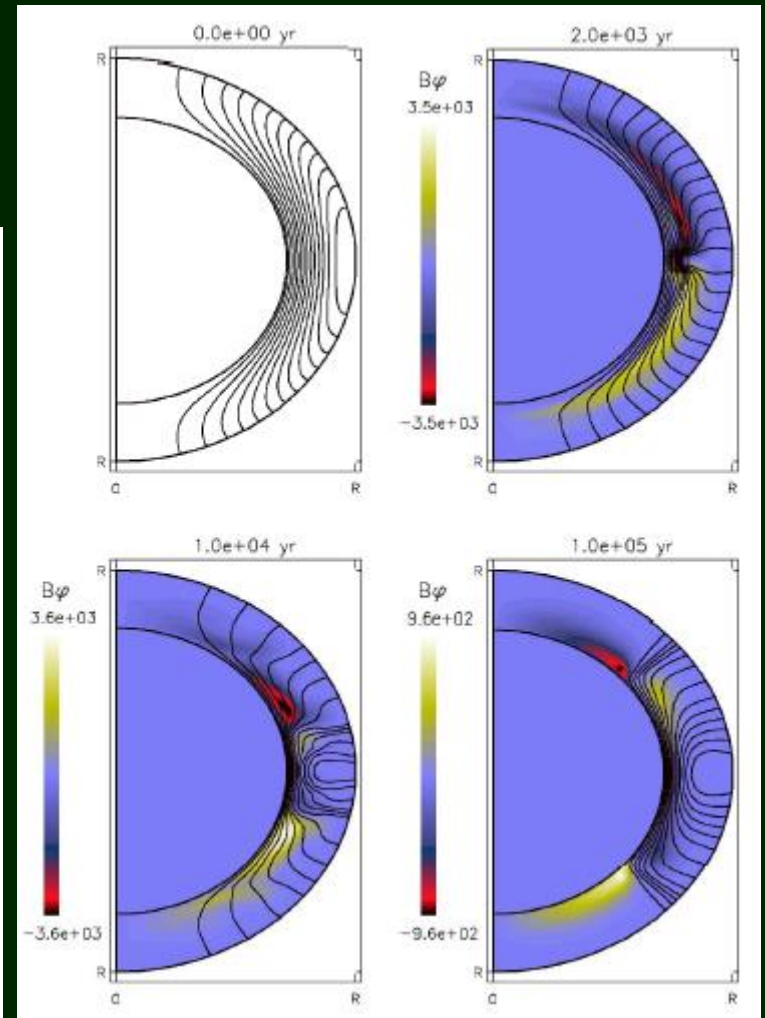
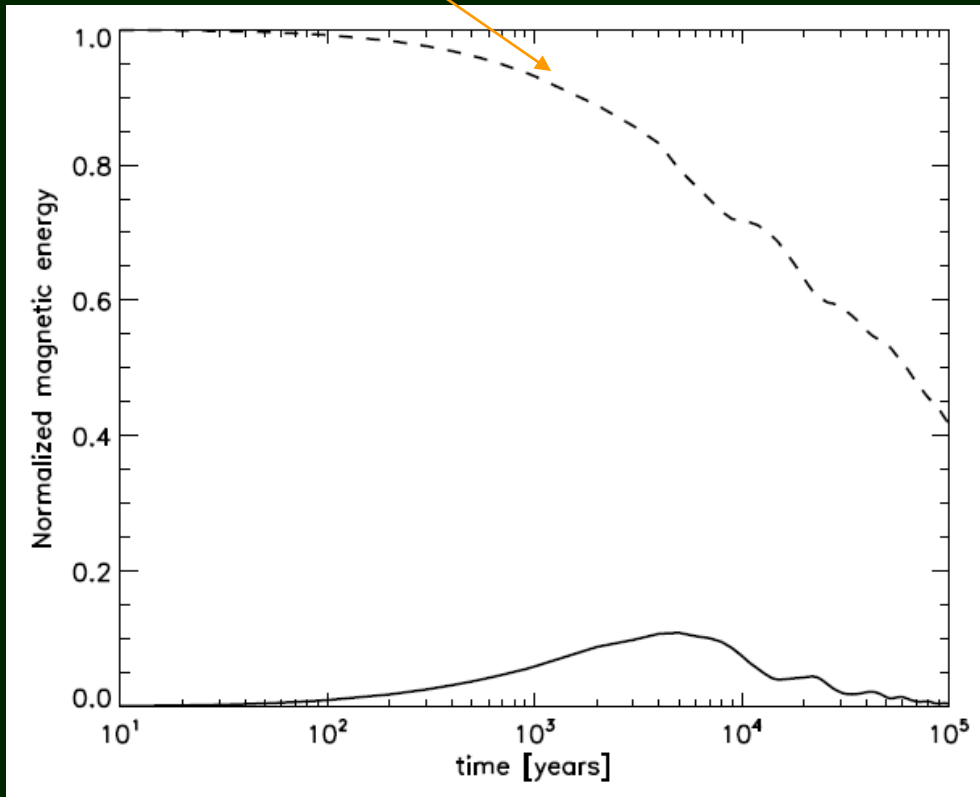
In the field decay model it is possible to study burst activity. Bursts occur due to crust cracking. The decaying field produce stresses in the crust that are not compensated by plastic deformations. When the stress level reaches a critical value the crust cracks, and energy can be released.

At the moment the model is very simple, but this just the first step.



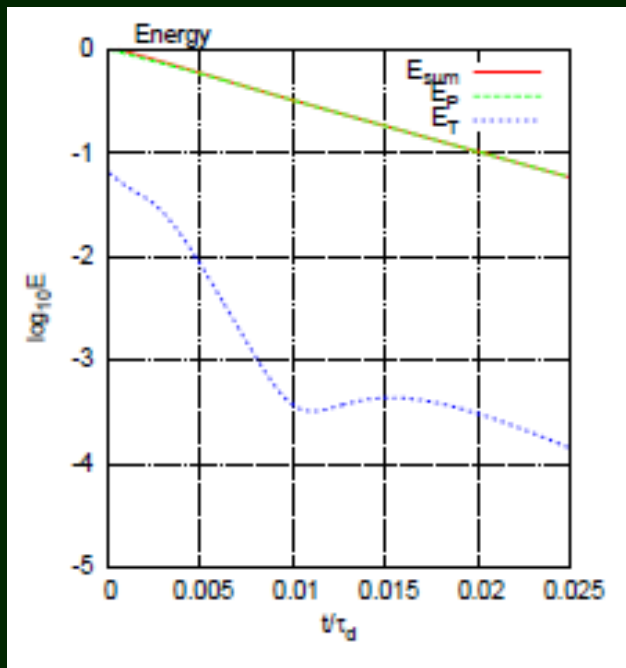
# An illustrative model

Poloidal

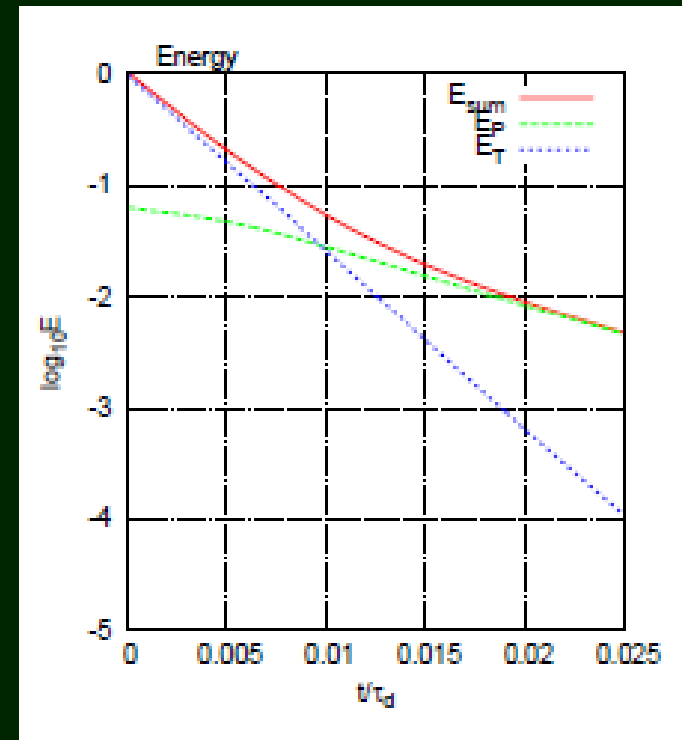


Test illustrates the evolution of initially purely poloidal field

# Another model



Initially the poloidal field is large.



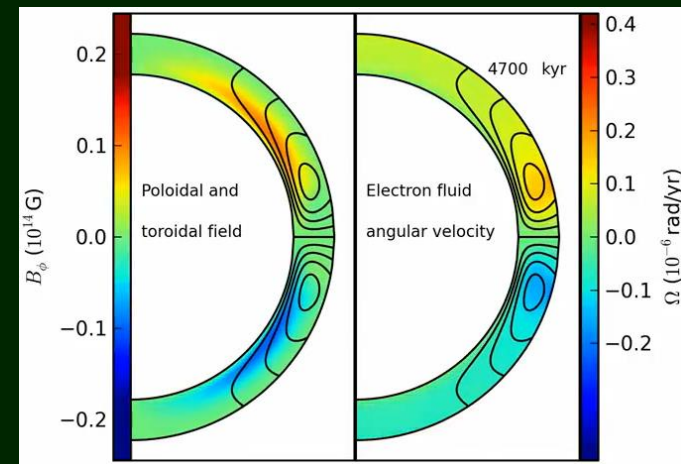
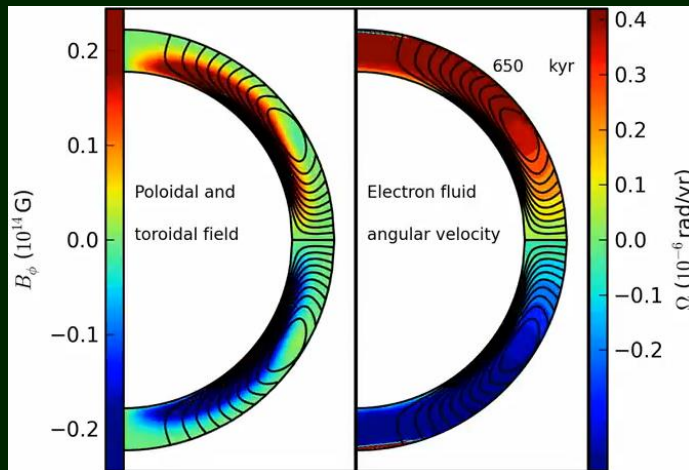
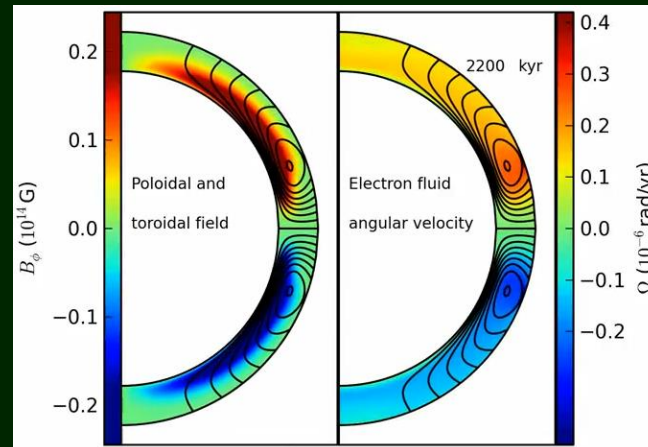
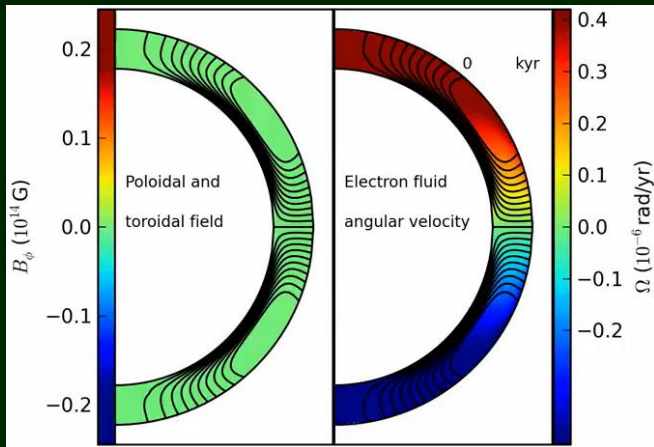
Initially the toroidal field is large.

If the toroidal field dominates initially then significant energy is transferred to the poloidal component during evolution.

In the opposite case, when the poloidal component initially dominates, energy is not transferred. The toroidal component decouples.



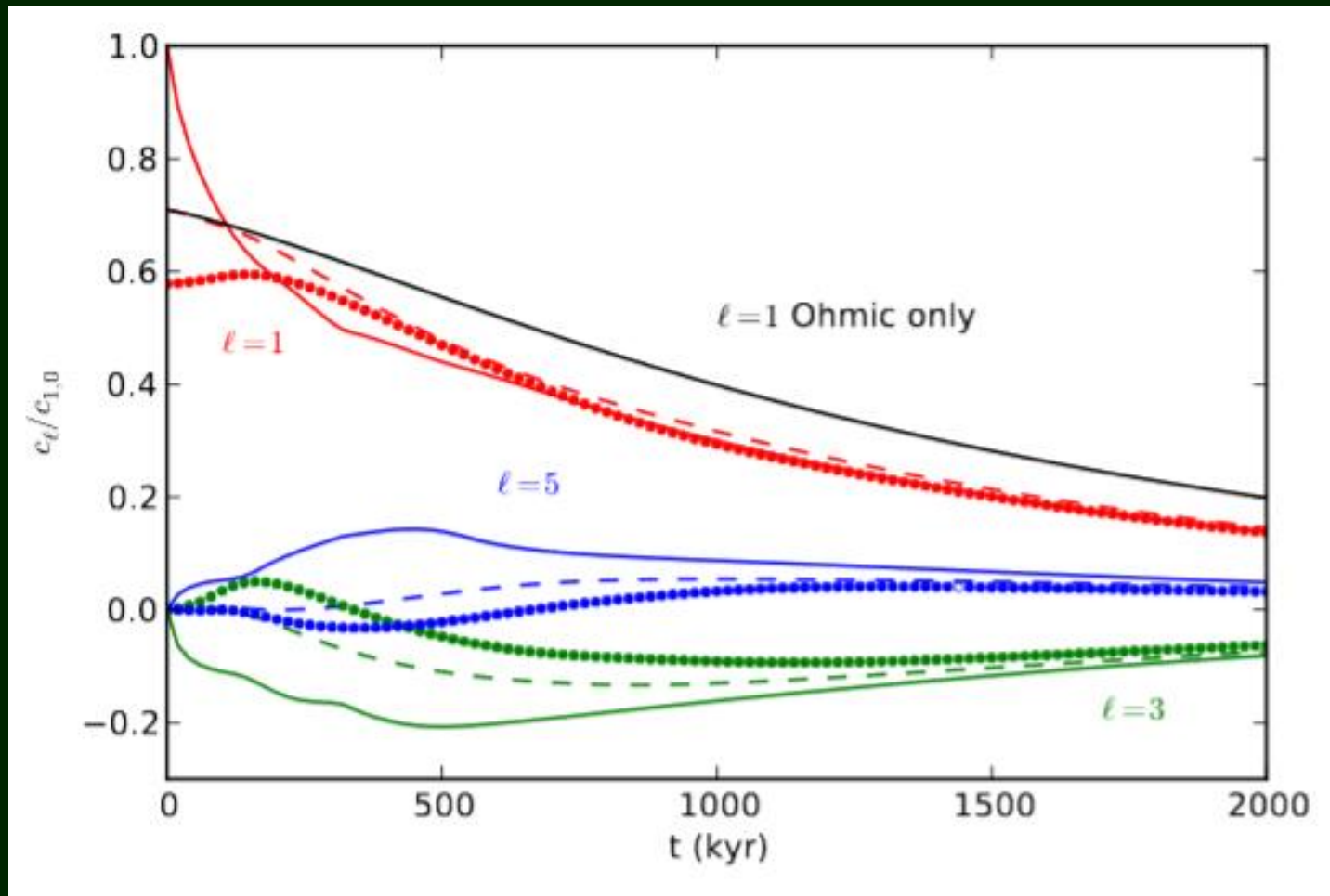
# Hall cascade and attractor



Hall cascade can reach the stage of so-called Hall attractor, where the field decay stalls for some time (Gourgoulatos, Cumming).

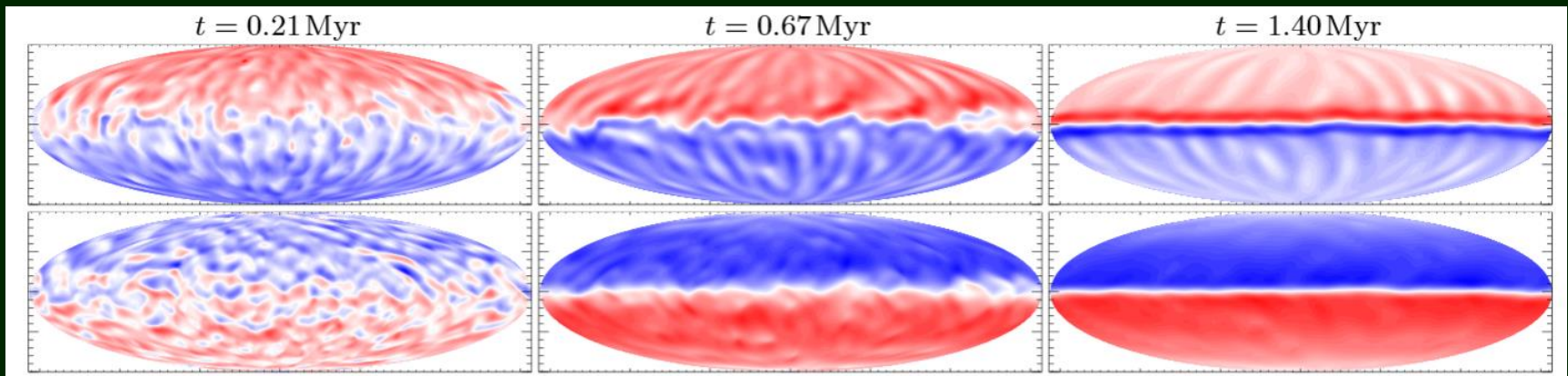
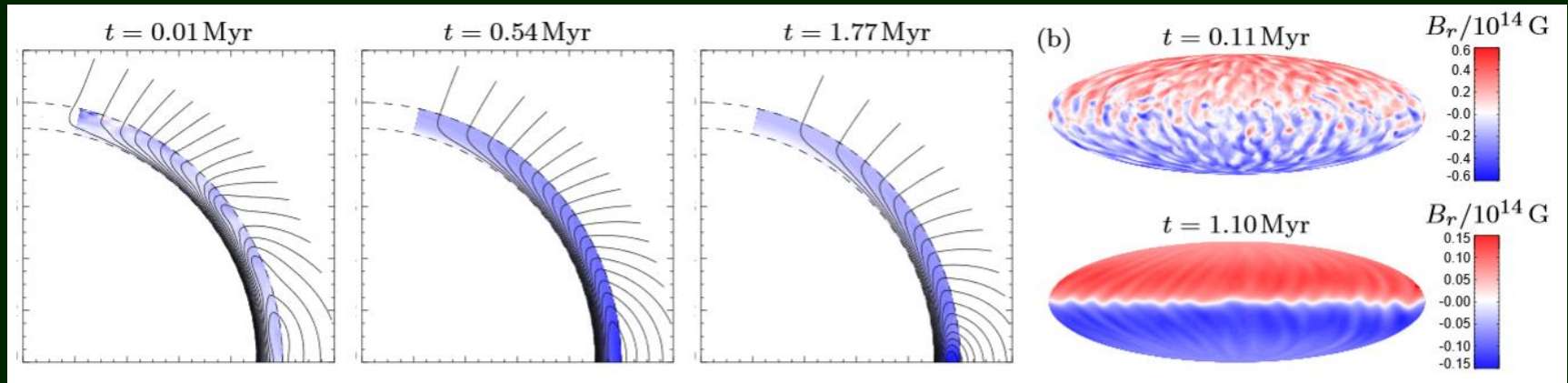


# Evolution of different components



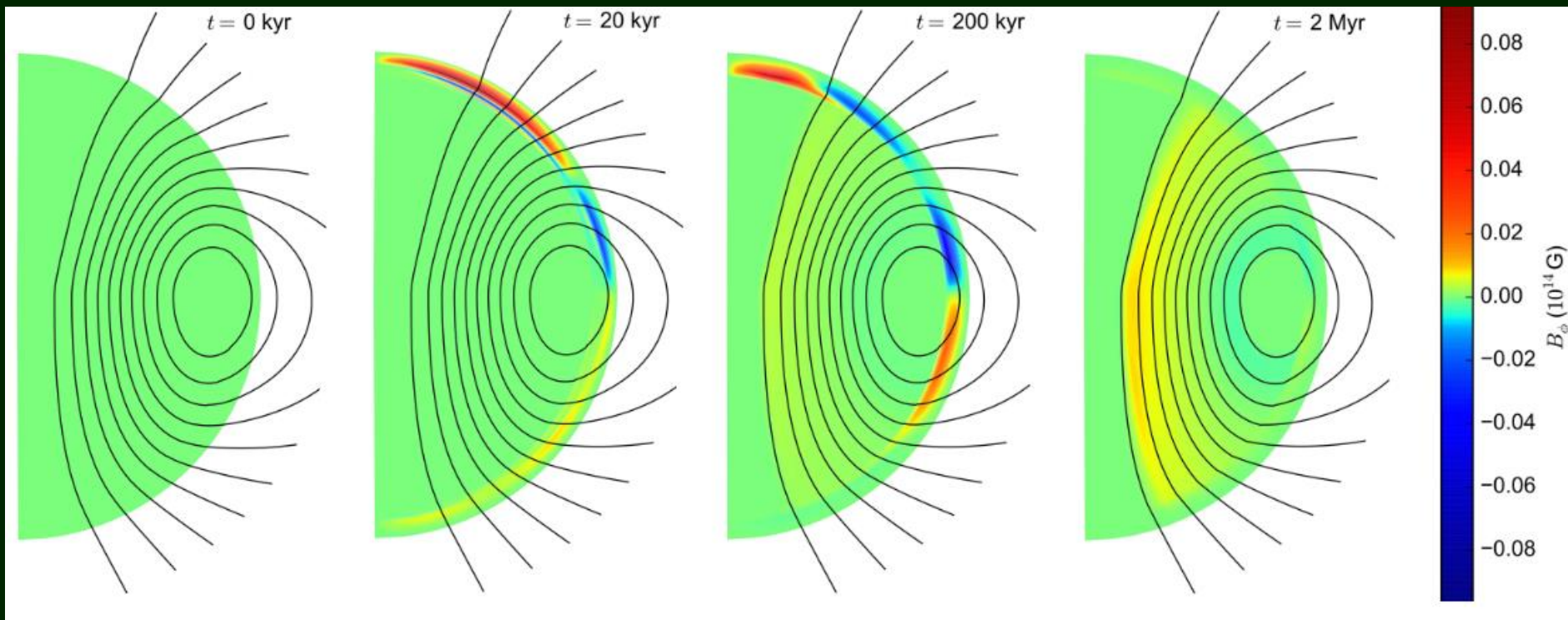
Hall attractor mainly consists of dipole and octupole (+15)

# New studies of the hall cascade



New calculations support the idea of a kind of stable configuration.

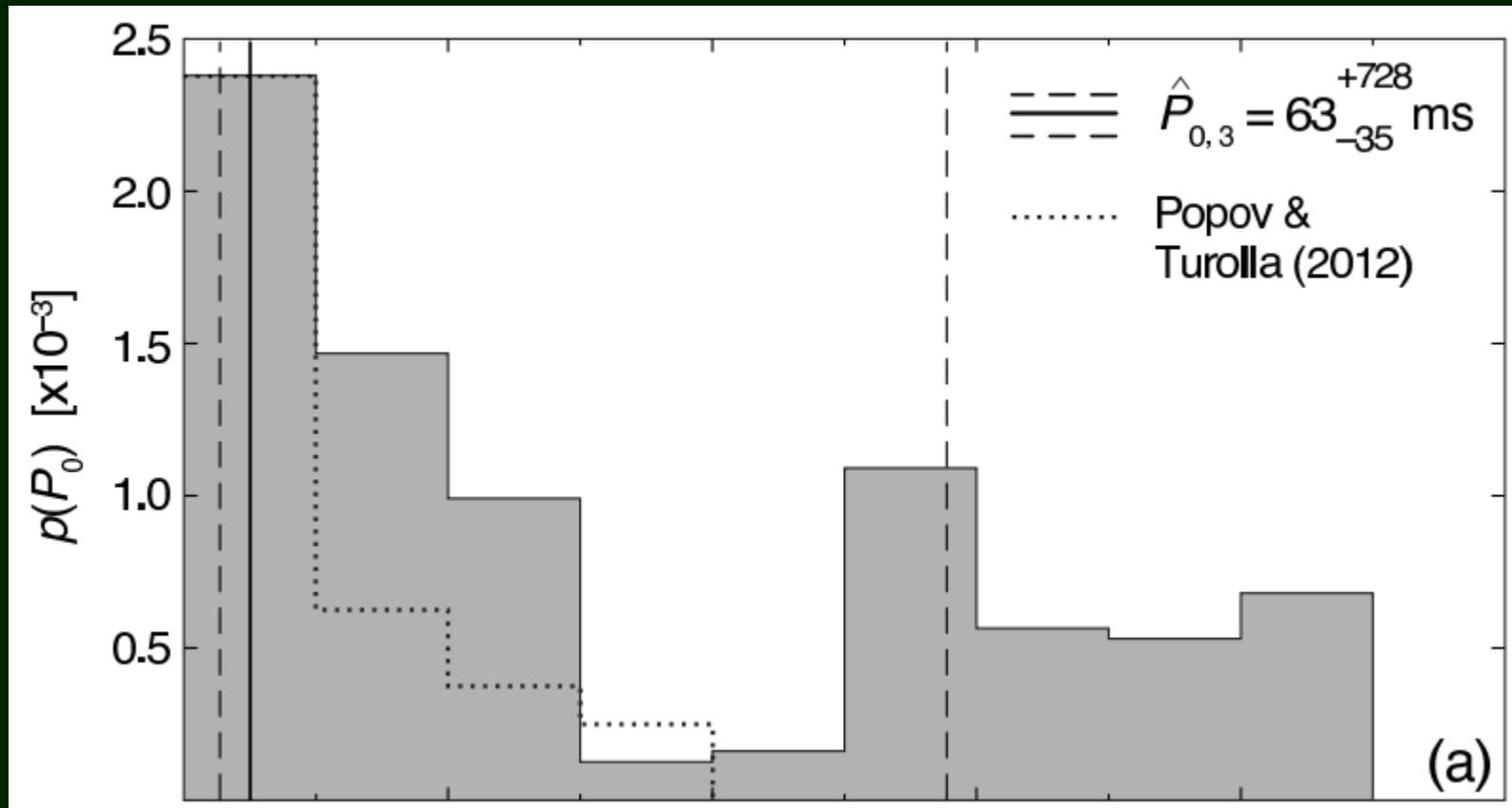
# Core and crust field evolution



Hall attractor is confirmed.

# Core field evolution

# Wide initial spin period distribution



Based on kinematic ages. Mean age – few million years.  
Note, that in Popov & Turolla (2012) only NSs in SNRs  
were used, i.e. the sample is much younger!  
Can it explain the difference?

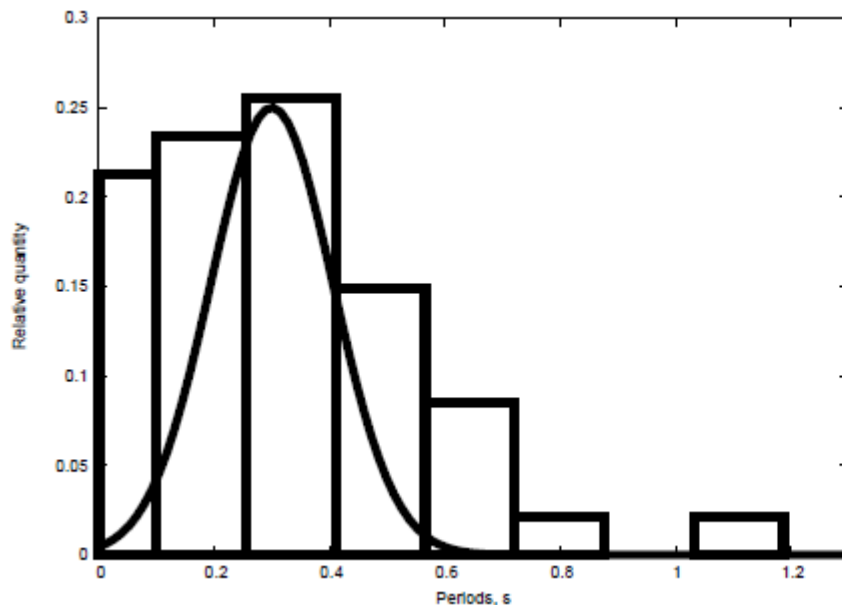
# Magnetic field decay and $P_0$

One can suspect that magnetic field decay can influence the reconstruction of the initial spin period distribution.

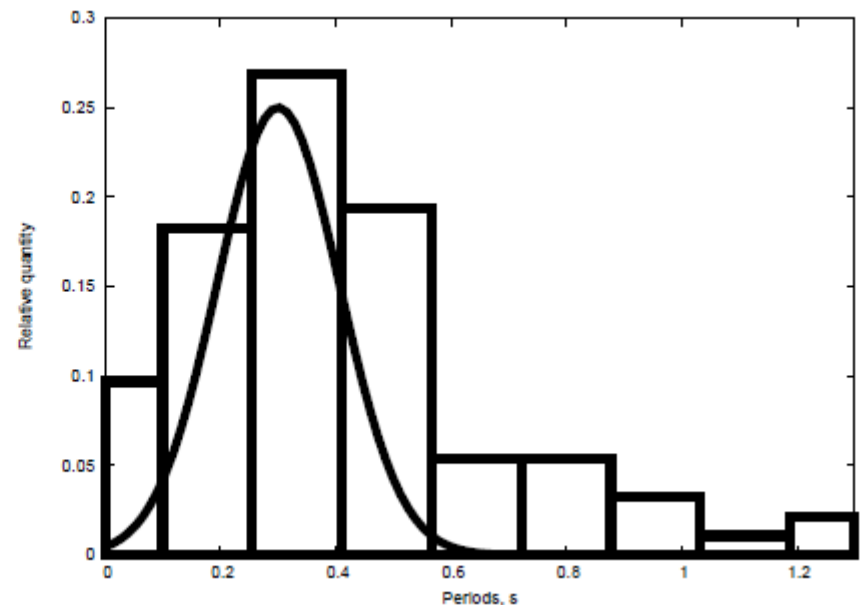
Exponential field decay with  $\tau=5$  Myrs.

$\langle P_0 \rangle = 0.3$  s,  $\sigma_P = 0.15$  s;  $\langle \log B_0/[G] \rangle = 12.65$ ,  $\sigma_B = 0.55$

$$P_0 = P \sqrt{1 - \frac{t}{\tau}}$$

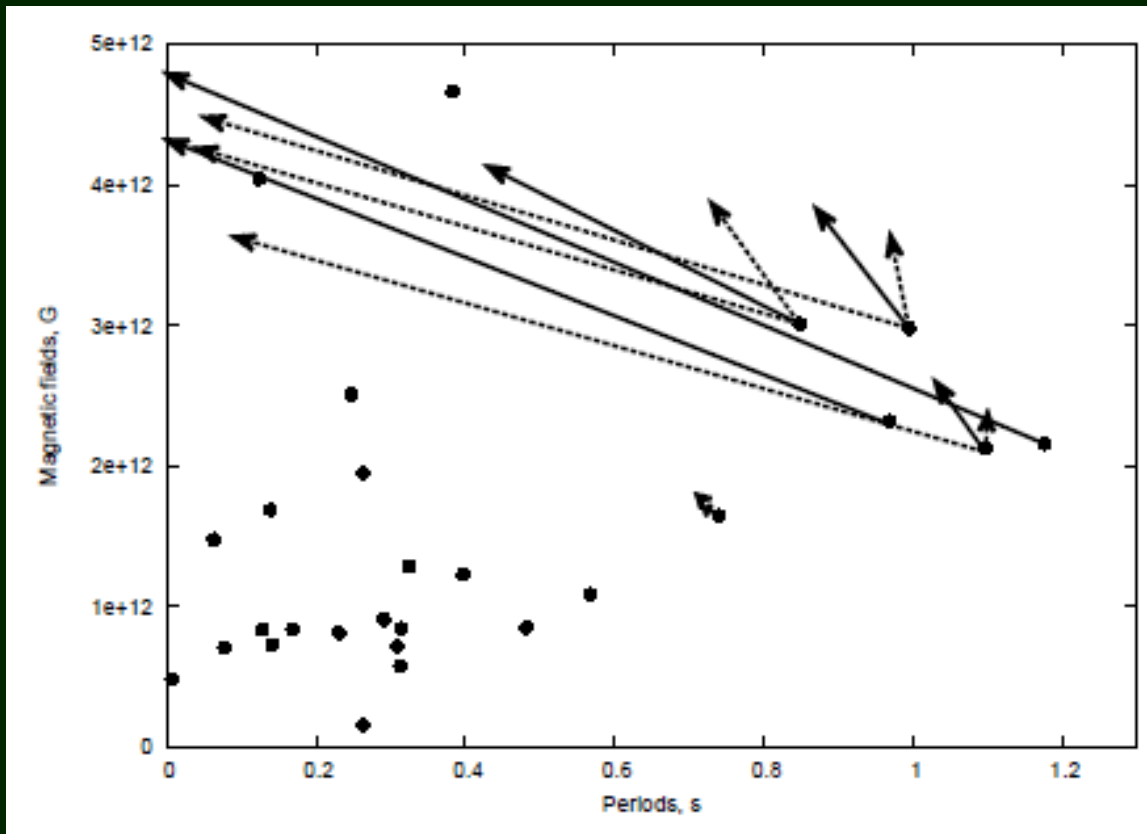


$\tau < 10^7$  yrs,  $10^5 < t$



$10^5 < t < 10^7$  yrs

# Real vs. reconstructed $P_0$



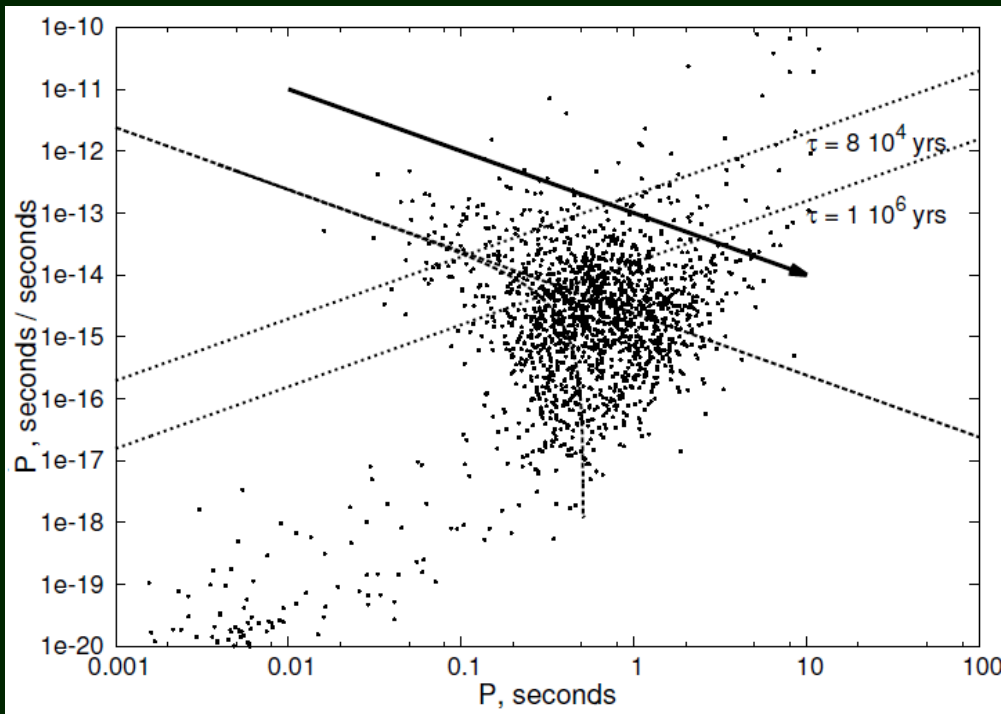
Arrows point to initial parameters of pulsars if the exponential magnetic field decay was operating.

How significantly the reconstructed initial periods changed due to not taking into account the exponential field decay



# Modified pulsar current

We perform a modified pulsar current analysis. In our approach we analyse the flow not along the spin period axis, as it was done in previous studies, but study the flow along the axis of growing characteristic age.



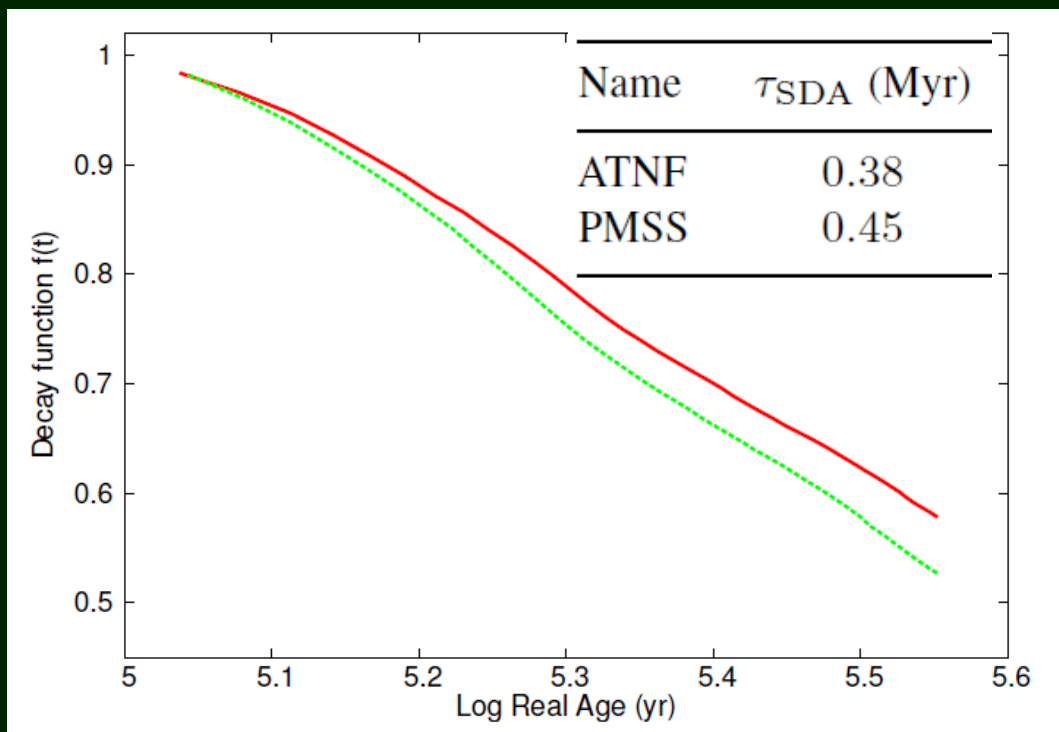
The idea is to probe magnetic field decay. Our method can be applied only in a limited range of ages.

We use distribution in characteristic ages to reconstruct the field evolution.

# Application to real data

We apply our methods to large observed samples of radio pulsars to study field decay in these objects. As we need to have as large statistics as possible, and also we need uniform samples, in the first place we study sources from the ATNF catalogue (Manchester et al. 2005).

Then we apply our methods to the largest uniform subsample of the ATNF — to the PMSS (stands for the Parkes Multibeam and Swinburne surveys) (Manchester et al. 2001).



We reconstruct the magnetic field decay in the range of true (statistical) ages:

$8 \cdot 10^4 < t < 3.5 \cdot 10^5$  yrs

which corresponds to characteristic ages

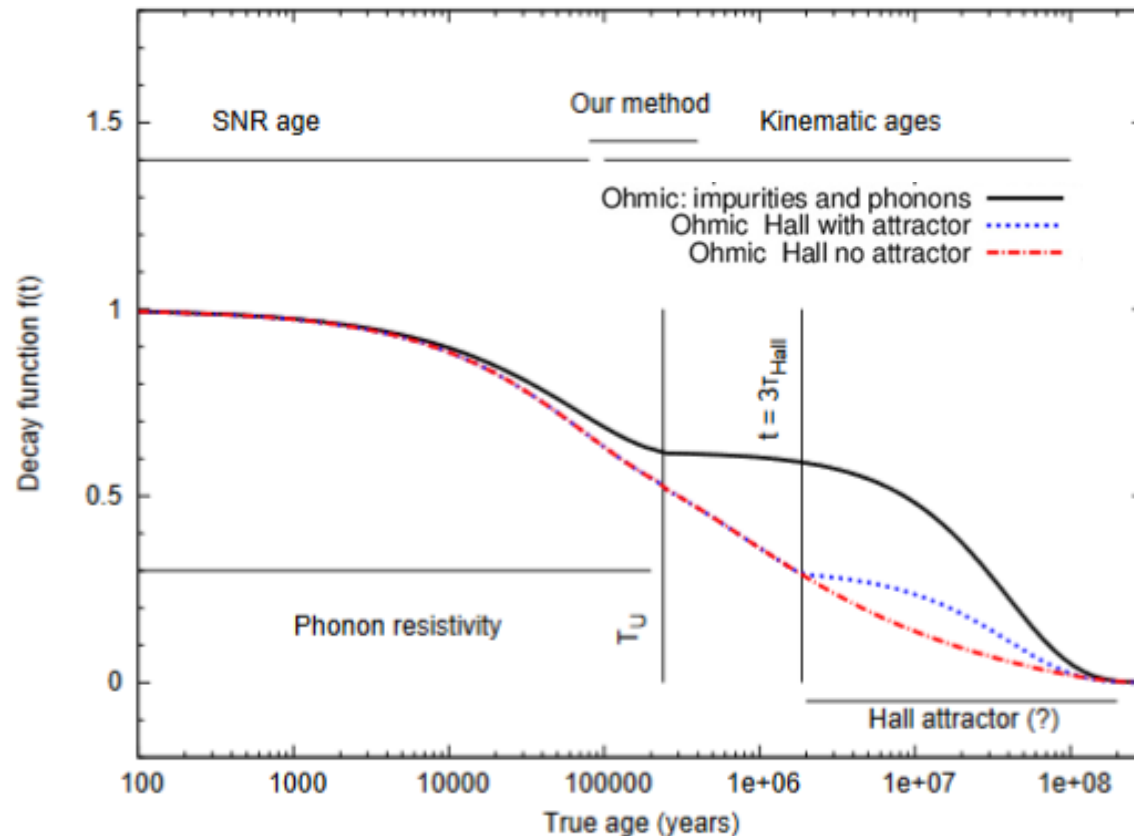
$8 \cdot 10^4 < \tau < 10^6$  yrs.

In this range, the field decays roughly by a factor of two.

With an exponential fit this corresponds to the decay time scale  $\sim 4 \cdot 10^5$  yrs.

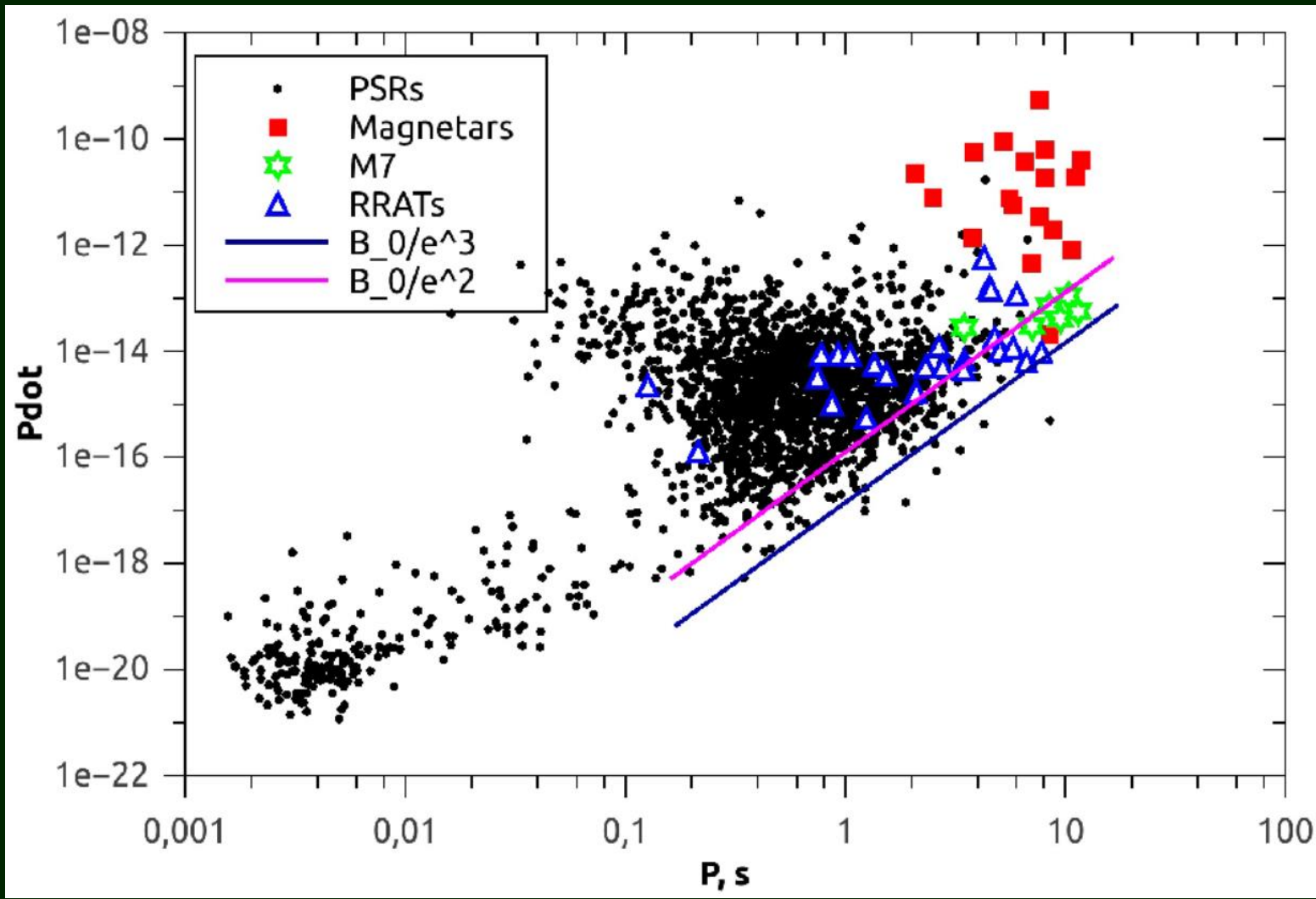
Note, this decay is limited in time.

# Comparison of different options

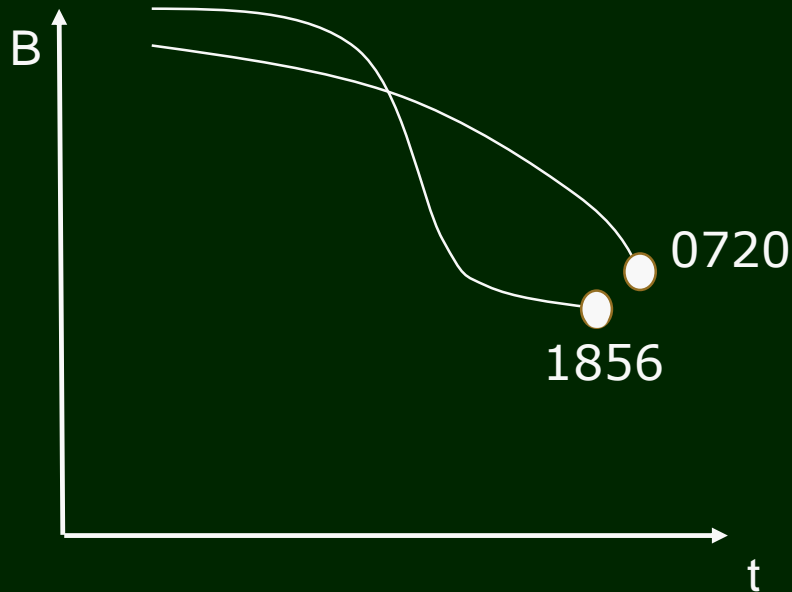


We think that at the ages  $\sim 10^5$  yrs and below for normal pulsars we see mostly Ohmic decay, which then disappears as NSs cool down below the critical T.

# Getting close to the attractor



# Tracks on the P-dot diagram



Kinematic age is larger for 0720, but characteristic age – for 1856.

It seems that 1856 is now on a more relaxed stage of the magneto-rotational evolution.

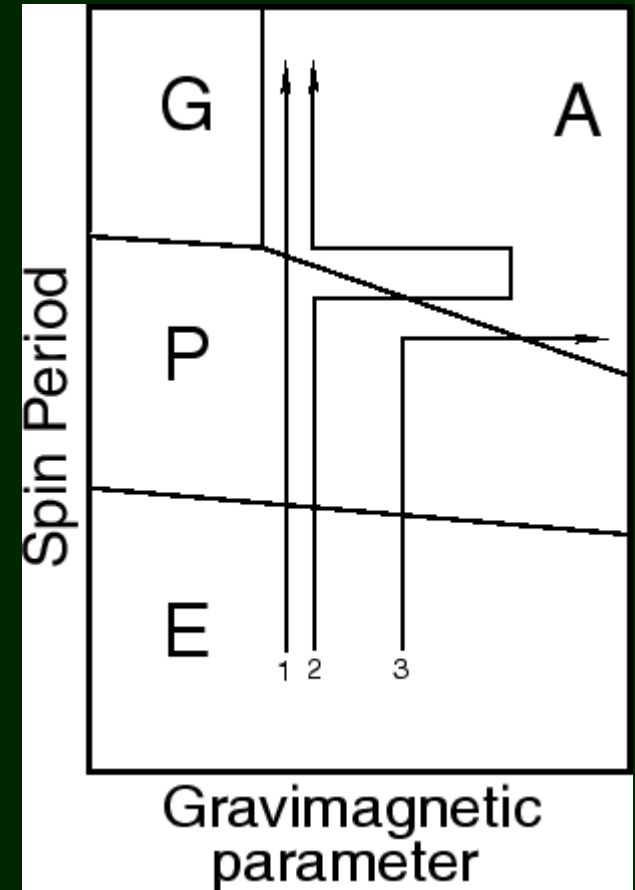
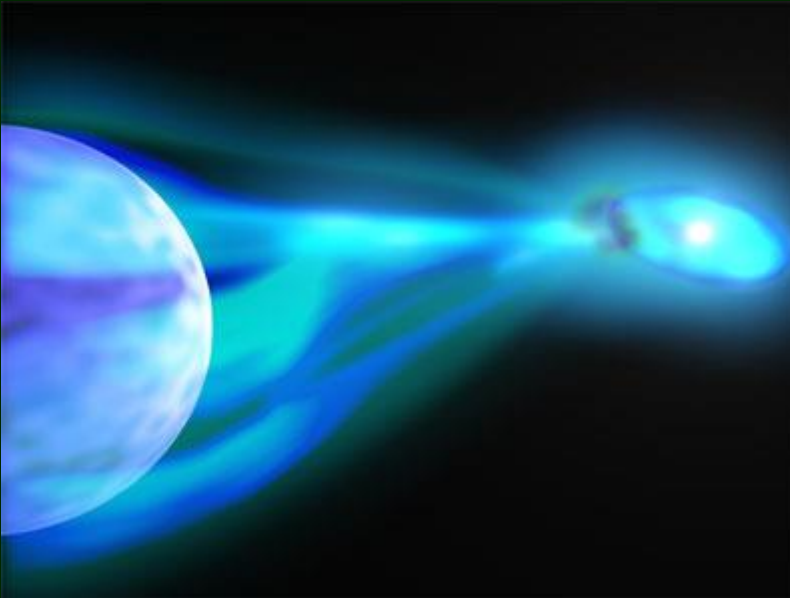
RX J0720 shows several types of activity, but RX J1856 is a very quiet source.

Non-monotonic evolution?

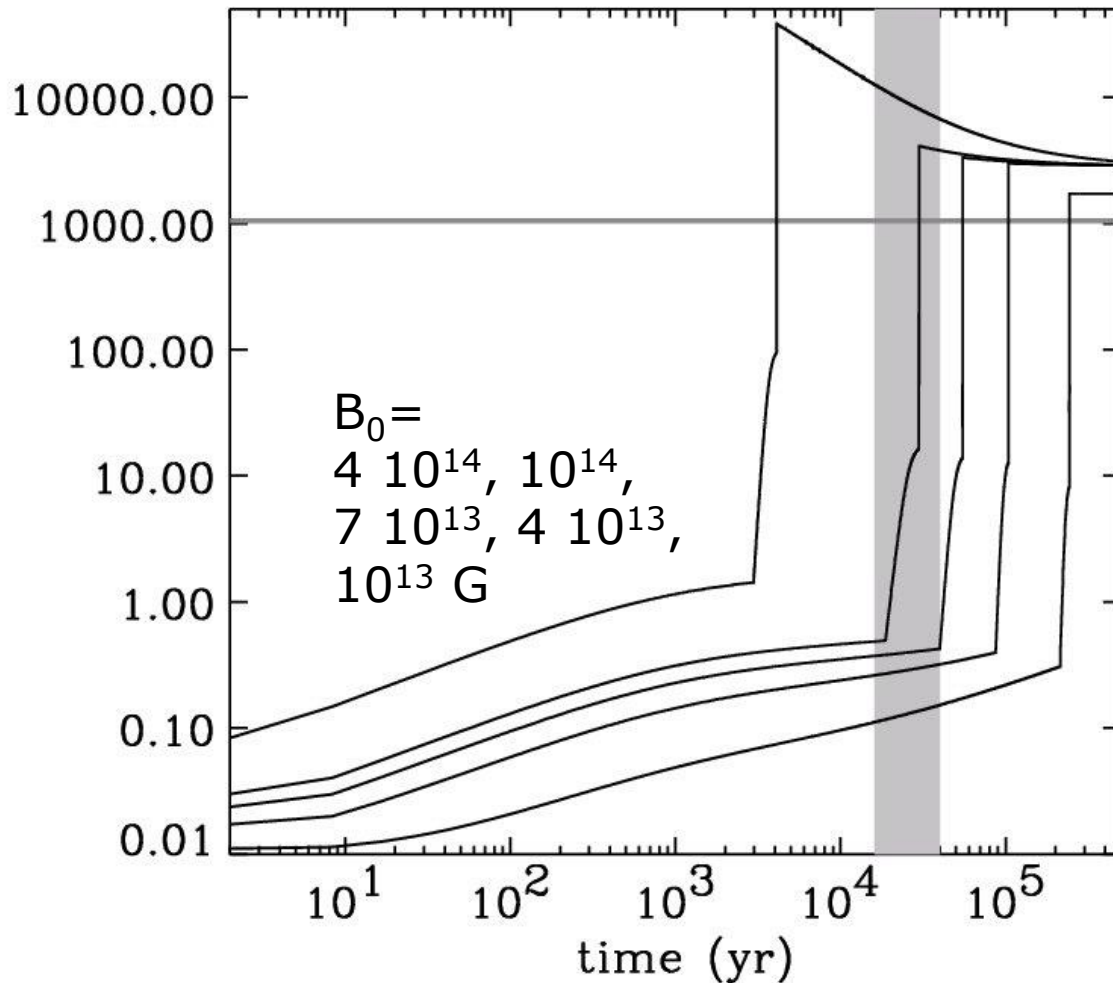
# SXP 1062

A peculiar source was discovered in SMC.  
Be/Xray binary,  $P=1062$  sec.  
A SNR is found. Age  $\sim 10^4$  yrs.  
(1110.6404; 1112.0491)

Typically, it can take  $\sim 1$  Myr for a NS  
with  $B \sim 10^{12}$  G to start accretion.



# Evolution of SXP 1062



A model of a NS with initial field  $\sim 10^{14}$  G which decayed down to  $\sim 10^{13}$  G can explain the data on SXP 1062.

1112.2507

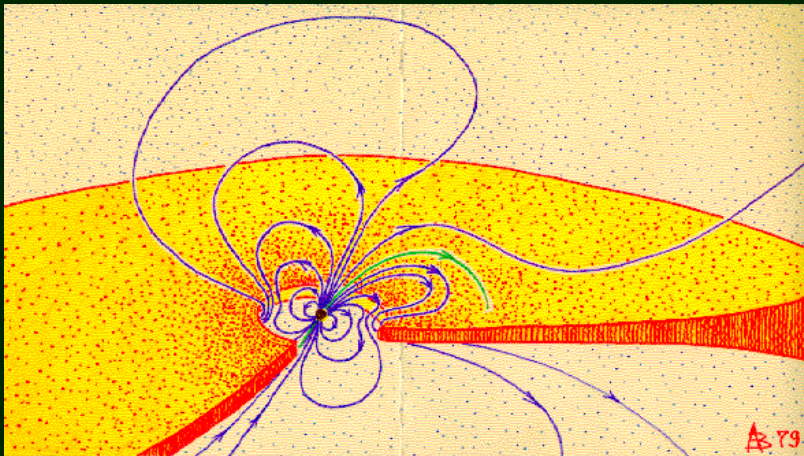
Some new data in  
1304.6022

Many other scenarios have been proposed.  
We need new observational data.

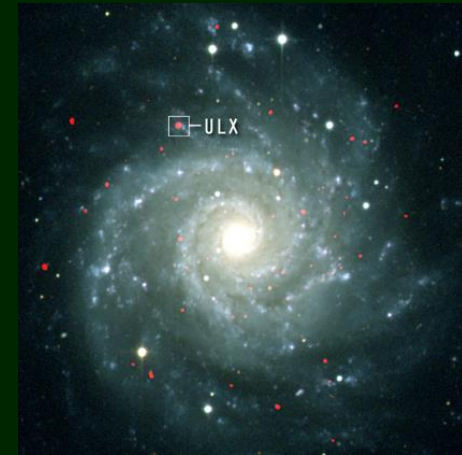


# Accreting magnetars

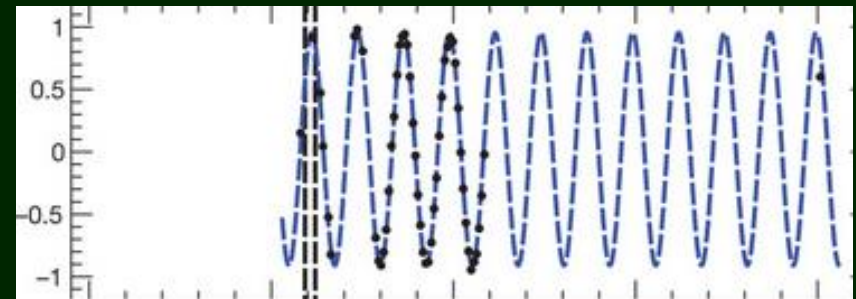
Typically magnetic fields of neutron stars in accreting X-ray binaries are estimated with indirect methods.



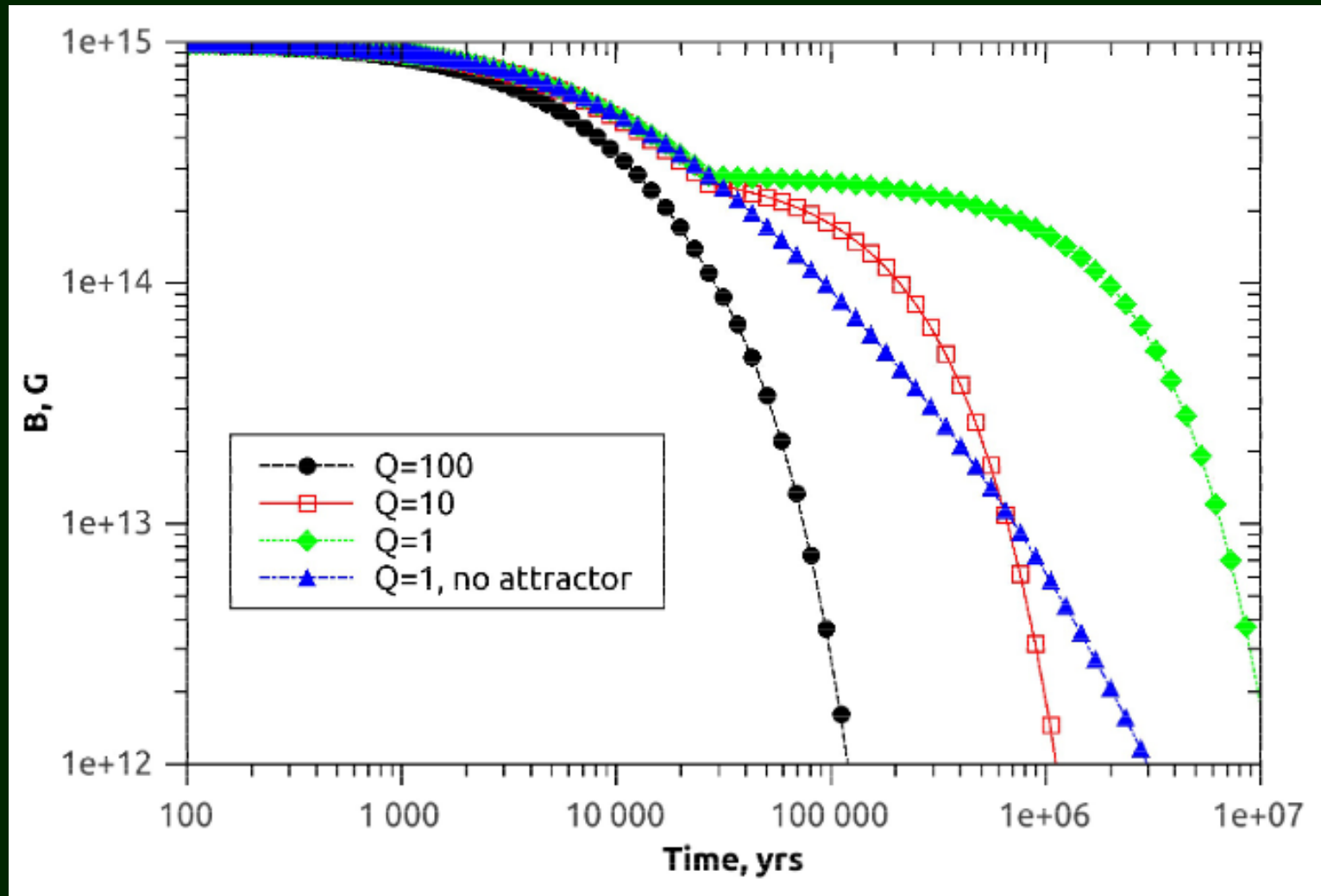
- Spin-up
- Spin-down
- Equilibrium period
- Accretion model
- .....



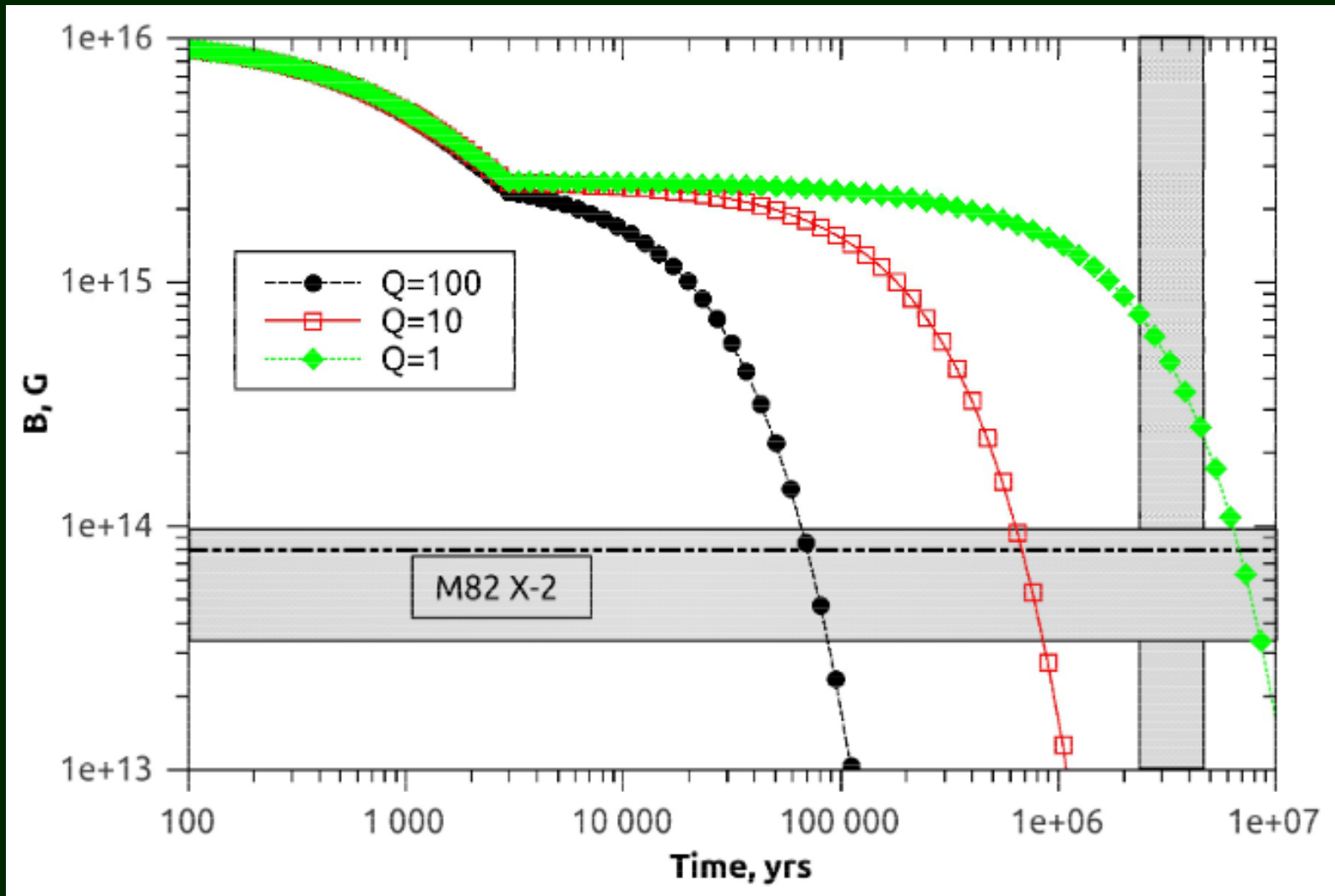
- ULX. NuSTAR J095551+6940.8 (M82 X-2). [Ekşi et al. \(2015\)](#).
- ULX. NGC 5907. [Israel et al. \(2017a\)](#)
- ULX. NGC 7793 P13. [Israel et al. \(2017b\)](#).
- 4U0114+65. [Sanjurjo et al. \(2017\)](#).
- 4U 2206+54. [Ikhsanov & Beskrovnaya \(2010\)](#).
- SXP1062. [Fu & Li \(2012\)](#)
- Swift J045106.8-694803. [Klus et al. \(2013\)](#).



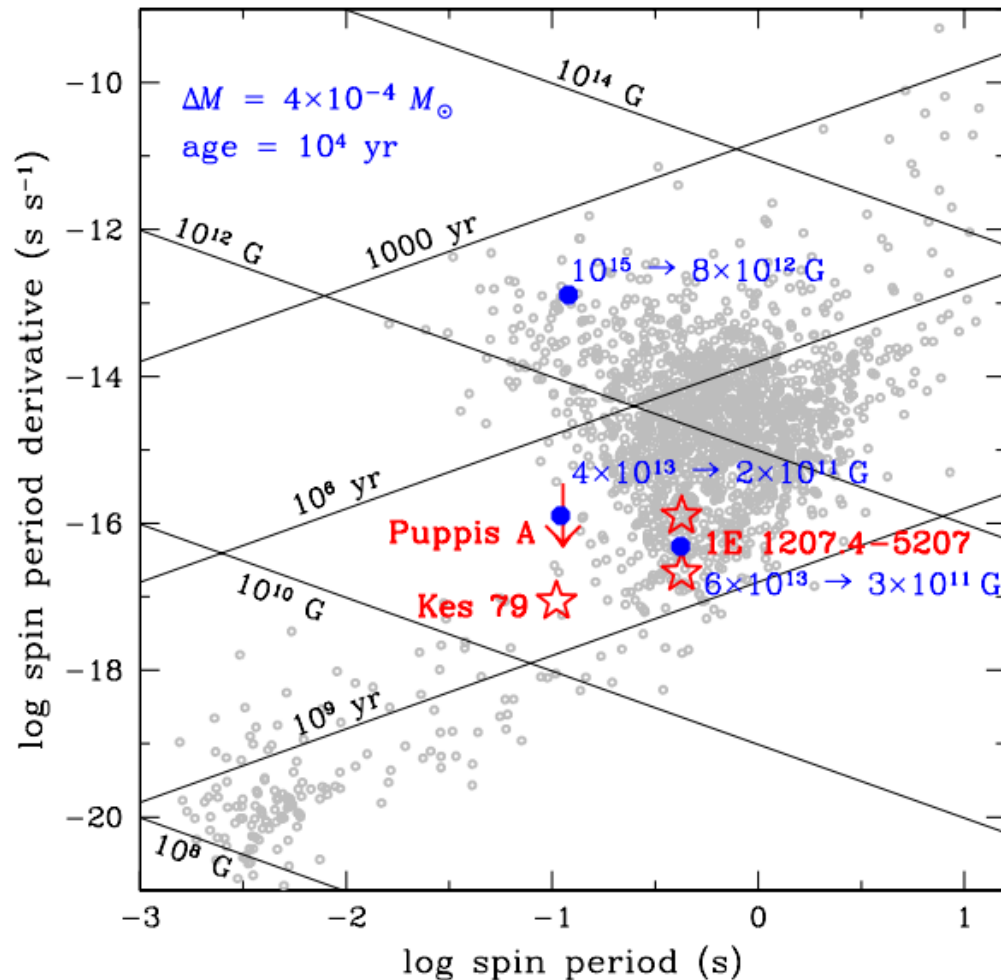
# Field evolution in a magnetar



# Parameters of ULX M82 X-2



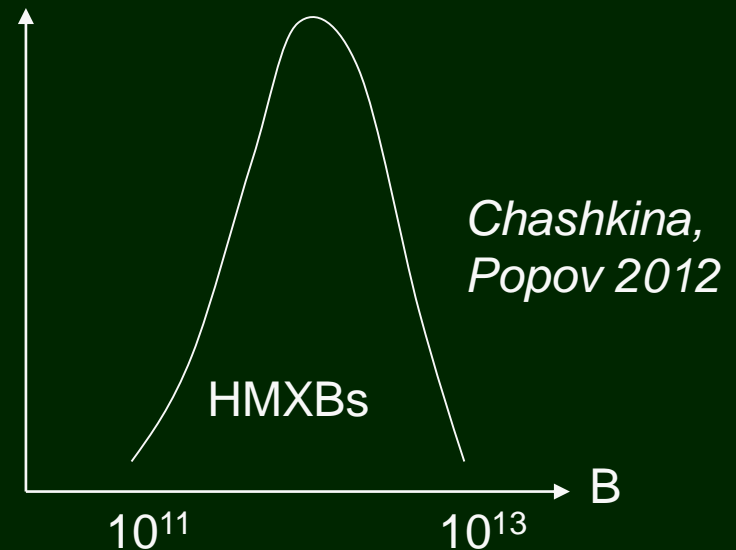
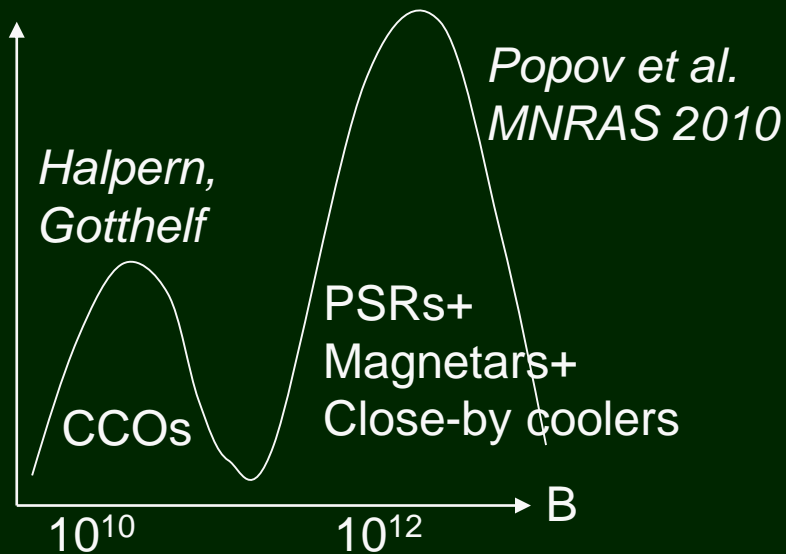
# Anti-magnetars



Note, that there is no room for antimagnetars from the point of view of birthrate in many studies of different NS populations.

New results 1301.2717  
 Spins and derivative are measured for  
 PSR J0821-4300 and  
 PSR J1210-5226

# Evolution of CCOs



Among young isolated NSs about 1/3 can be related to CCOs. If they are anti-magnetars, then we can expect that 1/3 of NSs in HMXBs are also low-magnetized objects.

They are expected to have short spin periods  $< 1$  sec.

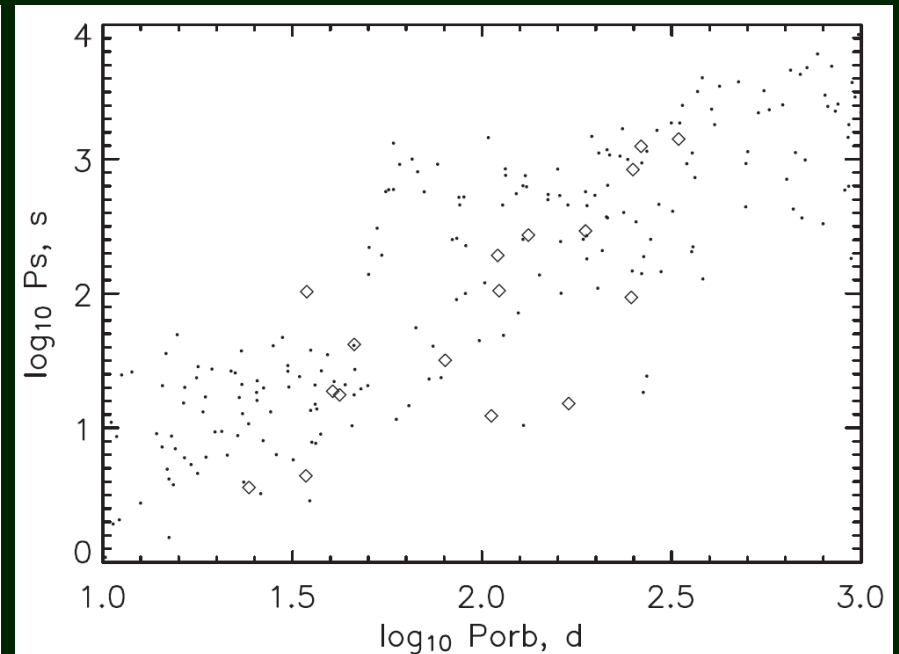
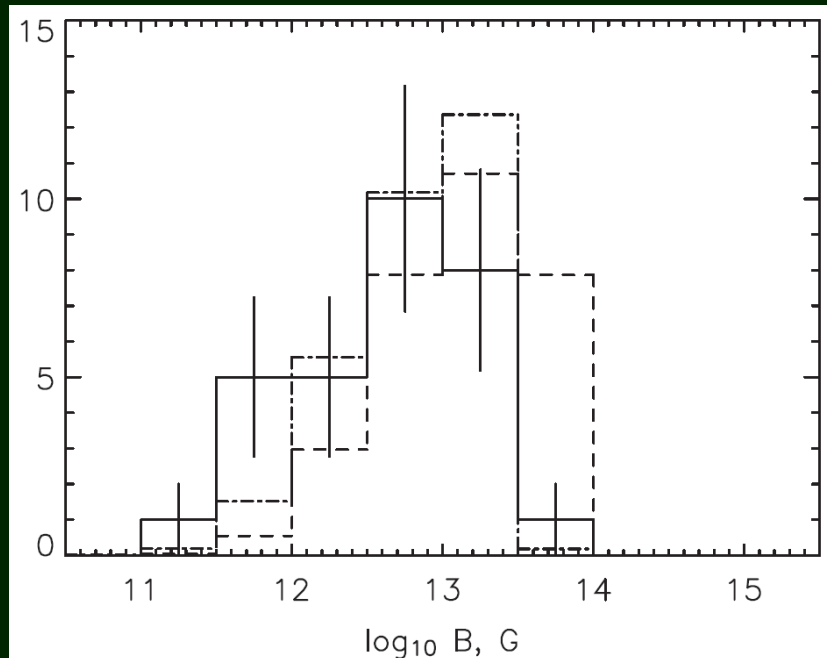
However, there are no many sources with such properties.

The only good example - SAX J0635+0533. An old CCO?

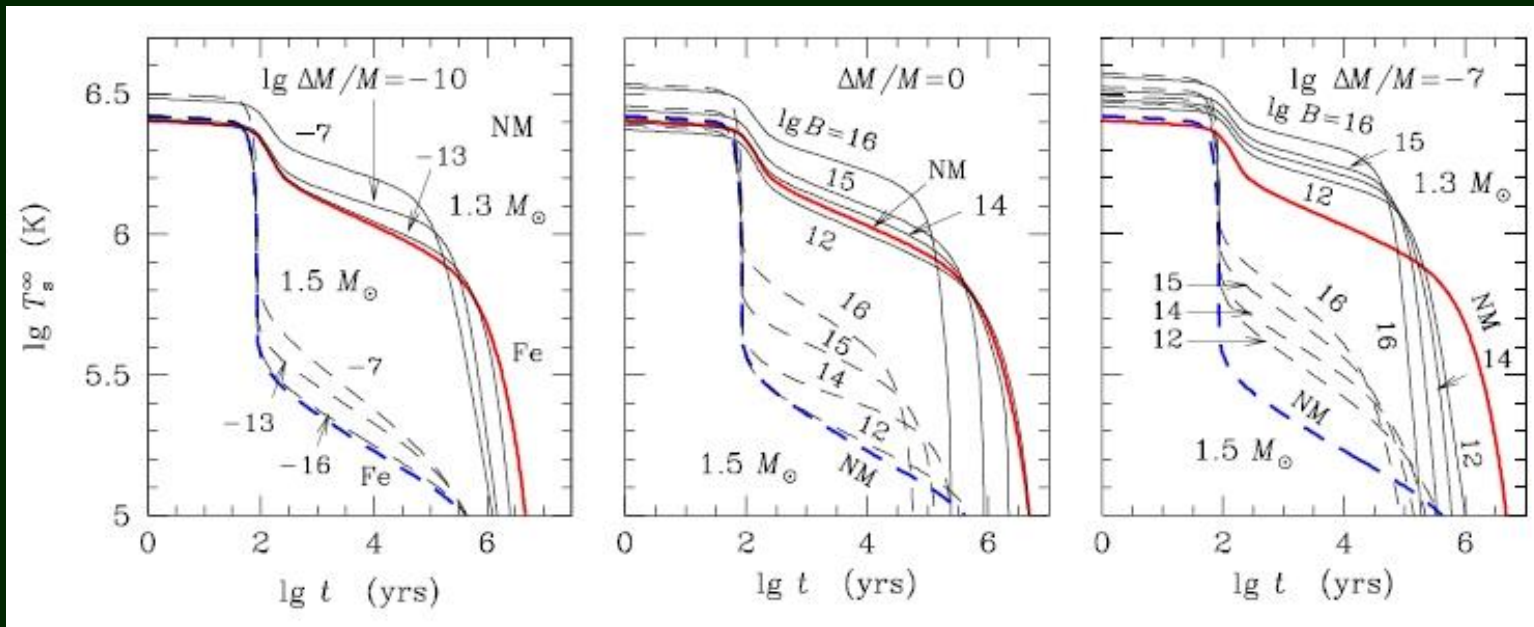
Possible solution: emergence of magnetic field (see physics in Ho 2011, Vigano, Pons 2012).

# Observations vs. theory

We use observations of Be/X-ray binaries in SMC to derive magnetic field estimates, and compare them with prediction of the Pons et al. model.



# Where are old CCOs?



Yakovlev, Pethick 2004

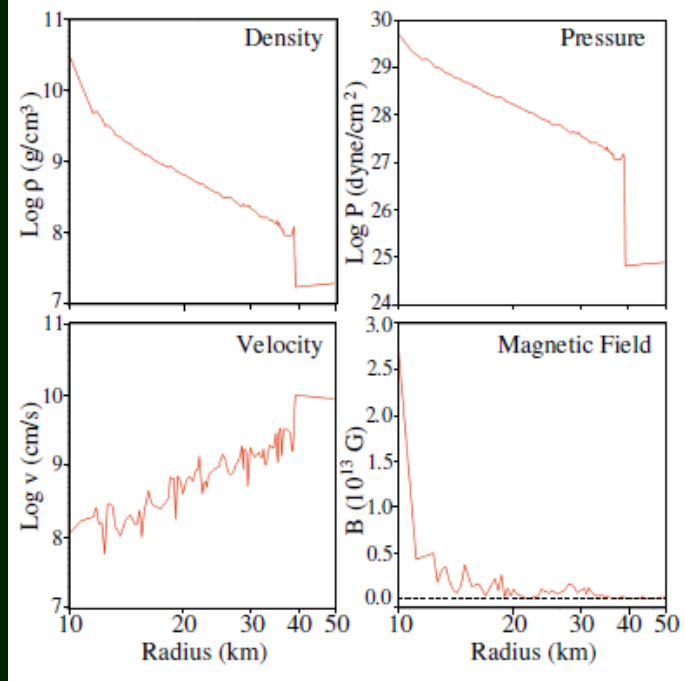
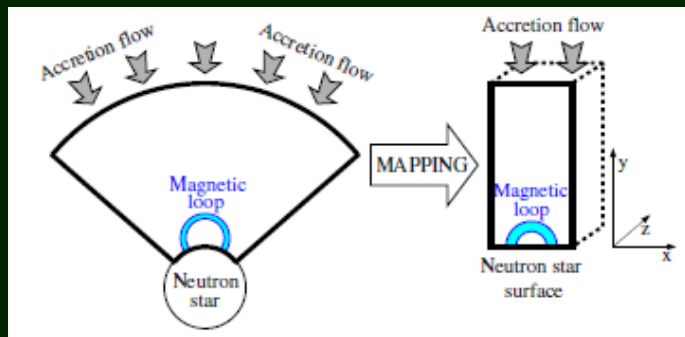
According to cooling studies they have to be bright till at least  $10^5$  years. But only one candidate (2XMM J104608.7-594306 Pires et al.) to be a low-B cooling NS is known (Calvera is also a possible candidate).

We propose that a large set of data on HMXBs and cooling NSs is in favour of field emergence on the time scale  $10^4 \leq \tau \leq 10^5$  years (arXiv:1206.2819).

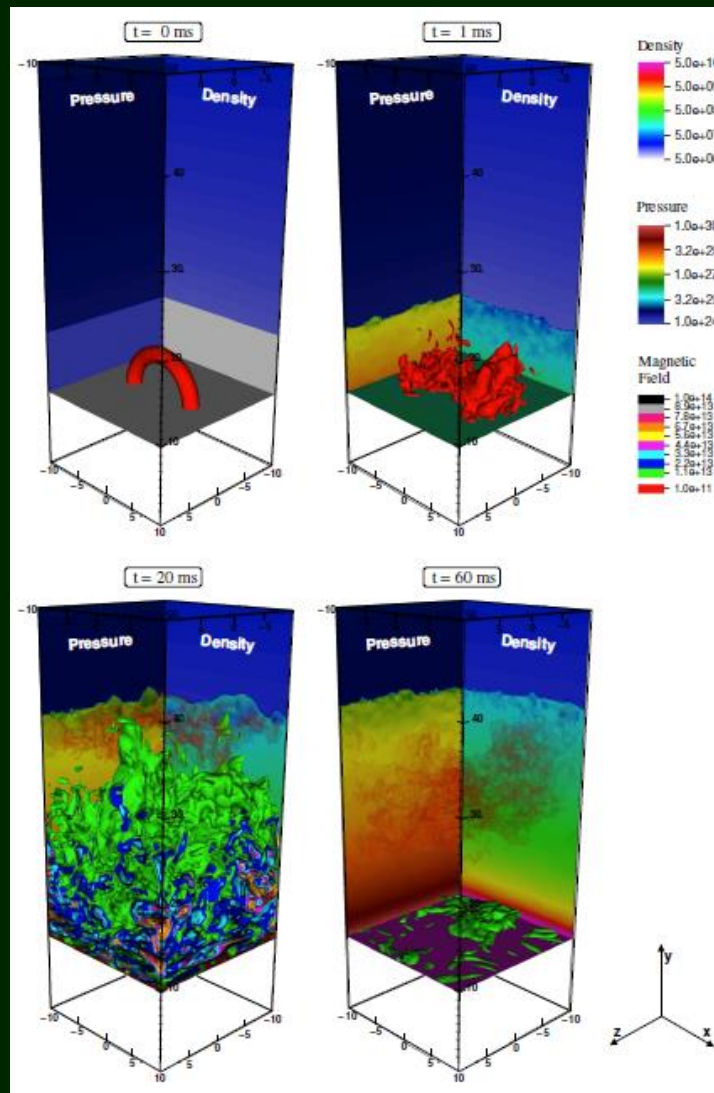
Some PSRs with thermal emission for which additional heating was proposed can be descendants of CCOs with emerged field.



# How the field is buried



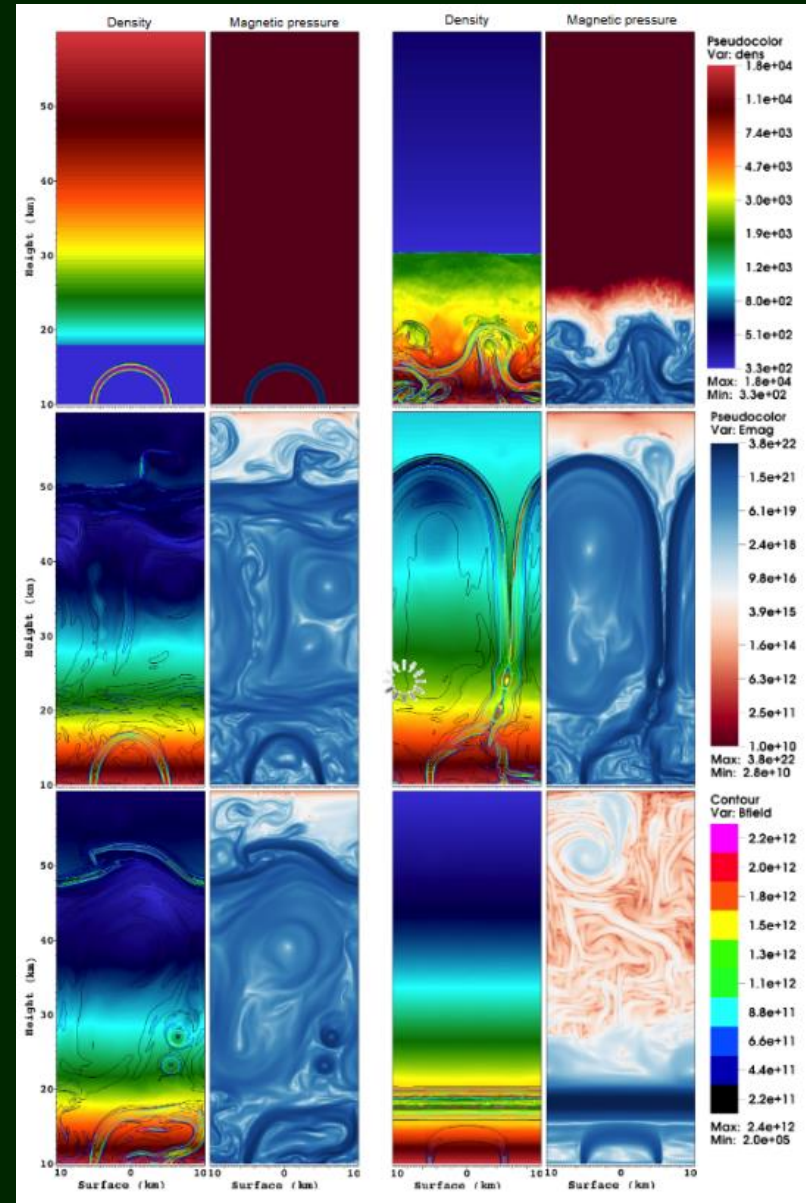
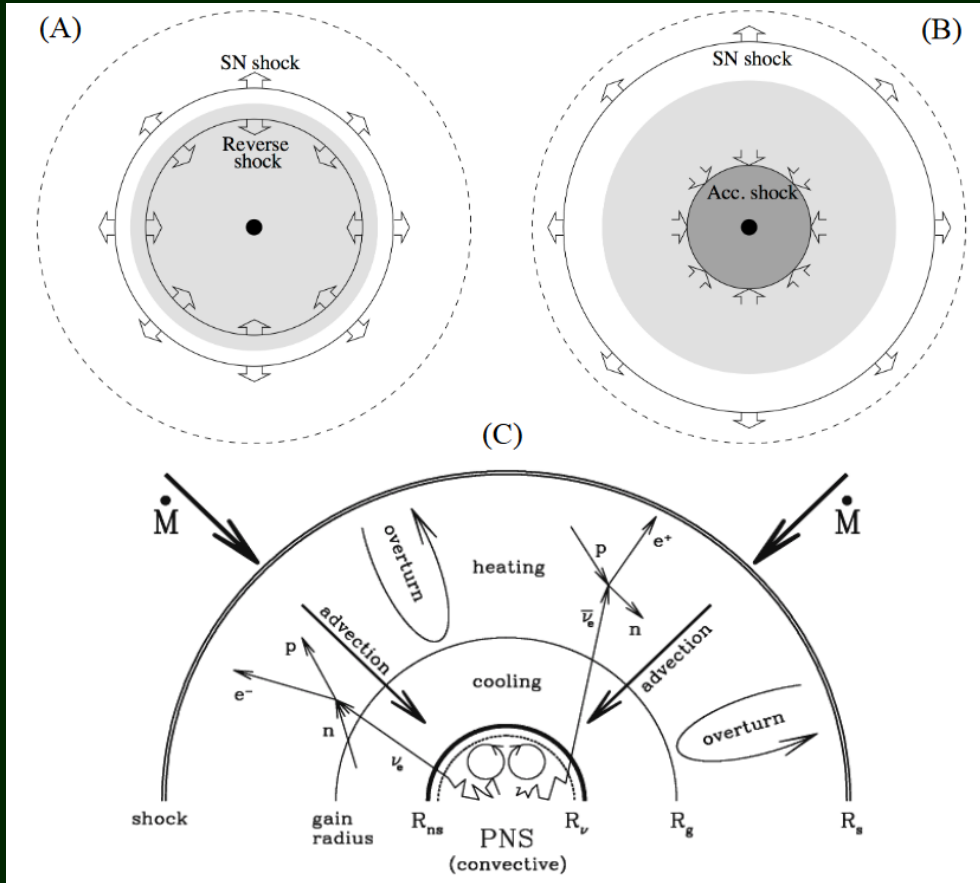
For  $t=60$  msec



1212.0464

See 1210.7112 for a review of CCOs magnetic fields

# Recent model

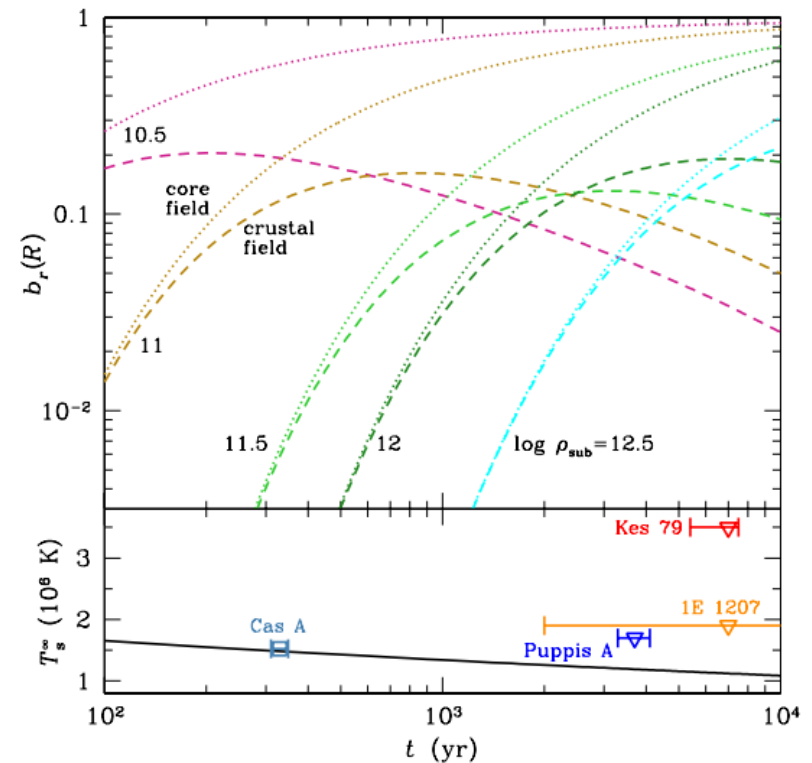
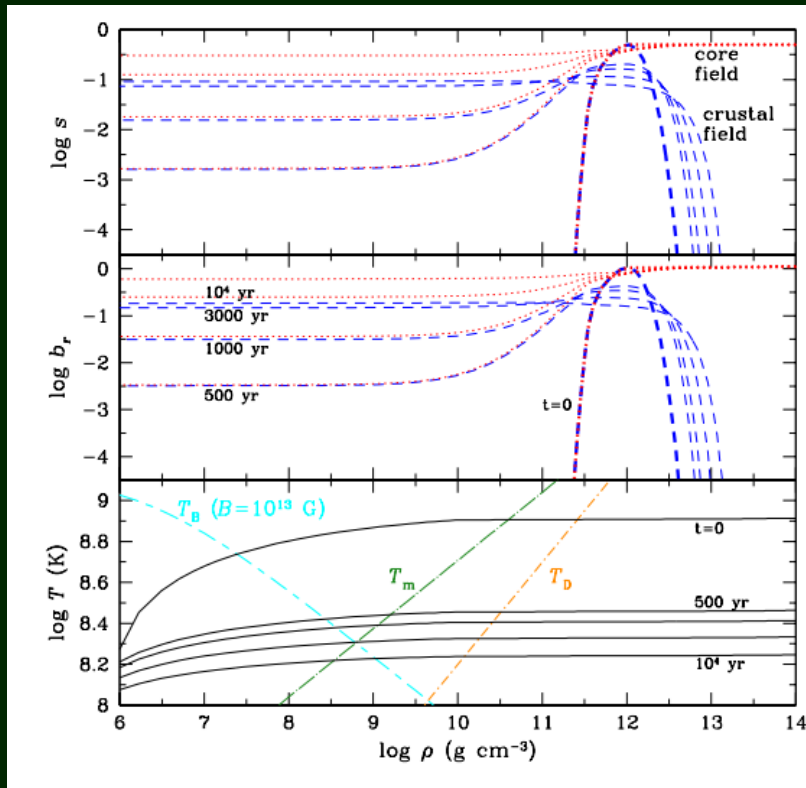


1809.07057

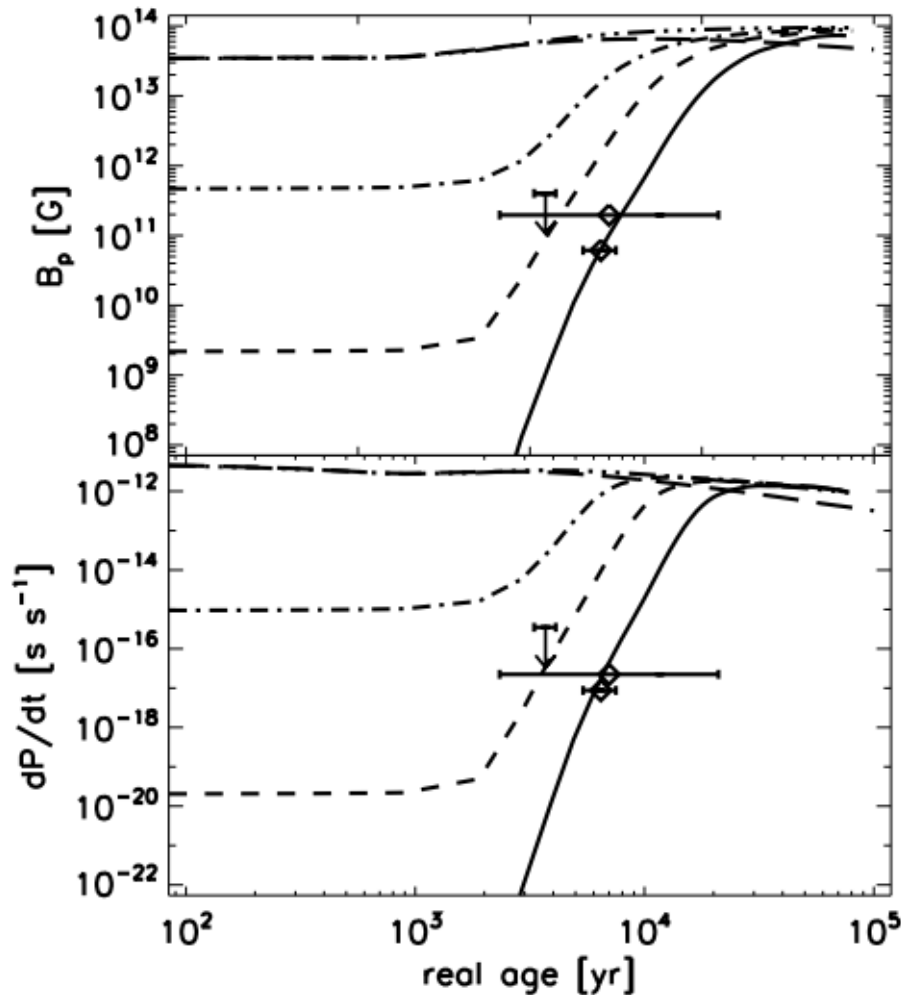
# Emerging field: modeling

## 1D model of field emergence

Dashed – crustal, dotted – core field



# Another model



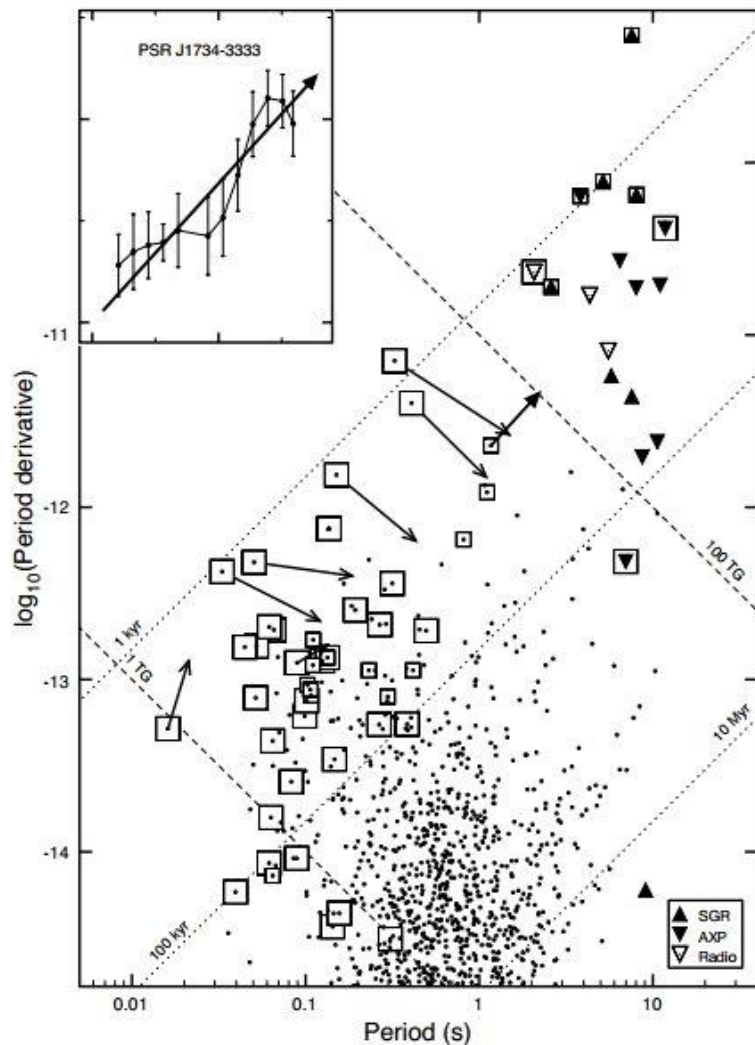
2D model with field decay

Ohmic diffusion dominates in field emergence, but Hall term also can be important.

Calculations confirm that emergence on the time scale  $10^3$ - $10^5$  years is possible.

$$B_{0p} = 10^{14} \text{ G}$$

# Emerged pulsars in the P-Pdot diagram



Emerged pulsars are expected to have  
 $P \sim 0.1-0.5$  sec

$B \sim 10^{11}-10^{12}$  G

Negative braking indices or at least  $n < 2$ .  
About 20-40 of such objects are known.

Parameters of emerged PSRs:  
similar to “injected” PSRs  
(Vivekanand, Narayan, Ostriker).

The existence of significant fraction  
of “injected” pulsars formally  
do not contradict recent pulsar current studies  
(Vranesevic, Melrose 2011).

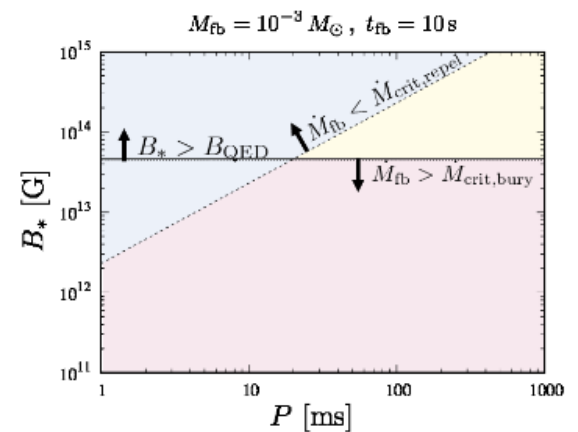
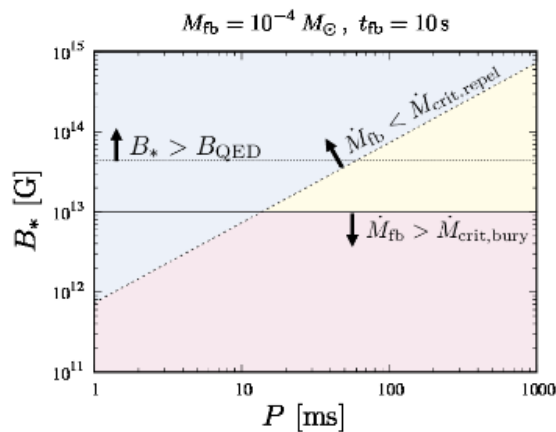
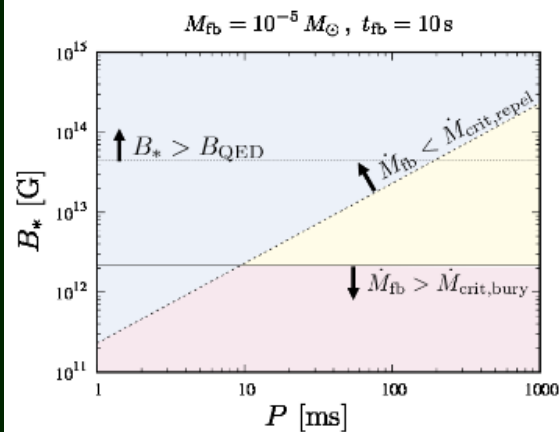
Part of PSRs supposed to be born with  
long (0.1-0.5 s) spin periods can be  
matured CCOs.

# Field, rotation, fallback

$$\dot{M}_{\text{crit,repul}} \sim 3 \times 10^{-5} M_{\odot} \text{s}^{-1} \frac{\xi_{\text{s,crit}}}{0.2} \frac{(4\pi D_{\text{fb}} \sqrt{\xi_{\text{s}}})_{\text{crit}}}{5.3} \left( \frac{B_{*}}{10^{13} \text{G}} \right)^2 \left( \frac{P}{10 \text{ms}} \right)^{-2} \left( \frac{t_{\text{fb}}}{20 \text{s}} \right)^{2/3}$$

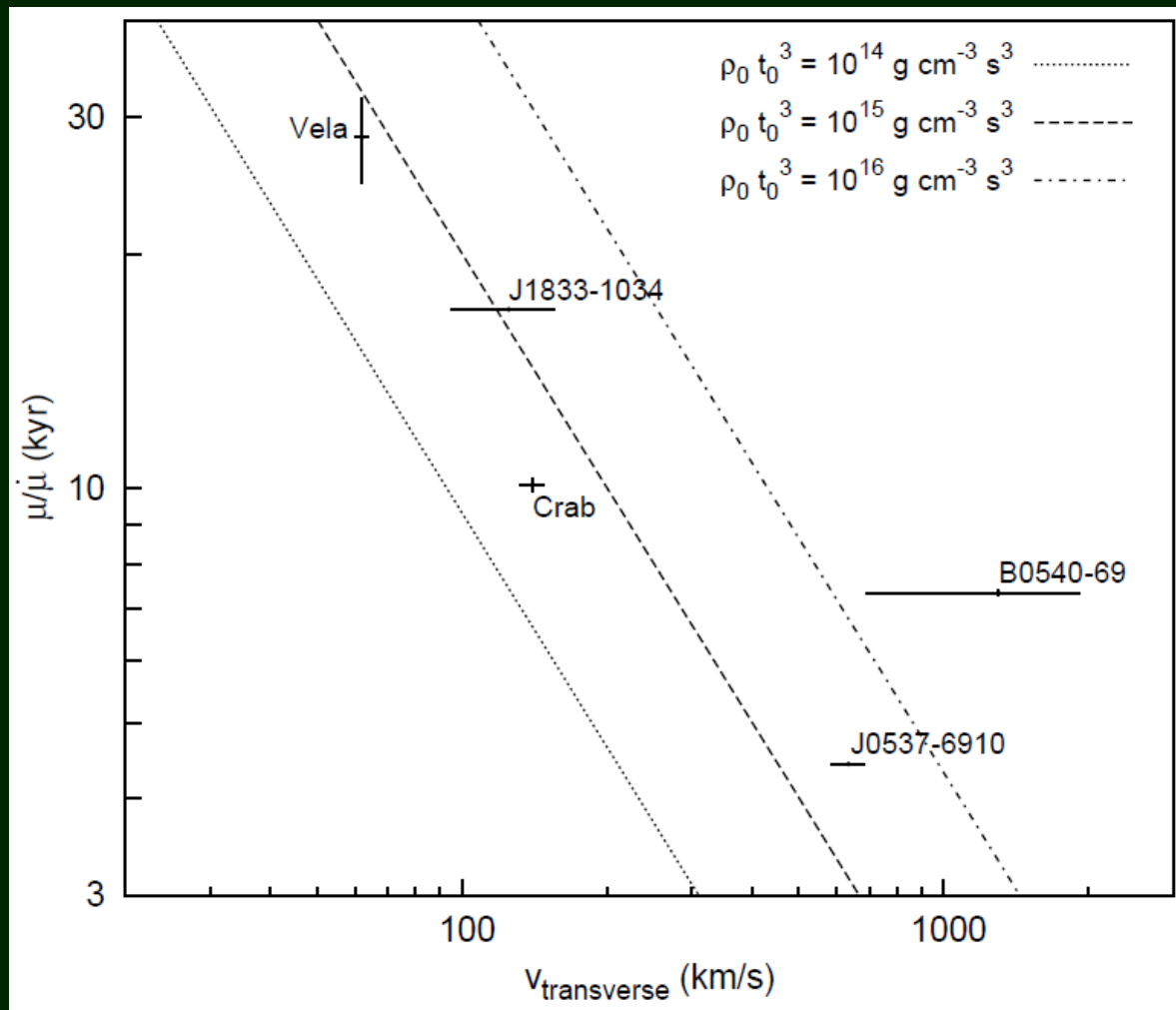
$$\frac{B_{*}^2}{8\pi} \lesssim \rho v^2 \sim \frac{\dot{M}}{4\pi R_{*}^2} \sqrt{\frac{GM_{\text{c}}}{R_{*}}}$$

$$\dot{M}_{\text{crit,bury}} \sim 3 \times 10^{-6} M_{\odot} \text{s}^{-1} \left( \frac{B_{*}}{10^{13} \text{G}} \right)^2$$





# Growing field and kick velocities?

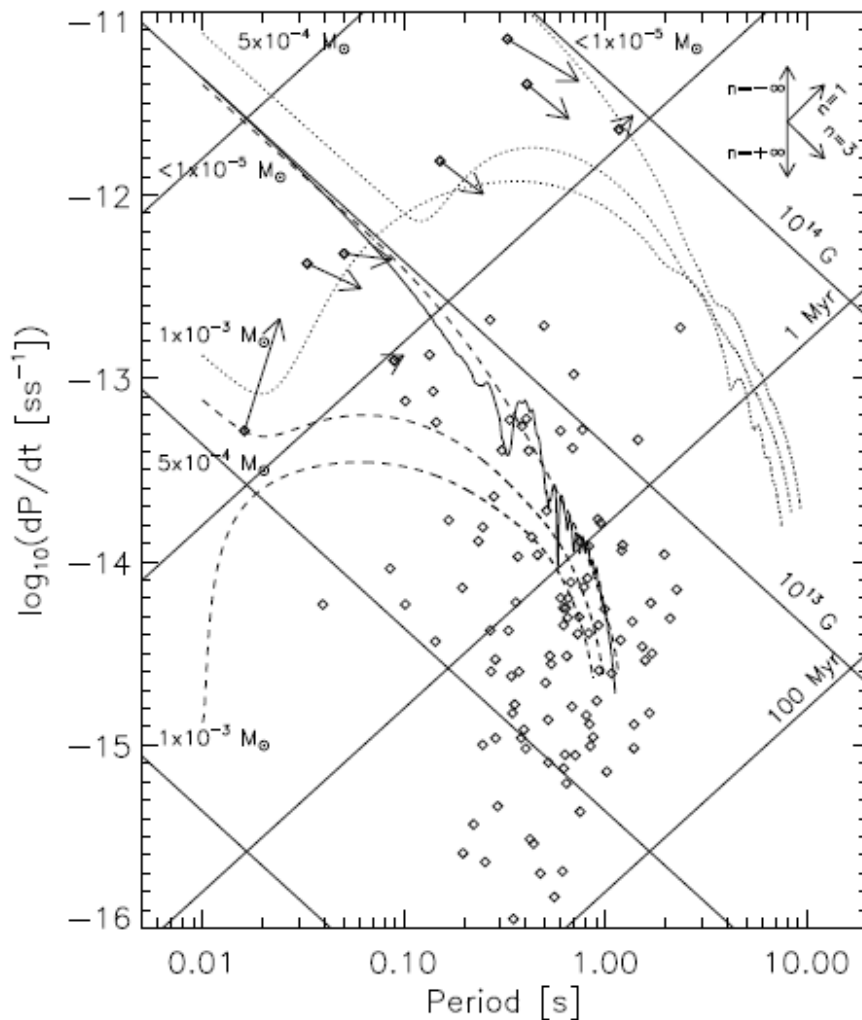


The idea is that  $n < 3$  are explained as due to growing field. Then it is possible to estimate the timescale for growing and plot it vs. velocity.

Larger kick –  
- smaller fallback –  
- faster field growing



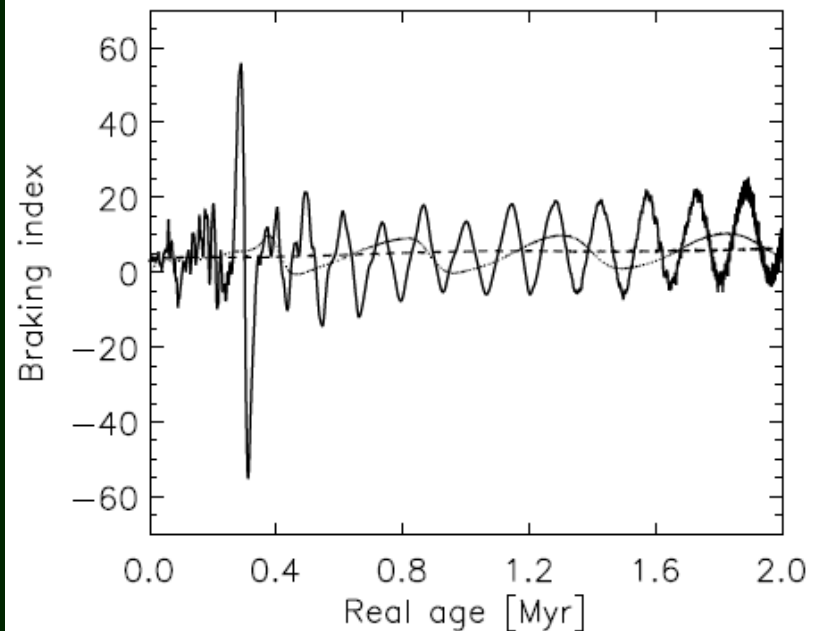
# Evolution of PSRs with evolving field



Three stages:

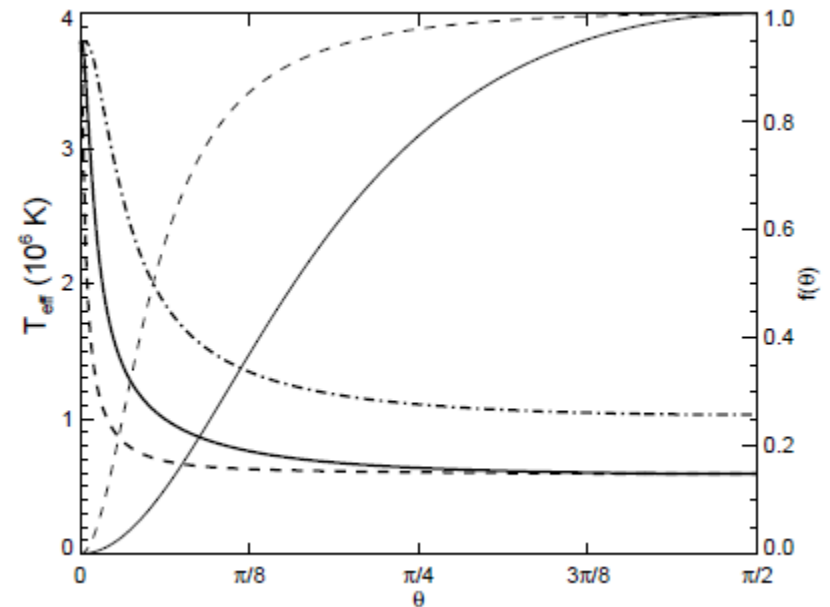
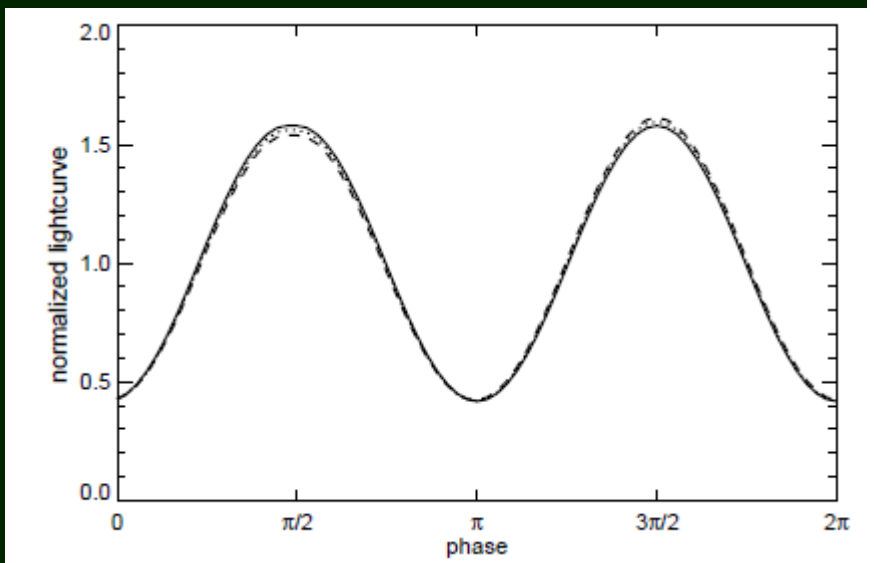
1.  $n \leq 3$  Standard + emerging field
2.  $n > 3$  Ohmic field decay
3. oscillating and large  $n$  – Hall drift

$$n = 3 - 4 \frac{\dot{B}_0}{B_0} \tau_c \equiv 3 - 4 \frac{\tau_c}{\tau_B},$$



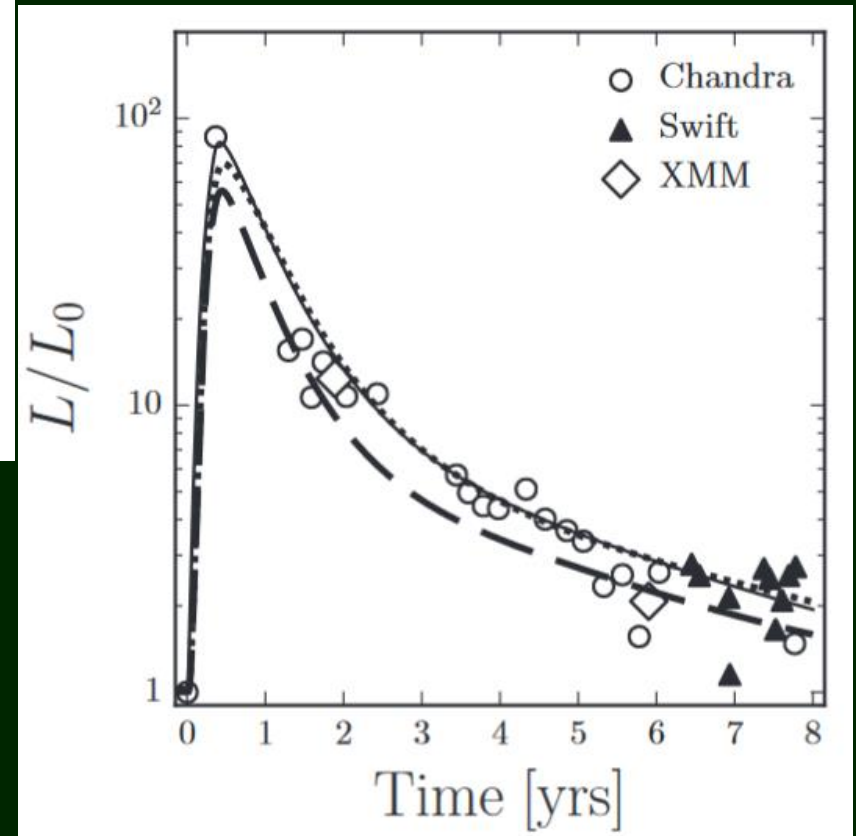
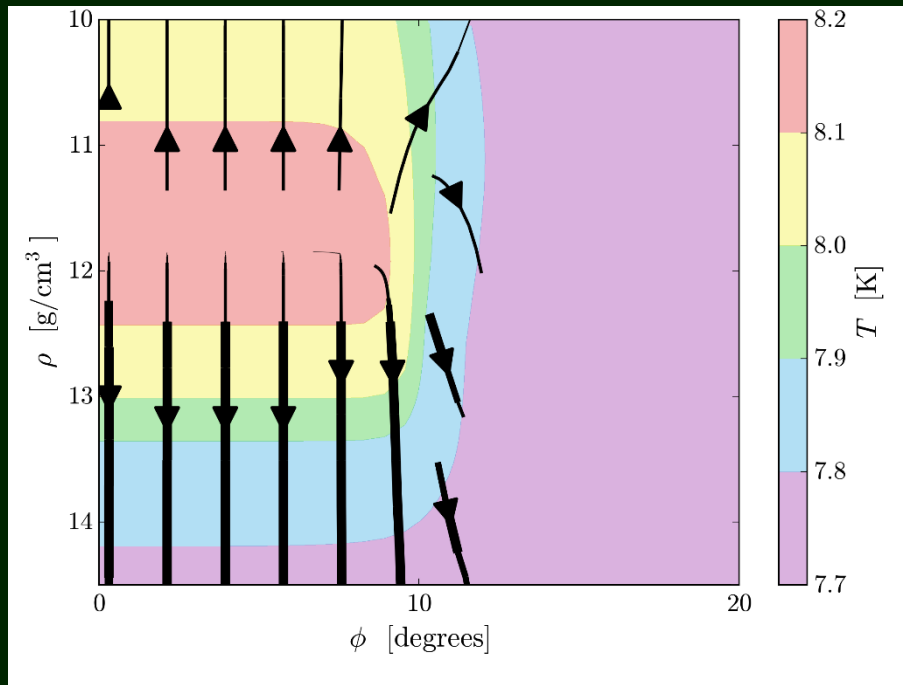
# Buried field in Kes79?

The idea is to reconstruct surface temperature distribution, and then calculate which field configuration can produce it.

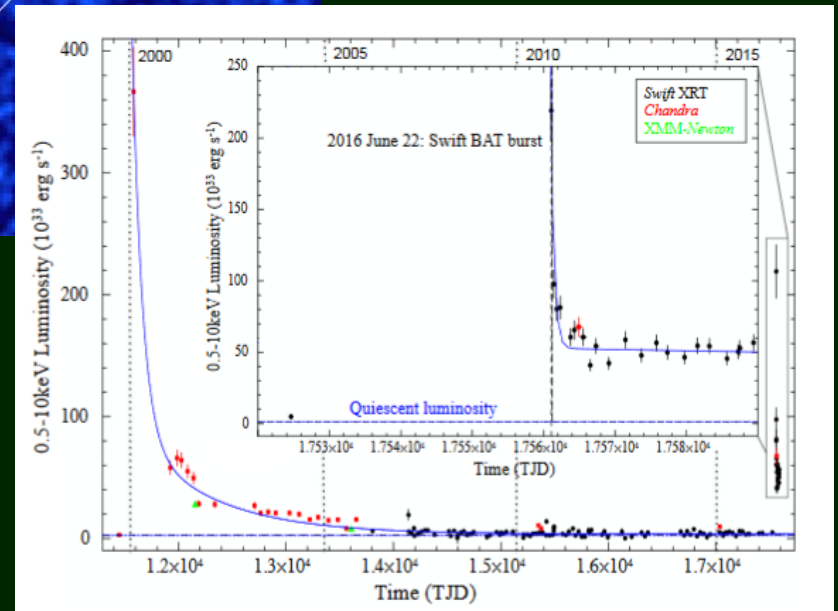
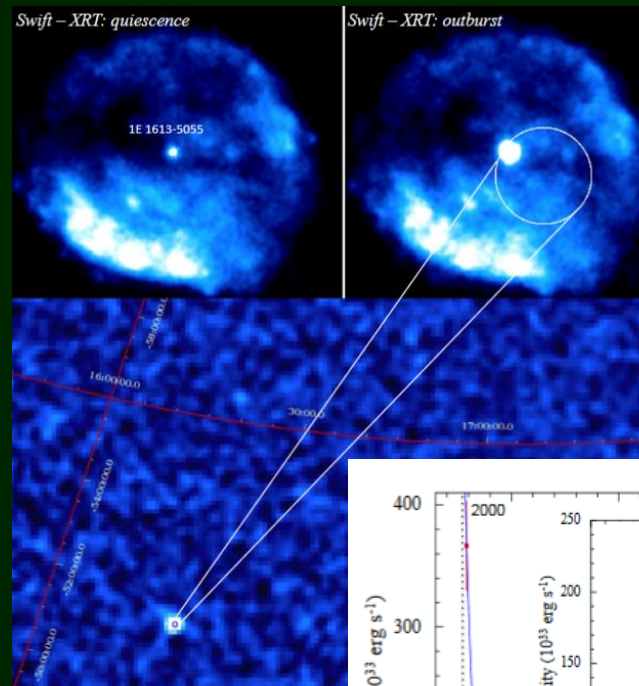
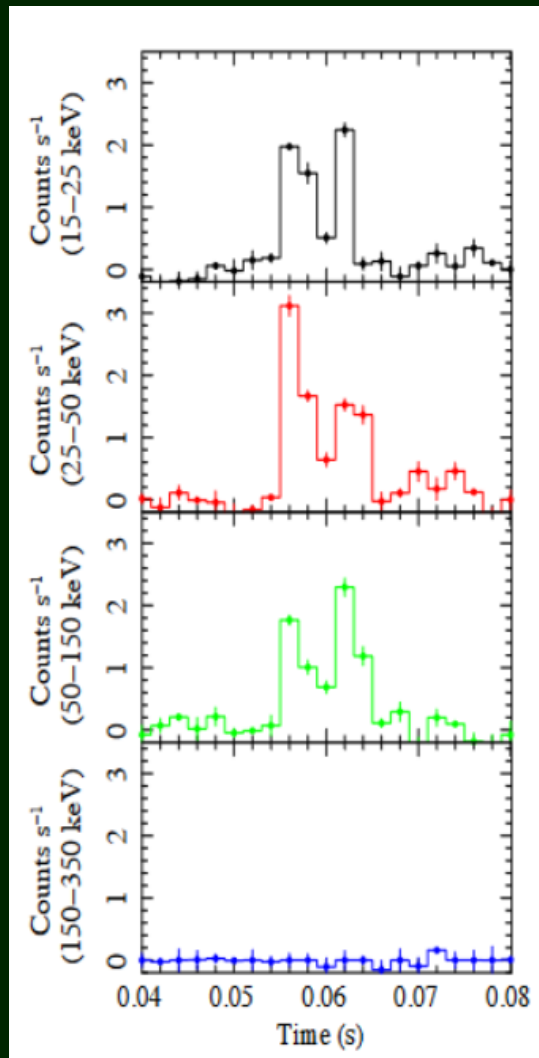


Very large pulse fraction (64%) in the anti-magnetar Kes 79. Large sub-surface magnetic field can explain the existence of compact hot spots. Then the field must have been buried in a fall-back episode.

# Hidden magnetar in RCW103



# Not so hidden!



1607.04107

1607.04264

# GRBs and fallback onto magnetars

Giant X-ray flares in GRB happen after  $\sim 30\text{-}10^5$  s.

Rotational energy  $\sim 2 \cdot 10^{52} \text{ erg } P_{\text{ms}}^{-2}$

$$\dot{M}_{\text{D}}(t) = \dot{M}_{\text{fb}} - \dot{M}_{\text{acc}} - \dot{M}_{\text{prop}},$$

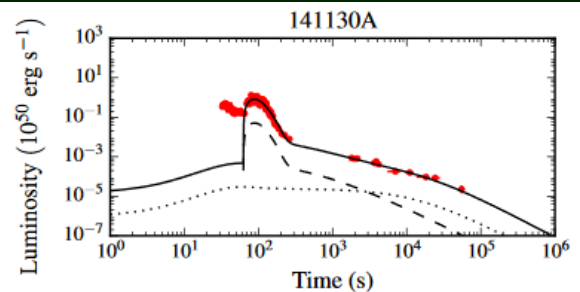
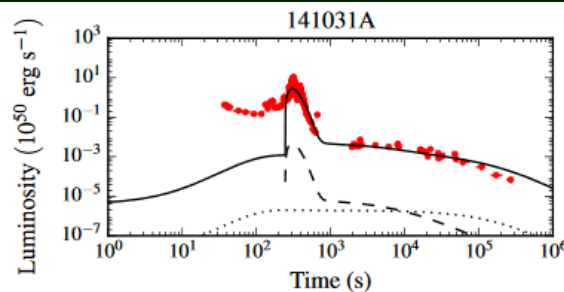
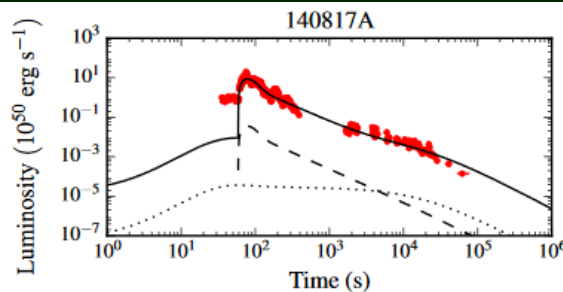
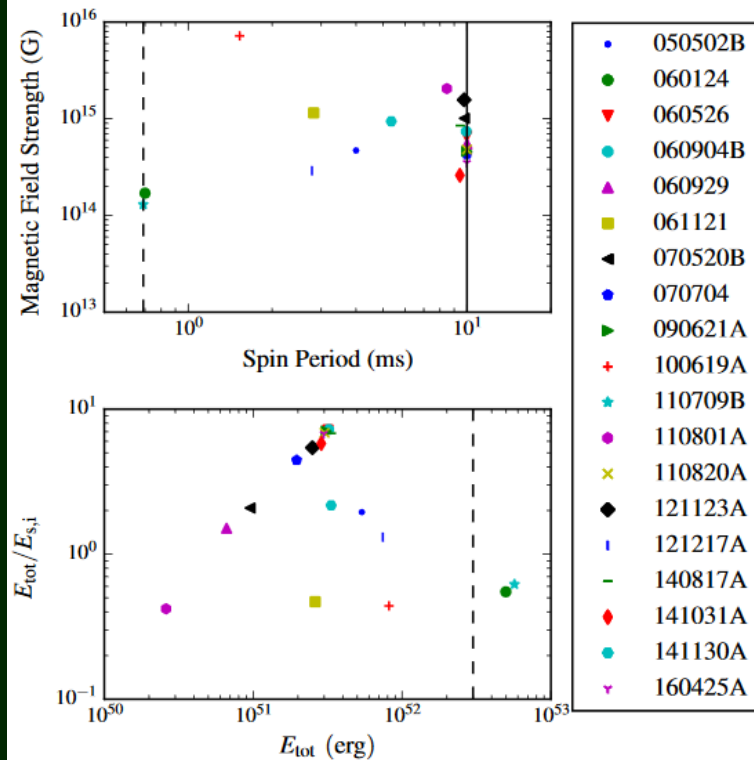
$$\dot{M}_{\text{fb}}(t) = \frac{M_{\text{fb}}}{t_{\text{fb}}} \left( \frac{t + t_{\text{fb}}}{t_{\text{fb}}} \right)^{-\frac{5}{3}}$$

$$L_{\text{dip}} = -\tau_{\text{dip}}\omega$$

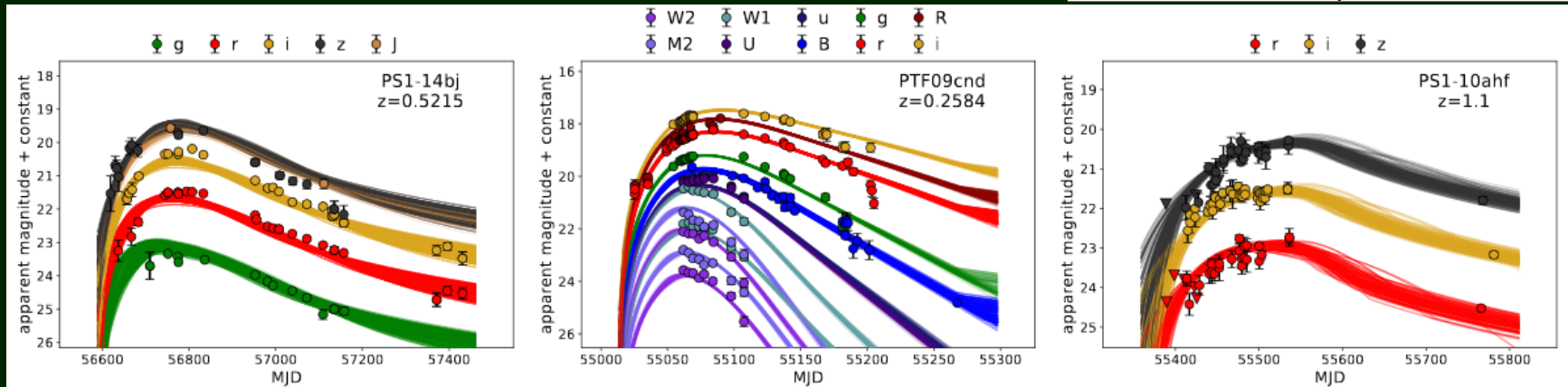
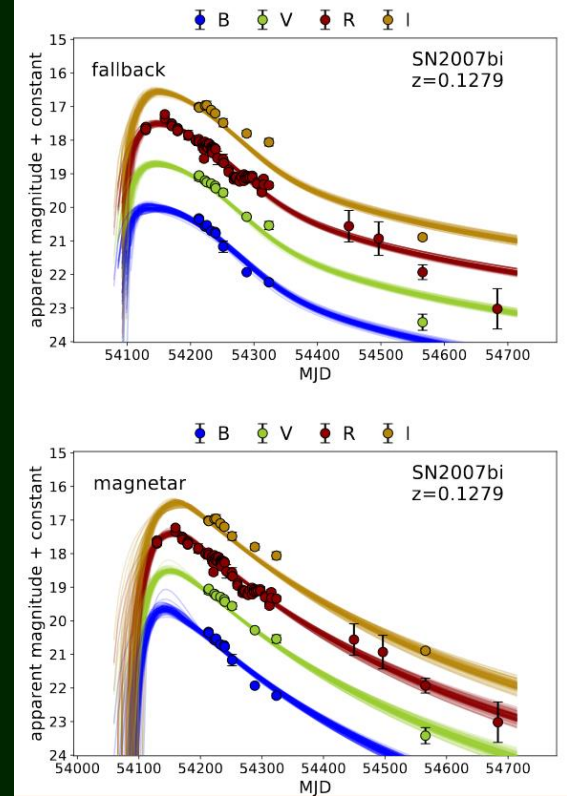
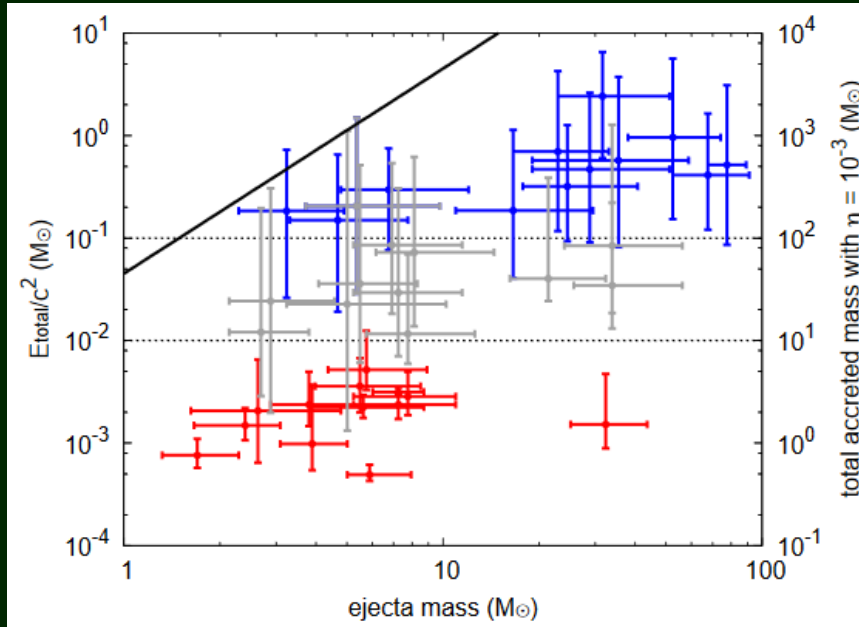
$$L_{\text{prop}} = -\tau_{\text{acc}}\omega$$

$\tau_{\text{acc}}$  and  $\tau_{\text{dip}}$  are the accretion and dipole torques

$$L_{\text{tot}} = \frac{1}{f_{\text{B}}} \left( \eta_{\text{prop}} L_{\text{prop}} + \eta_{\text{dip}} L_{\text{dip}} \right)$$



# Fallback to power SN



1806.00090

$$L_{\text{fallback}}(t) = \begin{cases} L_1 \left( \frac{t_{\text{tr}}}{1 \text{ sec}} \right)^{-\frac{5}{3}} \equiv L_{\text{flat}} & (t < t_{\text{tr}}) \\ L_1 \left( \frac{t}{1 \text{ sec}} \right)^{-\frac{5}{3}} & (t \geq t_{\text{tr}}) \end{cases}$$

# Conclusions

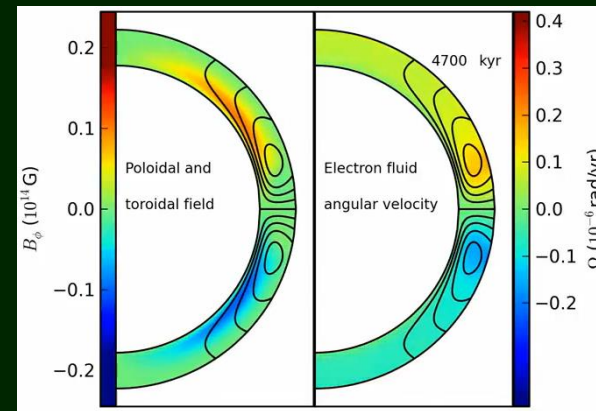
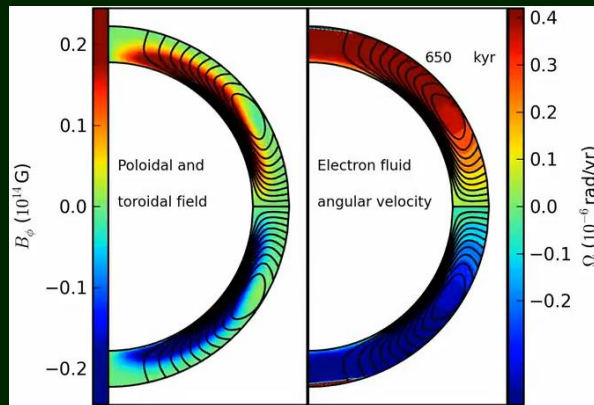
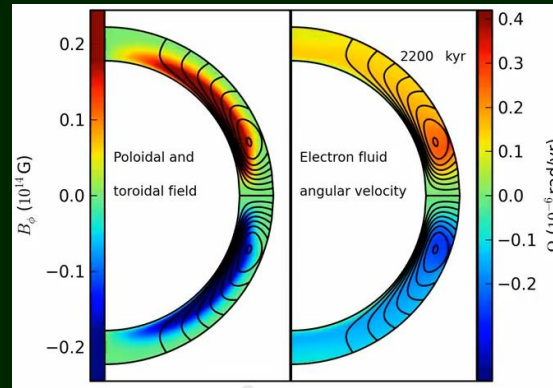
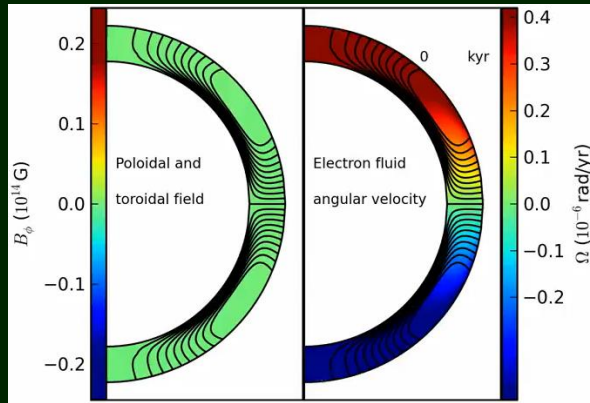
- Decaying magnetic field results in additional heating of a NS and decreasing its spin-down rate
- Field decay can be more important for large initial fields, for “standard” fields ( $\sim 10^{12}$  G) it is not important
- It is possible to describe different types of young NSs (PSRs, magnetars, M7 etc.) in the model with decaying magnetic field
- Re-merging magnetic field can be an important ingredient
- With re-emerging field we can add to the general picture also CCOs.
- Recent studies indicate that in the life of normal radio pulsars there is a period when their magnetic field decay
- Hall cascade (and attractor) can be an important ingredient of the field evolution.
- At the moment we cannot state that we see the Hall attractor in the population of normal radio pulsars
- Also, we do not see that any of the M7 NSs are at the attractor stage, as its properties are predicted by GC2013
- Probably, the attractor stage is reached later, or its properties are different from the predicted ones.



# Papers to read

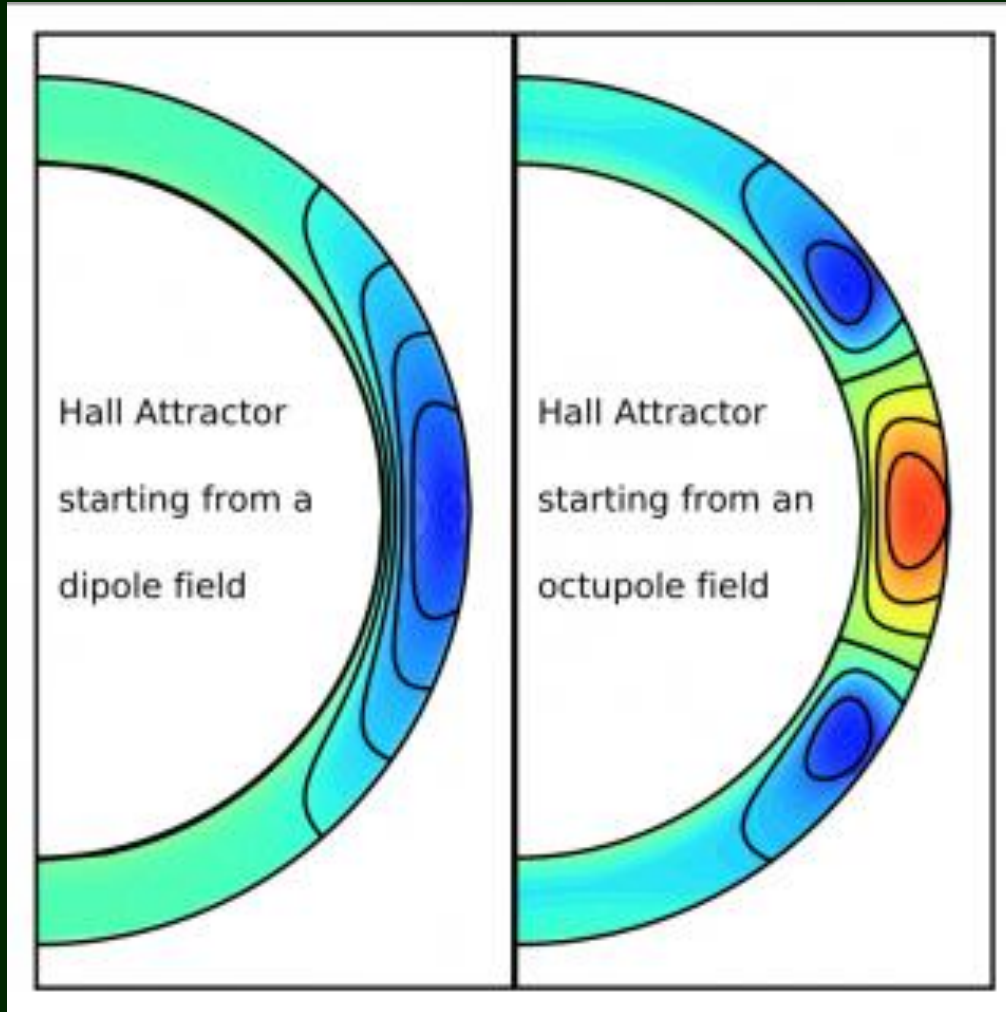
- Pons, Geppert “Magnetic field dissipation in neutron star crusts: from magnetars to isolated neutron stars ” astro-ph/0703267
- Popov et al. “Population synthesis studies of isolated neutron stars with magnetic field decay” MNRAS (2009) arXiv: 0910.2190
- Ho “Evolution of a buried magnetic field in the central compact object neutron stars ” arXiv:1102.4870
- Pons et al. “Pulsar timing irregularities and the imprint of magnetic field Evolution” arXiv: 1209.2273
- Cumming et al. “MAGNETIC FIELD EVOLUTION IN NEUTRON STAR CRUSTS DUE TO THE HALL EFFECT AND OHMIC DECAY” astro-ph/0402392

# Hall cascade and attractor



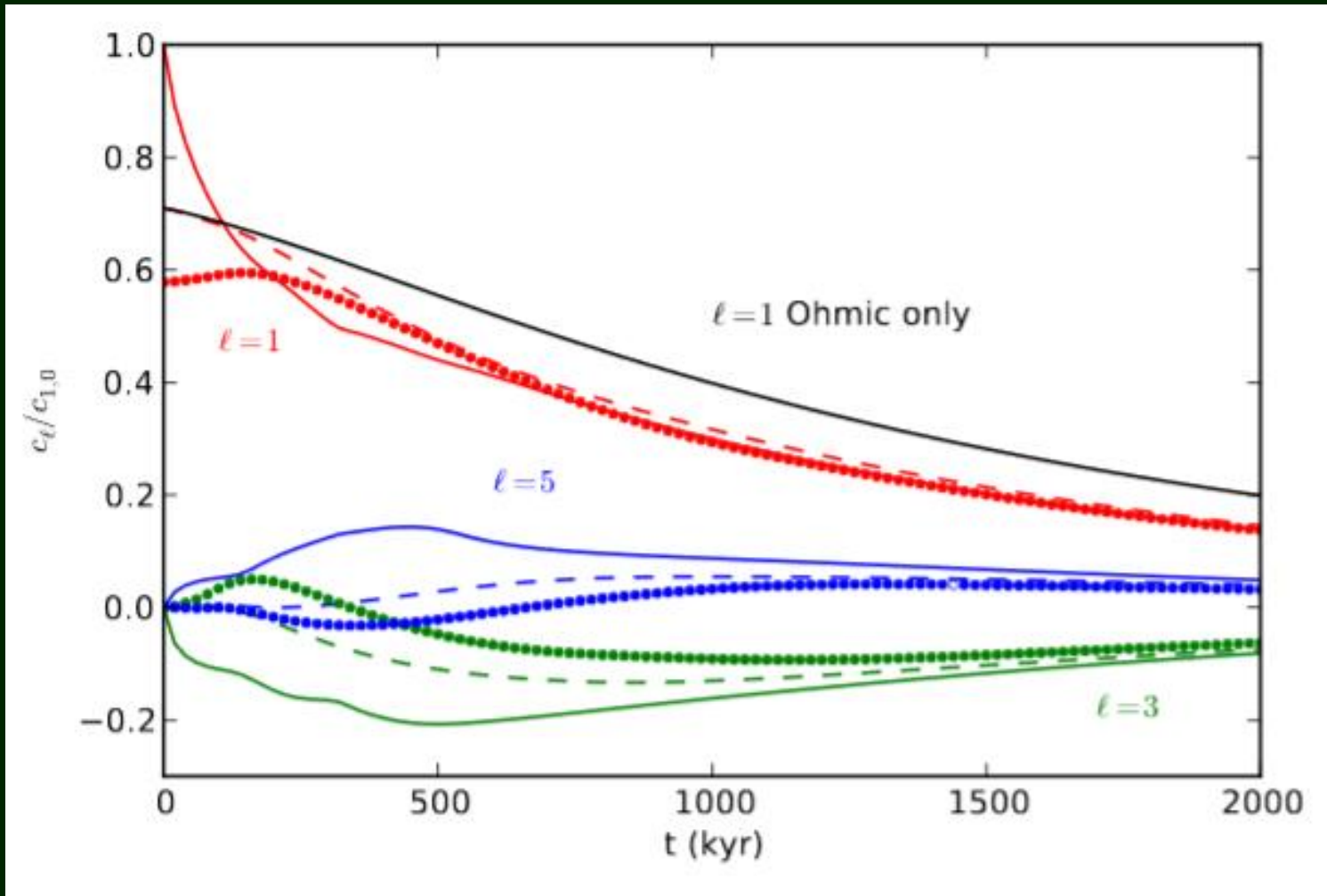
Hall cascade can reach the stage of so-called Hall attractor, where the field decay stalls for some time (Gourgoulatos, Cumming).

# Hall attractor



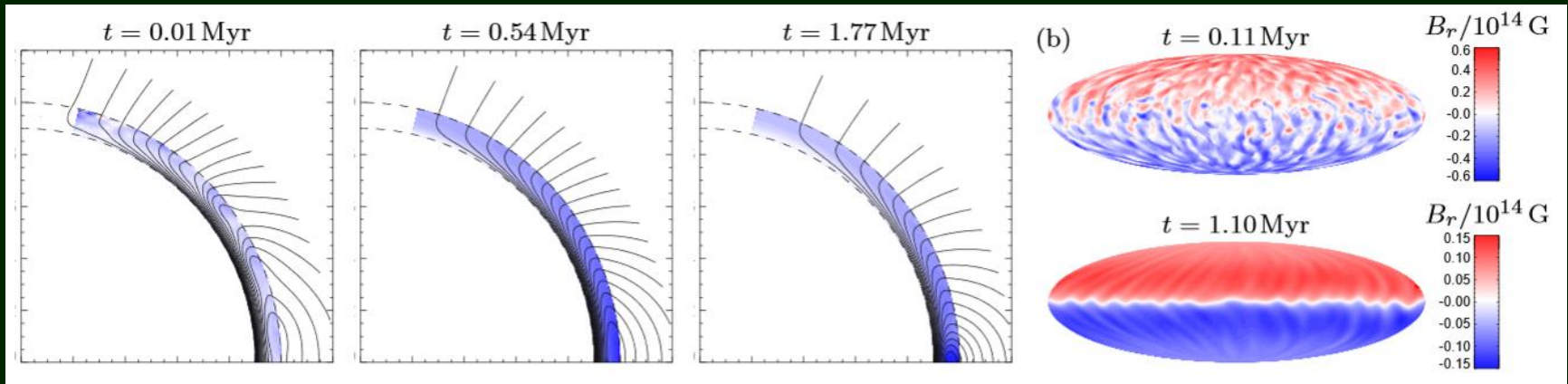
After some time the Hall cascade decays as the field finds a new| stable configuration.

# Evolution of different components

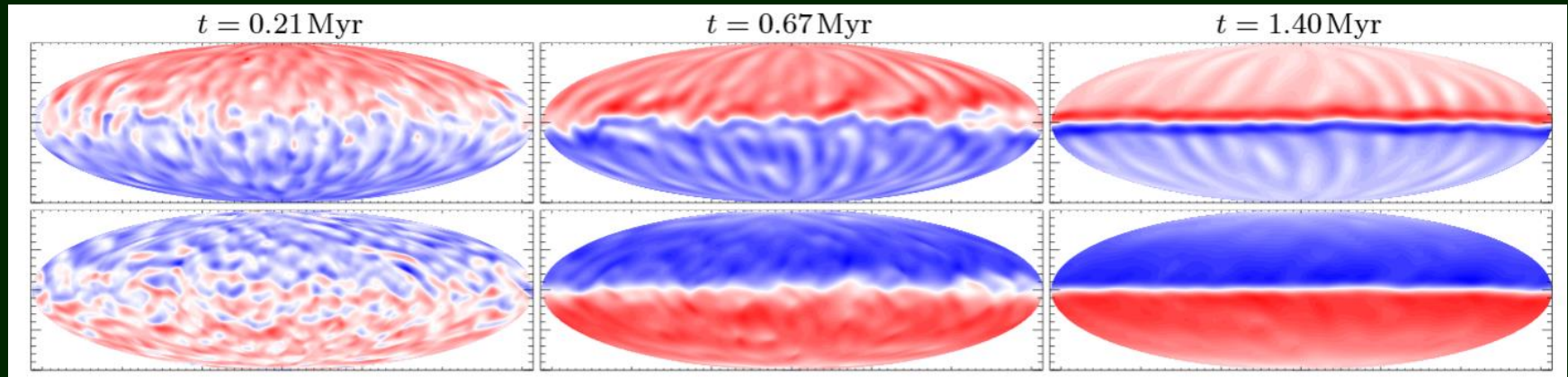


Hall attractor mainly consists of dipole and octupole

# New studies of the hall cascade



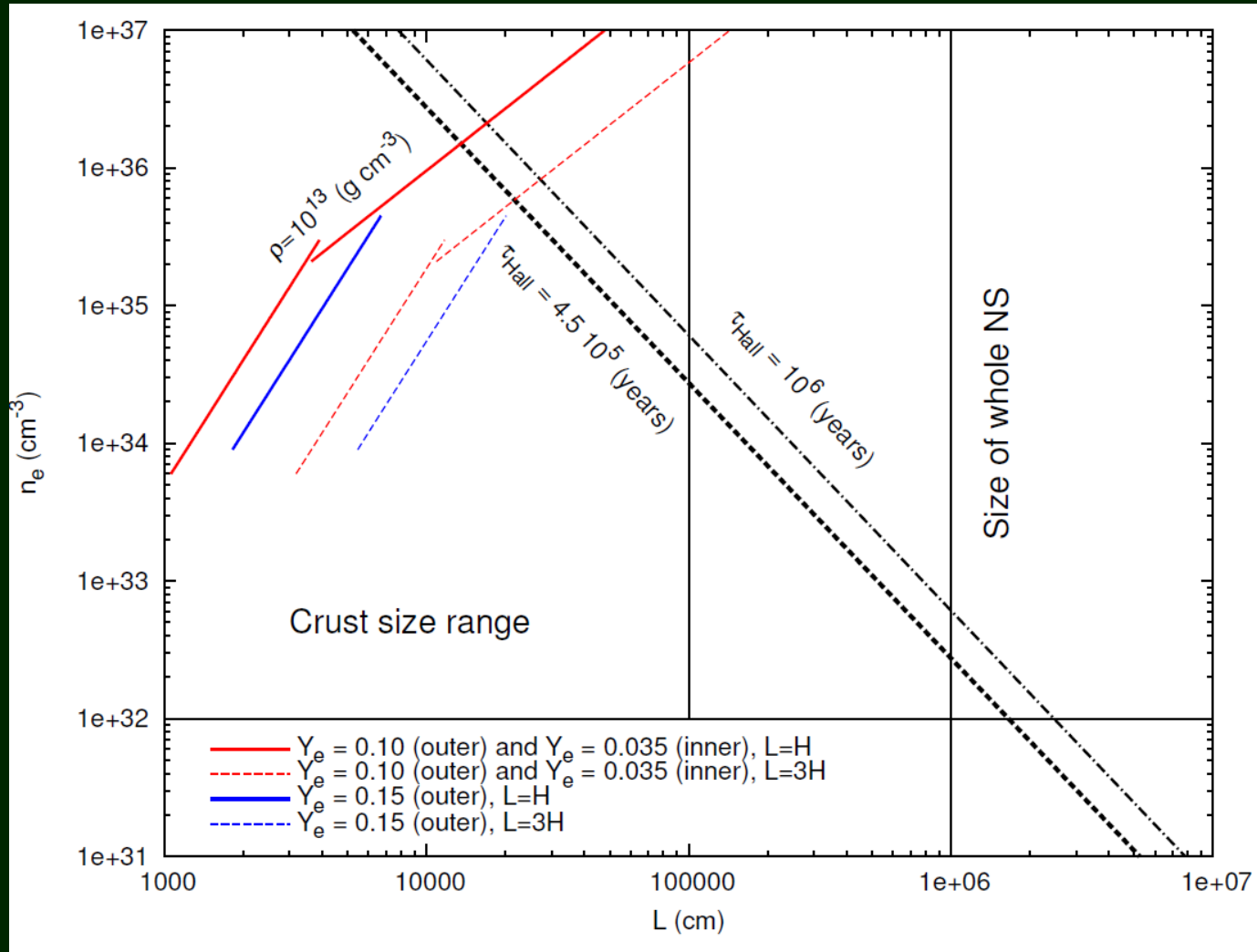
Toroidal  
Poloidal



New calculations support the idea of a kind of stable configuration.

1501.05149

# Where the currents are located?



$$\tau_{Hall} \approx \frac{4\pi e L^2 n_e}{cB}$$

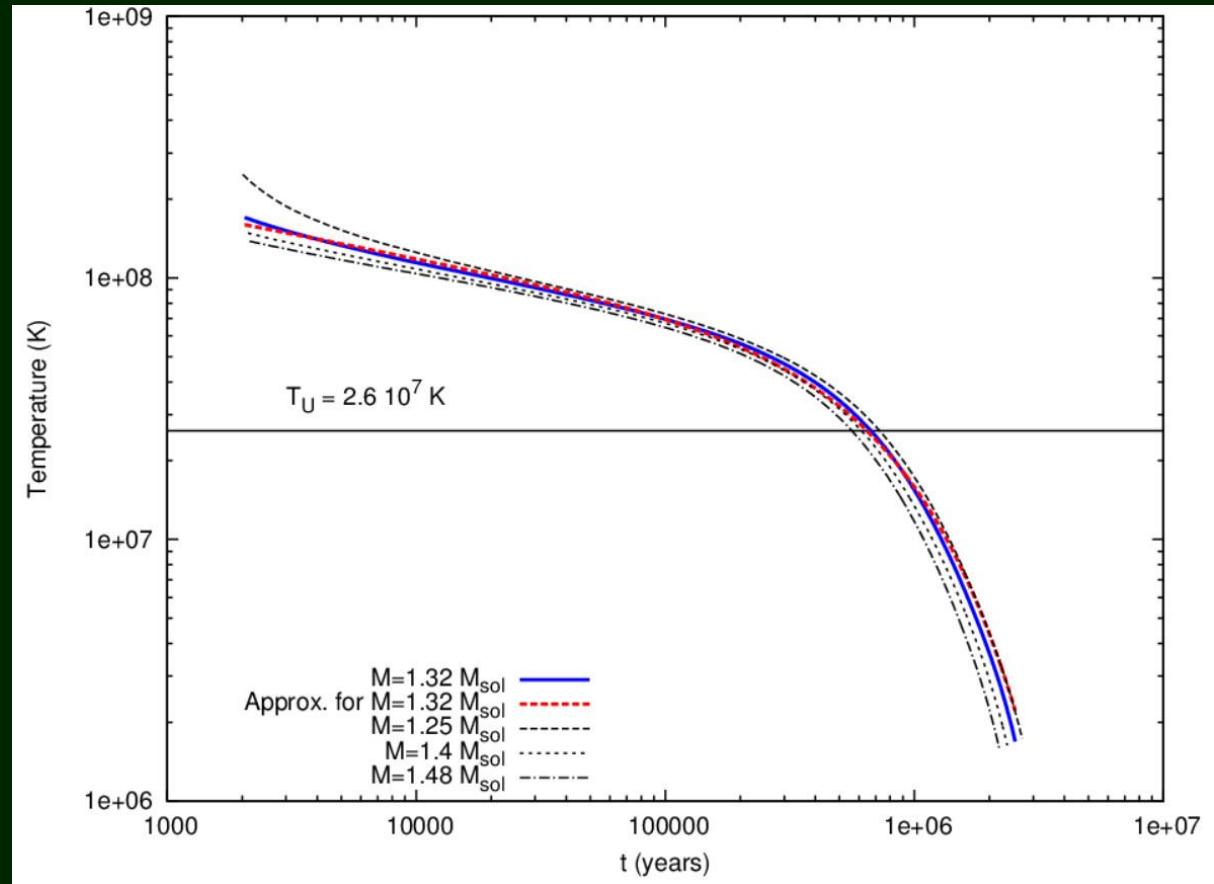
$$L \approx H = P(\rho)/(\rho g)$$



# Thermal evolution

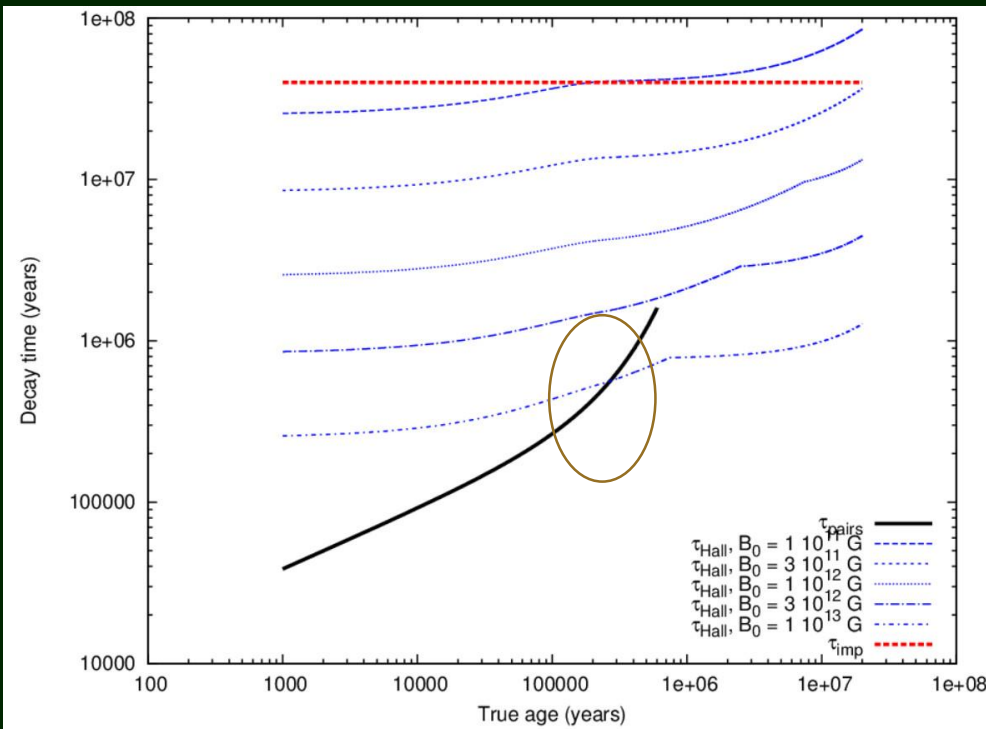
Calculations are made by Shternin et al.

We fit the numerical results to perform a population synthesis of radio pulsars with decaying field.





# Different decay time scales



In the range of ages interesting for us the Hall rate is about the same value as the rate of Ohmic dissipation due to phonons.

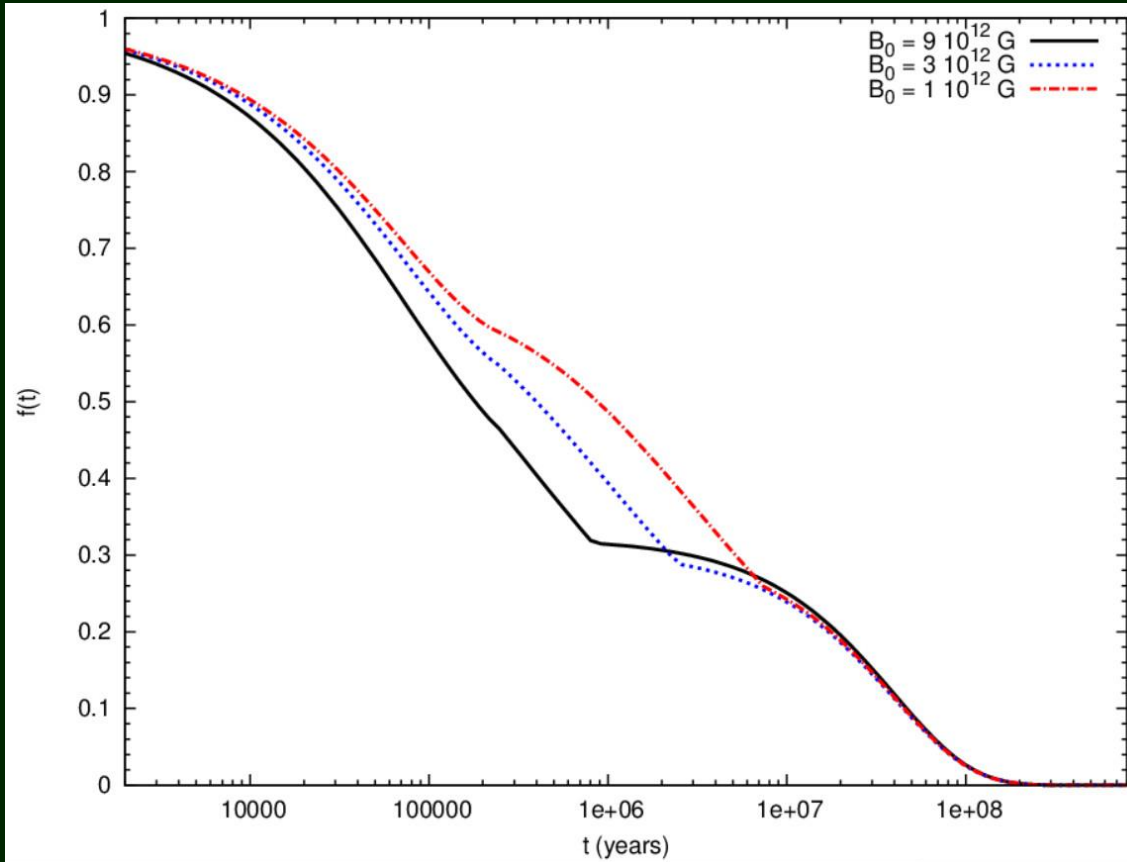
$$B = B_0 \frac{\exp(-t/\tau_{\text{Ohm}})}{1 + \frac{\tau_{\text{Ohm}}}{\tau_{\text{Hall}}}(1 - \exp(-t/\tau_{\text{Ohm}}))}$$

$$\tau_{\text{imp}} = 5.7 \frac{\rho_{14}^{5/3}}{Q} \left( \frac{Z}{30} \right) \left( \frac{Y_e}{0.05} \right)^{1/3} \left( \frac{Y_n}{0.8} \right)^{10/3} \times \left( \frac{f}{0.5} \right)^2 \left( \frac{g_{14}}{2.45} \right) \text{ Myrs},$$

$$\tau_{\text{phonon}} = 2.2 \frac{\rho_{14}^{15/6}}{T_8^2} \left( \frac{Y_e}{0.05} \right)^{5/3} \left( \frac{Y_n}{0.8} \right)^{10/3} \times \left( \frac{f}{0.5} \right)^2 \left( \frac{g_{14}}{2.45} \right)^{-2} \text{ Myrs},$$

# Magnetic field evolution

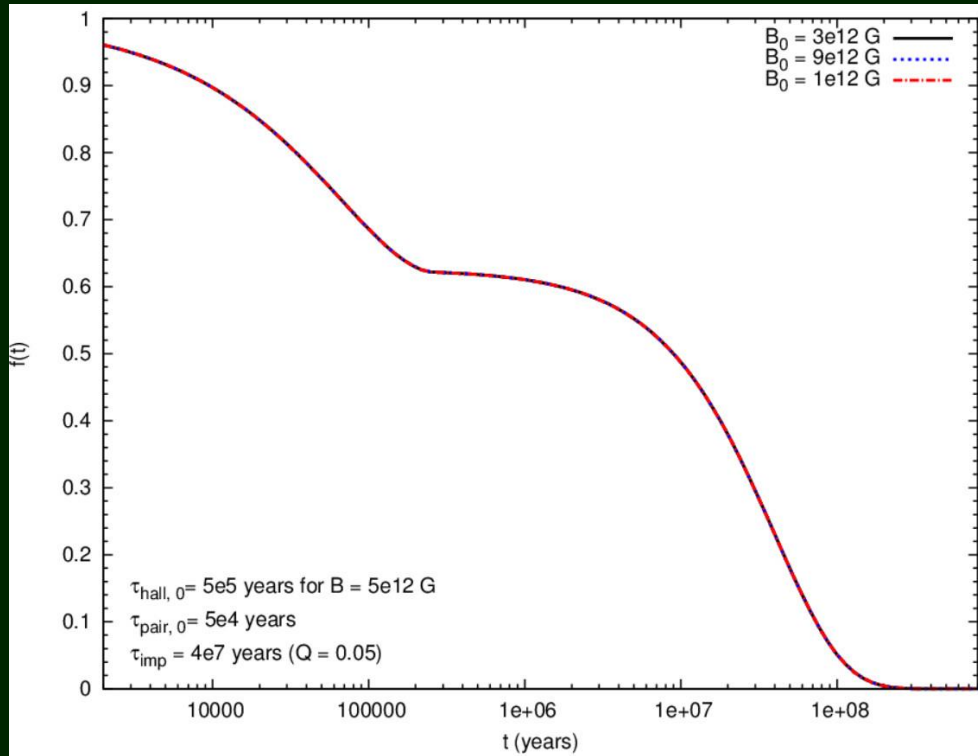
$$B = B_0 \frac{\exp(-t/\tau_{\text{Ohm}})}{1 + \frac{\tau_{\text{Ohm}}}{\tau_{\text{Hall}}}(1 - \exp(-t/\tau_{\text{Ohm}}))}$$



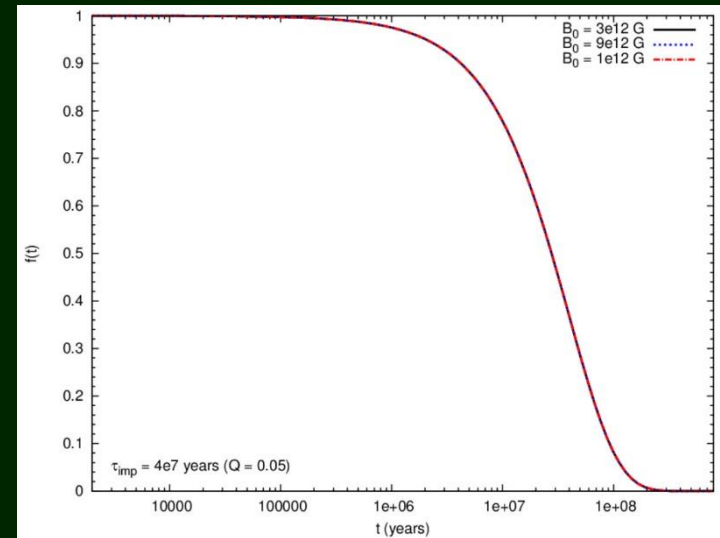
All inclusive:

- Hall
- Phonons
- Impurities

# Only Ohmic decay

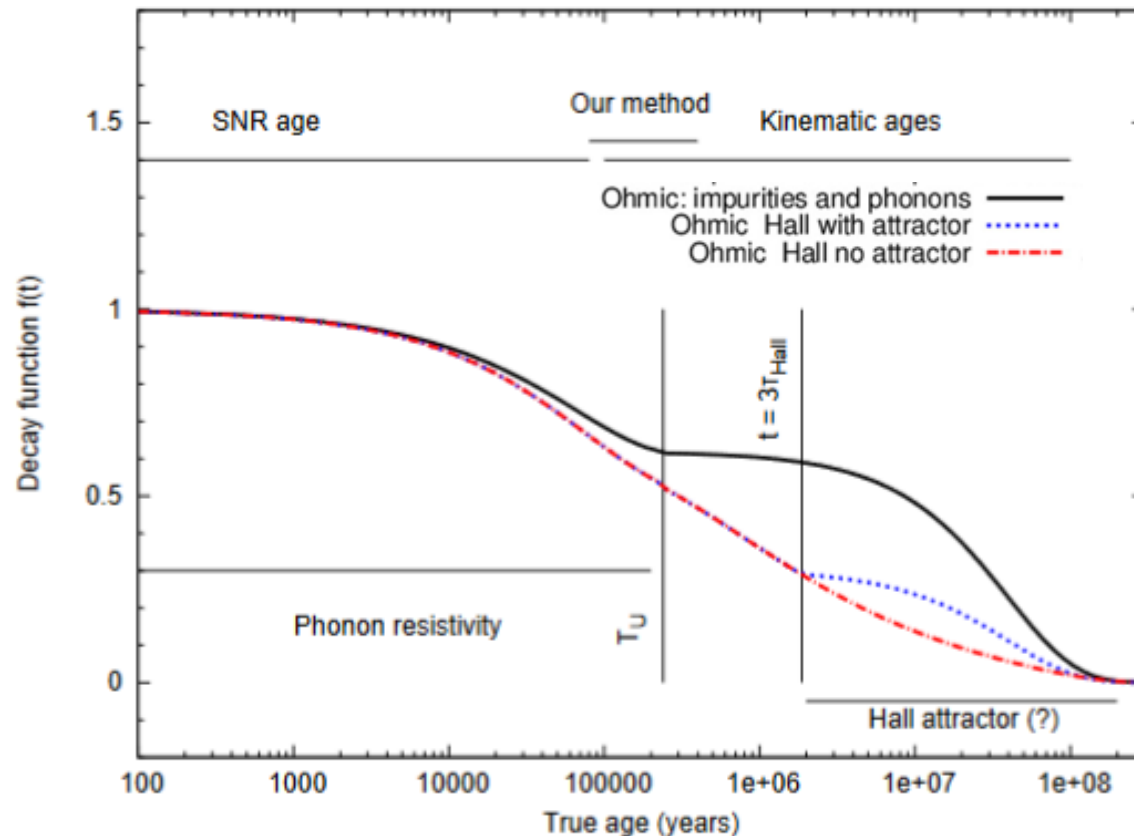


Here the Hall cascade is switched off



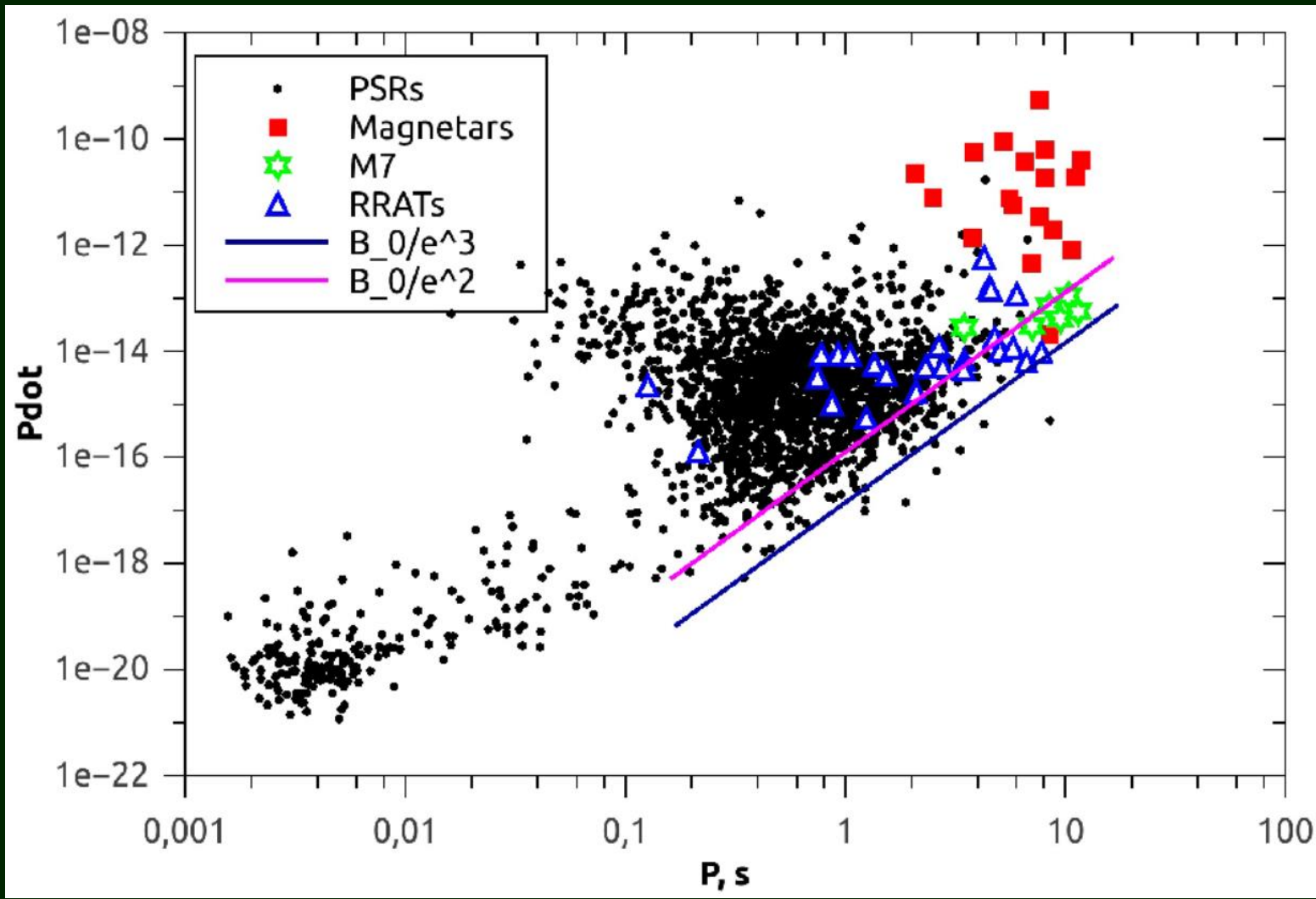
In one figure we have Ohmic decay only due to impurities, on another one – phonons are added.

# Comparison of different options



We think that at the ages  $\sim 10^5$  yrs and below for normal pulsars we see mostly Ohmic decay, which then disappears as NSs cool down below the critical T.

# Getting close to the attractor



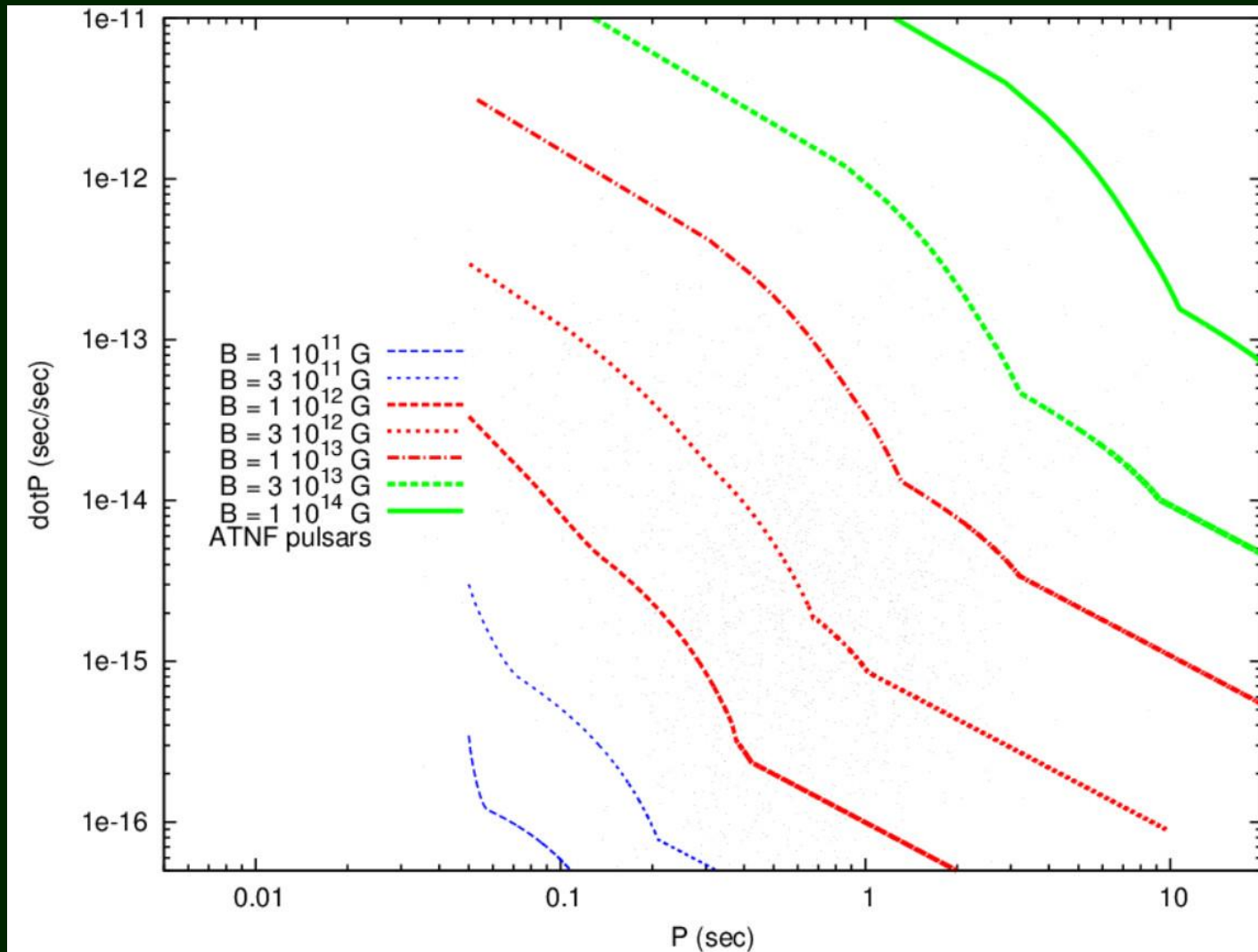
# Tests

We make extensive tests of the method and obtain that in most of the cases it is able to uncover non-negligible magnetic field decay (more than a few tens of per cent during the studied range of ages) in normal radio pulsars for realistic initial properties of neutron stars.

Name	$\log \mu_{B_0}$ [G]	$\log \sigma_{B_0}$ [G]	$\mu_{P_0}$ [s]	$\sigma_{P_0}$ [s]	$\alpha$	$\tau_D$ [Myr]	$\tau_{SDA}$ [Myr]
A1	12.60	0.47	0.33	0.23	0.50	$\infty$	$\infty$
A2	12.95	0.55	0.30	0.15	0.50	$\infty$	10
B1	12.60	0.47	0.33	0.23	0.50	0.5	1.00
B2	12.95	0.55	0.30	0.15	0.50	0.5	0.690
C1	12.60	0.47	0.33	0.23	0.50	1	1.15
C2	12.95	0.55	0.30	0.15	0.50	1	0.560
D1	12.60	0.47	0.33	0.23	0.50	5	2.00
D2	12.95	0.55	0.30	0.15	0.50	5	0.80
E	13.04	0.55	0.22	0.32	0.44	$\sim 0.8$	0.880

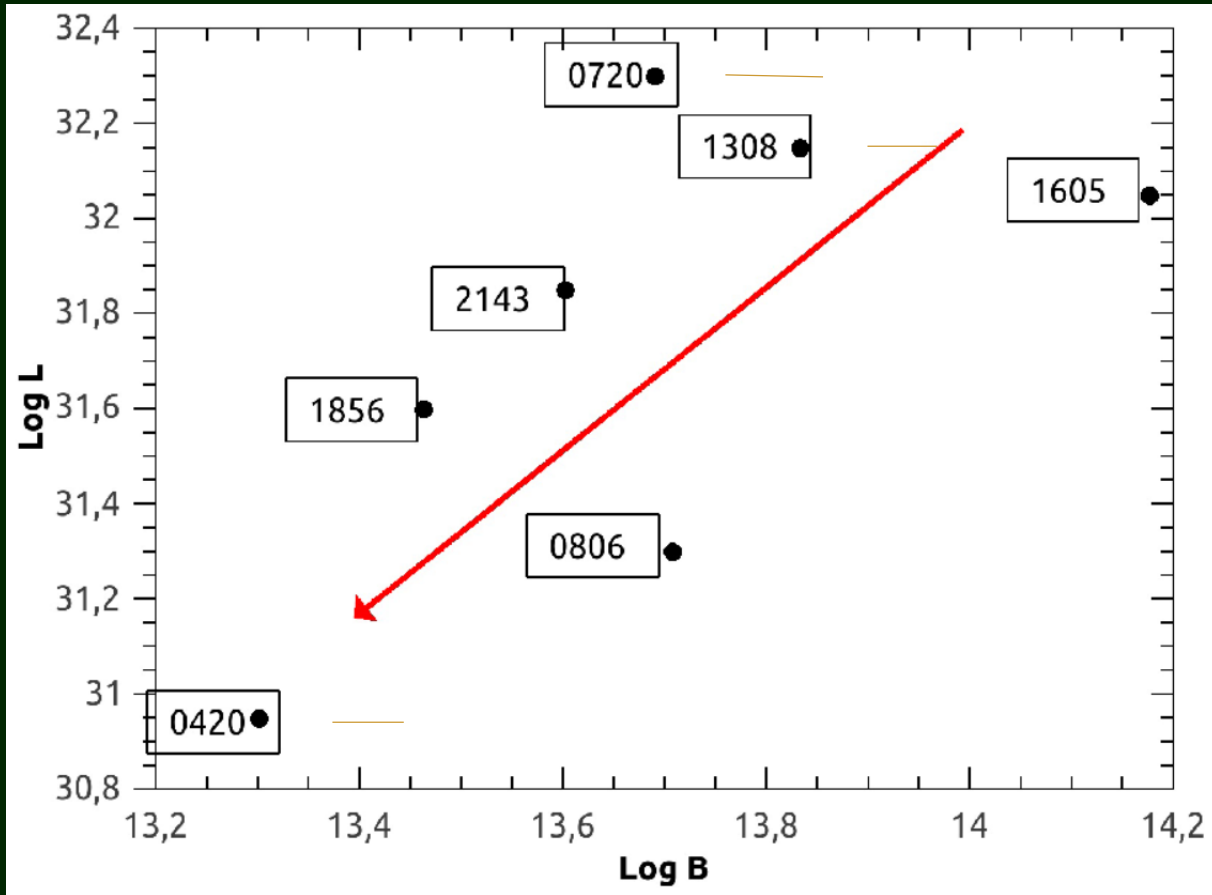
(Synthetic samples are calculated by Gullon, Pons, Miralles)

# Evolution with field decay

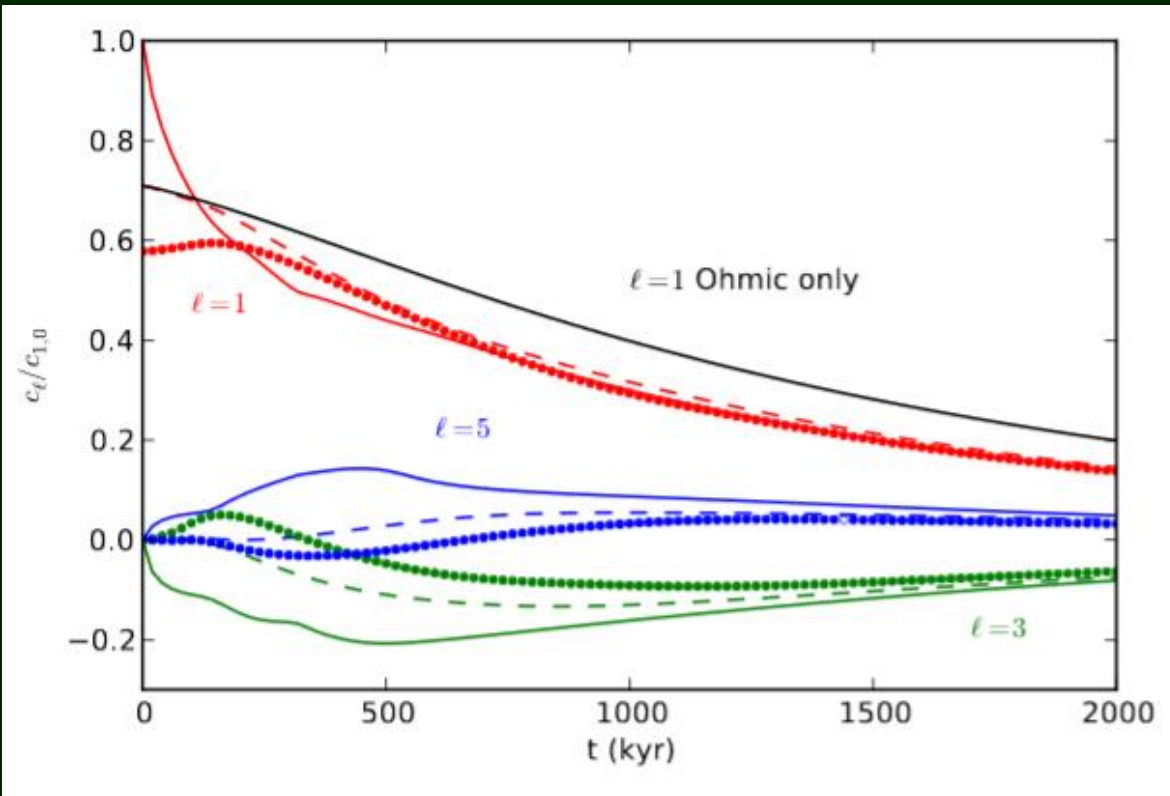




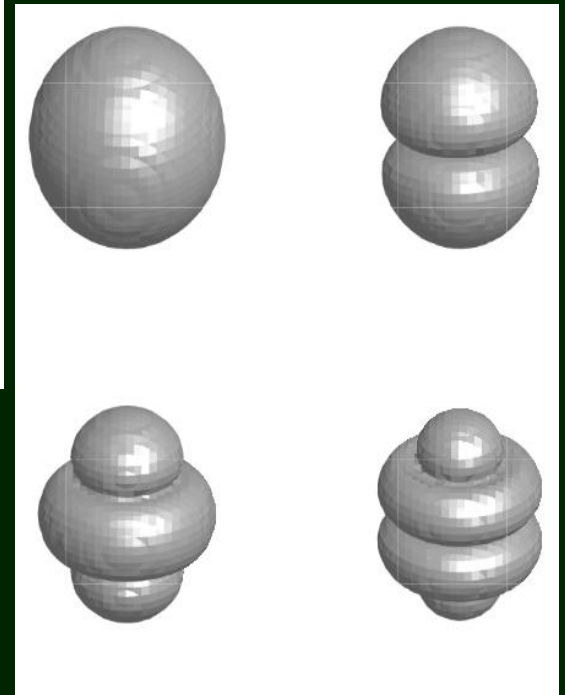
# Who is closer to the attractor stage?



# Evolution of different components

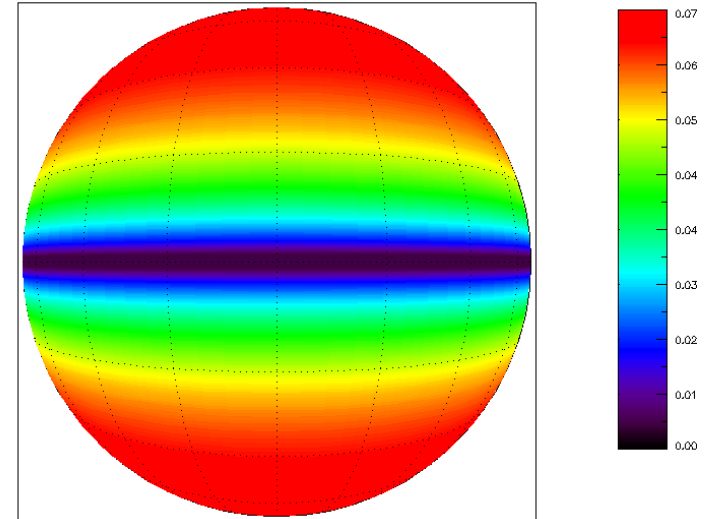
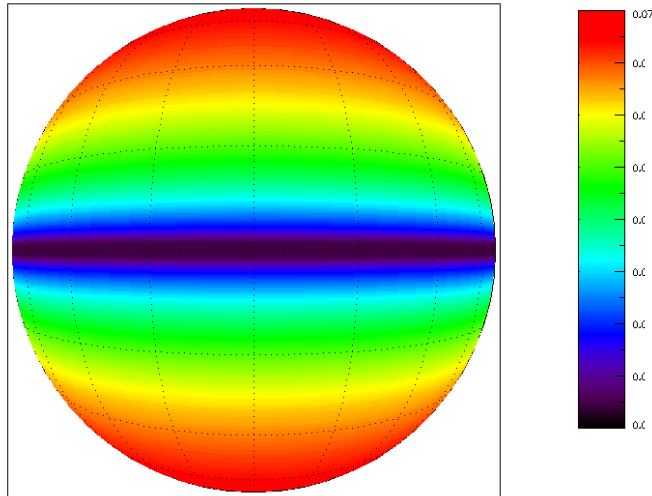


Hall attractor mainly consists of dipole and octupole



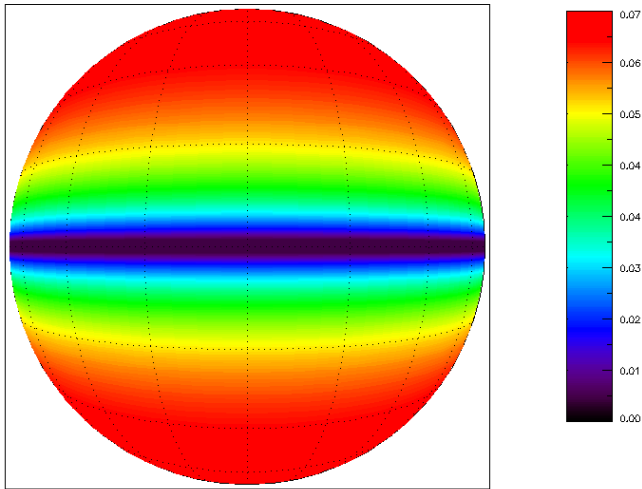
# Temperature maps

Pure dipole

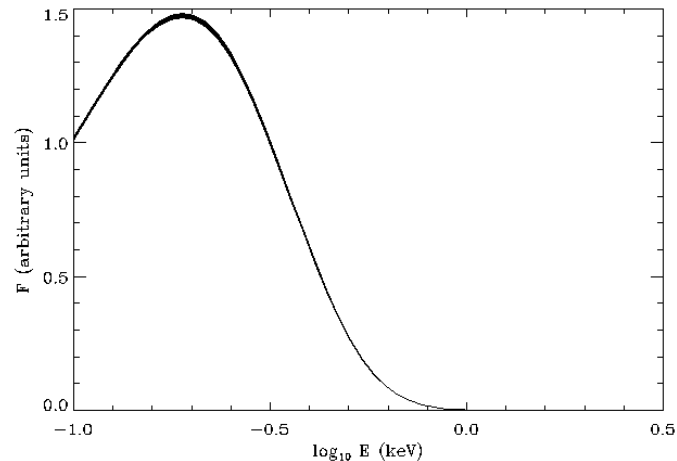
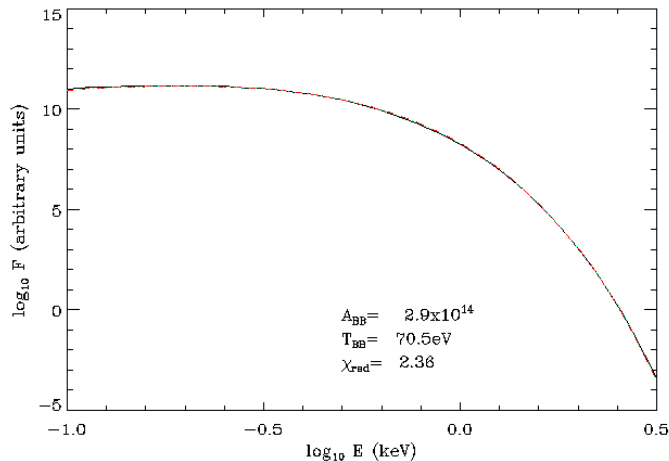


Dipole + octupole

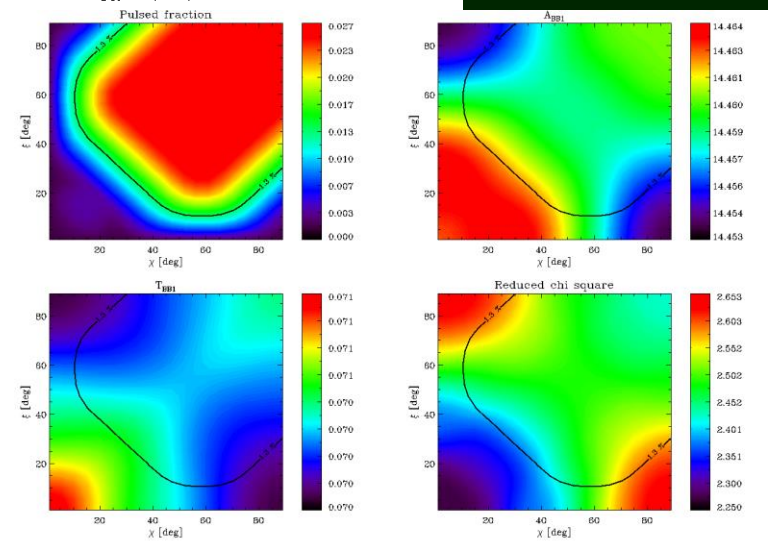
Dipole+octupole+15



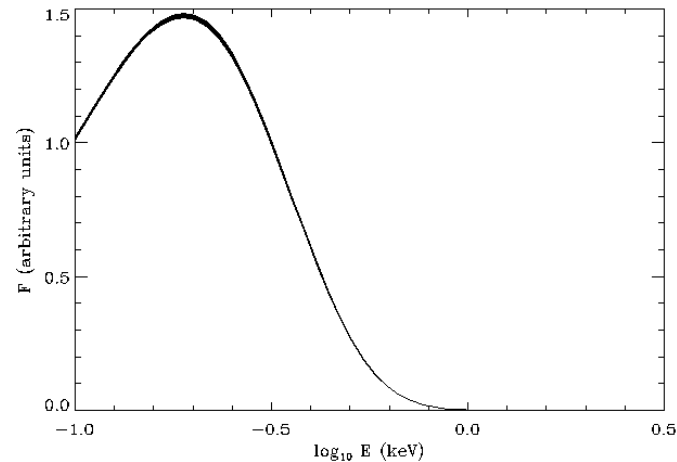
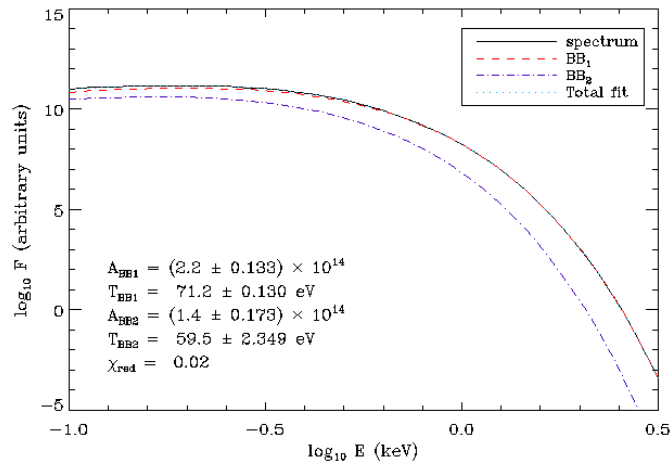
# Spectral fits: single blackbody



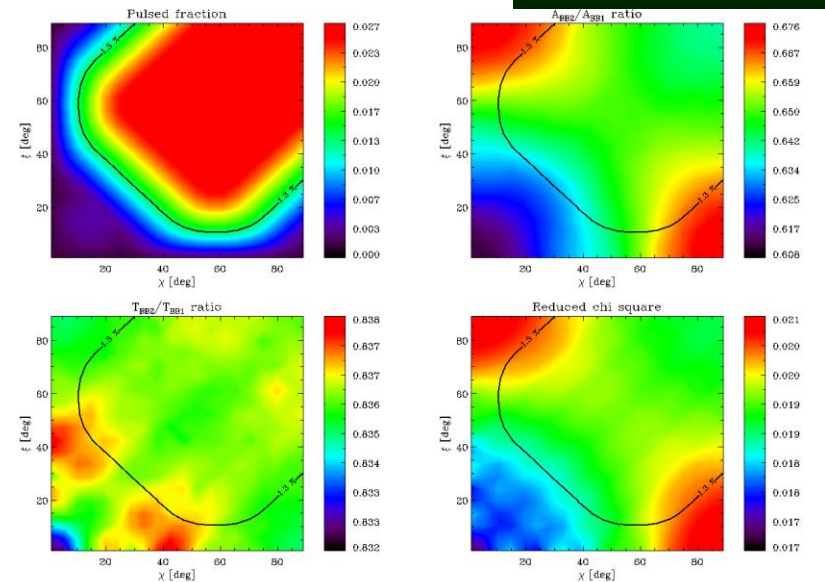
Single black body  
does not provide  
a good fit, even using,  
in addition, a line,  
or condensed surface.



# Spectral fits: two black bodies



Formally, two black bodies is the best fit for 1856. And for dipole+octupole we can obtain a very good fit. But ....



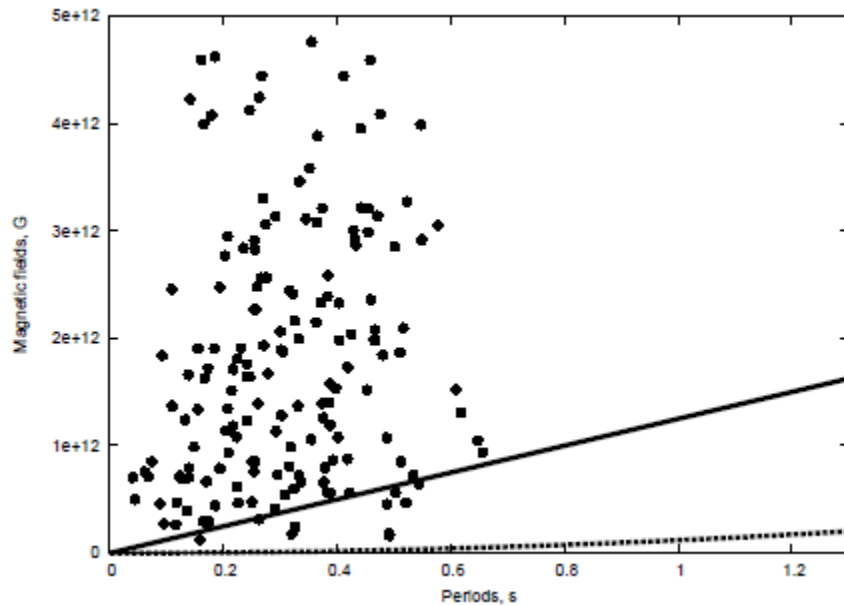
# Observational data

Parameter	Single BB	Two BB
$N_H$ [ $10^{19} \text{ cm}^{-2}$ ]	$4.8^{+0.2}_{-0.2}$	$12.9^{+2.2}_{-2.3}$
$kT_h^\infty$ [eV]	$61.5^{+0.1}_{-0.1}$	$62.4^{+0.6}_{-0.4}$
$R_h^\infty$ [km]	$5.0^{+0.1}_{-0.1}$	$4.7^{+0.2}_{-0.3}$
$kT_s^\infty$ [eV]	-	$38.9^{+4.9}_{-2.9}$
$R_s^\infty$ [km]	-	$11.8^{+5.0}_{-0.4}$
$\sigma_{\text{sys}}$	1.5%	0.6%
$\chi^2_\nu$	1.12	1.11

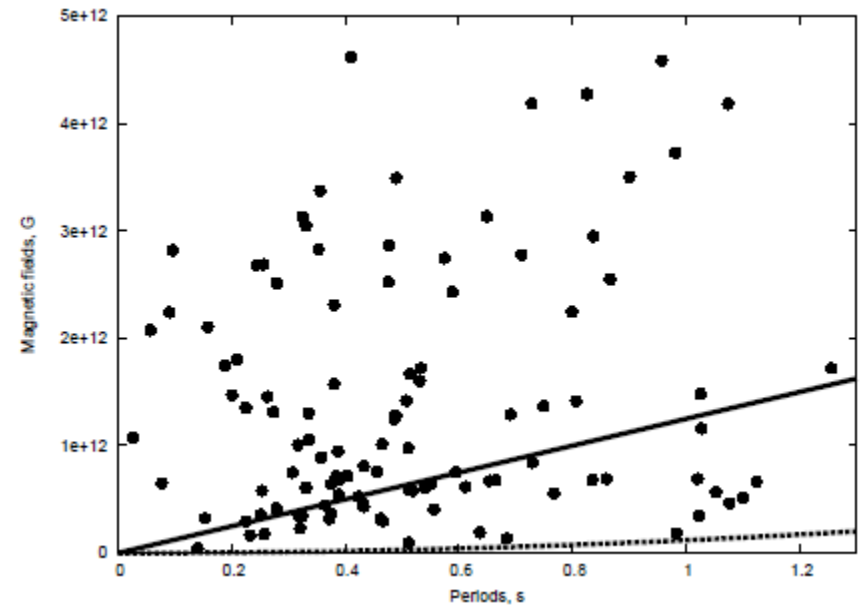
Two black bodies is the best fit.  
The colder component corresponds to larger surface area.  
This is in contrast with our results for the Hall attractor proposed by GC2013 (dipole + octupole).

# Synthetic populations

Constant field



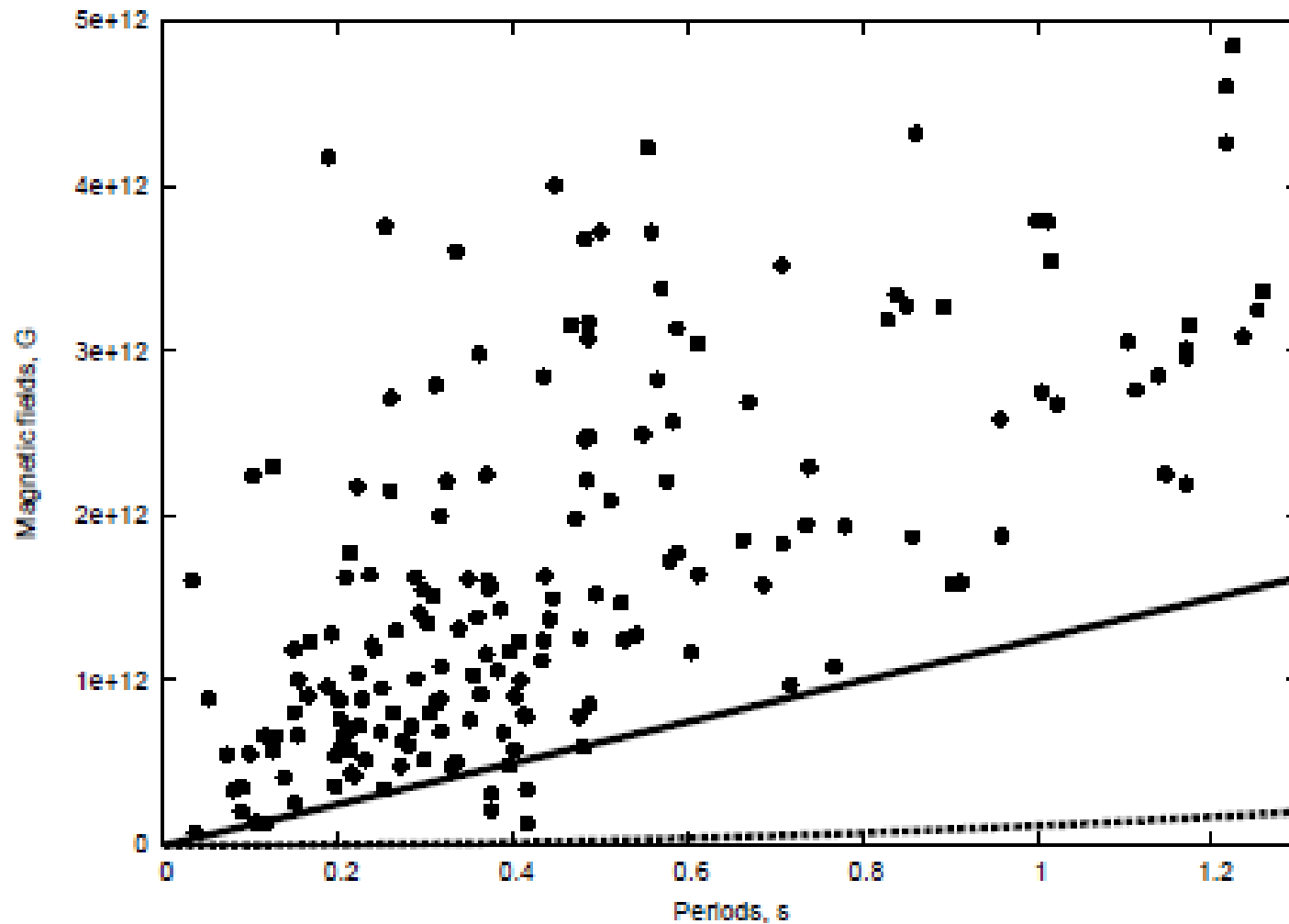
Exponential decay



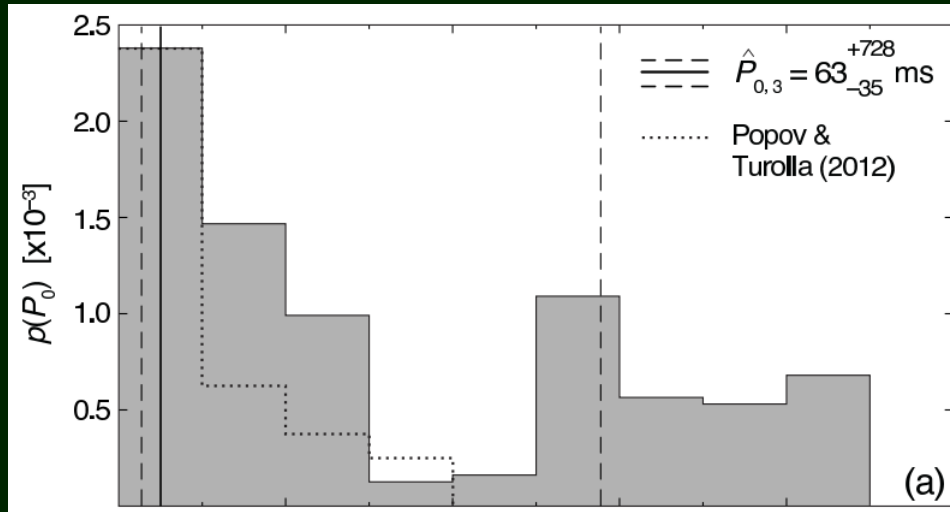
$$P_0 = P \sqrt{1 - \frac{t_{\text{true}}}{\tau}}.$$



# Fitting the field decay



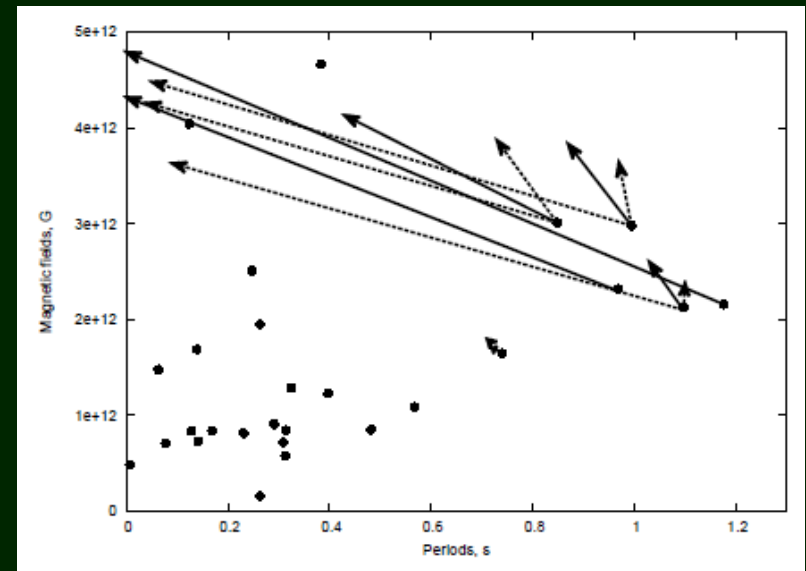
# Another option: emerging field



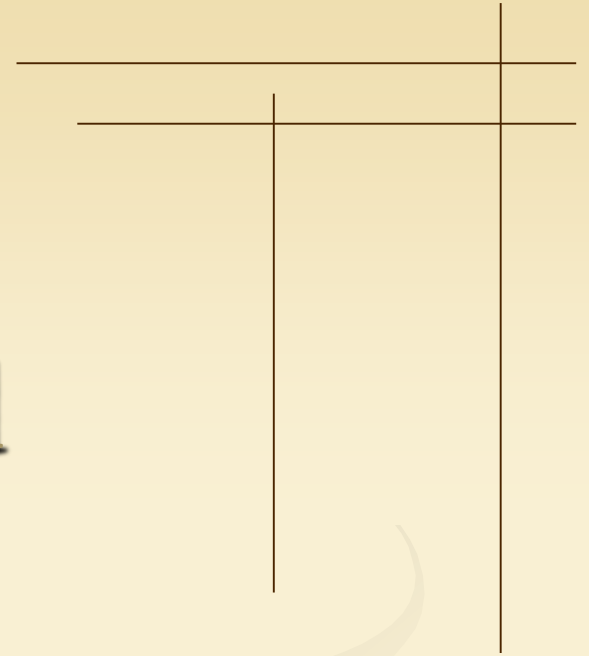
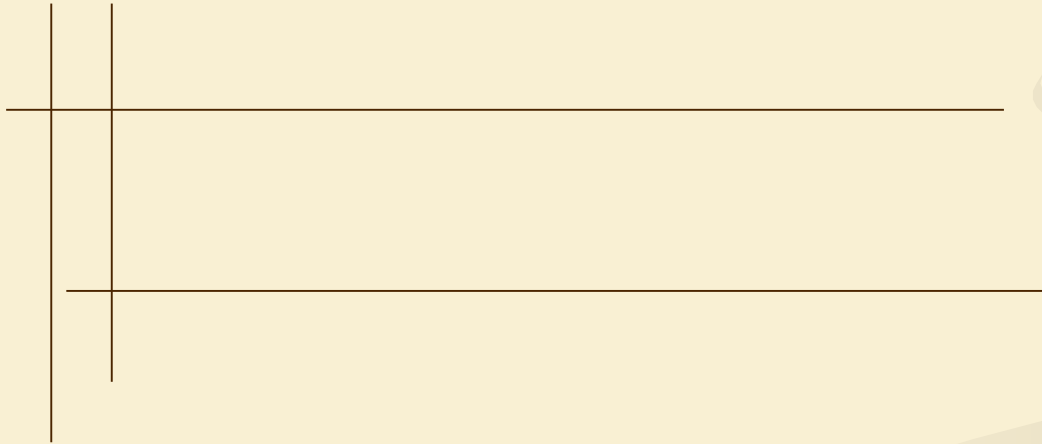
**Yes! Emerging magnetic field!!!**

Then we need correlations between different parameters.

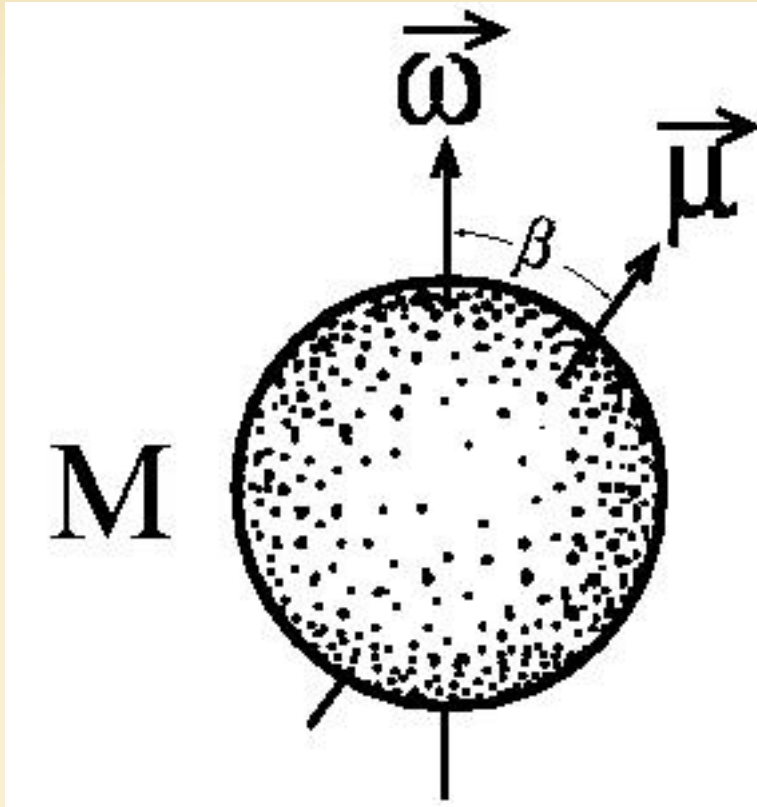
The problem is just with few (6) most long-period NSs. Is it possible to hide them when they are young, and make them visible at the age ~few million years?



# Accreting isolated neutron stars



# Magnetic rotator



Observational appearances of NSs (if we are not speaking about cooling) are mainly determined by  $P$ ,  $\dot{P}$ ,  $V$ ,  $B$ , (also, probably by the inclination angle  $\beta$ ), and properties of the surrounding medium.  $B$  is not evolving significantly in most cases, so it is important to discuss spin evolution.

Together with changes in  $B$  (and  $\beta$ )  
one can speak about

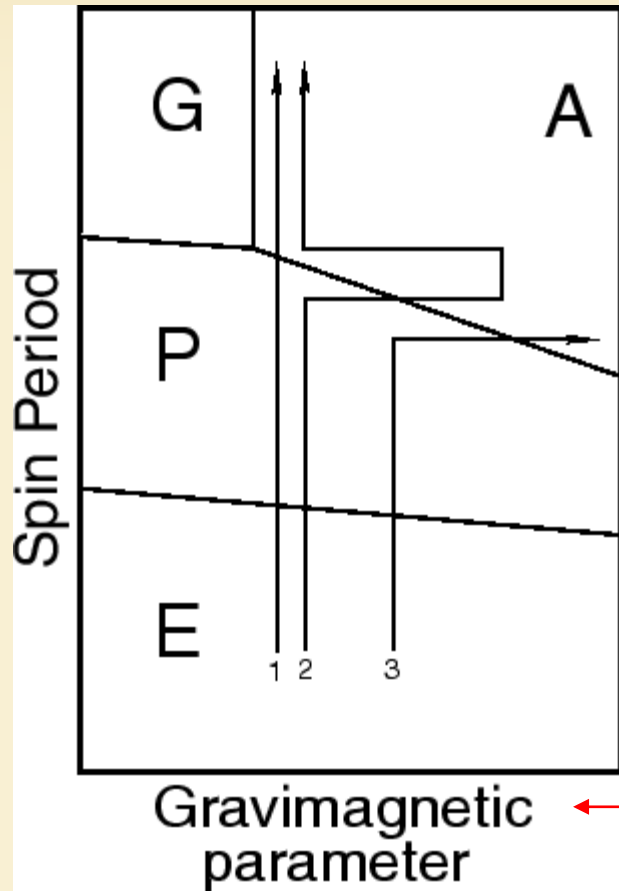
**magneto-rotational evolution**

We are going to discuss the main stages of this evolution, namely:

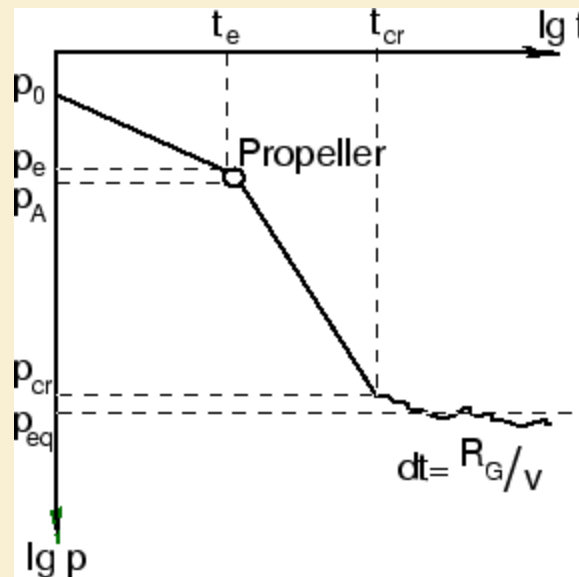
*Ejector*, *Propeller*, *Accretor*, and *Georotator* following the classification by Lipunov

# Evolution of neutron stars: rotation + magnetic field

Ejector → Propeller → Accretor → Georotator



- 1 – spin down
- 2 – passage through a molecular cloud
- 3 – magnetic field decay



[astro-ph/0101031]

$\dot{M}/\mu^2$

See the book by Lipunov (1987, 1992)

# Accreting isolated neutron stars

## Why are they so important?

- Can show us how old NSs look like
  1. Magnetic field decay
  2. Spin evolution
- Physics of accretion at low rates
- NS velocity distribution
- New probe of NS surface and interiors
- ISM probe

# Critical periods for isolated NSs

$$P_E(E \rightarrow P) \simeq 10 \mu_{30}^{1/2} n^{-1/4} v_{10}^{1/2} \text{ s}$$

Transition from Ejector to Propeller (supersonic)

$$t_E \simeq 10^9 \mu_{30}^{-1} n^{-1/2} v_{10} \text{ yr}$$

Duration of the ejector stage

$$P_A(P \rightarrow A) \simeq 420 \mu_{30}^{6/7} n^{-3/7} v_{10}^{9/7} \text{ s}$$

Transition from supersonic Propeller to subsonic Propeller or Accretor

$$P_{eq} = 2.6 \times 10^3 v_{(t)10}^{-2/3} \mu_{30}^{2/3} n^{-2/3} v_{10}^{13/3} \text{ s}$$

A kind of equilibrium period for the case of accretion from turbulent medium

$$v < 410 n^{1/10} \mu_{30}^{-1/5} \text{ km s}^{-1}$$

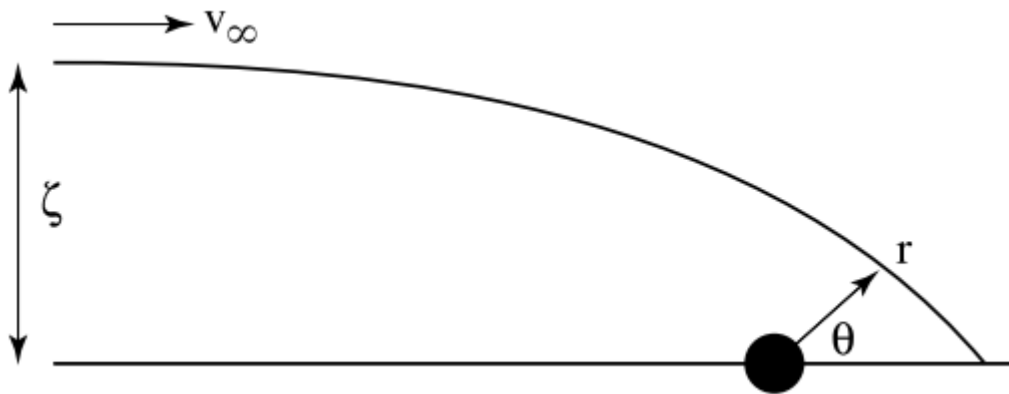
Condition for the Georotator formation (instead of Propeller or Accretor)

(see, for example, astro-ph/9910114)

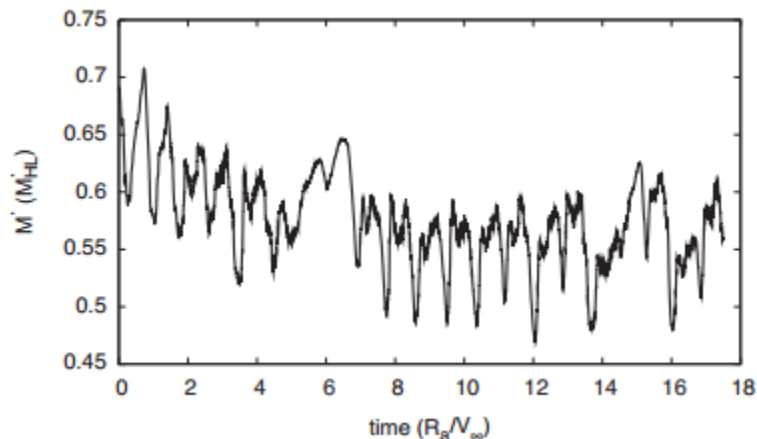


# Accretion dynamics

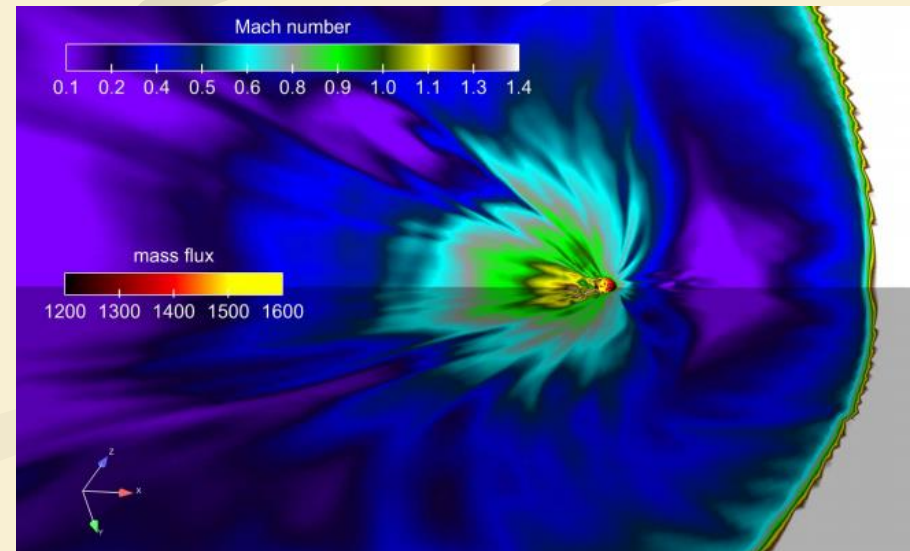
Bondi-Hoyle-Littleton accretion (astro-ph/0406166)



$$\dot{M}_{\text{HL}} = \pi \zeta_{\text{HL}}^2 v_\infty \rho_\infty = \frac{4\pi G^2 M^2 \rho_\infty}{v_\infty^3}$$



Recently BHL accretion for non-magnetized accretors have been studied in 1204.0717. Still, resolution in 3D is not high enough.



# Expected properties

## 1. Accretion rate

An upper limit can be given by the Bondi formula:

$$\dot{M} = \pi R_G^2 \rho v, R_G \sim v^{-2}$$

$$\dot{M} = 10^{11} \text{ g/s } (v/10 \text{ km/s})^{-3} n$$

$$L = 0.1 \dot{M} c^2 \sim 10^{31} \text{ erg/s}$$

However, accretion can be smaller due to the influence of a magnetosphere of a NS  
(see numerical studies by Toropina et al. 1111.2460 ).

## 2. Periods

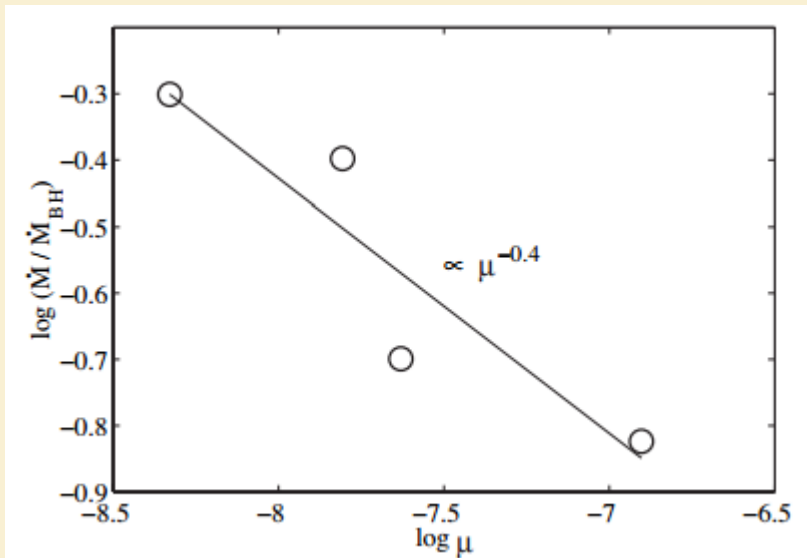
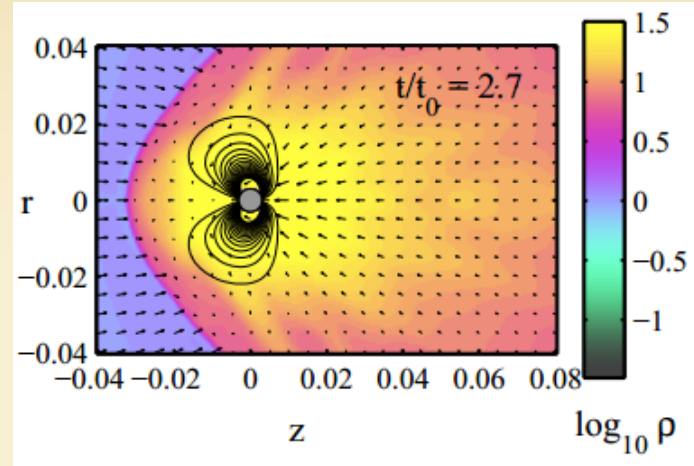
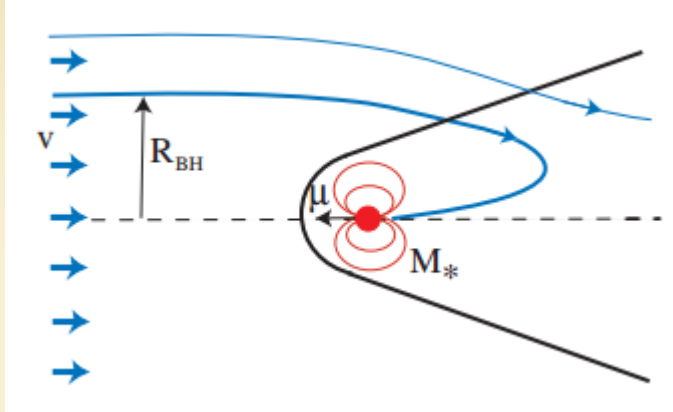
Periods of old accreting NSs are uncertain, because we do not know evolution well enough.

$$p_A = 2^{5/14} \pi (GM)^{-5/7} (\mu^2 / \dot{M})^{3/7} \simeq$$

$$R_A = R_{\text{co}}$$

$$300 \mu_{30}^{6/7} (v/10 \text{ km s}^{-1})^{9/7} n^{-3/7} \text{ s.}$$

# Reduction of the accretion rate



Surface accretion accretion rate can be much reduced due to the presence of large magnetosphere.

# Subsonic propeller

Even after  $R_{\text{co}} > R_A$  accretion can be inhibited.

This have been noted already in the pioneer papers by Davies et al.

Due to rapid (however, subsonic) rotation a hot envelope is formed around the magnetosphere. So, a new critical period appear.

$$P_{\text{br}} \simeq 450 \mu_{30}^{16/21} \dot{M}_{15}^{-5/7} m^{-4/21} \text{ s.}$$

(Ikhsanov astro-ph/0310076)

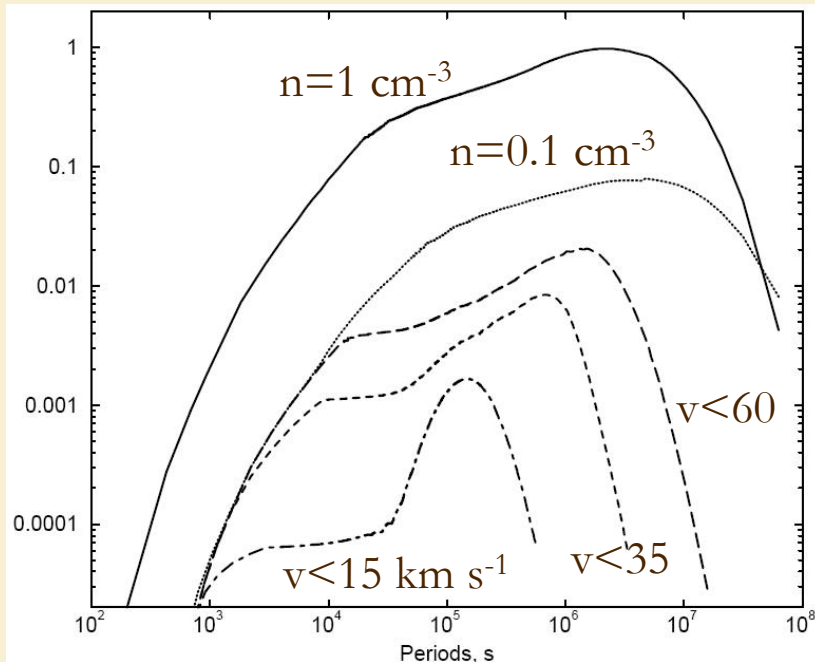
If this stage is realized (inefficient cooling) then

- accretion starts later
- accretors have longer periods

# Equilibrium period

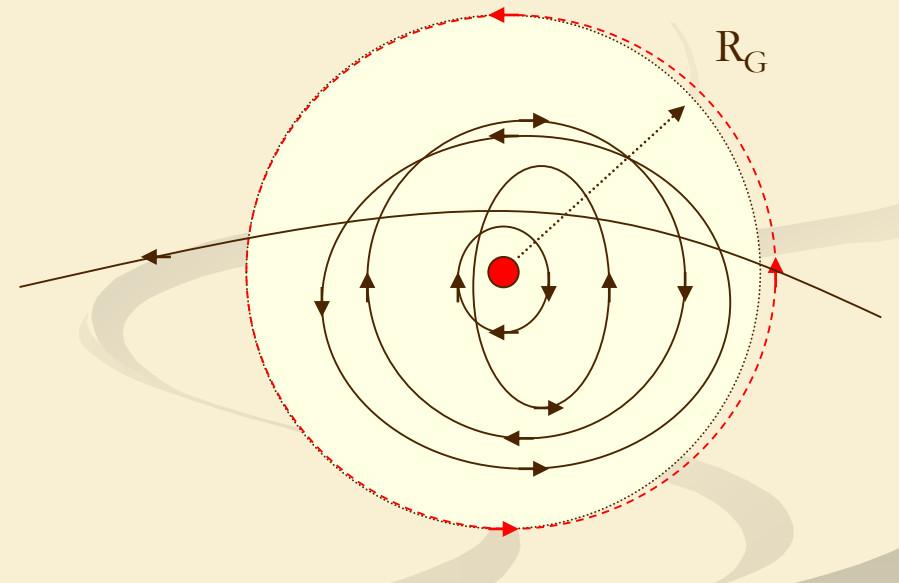
Interstellar medium is turbulized. If we put a non-rotating NS in the ISM, then because of accretions of turbulized matter it'll start to rotate.

This clearly illustrates, that a spinning-down accreting isolated NS in a realistic ISM should reach some equilibrium period.



[A&A 381, 1000 (2002)]

$$P_{eq} = 2.6 \times 10^3 v_{(t)10}^{-2/3} \mu_{30}^{2/3} n^{-2/3} v_{10}^{13/3} \text{ s}$$



A kind of equilibrium period for the case of accretion from turbulent medium

# Disc formation

Accretion of turbulized matter can result in a transient accretion disc formation.

$$\dot{j}_t = v_t(R_G) \cdot R_G = v_t(R_t) R_t^{-1/3} R_G^{4/3}$$

If  $\dot{j}_t$  is larger than the keplerian momentum at the magnetospheric boundary (Alfven radius) then a disc can be formed.

$$\dot{j}_K = v_K(R_A) \cdot R_A$$

Might happen for low magnetic fields  
and low spatial velocities.

More prominent for accreting isolated BHs.

# Expected properties-2

## 3. Temperatures

Depend on the magnetic field. The size of polar caps depends on the field and accretion rate:  $\sim R (R/R_A)^{1/2}$

## 4. Magnetic fields

Very uncertain, as models of the field decay cannot give any solid predictions for very long time scales (billions of years).

## 5. Flux variability.

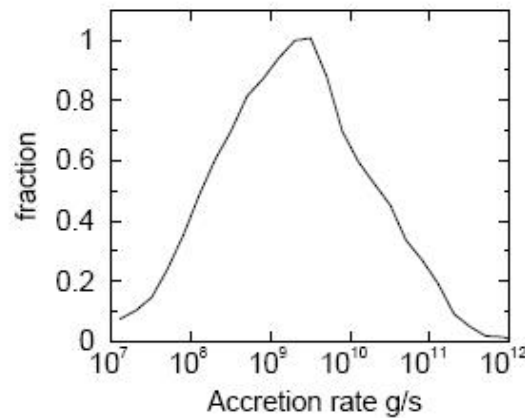
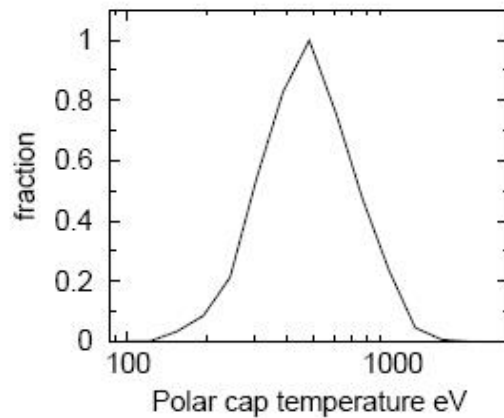
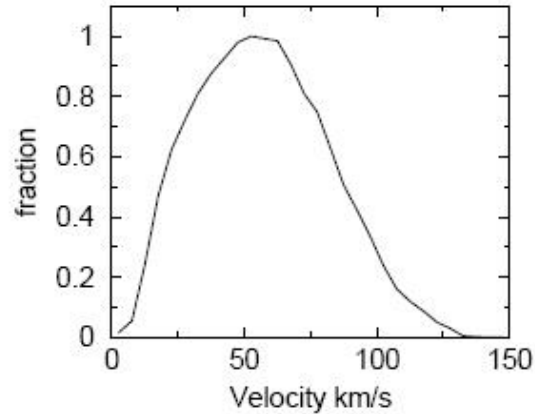
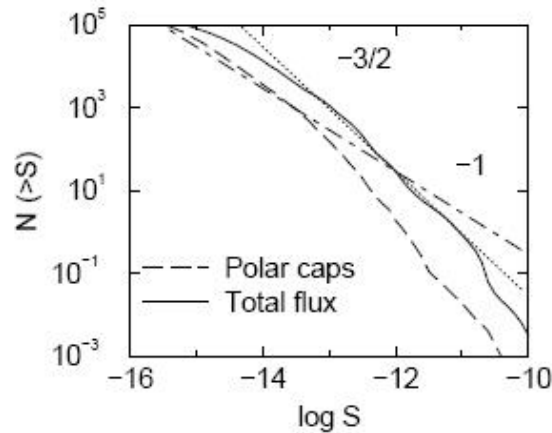
Due to fluctuations of matter density and turbulent velocity in the ISM it is expected that isolated accretors are variable on a time scale  $\sim R_G/v \sim \text{days} - \text{months}$

---

Still, isolated accretors are expected to be numerous at low fluxes (their total number in the Galaxy is large than the number of coolers of comparable luminosity). They should be hotter than coolers, and have much longer spin periods.



# Properties of accretors



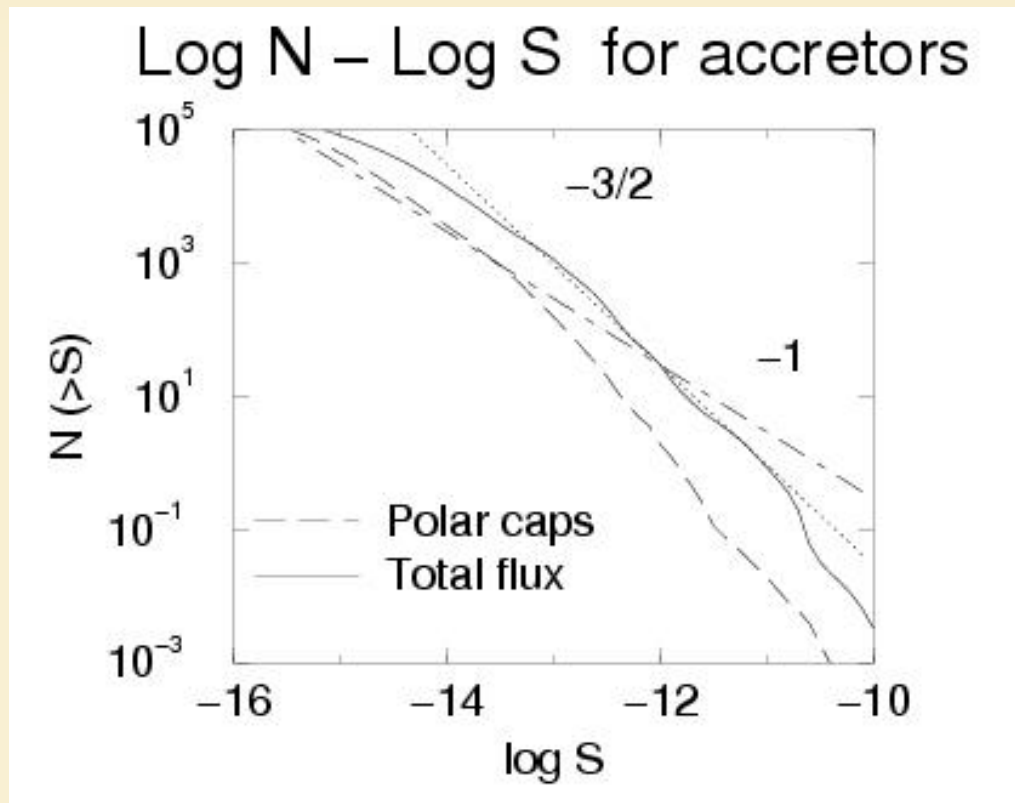
In the framework of a simplified model (no subsonic propeller, no field decay, no accretion inhibition, etc.) one can estimate properties of isolated accretors.

Slow, hot, dim,  
numerous at low fluxes  
( $< 10^{-13}$  erg/cm<sup>2</sup>/s)

Reality is more uncertain.

# Accreting isolated NSs

At small fluxes  $<10^{-13}$  erg/s/cm<sup>2</sup> accretors can become more abundant than coolers. Accretors are expected to be slightly harder: 300-500 eV vs. 50-100 eV. Good targets for eROSITA!



From several hundreds up to several thousands objects at fluxes about  $\text{few} \cdot 10^{-14}$ , but difficult to identify.

Monitoring is important.

Also isolated accretors can be found in the Galactic center (Zane et al. 1996, Deegan, Nayakshin 2006).

# Where and how to look for

**As sources are dim even in X-rays,  
and probably are extremely dim in other bands  
it is very difficult to find them.**

In an optimistic scenario they outnumber cooling NSs at low fluxes.  
Probably, for ROSAT they are too dim.  
We hope that eROSITA will be able to identify accreting INSs.

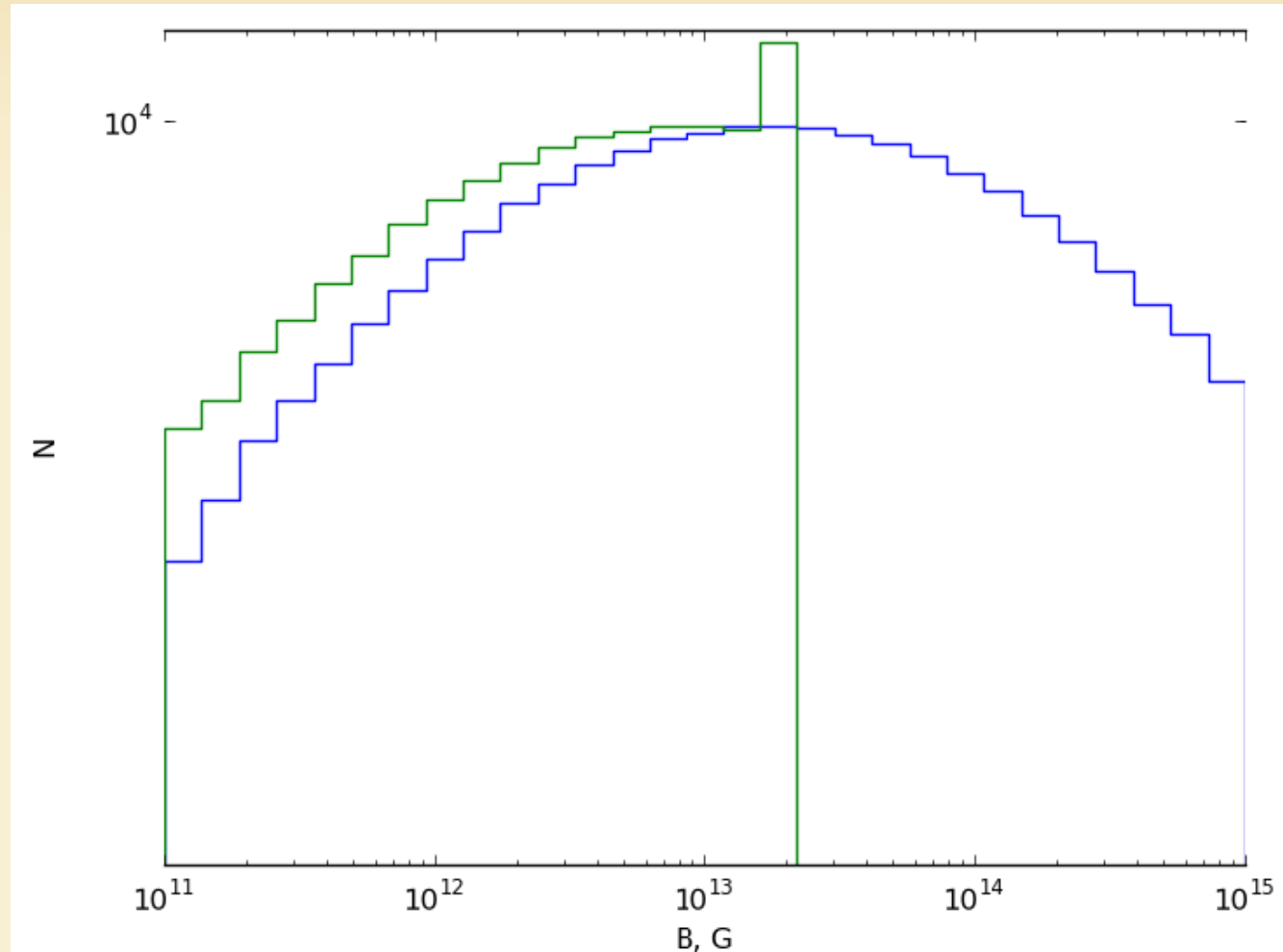
Their spatial density at fluxes  $\sim 10^{-15}$  erg/cm<sup>2</sup>/s is expected to be  $\sim$ few per sq.degree  
in directions close to the galactic plane.

It is necessary to have an X-ray survey at  $\sim 100$ -500 eV with good resolution.

In a recent paper by Munro et al. the authors put interesting limits on the  
number of unidentified magnetars. The same results can be rescaled to  
give limits on the M7-like sources.

# “Decayed” field distribution

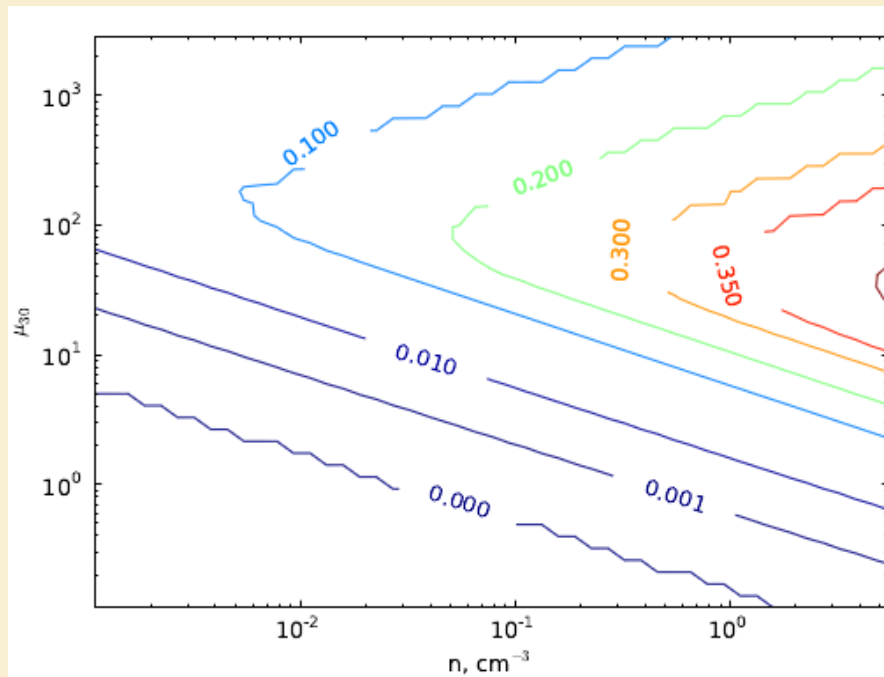
We assume the field to be constant, but as an initial we use the “decayed” distribution, following Popov et al. 2010.



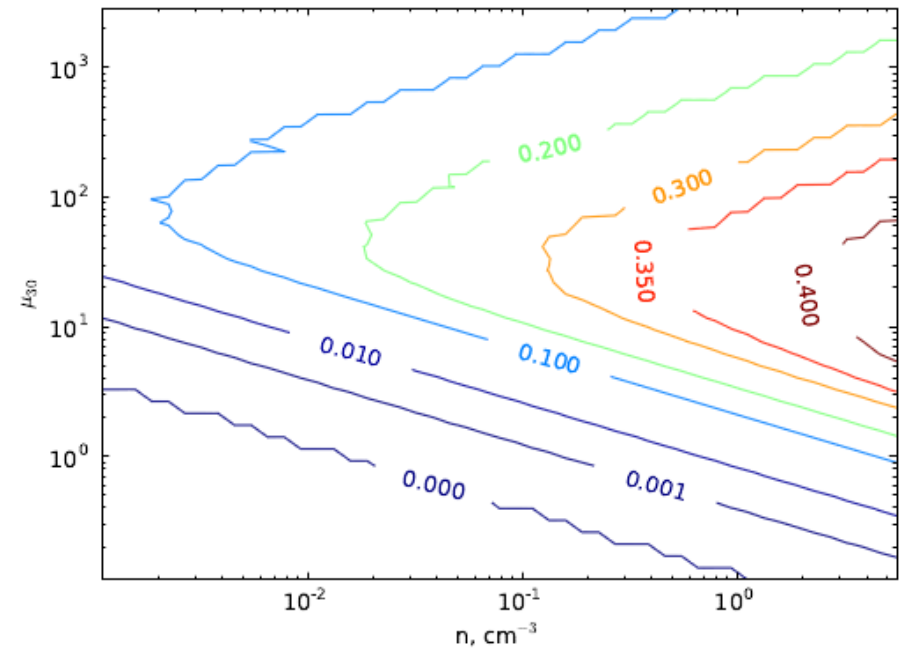
# Simple semianalytical model

Fraction of accretors for different magnetic fields and ISM density.

Kick velocity distribution is taken following Arzoumanian et al. (2002).



With subsonic



Without subsonic

# Individual tracks

Individual tracks in the semianalytical model.

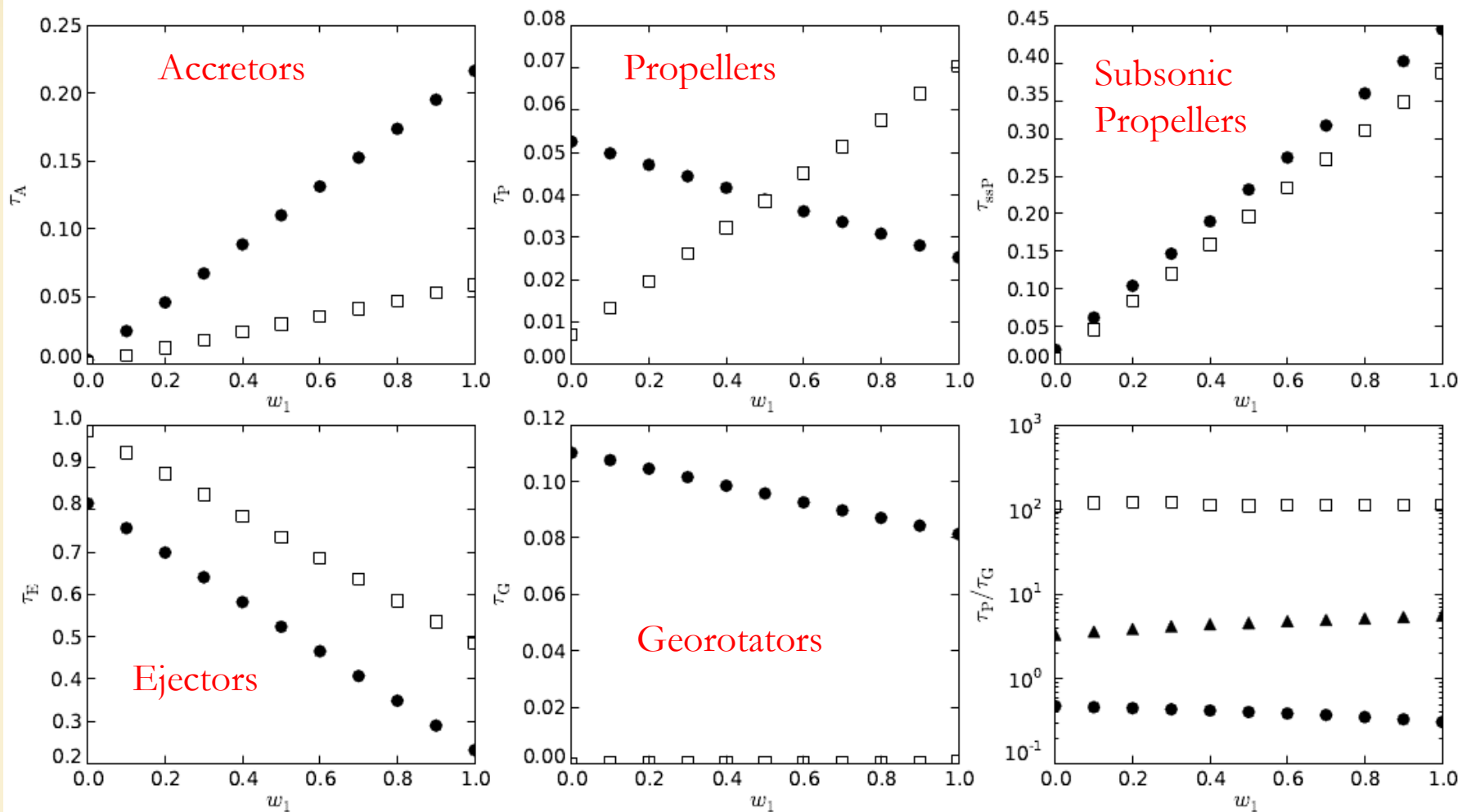
Clearly, even with long subsonic propeller stage highly magnetized NSs (like the M7) can become accretors relatively soon.

Track	$n, \text{cm}^{-3}$	$\mu_{30}$	$v_{10}$	$\tau_E$	$P_E, \text{s}$	$\tau_P$	$P_P, \text{s}$	$\tau_{\text{ssP}}$	$P_{\text{break}}, \text{s}$
Track I	0.5	1	5	0.419	16.051	0.423	$3.163 \times 10^3$	0.850	$2.278 \times 10^6$
Track II	0.5	1	20	—	—	—	—	—	—
Track III	0.5	1	40	—	—	—	—	—	—
Track IV	0.5	10	5	0.042	50.758	0.042	$2.276 \times 10^4$	0.067	$1.317 \times 10^7$
Track V	0.5	10	20	0.168	101.517	0.170	$1.353 \times 10^5$	0.651	$2.568 \times 10^8$
Track VI	0.5	10	40	0.163	100.091	0.169	$1.523 \times 10^5$	Georotator	
Track VII	2.0	1	5	0.209	11.350	0.212	$1.746 \times 10^3$	0.370	$8.464 \times 10^5$
Track VIII	2.0	1	20	0.838	22.700	0.854	$1.038 \times 10^4$	—	—
Track IX	2.0	1	40	—	—	—	—	—	—
Track X	2.0	10	5	0.021	35.892	0.021	$1.257 \times 10^4$	0.030	$4.892 \times 10^6$
Track XI	2.0	10	20	0.084	71.783	0.085	$7.469 \times 10^4$	0.264	$9.541 \times 10^7$
Track XII	2.0	10	40	0.103	79.442	0.106	$1.077 \times 10^5$	Georotator	

# Final distributions

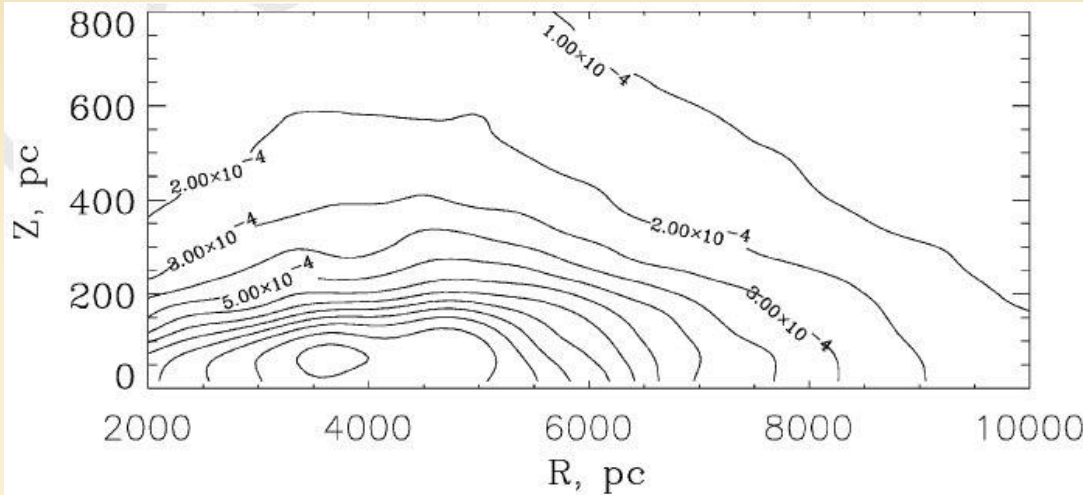
Filled symbols – “decayed distribution”.

Open squares – delta-function  $\mu_{30}=1$ .





# Spatial density of NSs

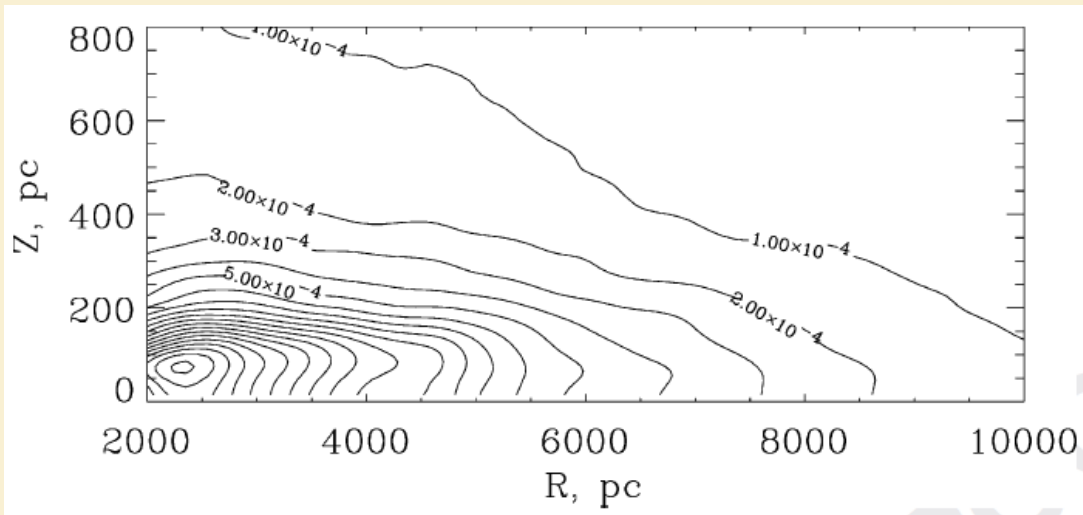


In both models  $N = 5 \times 10^8$ .

Kick: ACC02.

Potential: Paczynski 1990

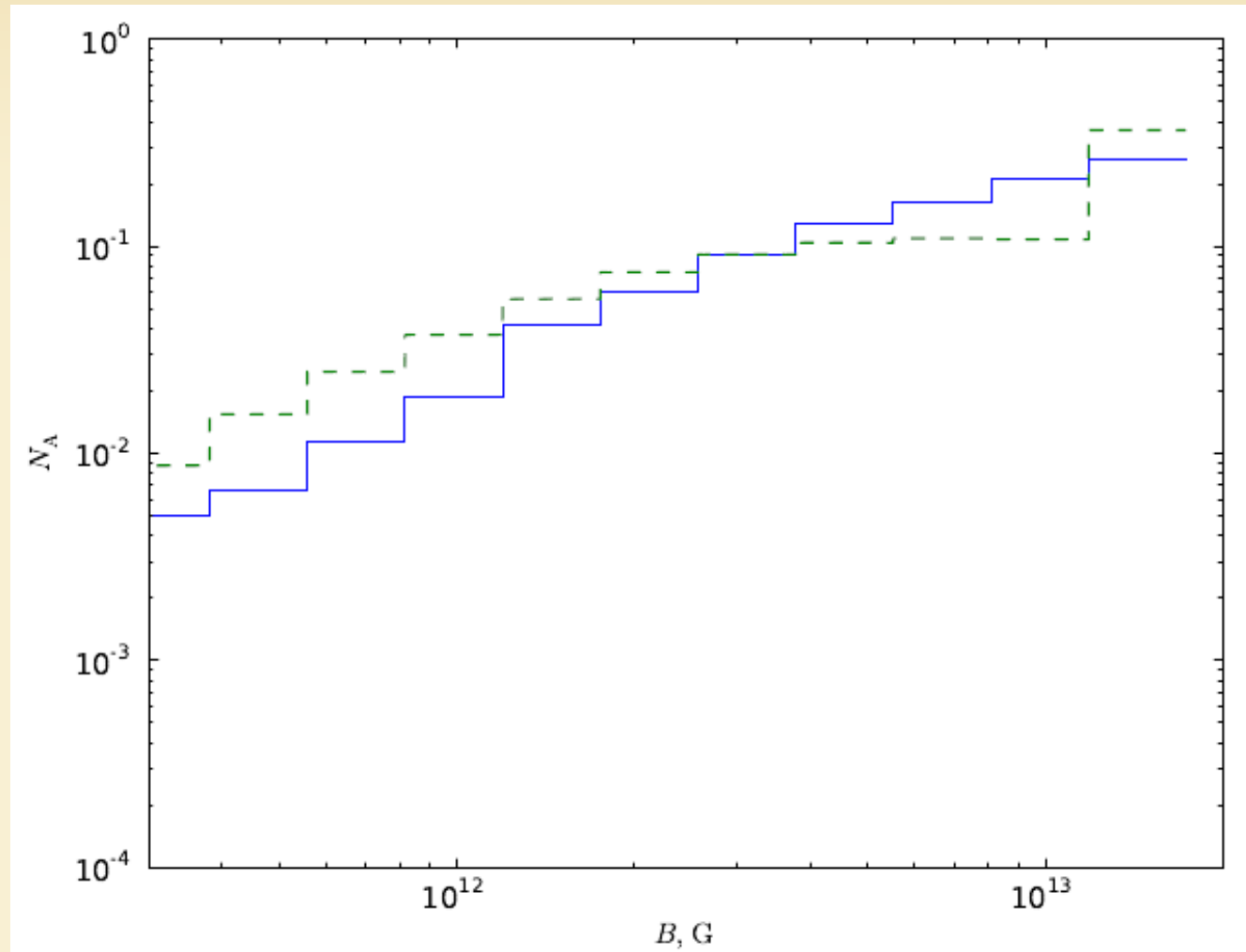
NS formation rate is assumed to be proportional to the square of the ISM density at the birthplace.



Formation rate is proportional to  $[\exp(-z/75 \text{ pc}) \exp(-R/4 \text{ kpc})]$ .

# Who forms accretors?

NSs with stronger fields form more accretors, unless their field *and* velocities are so high, that they become Georotators.

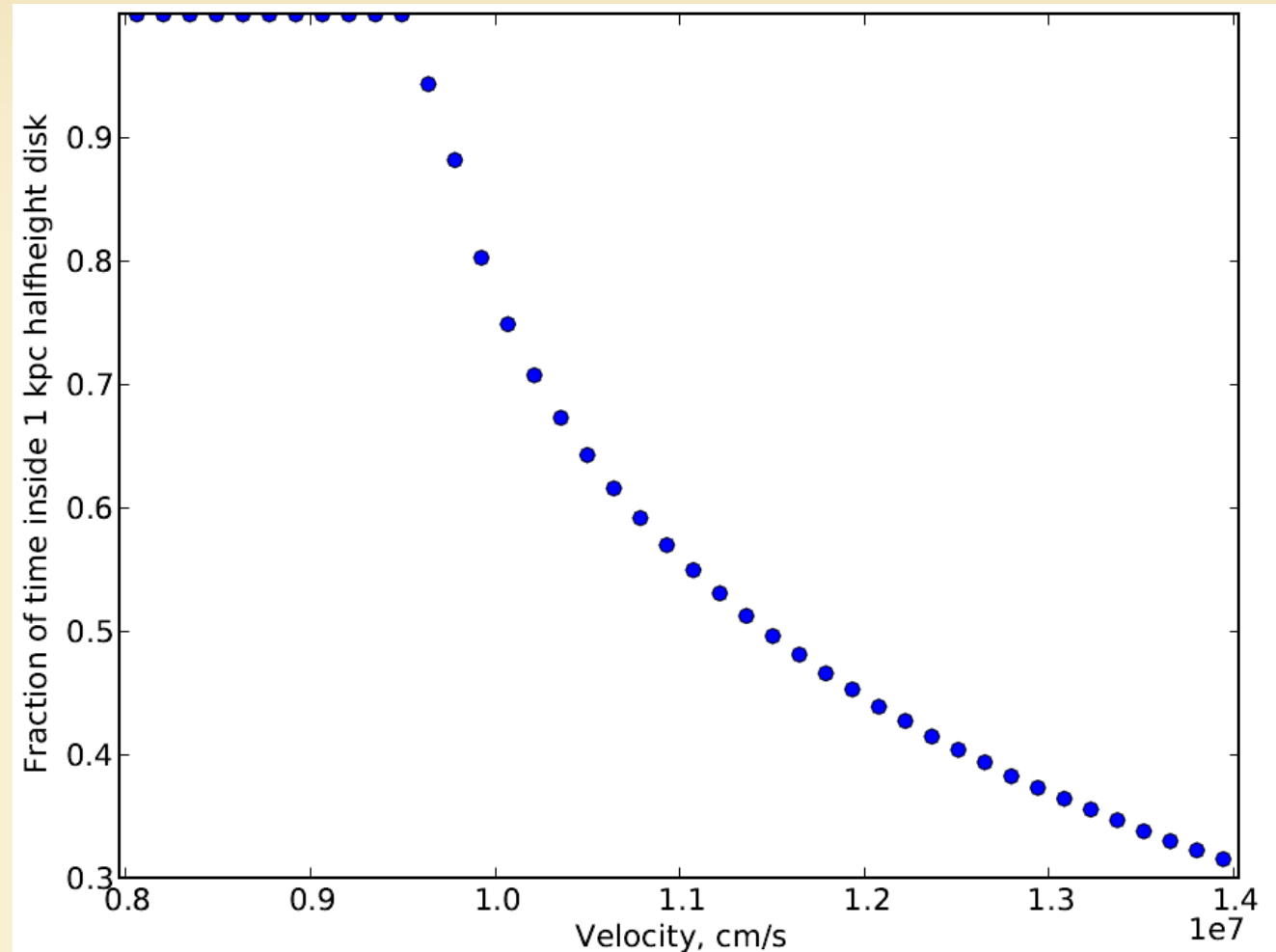


# Running out of the Galaxy

2/3 of NSs leave the Galaxy.  
Mostly, they stay as Ejectors, or become Georotators.

In the solar vicinity fractions of INSs at different evolutionary stages are:

- Ejectors: 18-20%
- Propellers: negligible
- subsonic P.: 40-45%
- Accretors: 35-40%
- Georotators: negligible



# Some conclusions

- Highly magnetized INS (as the M7)  
can become Accretors even taking  
into account long subsonic Propeller stage.
- In the solar vicinity fractions of INSs at  
different evolutionary stages are:
  - Ejectors: 18-20%
  - Propellers: negligible
  - subsonic P.: 40-45%
  - Accretors: 35-40%
  - Georotators: negligible

# Settling accretion onto INs

At low X-ray luminosities the captured matter, heated in the bow-shock, has no time to cool down and remains hot, which prevents it from entering the NS magnetosphere via Rayleigh-Taylor (RT) instability.

$$\dot{M}_x \simeq (t_{\text{ff}}/t_{\text{cool}})^{1/3} \dot{M}_B$$

$$\dot{M}_B = \rho_\infty v (\pi R_G^2) \sim 1.9 \times 10^9 n v_7^{-3} \text{ g s}^{-1}$$

$$t_{\text{ff}} = R^{3/2} / \sqrt{GM}.$$

$$t_{\text{cool}} = 3 \times 10^8 \left( \frac{R_A}{10^{10} \text{ cm}} \right) \times$$

$$\left( \frac{L_x}{10^{30} \text{ erg s}^{-1}} \right)^{-1} \frac{f(u)}{0.01} (1 + X)^{-1} \text{ s.}$$

$$f(u) = u_r / u_{\text{ff}} < 1$$

$$R_A \approx 2.2 \times 10^{10} L_{30}^{-2/9} \mu_{30}^{16/27} \text{ cm.}$$

Thus, steady accretion luminosity is expected to be low, however, flares with duration hours-day are possible. Maximum luminosity can be  $\sim 10^{31}$  erg/s.

# Papers to read

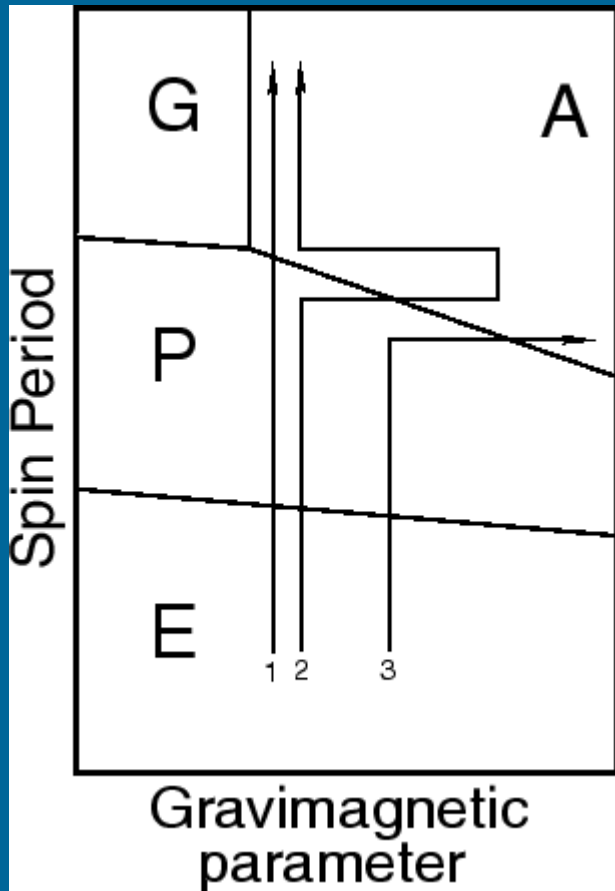
- Treves et al. PASP 112, 297 (2000)
- Popov et al. ApJ 530, 896 (2000)
- Popov, Prokhorov Physics Uspekhi 50, 1123 (2007) Ch. 5.4
- Boldin, Popov MNRAS vol. 407, pp. 1090-1097 (2010)
- Edgar astro-ph/0406166
- Popov et al. MNRAS 487, 2817 (2015)

# Thermal evolution of neutron stars

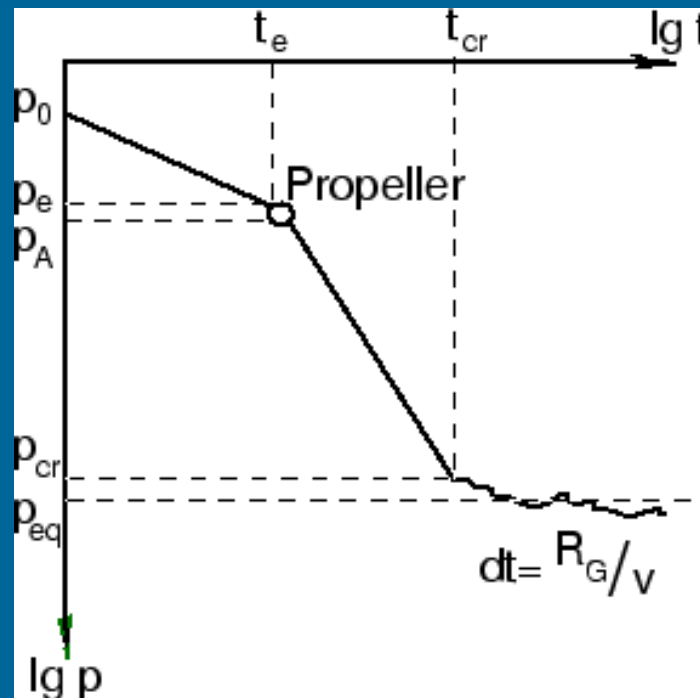


# Evolution of neutron stars. I.: rotation + magnetic field

Ejector → Propeller → Accretor → Georotator



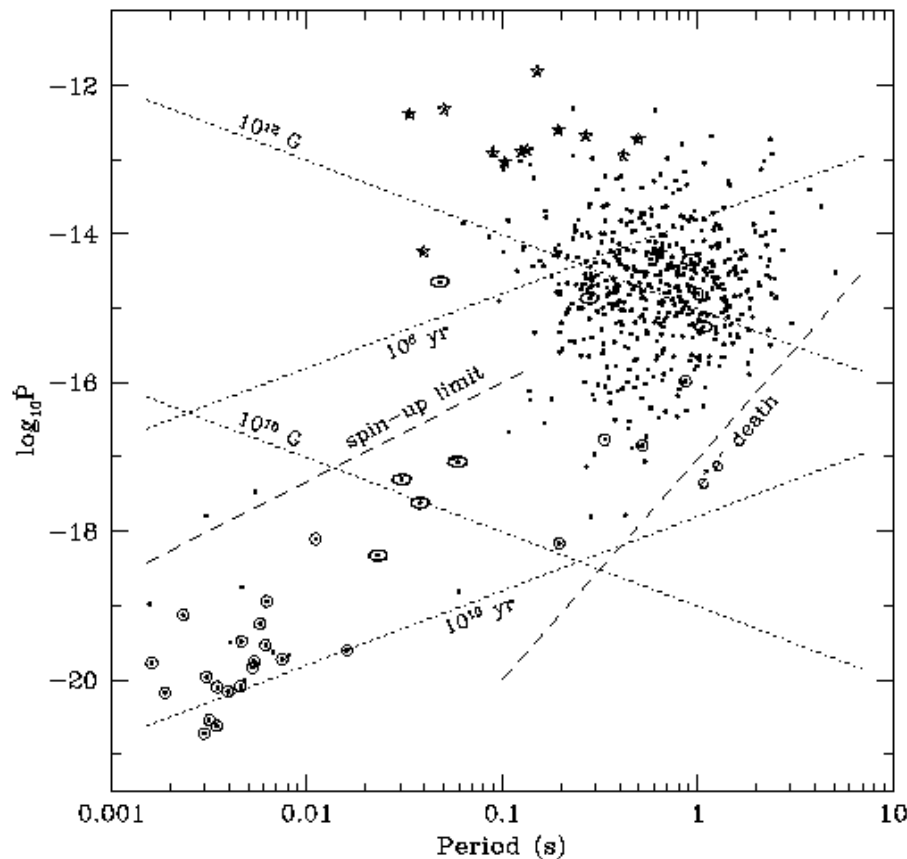
- 1 – spin down
- 2 – passage through a molecular cloud
- 3 – magnetic field decay



astro-ph/0101031

See the book by Lipunov (1987, 1992)

# Magnetorotational evolution of radio pulsars



$$L_m = \frac{2}{3} \frac{\mu^2 \omega^4}{c^3} \sin^2 \beta = \kappa_t \frac{\mu^2}{R_l^3} \omega,$$

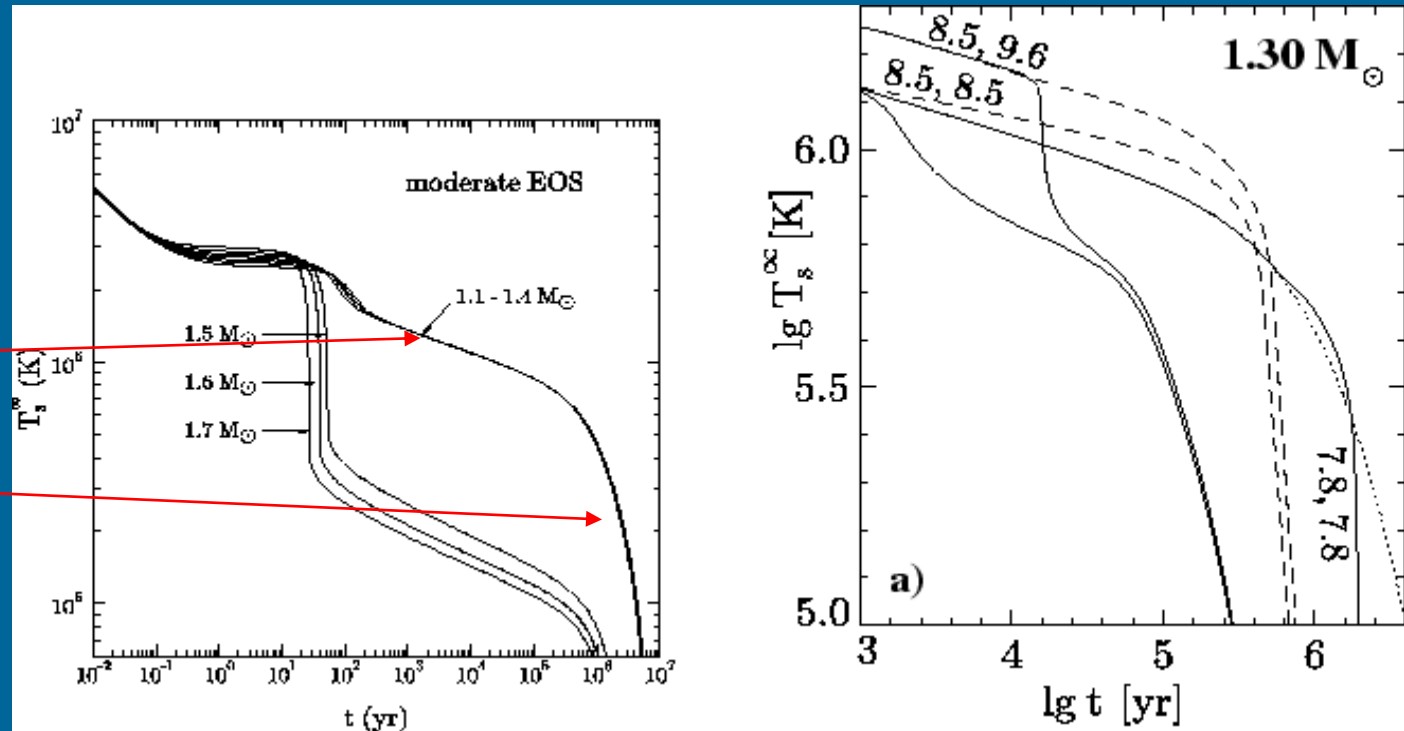
$$B \sim 3.2 \times 10^{19} (P dP/dt)^{1/2} \text{ G.}$$

Spin-down.  
Rotational energy is released.  
The exact mechanism is still unknown.

# Evolution of NSs. II.: temperature

Neutrino  
cooling stage

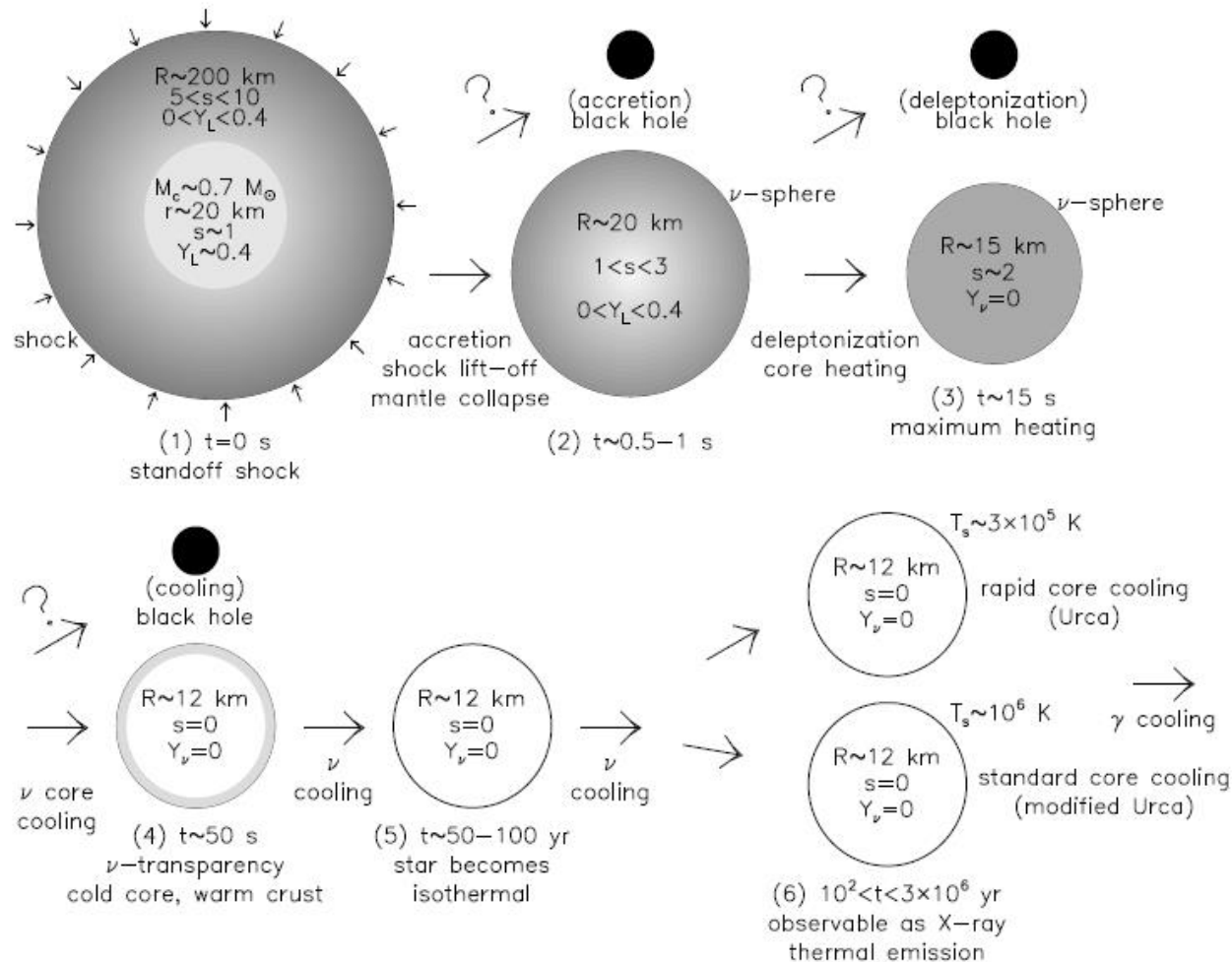
Photon  
cooling stage



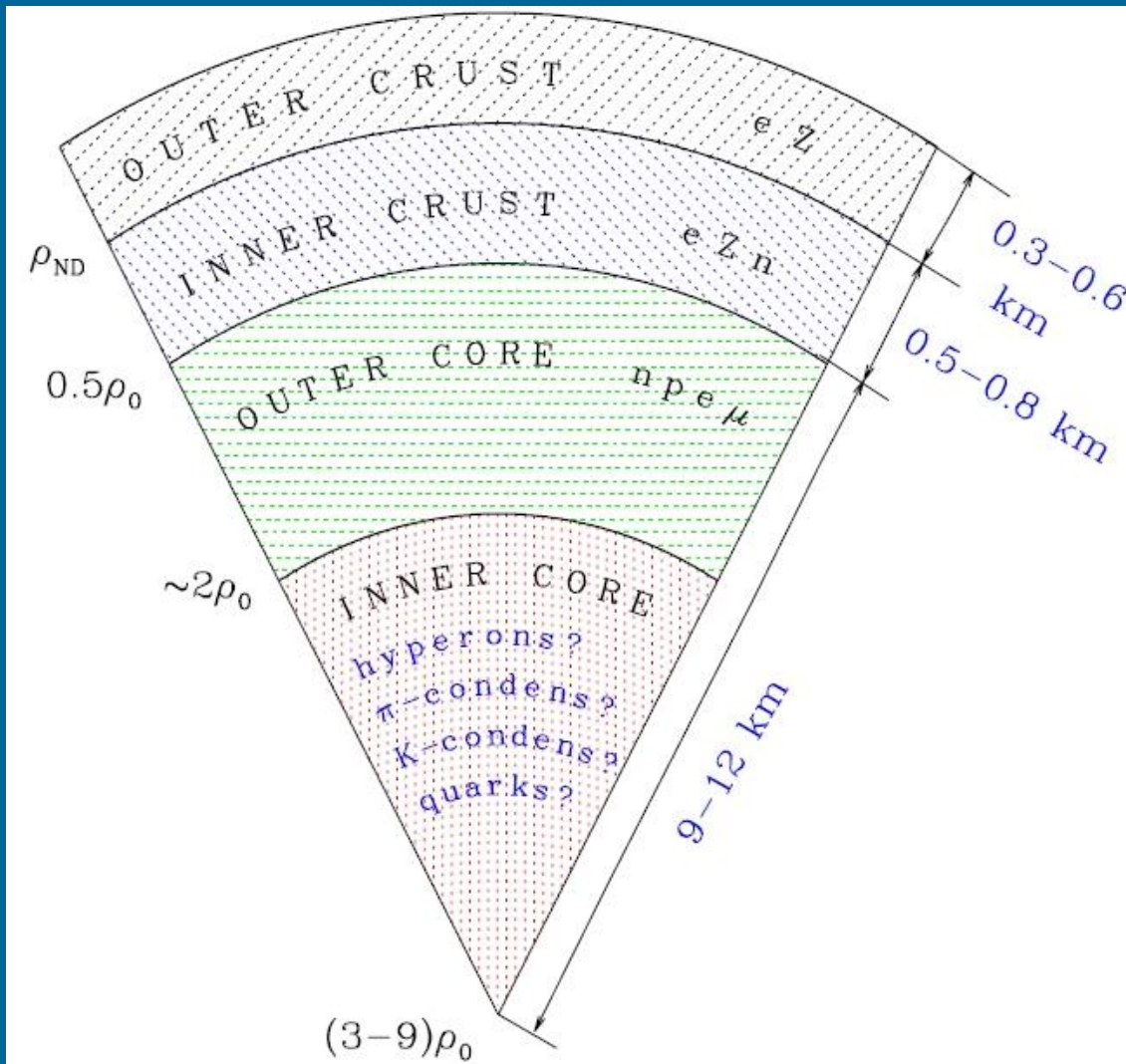
First papers on the thermal evolution appeared already in early 60s, i.e. before the discovery of radio pulsars.

[Yakovlev et al. (1999) Physics Uspekhi]

# Early evolution of a NS



# Structure and layers



Plus an atmosphere...

See Ch.6 in the book by Haensel, Potekhin, Yakovlev

$$\rho_0 \sim 2.8 \cdot 10^{14} \text{ g cm}^{-3}$$

The total thermal energy of a nonsuperfluid neutron star is estimated as  $U_T \sim 10^{48} T_9^2 \text{ erg}$ .

The heat capacity of an  $npe$  neutron star core with strongly superfluid neutrons and protons is determined by the electrons, which are not superfluid, and it is  $\sim 20$  times lower than for a neutron star with a nonsuperfluid core.

# NS Cooling

- NSs are born very hot,  $T > 10^{10}$  K
- At early stages neutrino cooling dominates (exotic is possible – axions 1205.6940)
- The core is isothermal

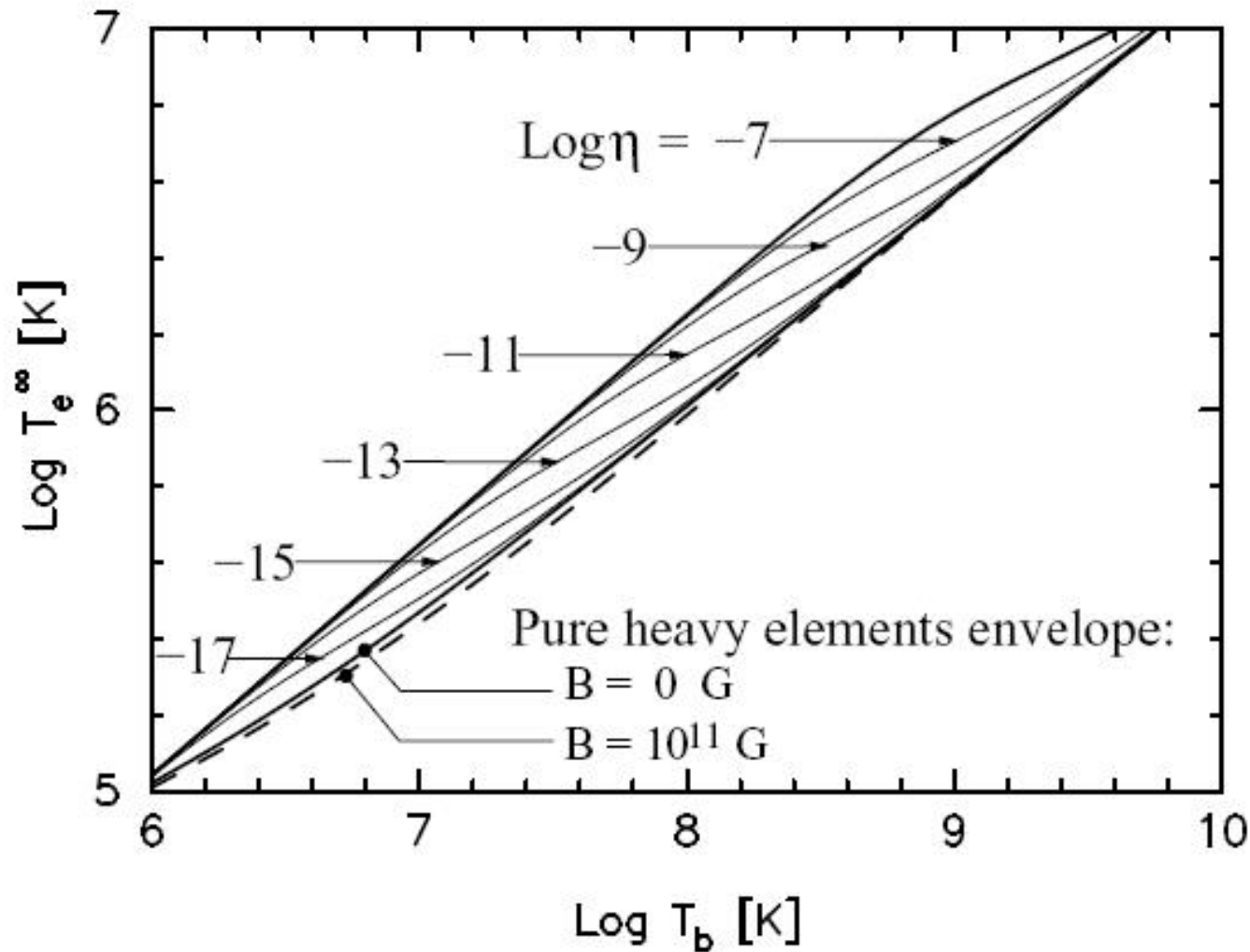
$$\frac{dE_{th}}{dt} = C_V \frac{dT}{dt} = -L_\nu - L_\gamma$$

Photon luminosity

Neutrino luminosity

$$L_\gamma = 4\pi R^2 \sigma T_s^4, \quad T_s \propto T^{1/2+\alpha} \quad (|\alpha| \ll 1)$$

# Core-crust temperature relation



Heat blanketing envelope.  
~100 meters  
density  $\sim 10^{10} \text{ gcm}^{-3}$

See a review about  
crust properties  
related to thermal  
evolution in  
1201.5602 and  
1507.06186



# Cooling depends on:

- 1. Rate of neutrino emission from NS interiors
  - 2. Heat capacity of internal parts of a star
  - 3. Superfluidity
  - 4. Thermal conductivity in the outer layers
  - 5. Possible heating
- } Depend on the EoS and composition

# Main neutrino processes

Model	Process	$Q_f, \text{ erg cm}^{-3} \text{ s}^{-1}$
Nucleon matter	$n \rightarrow pe\bar{\nu} \quad pe \rightarrow n\nu$	$10^{26} - 3 \times 10^{27}$
Pion condensate	$\tilde{N} \rightarrow \tilde{N}e\bar{\nu} \quad \tilde{N}e \rightarrow \tilde{N}\nu$	$10^{23} - 10^{26}$
Kaon condensate	$\tilde{B} \rightarrow \tilde{B}e\bar{\nu} \quad \tilde{B}e \rightarrow \tilde{B}\nu$	$10^{23} - 10^{24}$
Quark matter	$d \rightarrow ue\bar{\nu} \quad ue \rightarrow d\nu$	$10^{23} - 10^{24}$

Process	$Q_s, \text{ erg cm}^{-3} \text{ s}^{-1}$
Modified Urca $nN \rightarrow pNe\bar{\nu} \quad pNe \rightarrow nN\nu$	$10^{20} - 3 \times 10^{21}$
Bremsstrahlung $NN \rightarrow NN\nu\bar{\nu}$	$10^{19} - 10^{20}$

$$Q_{\text{slow}} = Q_s T_9^8, \quad Q_{\text{fast}} = Q_f T_9^6.$$

(Yakovlev & Pethick astro-ph/0402143)

## Fast Cooling (URCA cycle)

$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$p + e^- \rightarrow n + \nu_e$$

## Slow Cooling (modified URCA cycle)

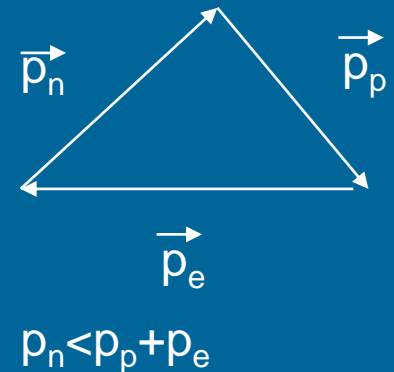
$$n + n \rightarrow n + p + e^- + \bar{\nu}_e$$

$$n + p + e^- \rightarrow n + n + \nu_e$$

$$p + n \rightarrow p + p + e^- + \bar{\nu}_e$$

$$p + p + e^- \rightarrow p + n + \nu_e$$

- Fast cooling possible only if  $n_p > n_n/8$
- Nucleon Cooper pairing is important
- Minimal cooling scenario (Page et al 2004):
  - no exotica
  - no fast processes
  - pairing included



[See the book Haensel, Potekhin, Yakovlev p. 265 (p.286 in the file)  
and Shapiro, Teukolsky for details: Ch. 2.3, 2.5, 11.]

# Equations

Neutrino emissivity

heating

$$\frac{e^{-\lambda-2\Phi}}{4\pi r^2} \frac{\partial}{\partial r} \left( e^{2\Phi} L_r \right) = -Q + Q_h - \frac{c_T}{e^\Phi} \frac{\partial T}{\partial t},$$

$$\frac{L_r}{4\pi \kappa T^2} = e^{-\lambda-\Phi} \frac{\partial}{\partial r} \left( T e^\Phi \right),$$

After thermal relaxation  
we have in the whole star:  
 $T_i(t) = T(r,t) e^{\Phi(r)}$

$$e^{-\lambda} = \sqrt{1 - 2Gm(r)/c^2 r},$$

At the surface we have:  $\Phi(R) = -\lambda(R)$

$$C(T_i) \frac{dT_i}{dt} = -L_\nu^\infty(T_i) + L_h^\infty - L_\gamma^\infty(T_s),$$

$$L_\nu^\infty(T_i) = \int dV Q(T) e^{2\Phi}, \text{ and } L_h^\infty = \int dV Q_h e^{2\Phi}, \quad C(T_i) = \int dV c_T(T),$$

$dV = 4\pi r^2 e^\lambda dr$  is the element of proper volume

$L_\nu^\infty$  is the total neutrino luminosity (for a distant observer)

$L_h^\infty$  is the total reheating power.

(Yakovlev & Pethick 2004)

Total stellar heat capacity

# Simplified model of a cooling NS

No superfluidity, no envelopes and magnetic fields, only hadrons.

**The most critical moment is the onset of direct URCA cooling.**

$$\rho_D = 7.851 \cdot 10^{14} \text{ g/cm}^3.$$

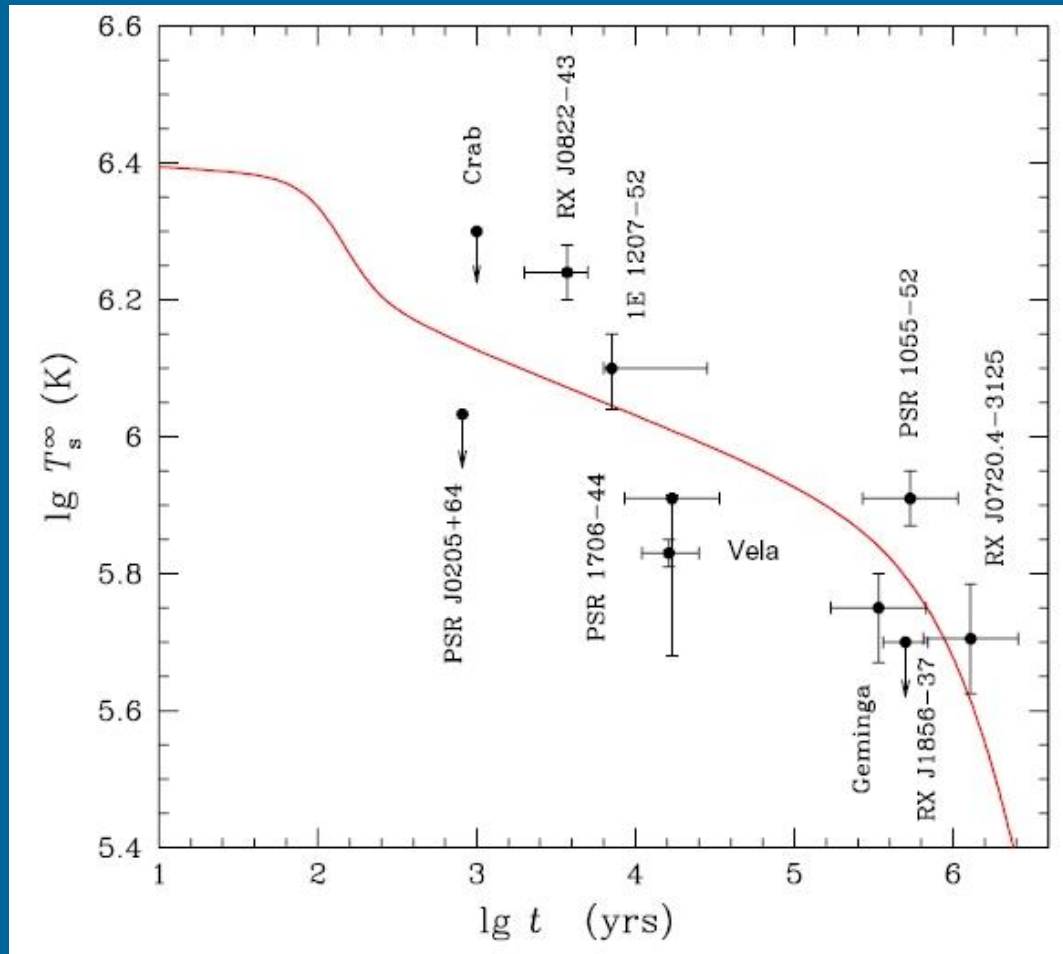
The critical mass  
depends on the EoS.  
For the examples below  
 $M_D = 1.358 M_{\text{solar}}$

$M$ ( $M_{\odot}$ )	$R$ (km)	$\rho_c$ ( $10^{14}$ $\text{g cm}^{-3}$ )	$M_{\text{crust}}$ ( $M_{\odot}$ )	$\Delta R_{\text{crust}}$ (km)	$\Delta M_D$ ( $M_{\odot}$ )	$R_D$ (km)
1.1	13.20	6.23	0.069	1.98	...	...
1.2	13.13	6.80	0.063	1.77	...	...
1.3	13.04	7.44	0.057	1.58	...	...
1.358 <sup>a</sup>	12.98	7.85	0.054	1.48	0.000	0.00
1.4	12.93	8.17	0.052	1.40	0.023	2.40
1.5	12.81	9.00	0.049	1.26	0.137	4.27
1.6	12.64	10.05	0.042	1.10	0.306	5.51
1.7	12.43	11.39	0.035	0.96	0.510	6.41
1.8	12.16	13.22	0.030	0.84	0.742	7.10
1.9	11.73	16.33	0.023	0.69	1.024	7.65
1.977 <sup>b</sup>	10.75	25.78	0.011	0.45	1.400	7.90

<sup>a</sup> Threshold configuration for the direct Urca process

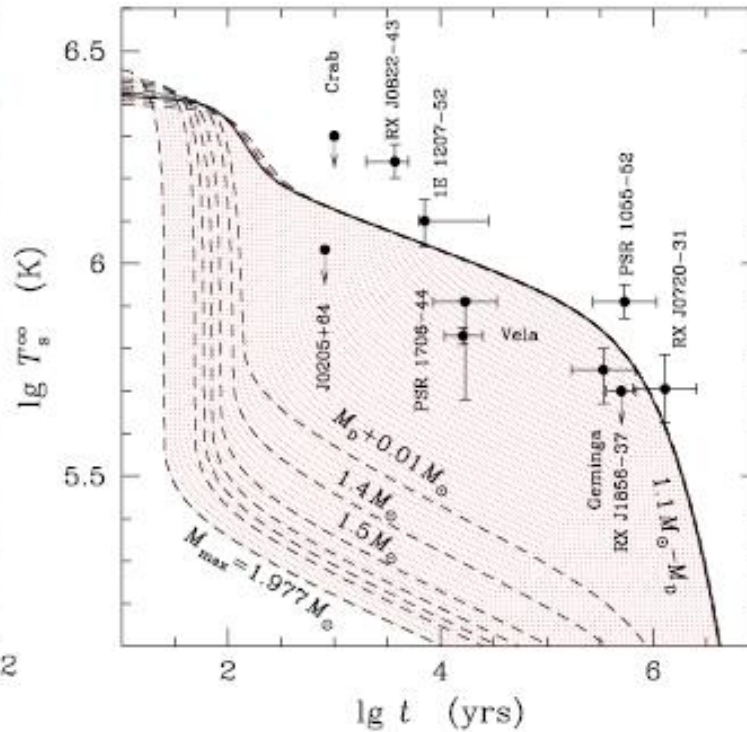
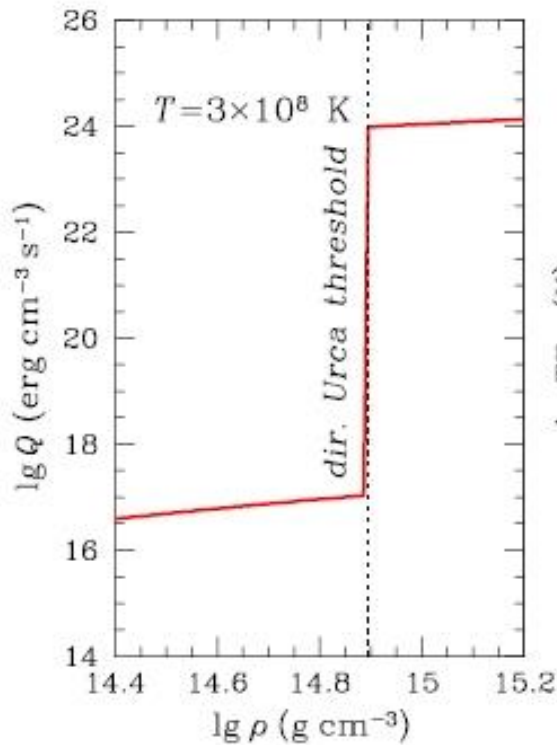
<sup>b</sup> Maximum-mass stable neutron star

# Simple cooling model for low-mass NSs.



Too hot .....  
Too cold ....

# Nonsuperfluid nucleon cores

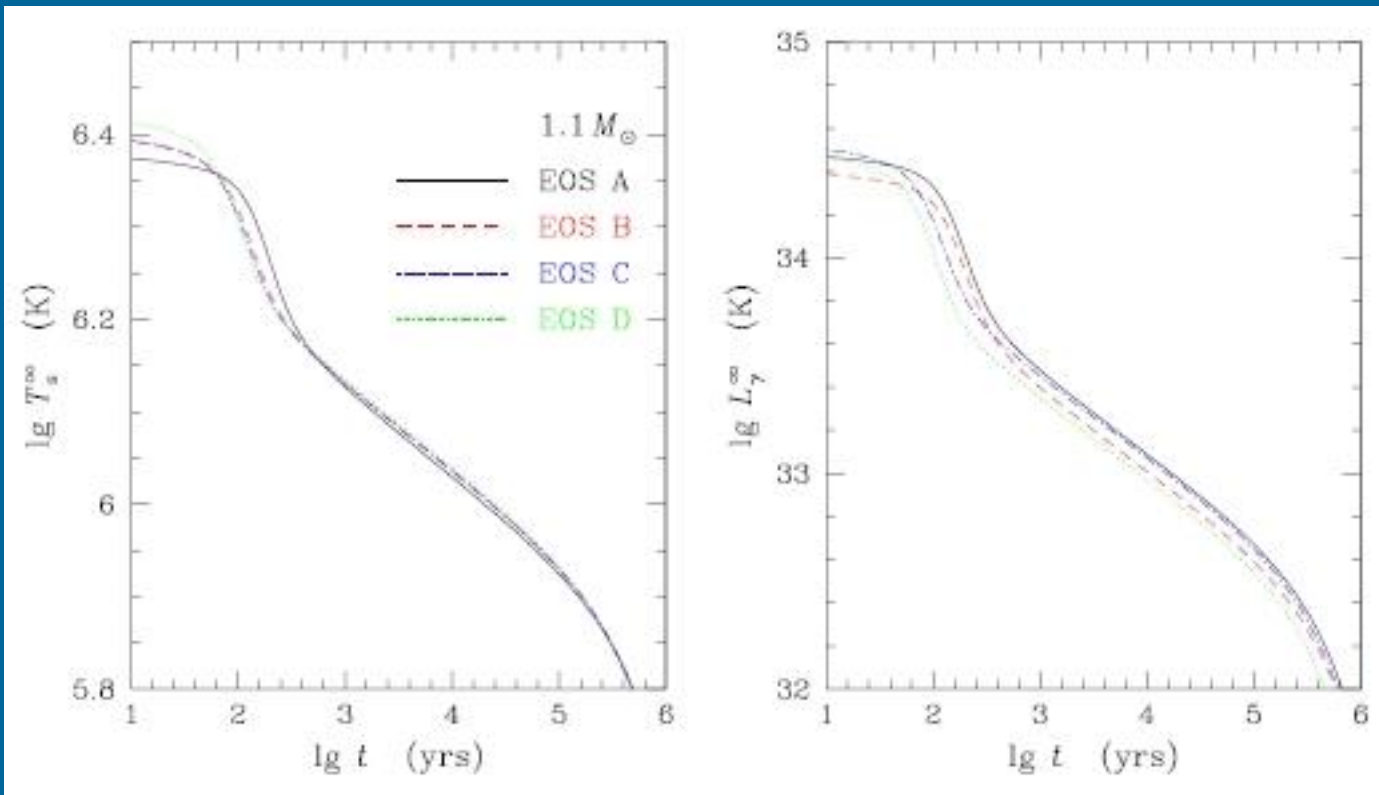


Note “population aspects” of the right plot: too many NSs have to be explained by a very narrow range of mass.

For slow cooling at the neutrino cooling stage  $t_{\text{slow}} \sim 1 \text{ yr} / T_{10}^6$   
 For fast cooling  $t_{\text{fast}} \sim 1 \text{ min} / T_{10}^4$

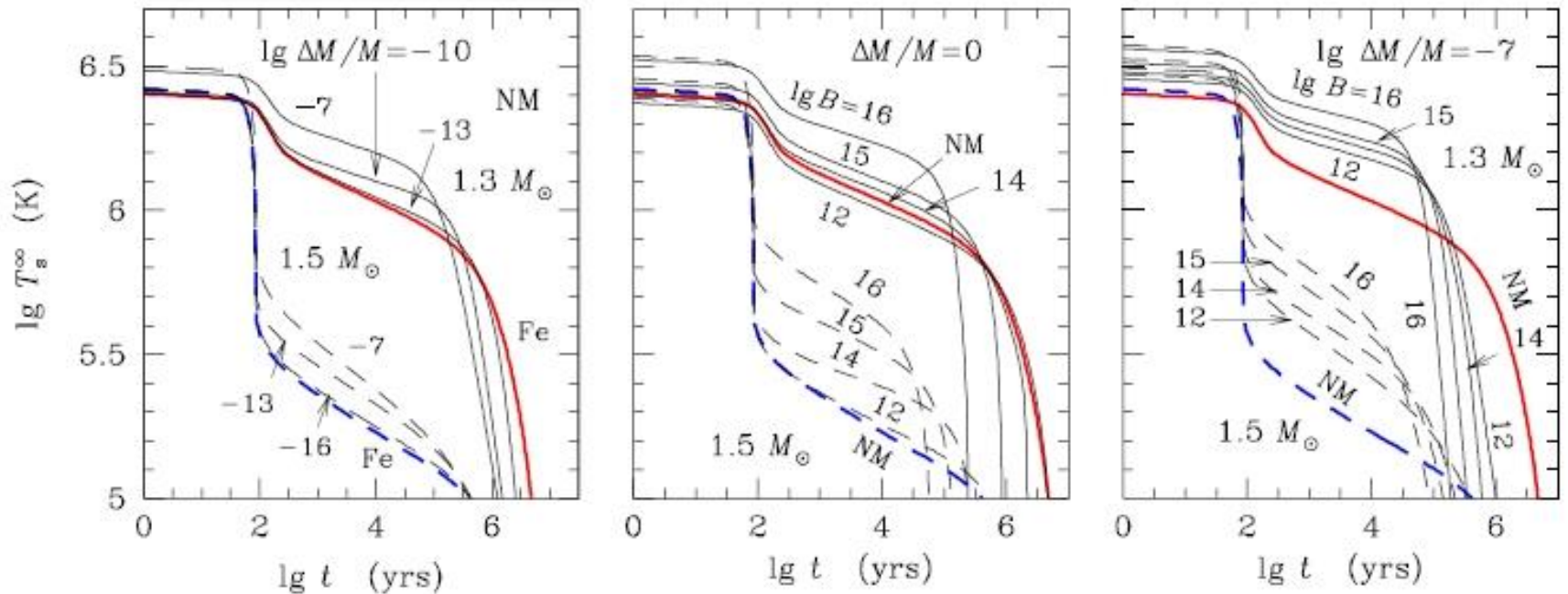


# Slow cooling for different EoS



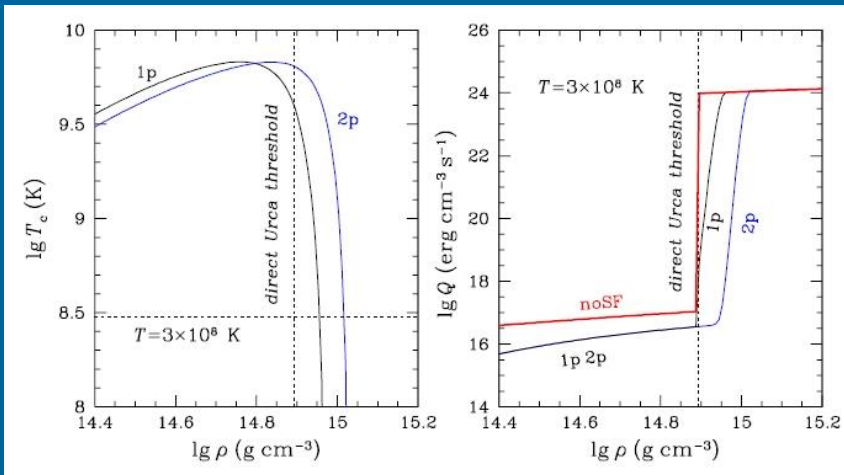
For slow cooling there is nearly no dependence on the EoS.  
The same is true for cooling curves for maximum mass for each EoS.

# Envelopes and magnetic field



Non-magnetic stars      No accreted envelopes,      Envelopes + Fields  
 Thick lines – no envelope      different magnetic fields.  
 Envelopes can be related to the fact that we see a subpopulation of hot NS  
 in CCOs with relatively long initial spin periods and low magnetic field, but  
 do not observed representatives of this population around us, i.e. in the Solar vicinity.  
 Solid line  $M=1.3 M_{\text{solar}}$ , Dashed lines  $M=1.5 M_{\text{solar}}$

# Simplified model: no neutron superfluidity



Superfluidity is an important ingredient of cooling models. It is important to consider different types of proton and neutron superfluidity.

There is no complete microphysical theory which can describe superfluidity in neutron stars.

If proton superfluidity is strong, but neutron superfluidity in the core is weak then it is possible to explain observations.

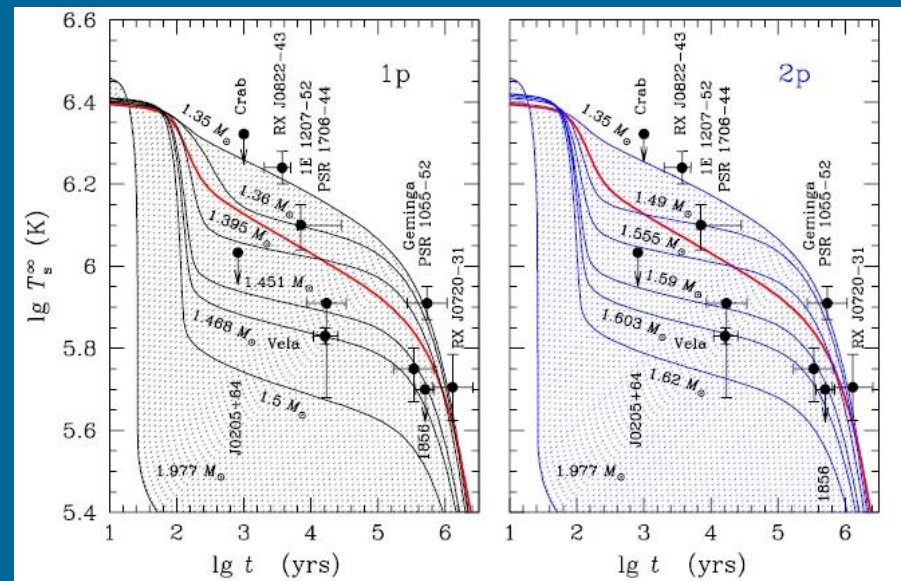
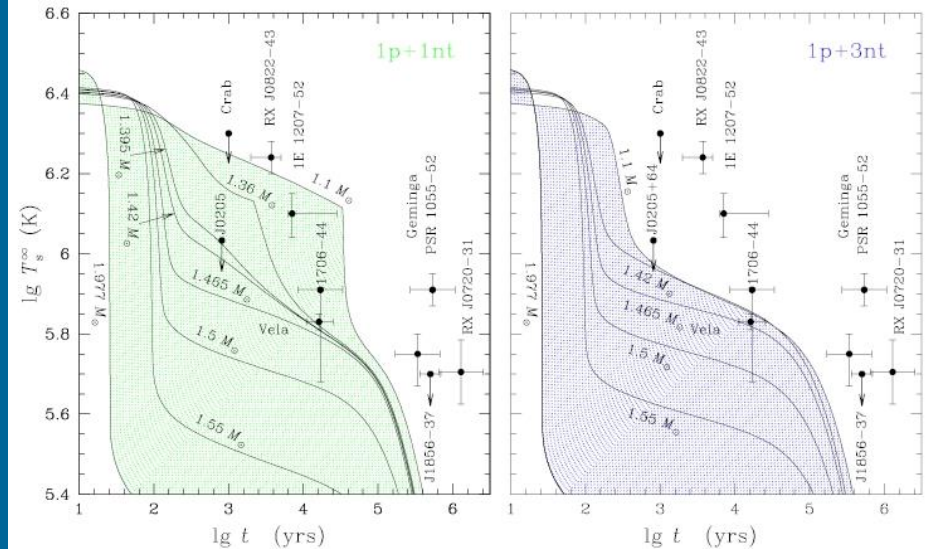


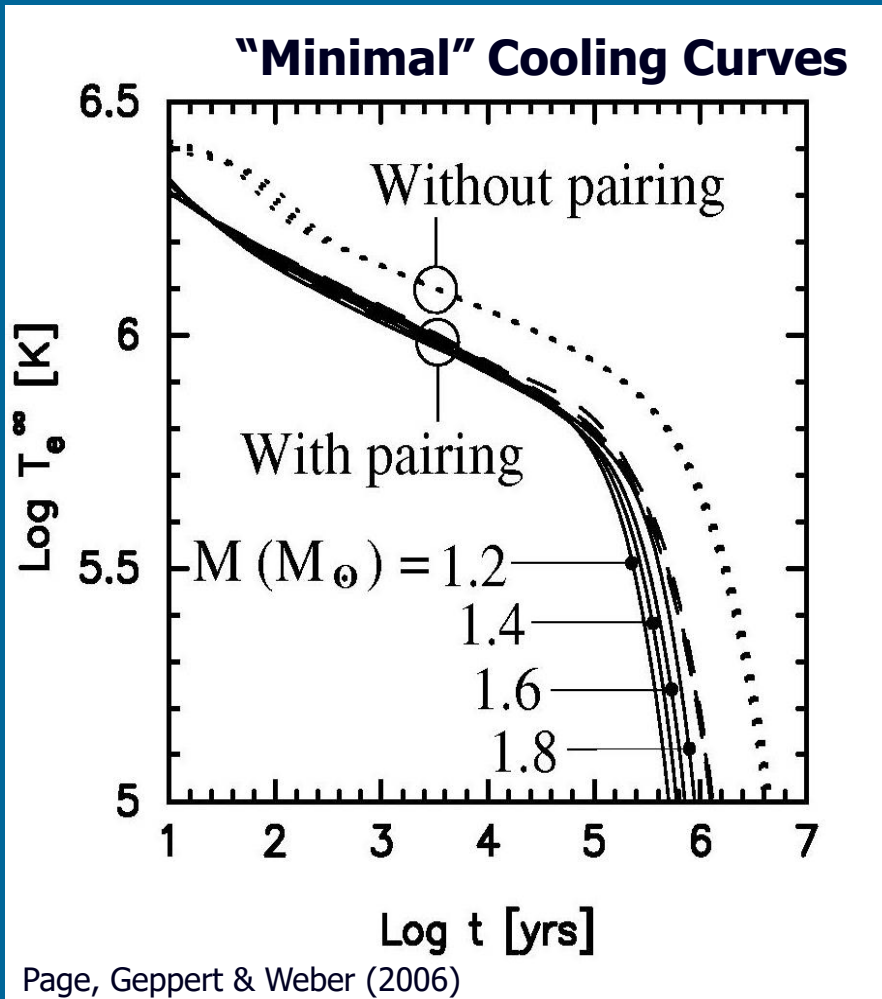
Figure 1 consists of two side-by-side plots. The left plot shows the central temperature  $\lg T_c$  (K) on the y-axis (ranging from 8 to 10) versus the central density  $\lg \rho$  ( $\text{g cm}^{-3}$ ) on the x-axis (ranging from 14.4 to 15.2). It features three curves: a black curve labeled '1p', a blue curve labeled '3nt', and a green curve labeled '1nt'. A horizontal dashed line is at  $\lg T_c \approx 8.45$ , labeled  $T = 3 \times 10^8 \text{ K}$ . A vertical dashed line is at  $\lg \rho \approx 14.9$ , labeled 'dir. Urca thresh.'. The right plot shows the central luminosity  $\lg Q$  ( $\text{erg cm}^{-3} \text{ s}^{-1}$ ) on the y-axis (ranging from 14 to 26) versus  $\lg \rho$  ( $\text{g cm}^{-3}$ ) on the x-axis (ranging from 14.4 to 15.2). It features four curves: a black curve labeled '1p', a blue curve labeled 'p+3nt', a green curve labeled 'p+1nt', and a red curve labeled 'noSF'. A vertical dashed line is at  $\lg \rho \approx 14.9$ , labeled 'direct Urca threshold'. A red curve segment at high luminosity is also labeled '1p'.

See a recent review about superfluidity and its relation to the thermal evolution of NSs in 1206.5011 and a very detailed review about superfluids in NSs in 1302.6626. A brief and more popular review in 1303.3282.



(Yakovlev & Pethick 2004)

# Minimal cooling model



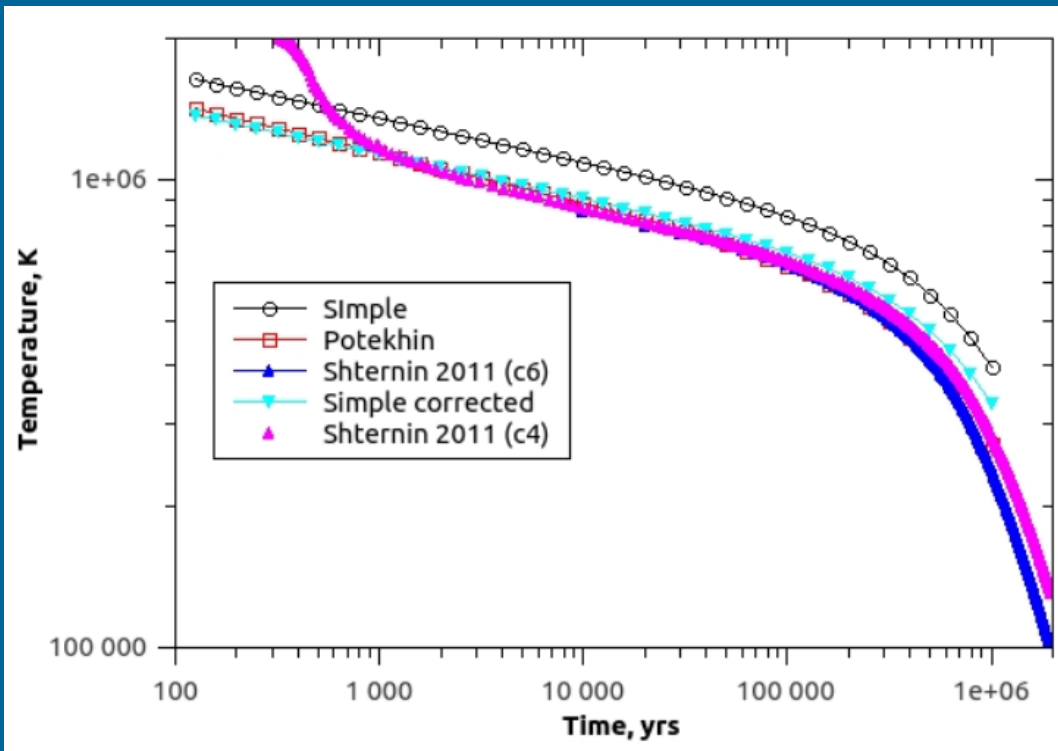
“minimal” means  
without additional cooling  
due to direct URCA  
and without additional heating

## Main ingredients of the minimal model

- EoS
- Superfluid properties
- Envelope composition
- NS mass



# Analytical fits



$$T_{\text{surface}} \sim T_{\text{core}}^{1/2}$$

$$T_{\text{eff6}} = T_* \equiv (7 T_{\text{b9}} \sqrt{g_{14}})^{1/2}.$$

(iron envelope)

$$T_{\text{eff6,a}}^4 = g_{14} (18.1 T_{\text{b9}})^{2.42},$$

(accreted envelope)

astro-ph/9706148

astro-ph/0105261

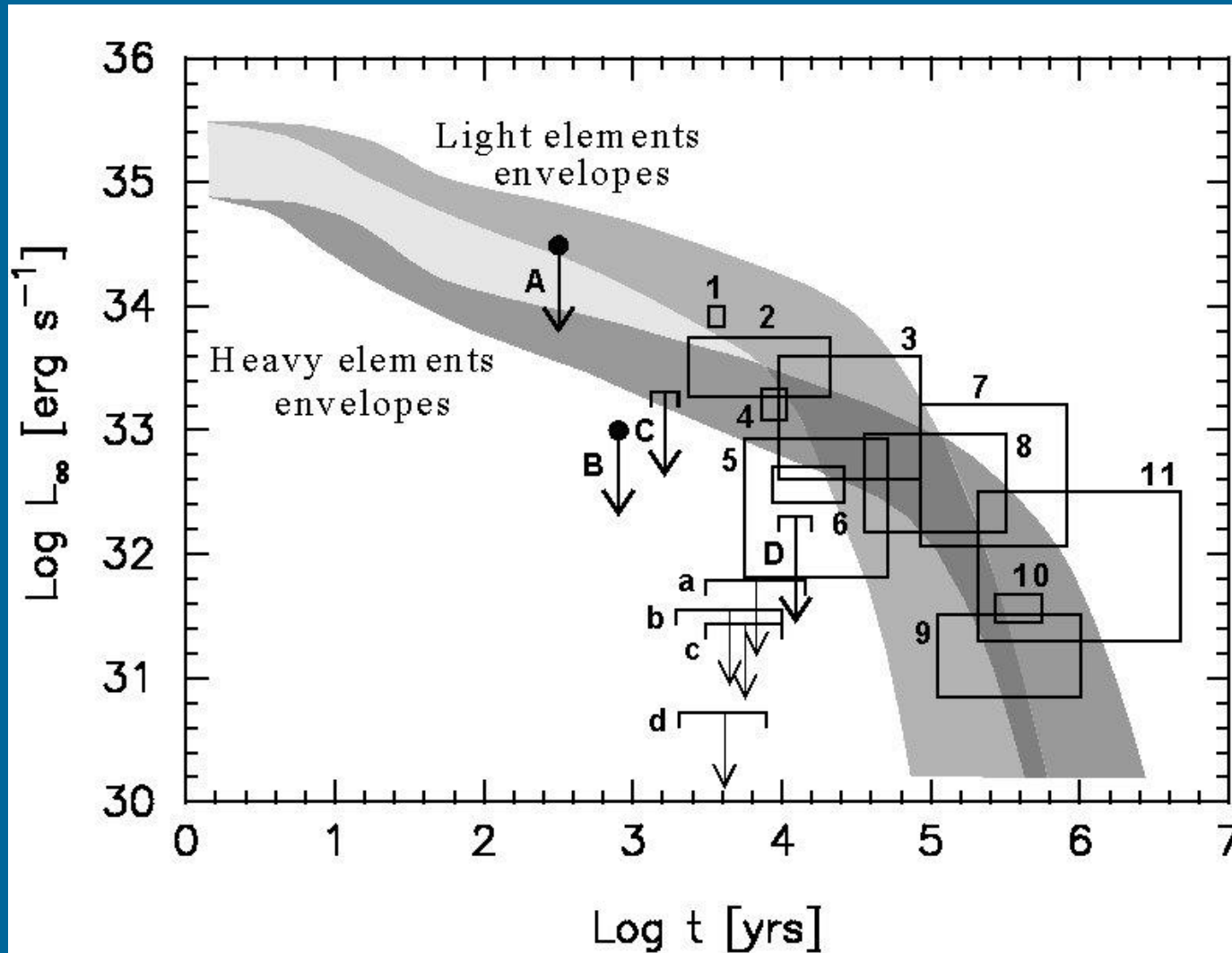
$$T = b \left( \frac{t}{1\text{yr}} \right)^a \exp(-t/\tau_c).$$

1709.10385

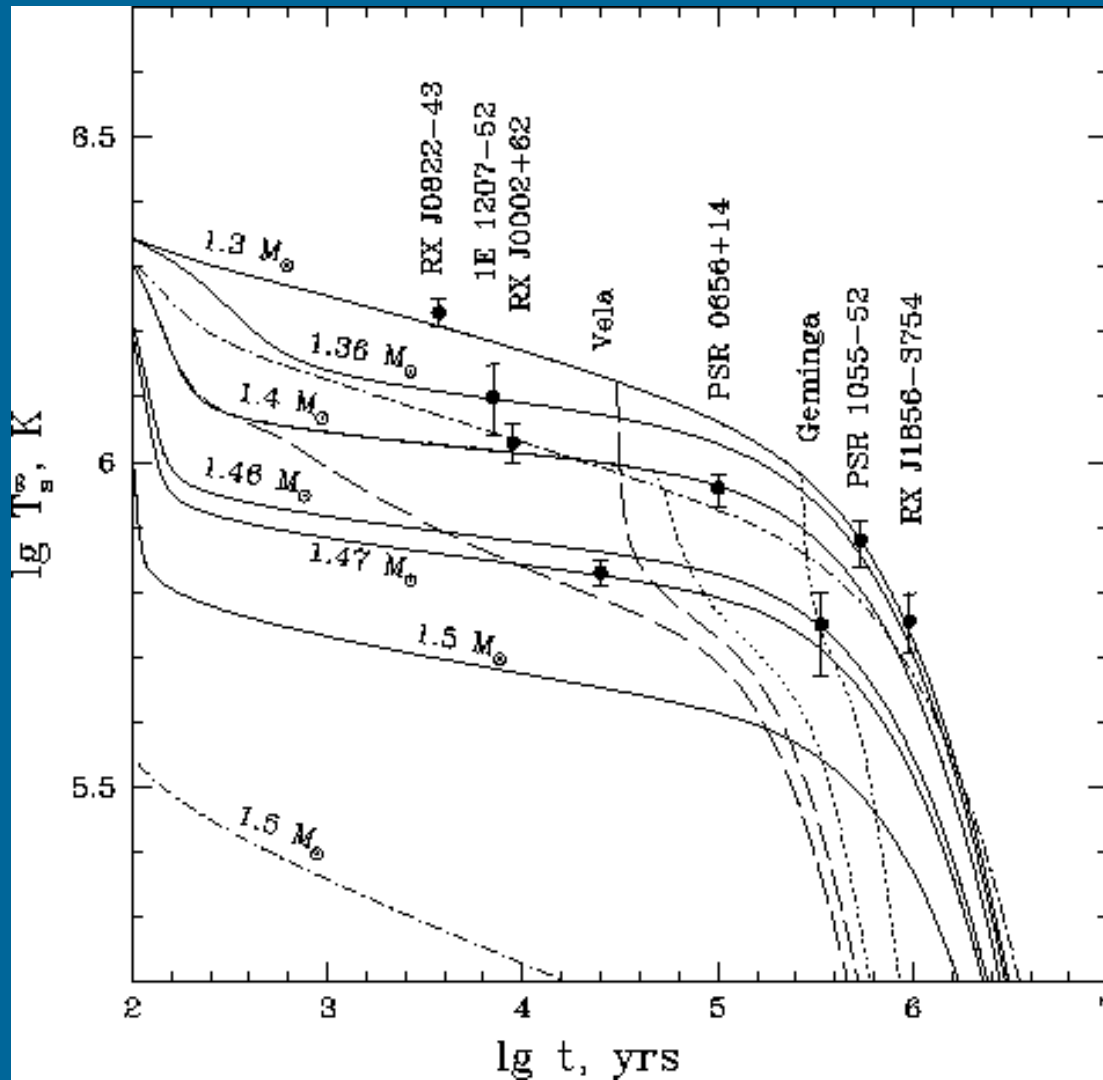
$$T_s^{(0)} \approx 10^6 g_{14}^{1/4} [(7\zeta)^{2.25} + (\zeta/3)^{1.25}]^{1/4} \text{ K}, \quad (27)$$

where  $\zeta \equiv T_{\text{int},9} - 0.001 g_{14}^{1/4} \sqrt{7 T_{\text{int},9}}$ ,  $T_{\text{int},9} \equiv T_{\text{int}}/(10^9 \text{ K})$  and  $g_{14} \equiv g/10^{14} \text{ cm s}^{-2}$ .

# Luminosity and age uncertainties



# Standard test: temperature vs. age



Kaminker et al. (2001)



# Data

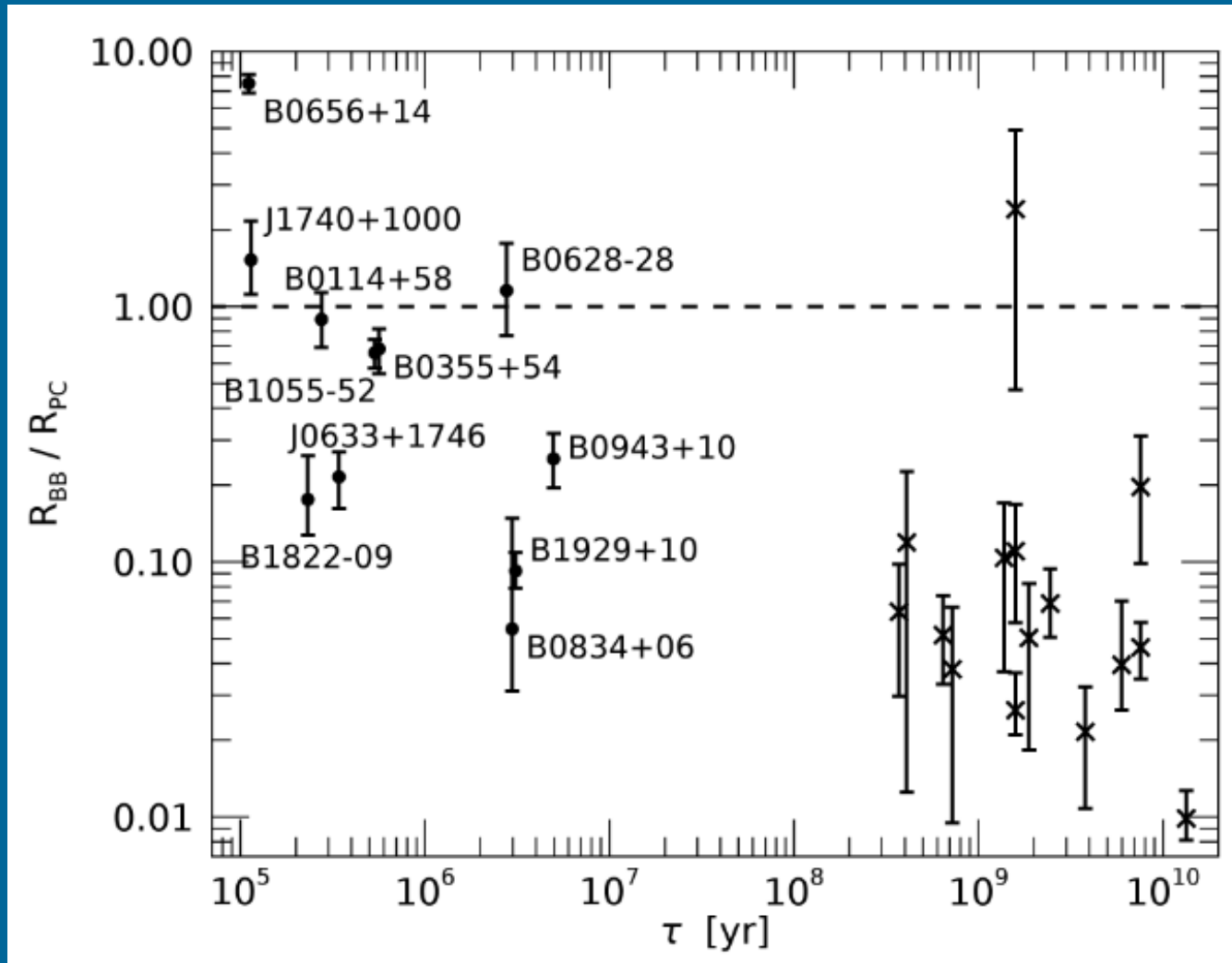
NEUTRON STAR PROPERTIES WITH HYDROGEN ATMOSPHERES

Star	$\log_{10} t_{sd}$ yr	$\log_{10} t_{kin}$ yr	$\log_{10} T_{\infty}$ K	$d$ kpc	$\log_{10} L_{\infty}$ erg/s
RX J0822-4247	3.90	$3.57^{+0.04}_{-0.04}$	$6.24^{+0.04}_{-0.04}$	1.9 – 2.5	33.85 – 34.00
1E 1207.4-5209	$5.53^{+0.44}_{-0.19}$	$3.85^{+0.48}_{-0.48}$	$6.21^{+0.07}_{-0.07}$	1.3 – 3.9	33.27 – 33.74
RX J0002+6246	–	$3.96^{+0.08}_{-0.08}$	$6.03^{+0.03}_{-0.03}$	2.5 – 3.5	33.08 – 33.33
PSR 0833-45 (Vela)	4.05	$4.26^{+0.17}_{-0.31}$	$5.83^{+0.02}_{-0.02}$	0.22 – 0.28	32.41 – 32.70
PSR 1706-44	4.24	–	$5.8^{+0.13}_{-0.13}$	1.4 – 2.3	31.81 – 32.93
PSR 0538+2817	4.47	–	$6.05^{+0.10}_{-0.10}$	1.2	32.6 – 33.6

NEUTRON STAR PROPERTIES WITH BLACKBODY ATMOSPHERES

Star	$\log_{10} t_{sd}$ yr	$\log_{10} t_{kin}$ yr	$\log_{10} T_{\infty}$ K	$R_{\infty}$ km	$d$ kpc	$\log_{10} L_{\infty}$ erg/s
RX J0822-4247	3.90	$3.57^{+0.04}_{-0.04}$	$6.65^{+0.04}_{-0.04}$	1 – 1.6	1.9 – 2.5	33.60 – 33.90
1E 1207.4-5209	$5.53^{+0.44}_{-0.19}$	$3.85^{+0.48}_{-0.48}$	$6.48^{+0.01}_{-0.01}$	1.0 – 3.7	1.3 – 3.9	32.70 – 33.88
RX J0002+6246	–	$3.96^{+0.08}_{-0.08}$	$6.15^{+0.11}_{-0.11}$	2.1 – 5.3	2.5 – 3.5	32.18 – 32.81
PSR 0833-45 (Vela)	4.05	$4.26^{+0.17}_{-0.31}$	$6.18^{+0.02}_{-0.02}$	1.7 – 2.5	0.22 – 0.28	32.04 – 32.32
PSR 1706-44	4.24	–	$6.22^{+0.04}_{-0.04}$	1.9 – 5.8	1.8 – 3.2	32.48 – 33.08
PSR 0656+14	5.04	–	$5.71^{+0.03}_{-0.04}$	7.0 – 8.5	0.26 – 0.32	32.18 – 32.97
PSR 0633+1748 (Geminga)	5.53	–	$5.75^{+0.04}_{-0.05}$	2.7 – 8.7	0.123 – 0.216	30.85 – 31.51
PSR 1055-52	5.43	–	$5.92^{+0.02}_{-0.02}$	6.5 – 19.5	0.5 – 1.5	32.07 – 33.19
RX J1856.5-3754	–	$5.70^{+0.05}_{-0.25}$	5.6 – 5.9	> 16	0.105 – 0.129	31.44 – 31.68
RX J0720.4-3125	$6.0 \pm 0.2$	–	5.55 – 5.95	5.0 – 15.0	0.1 – 0.3	31.3 – 32.5

# Not to mix with polar caps heating!



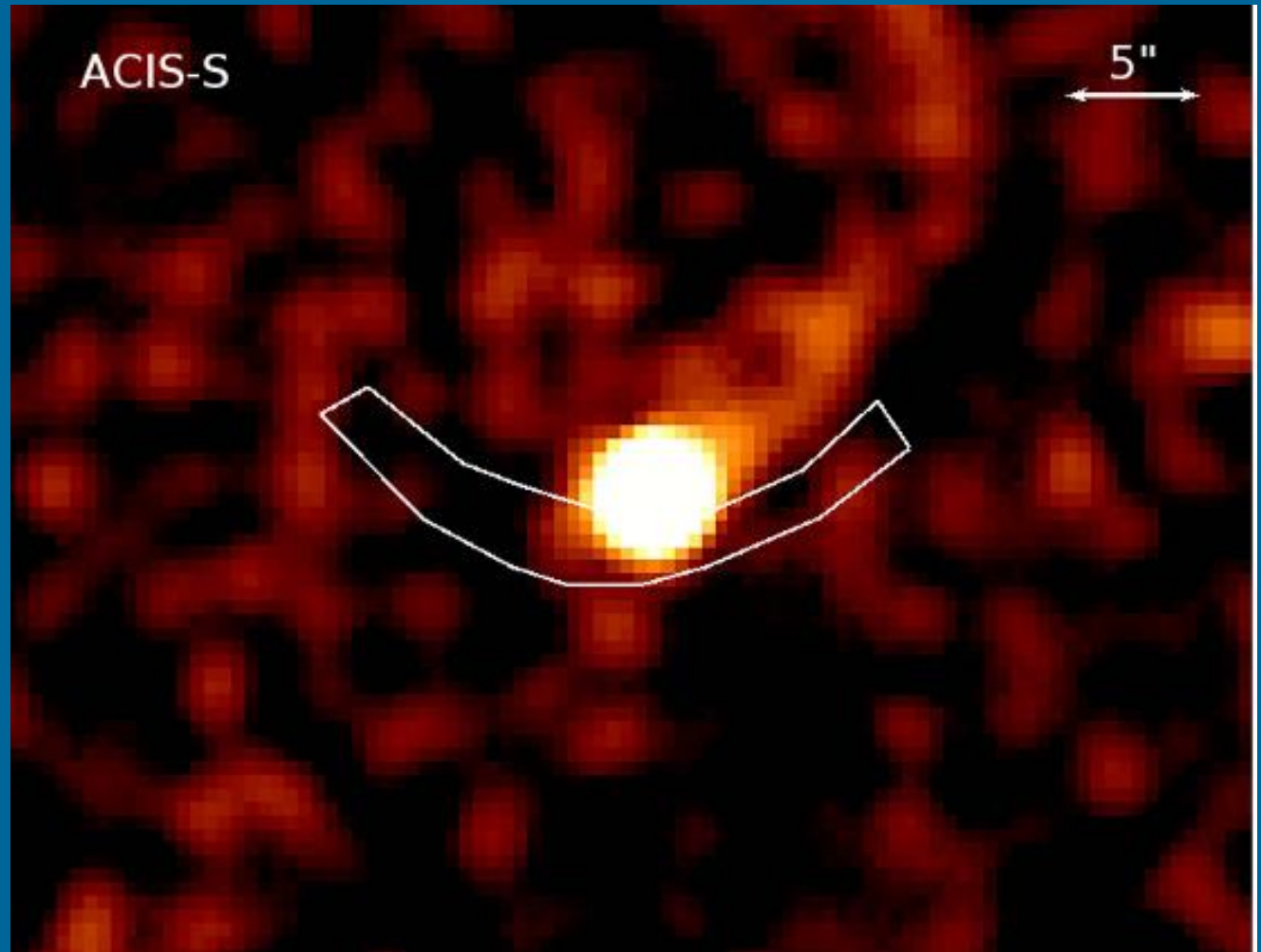
$$R_{\text{cap}} = R_{\text{NS}} (R_{\text{NS}} / R_{\text{I}})^{1/2}$$

# A puzzling source

Millisecond Pulsar  
J2124-3358

Characteristic  
age 3.4 Gyr

$T \sim (0.5-2) \times 10^5 \text{ K}$



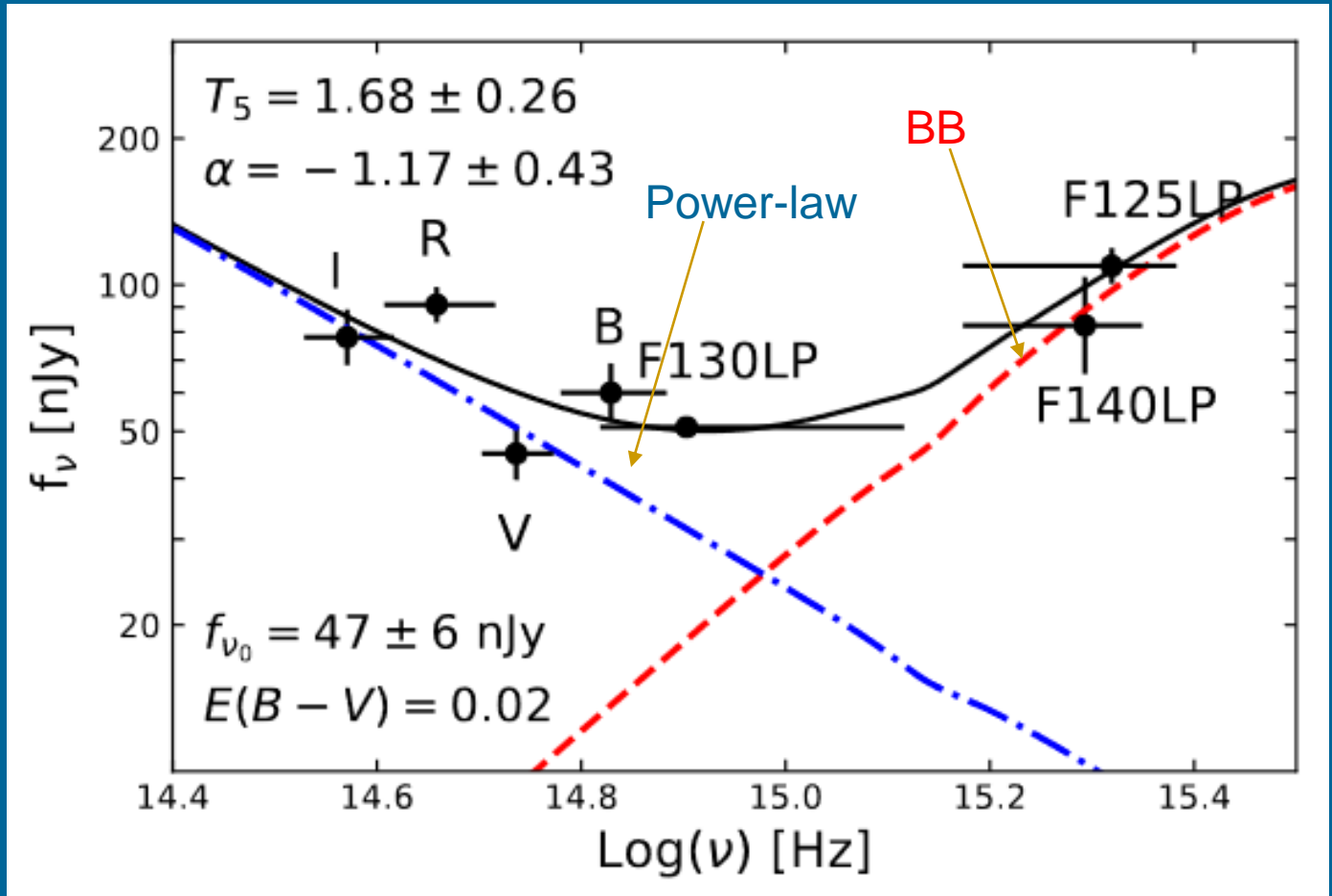
Rangelov et al. (2017)

# Another old, but hot

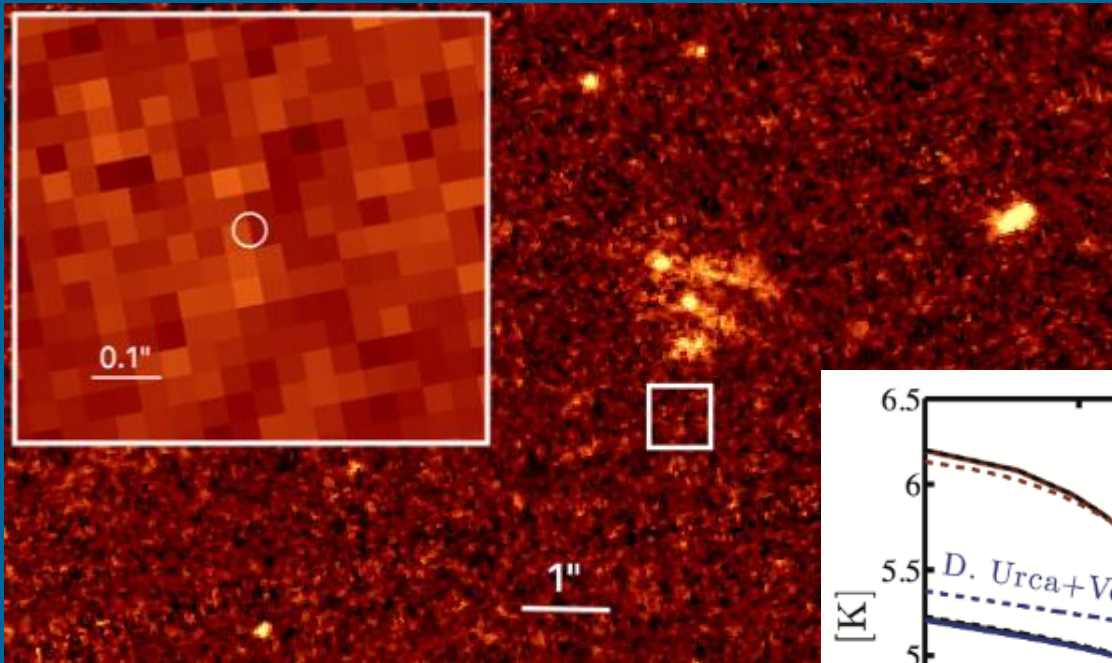
PSR B0950+08

Characteristic  
age 17.5 Myr

$T \sim (1-3) \times 10^5 \text{ K}$



# The coldest NS known



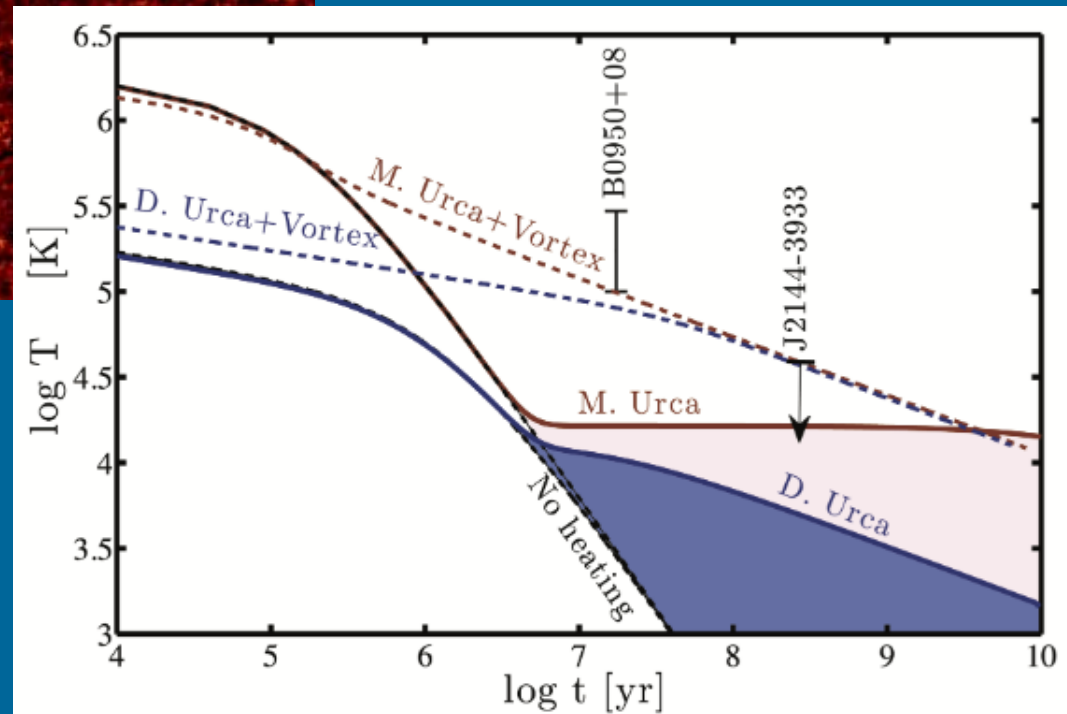
PSR J2144–3933

$P=8.5$  sec

$B=2 \times 10^{12}$  G

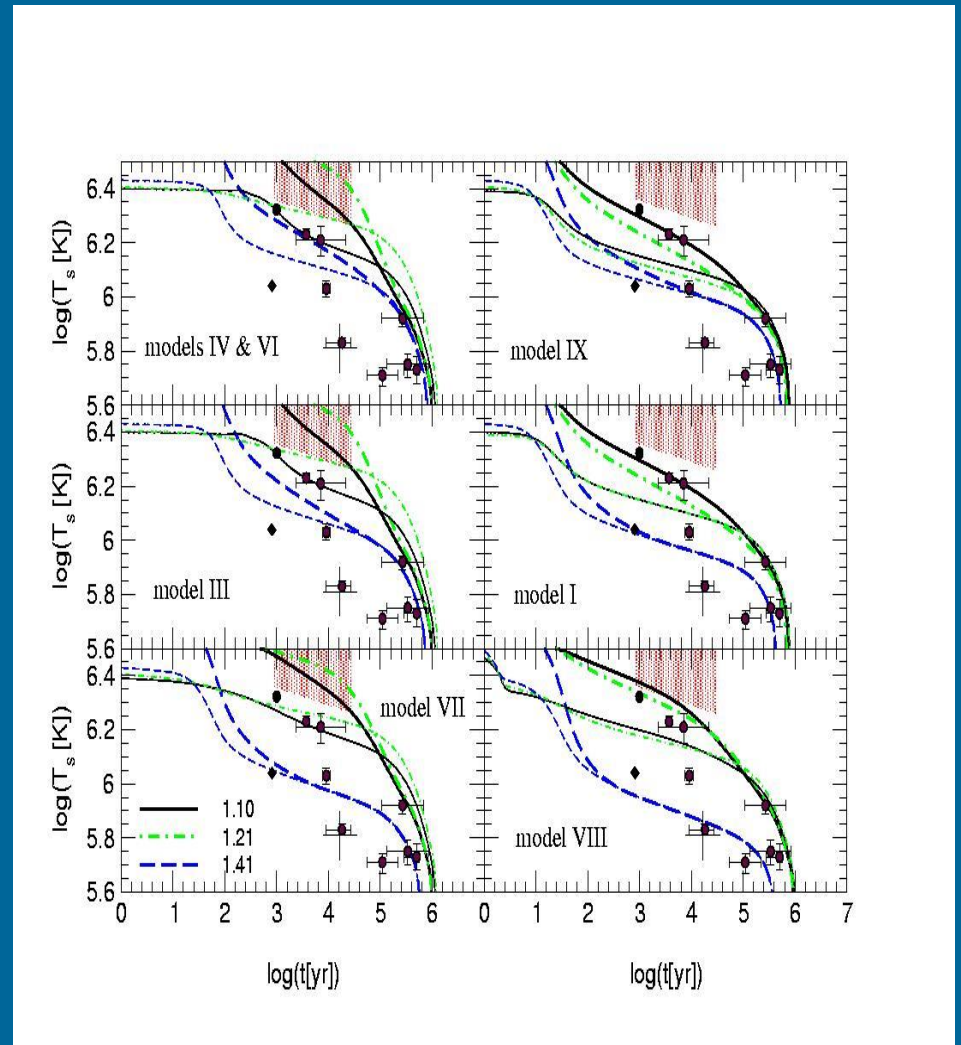
$d=160\text{--}200$  pc

Limit:  $T < 42000\text{K}$



# Brightness constraint

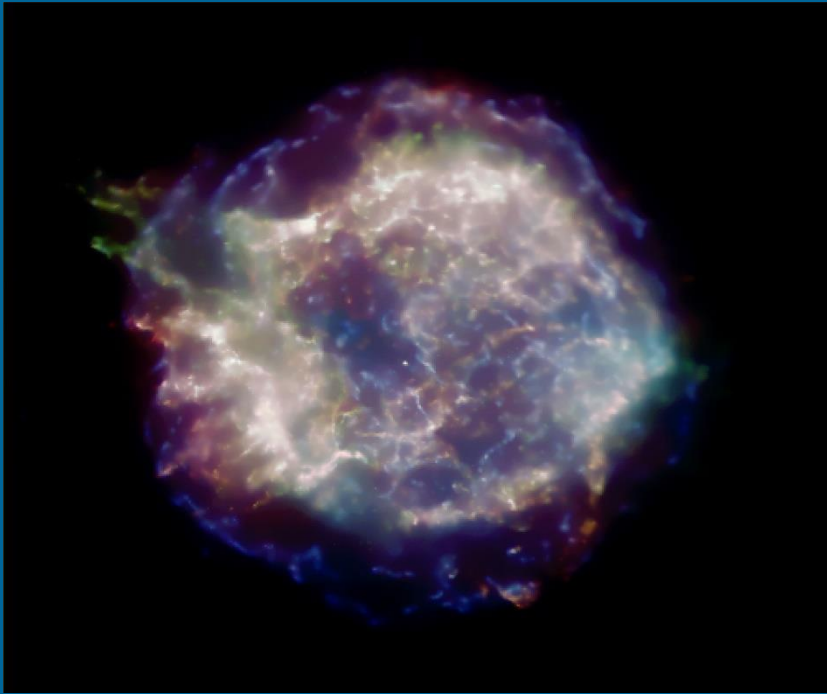
Different tests and constraints are sensitive to different parameters, so, typically it is better to use several different tests





# CCOs

1. Found in SNRs
2. Have no radio or gamma-ray counterparts
3. No pulsar wind nebula (PWN)
4. Have soft thermal-like spectra



# Known objects

Object	SNR	Age kyr	$d$ kpc	$P$	$F_{x,-12}$
J232327.9+584843	Cas A	0.32	3.3–3.7	...	0.8
J085201.4–461753	G266.1–1.2	1–3	1–2	...	1.4
J161736.3–510225(x)	RCW 103	1–3	3–7	6.4hr	0.9–60
J082157.5–430017	Pup A	1–3	1.6–3.3	...	4.5
J121000.8–522628	G296.5+10.0	3–20	1.3–3.9	424ms	2.3
J185238.6+004020(n)	Kes 79	~9	~10	...	0.2
J171328.4–394955(n)	G347.3–0.5	~10	~6	...	2.8
J000256 +62465 (n,x)	G117.9+0.6[?]	?	~ 3[?]	...	0.1

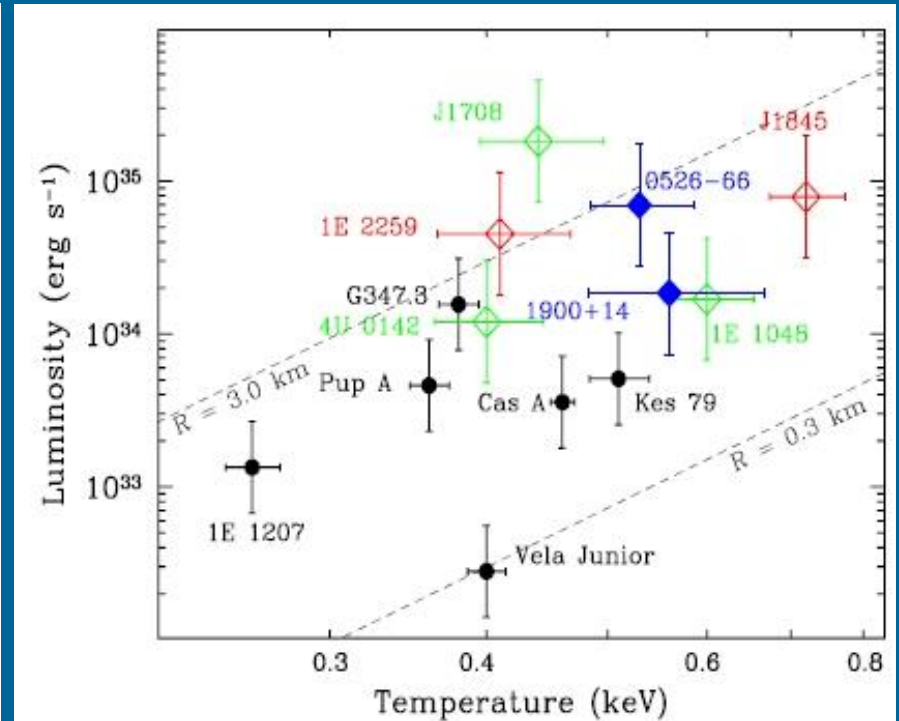
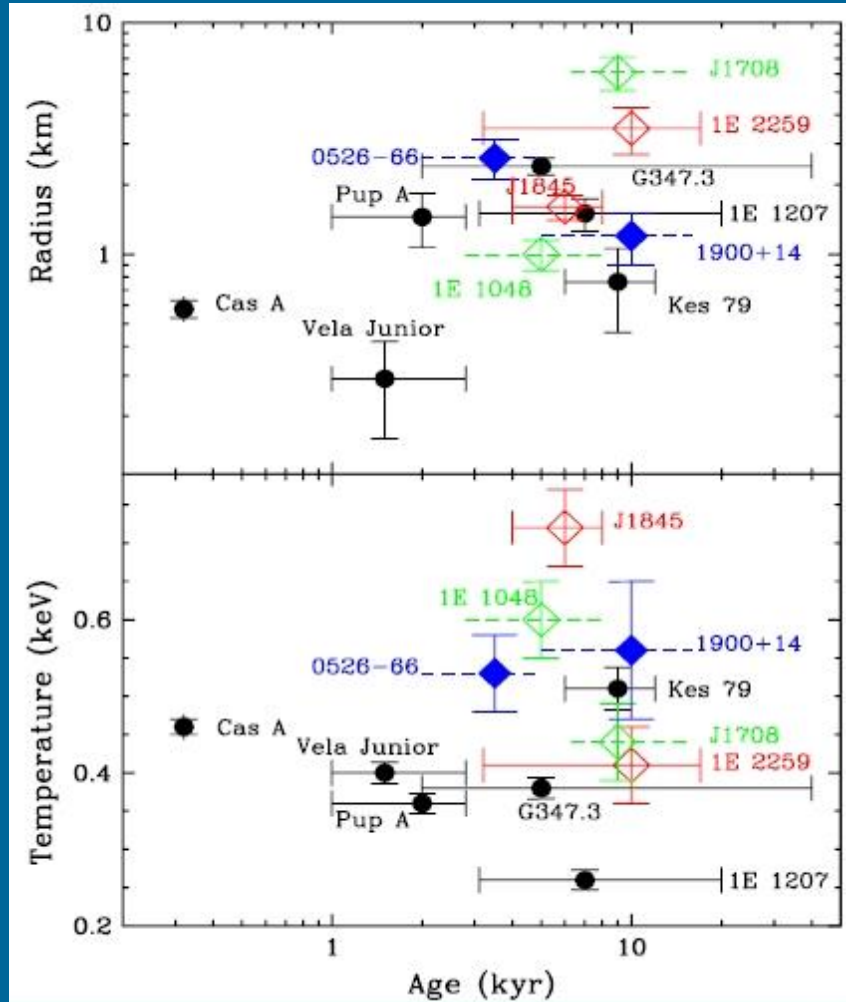
New candidates  
appear continuously.

Object	$kT$ keV	$R$ km	$L_{\text{bol},33}$	$\Gamma$	$L_{\text{pl},33}$	$n_{\text{H},22}$	$F^{\text{bb}}/F^{\text{pl}}$
J2323+5848	0.43	0.6	1.6	4.2	13	1.8	1.1
	0.43	0.7	1.9	2.5	0.2	[1.2]	4.5
J0852–4617	0.40	0.3	0.3	unconstr	...	0.4	...
J0821–4300	0.40	1.0	3.3	unconstr	...	0.3	...
J1210–5226	0.22	2.0	1.2	3.6	1.2	0.13	3.0
J1852+0040	0.50	1.0	8.0	unconstr	...	1.5	...
J1713–3949	0.38	2.4	15	3.9	72	0.8	0.9

(Pavlov et al. astro-ph/0311526)

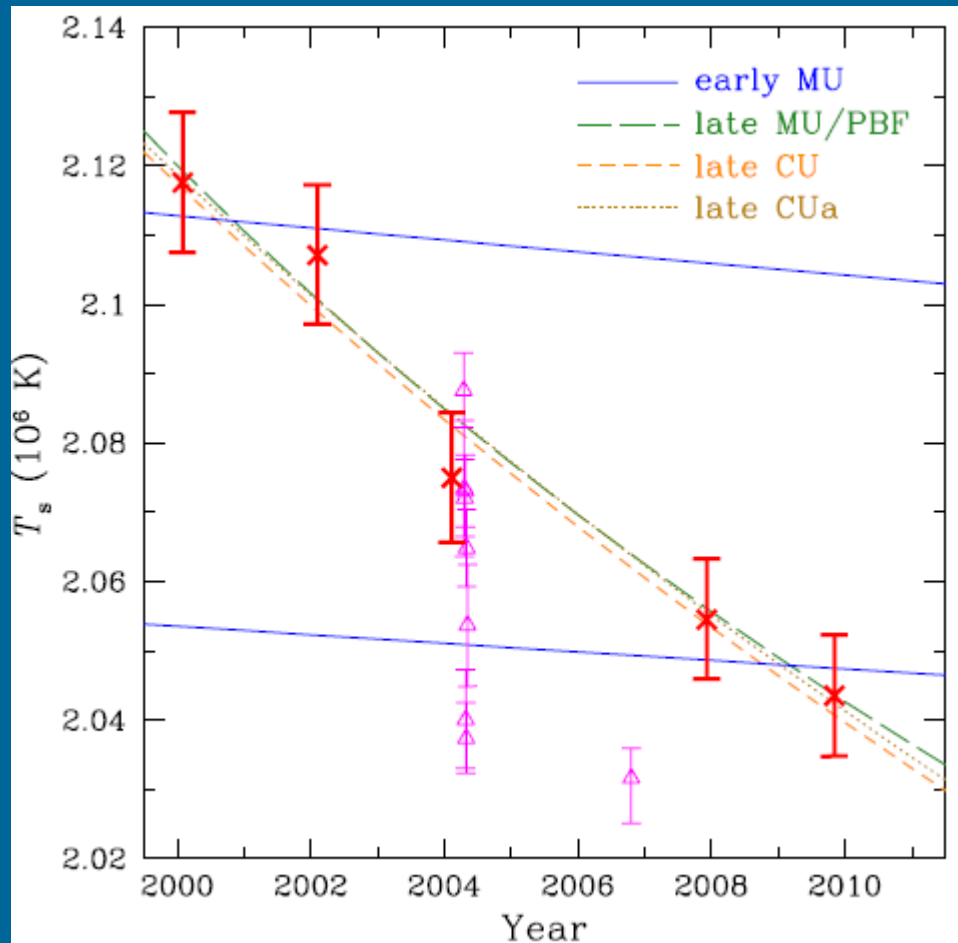


# Correlations



(Pavlov et al. astro-ph/0311526)

# Cas A peculiar cooling



330 years

~3.5 kpc

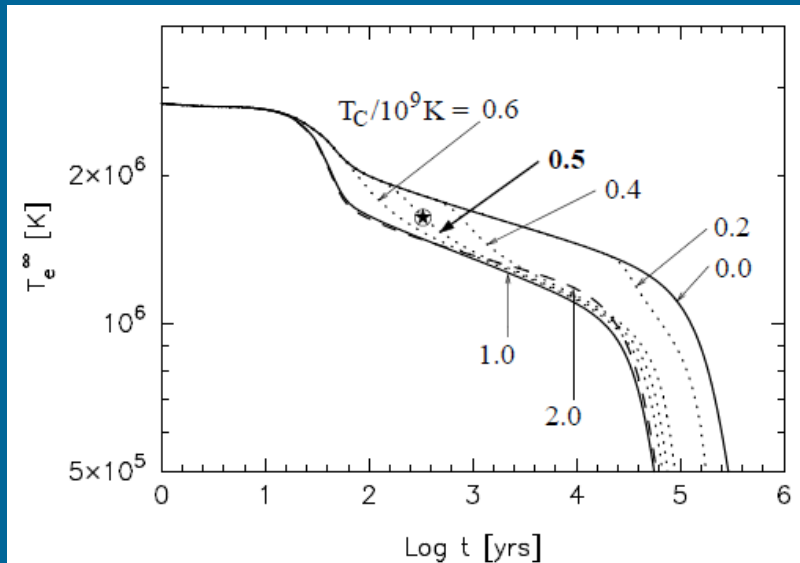
Carbon atmosphere

The youngest cooler known

Temperature steadily goes down  
by ~4% in 10 years:

2.12  $10^6$ K in 2000 – 2.04  $10^6$ K in 2009

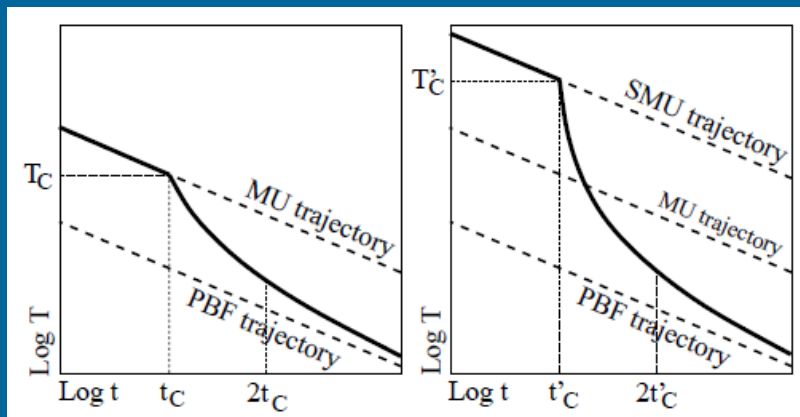
# Onset of neutron $^3\text{P}_2$ superfluidity in the core



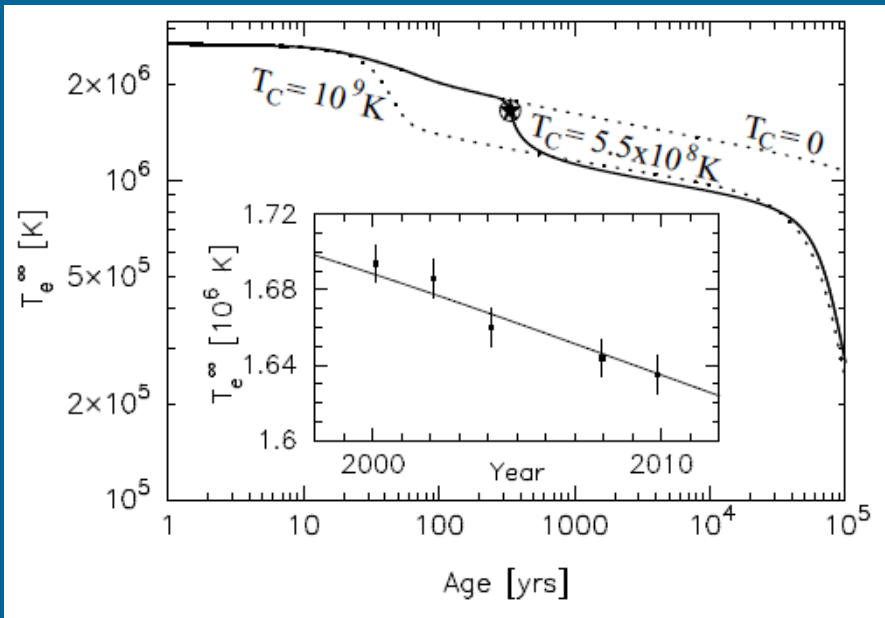
The idea is that we see the result of the onset of neutron  $^3\text{P}_2$  superfluidity in the core.

The NS just cooled down enough to have this type of neutron superfluidity in the core.

This gives an opportunity to estimate the critical temperature:  $0.5 \cdot 10^9 \text{ K}$



# The best fit model

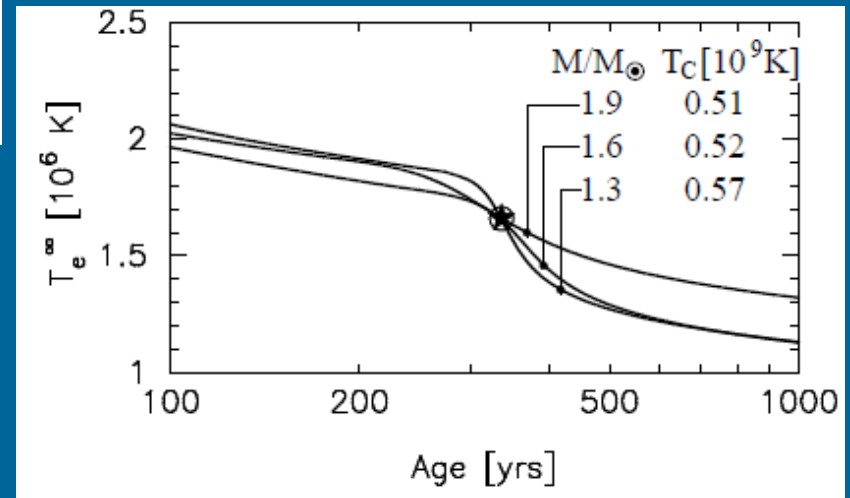


Cooling curves depend on masses, but the estimate of the critical temper. depends on  $M$  just slightly.

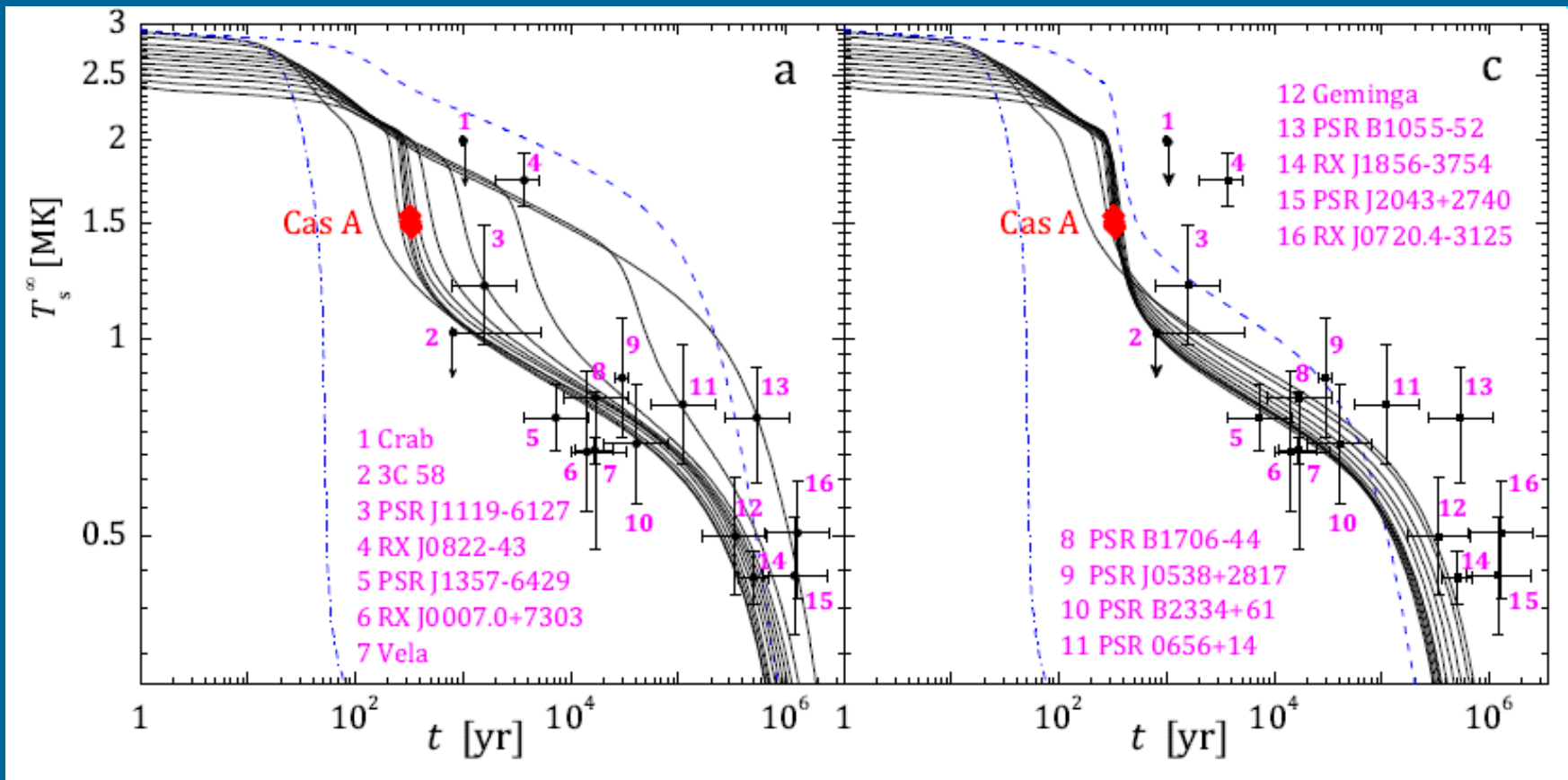
To explain a quick cooling it is necessary to assume suppression of cooling by proton  $^1S_0$  superfluidity in the core.

Rapid cooling will proceed for several tens of years more.

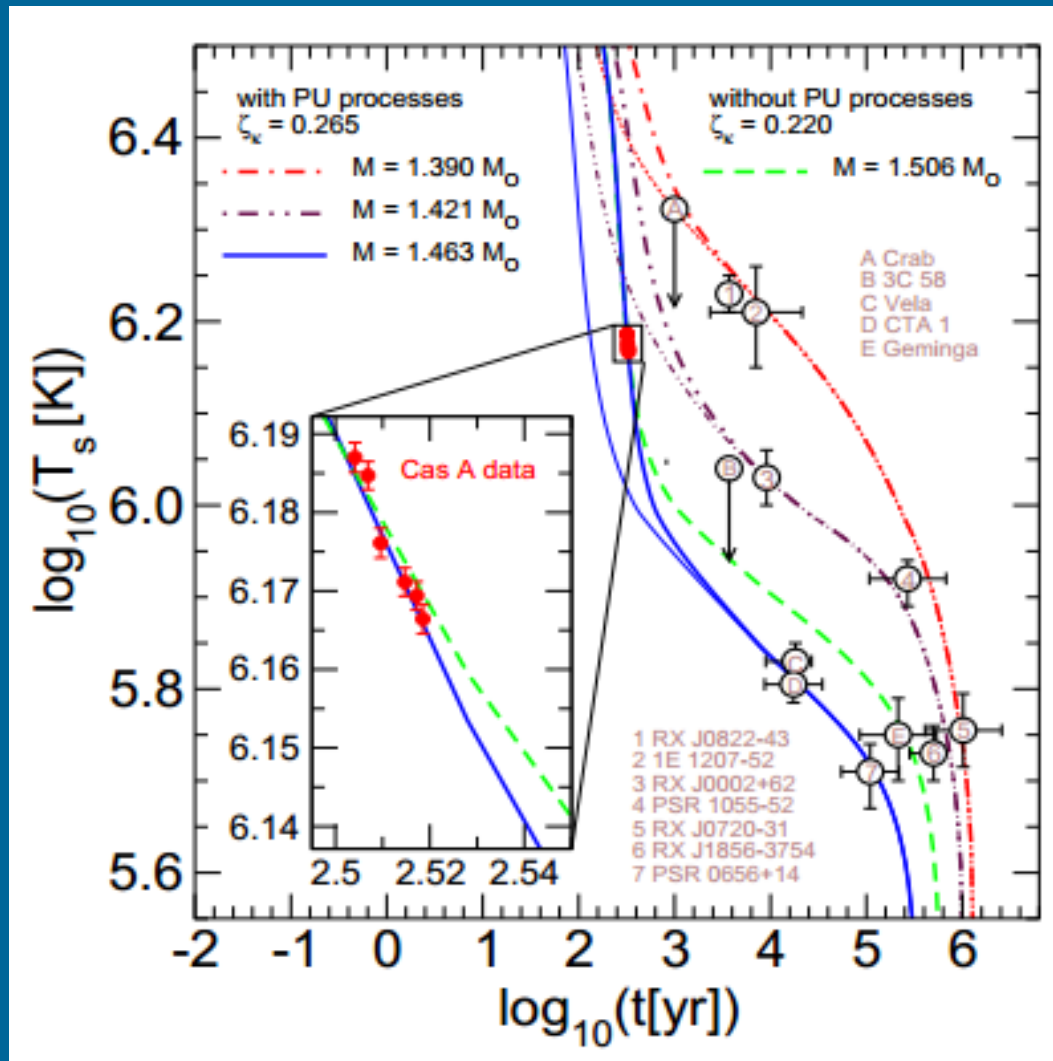
The plot is made for  $M=1.4M_\odot$



# Different superfluidity models

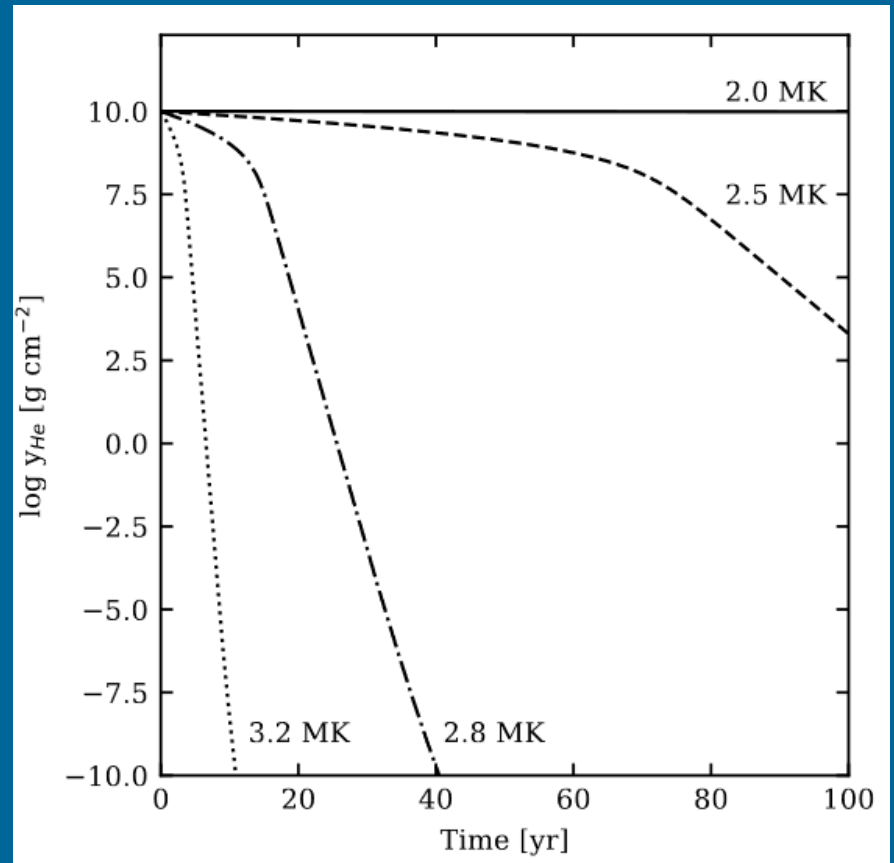
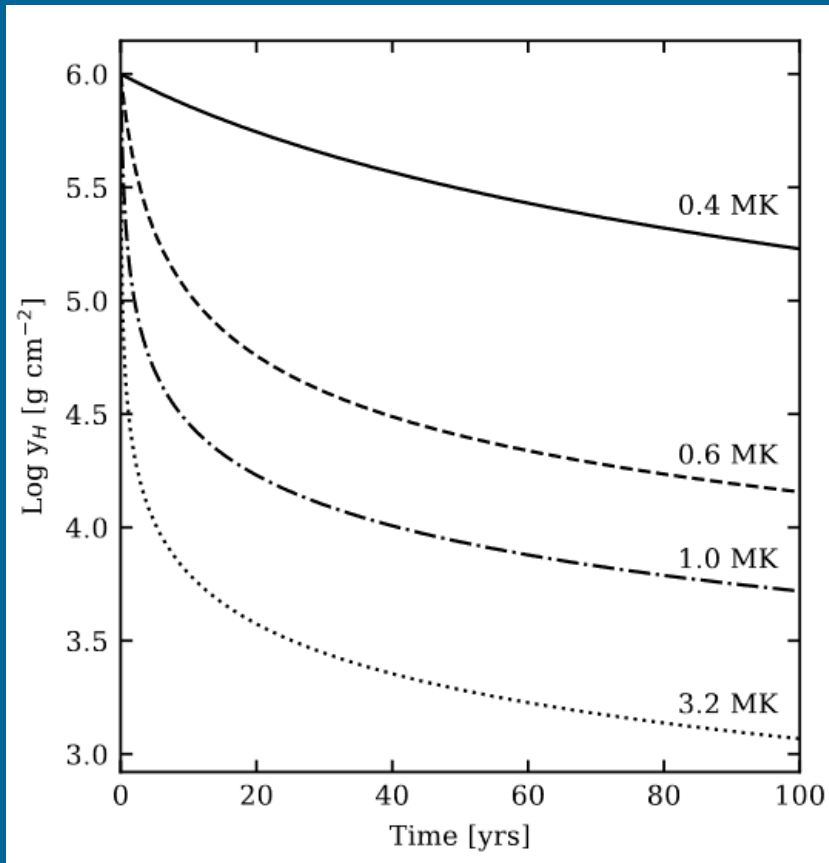


# Nuclear medium cooling

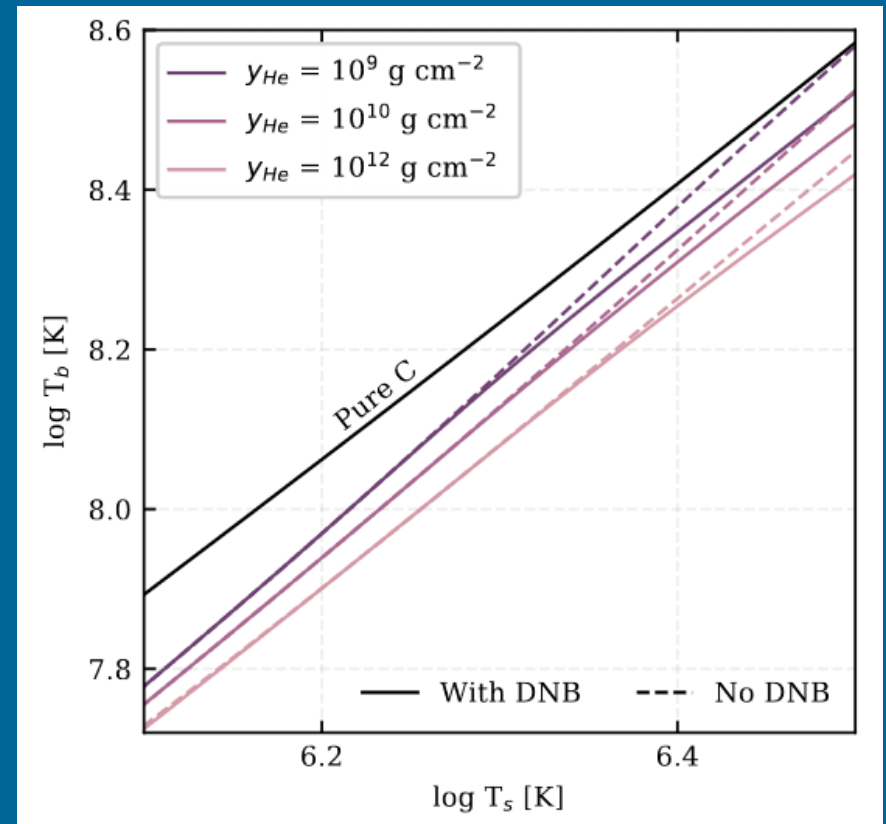
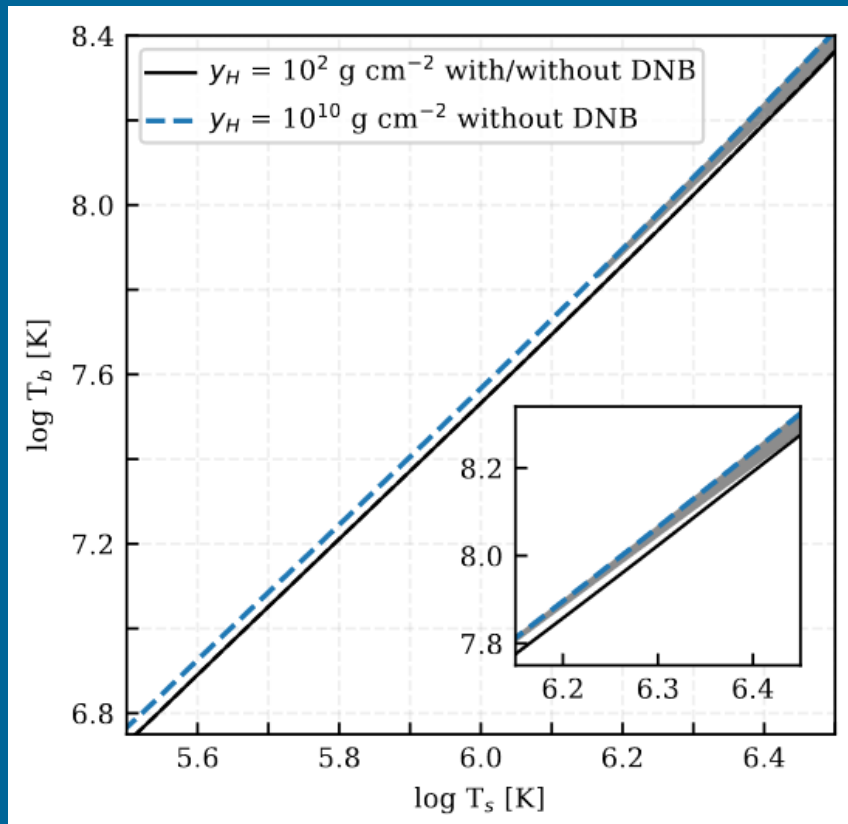


Crucial for the successful description of the observed data is a substantial reduction of the thermal conductivity, resulting from a suppression of both the electron and nucleon contributions to it by medium effects.

# Diffusive nuclear burning

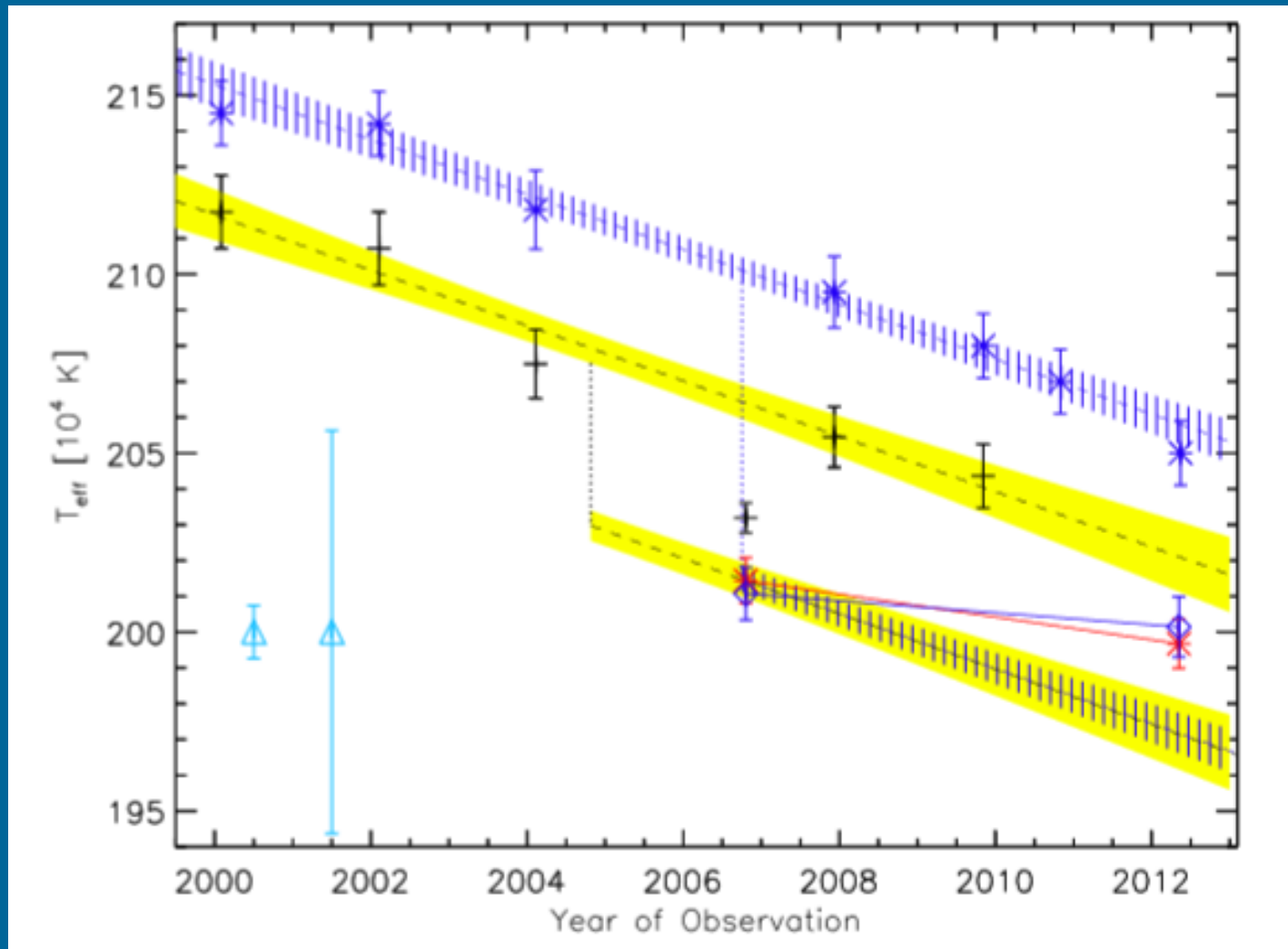


# Cooling with DNB

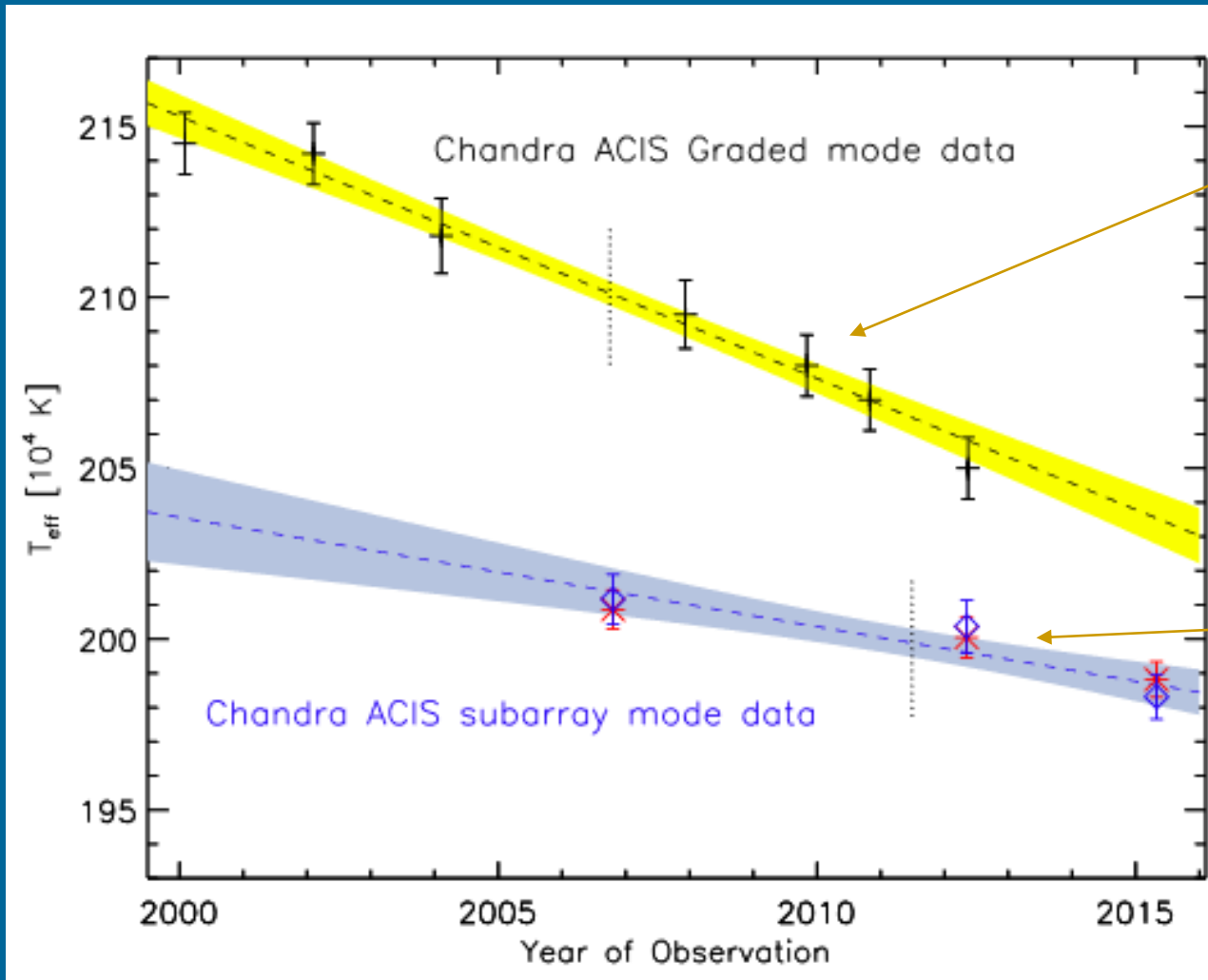




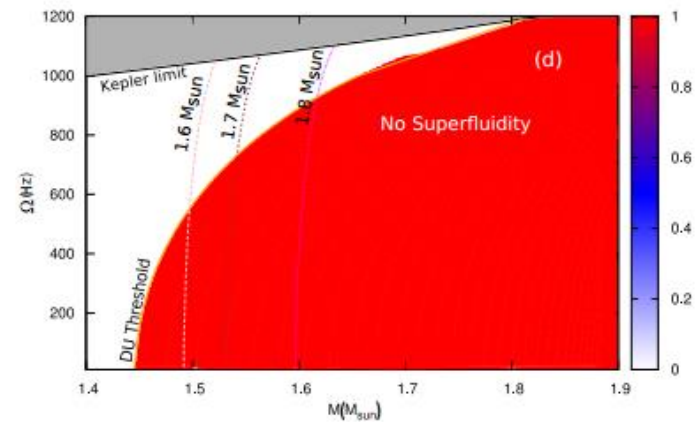
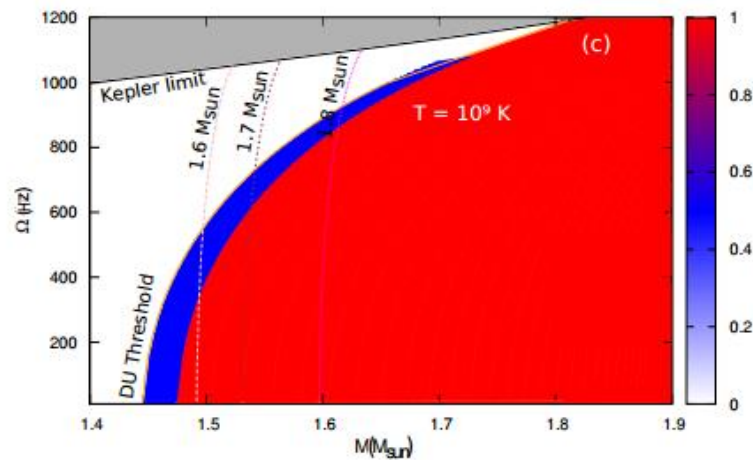
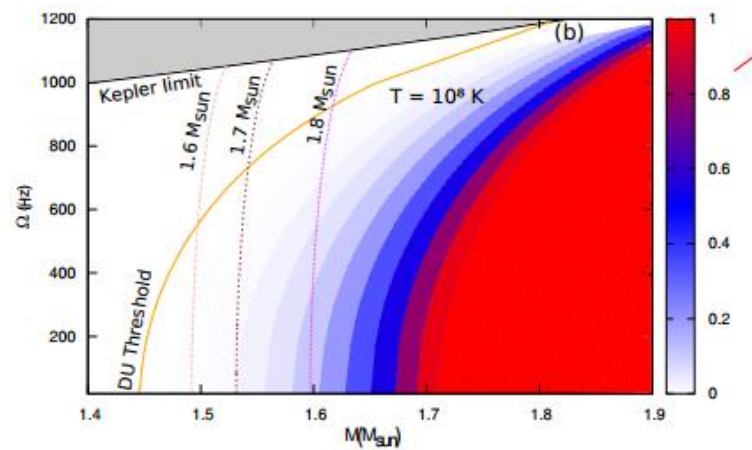
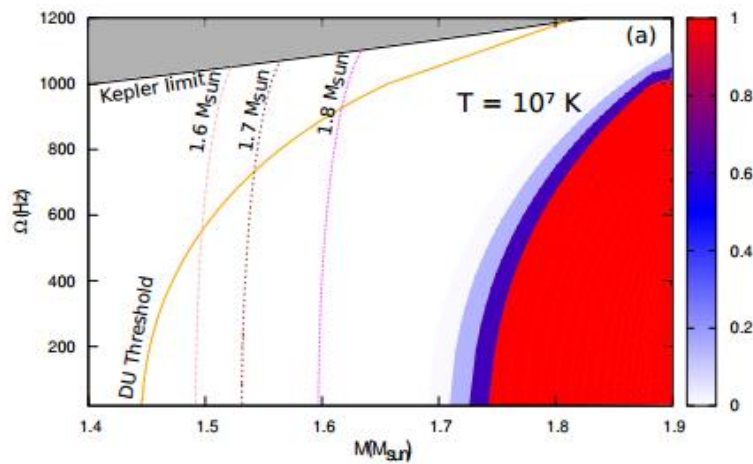
# New twist: no cooling!



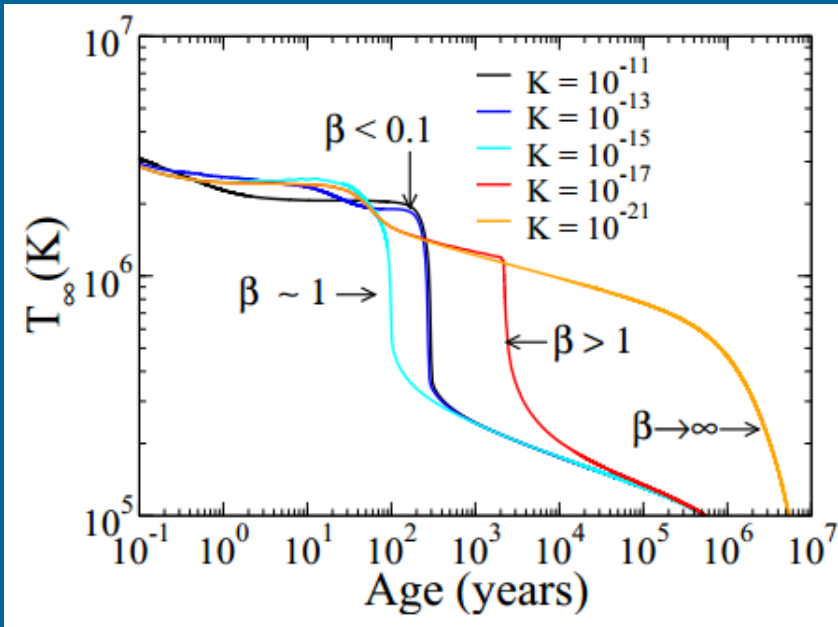
# New data: still no cooling?



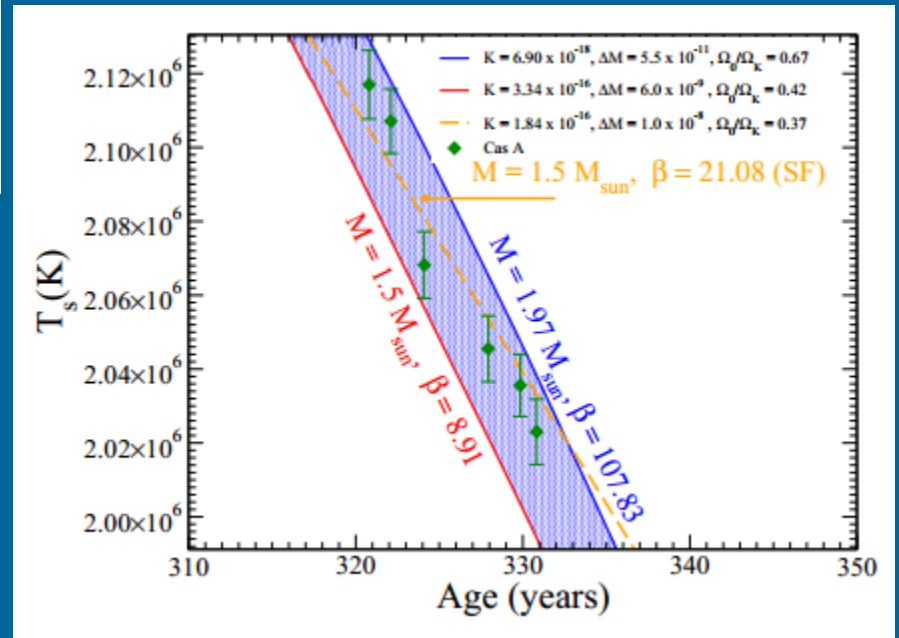
# Cooling and rotation



# Cas A case



$P_0 = 0.0025 - 0.00125$  sec  
 $B \sim 10^{11}$  G

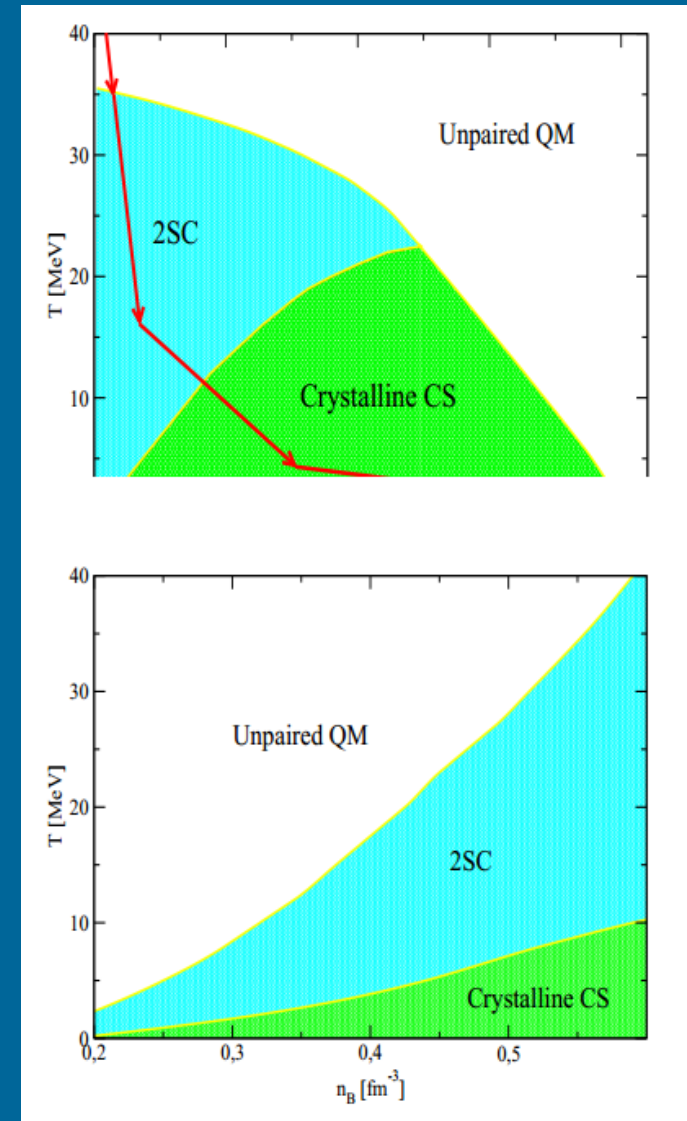
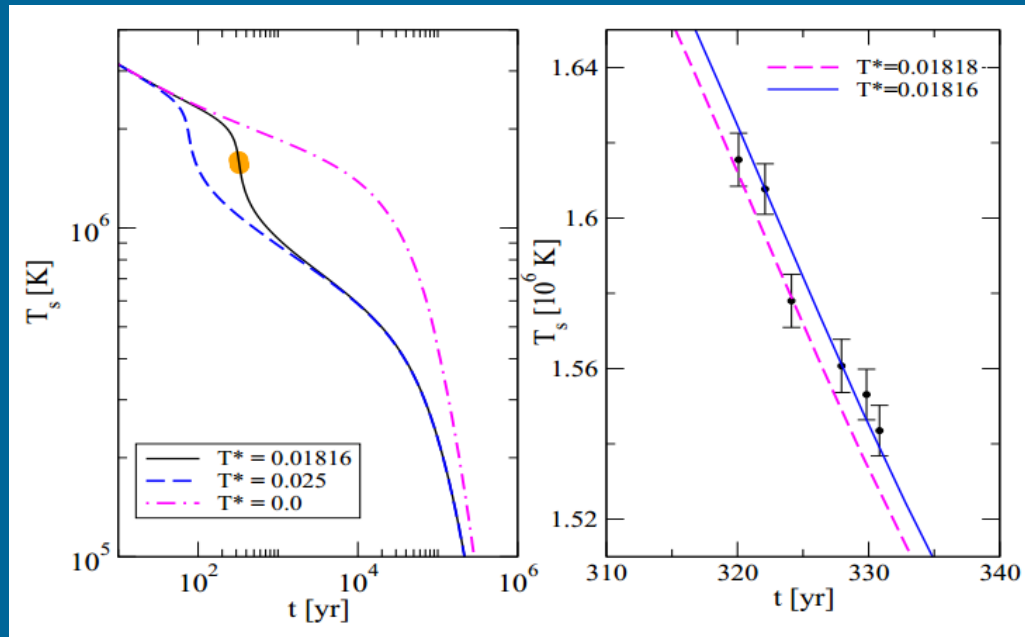


Other studies of the influence of effects of rotation see in 1201.2381

1103.3870

# Exotic phase transition

Rapid cooling of Cas A can be understood as a phase transition from the perfect 2SC phase to a crystalline/gapless color-superconducting state



# Cooling of X-ray transients

“Many neutron stars in close X-ray binaries are transient accretors (transients);

They exhibit X-ray bursts separated by long periods (months or even years) of quiescence.

It is believed that the quiescence corresponds to a lowlevel, or even halted, accretion onto the neutron star.

During high-state accretion episodes, the heat is deposited by nonequilibrium processes in the deep layers ( $10^{12}$  -  $10^{13}$  g cm<sup>-3</sup>) of the crust.

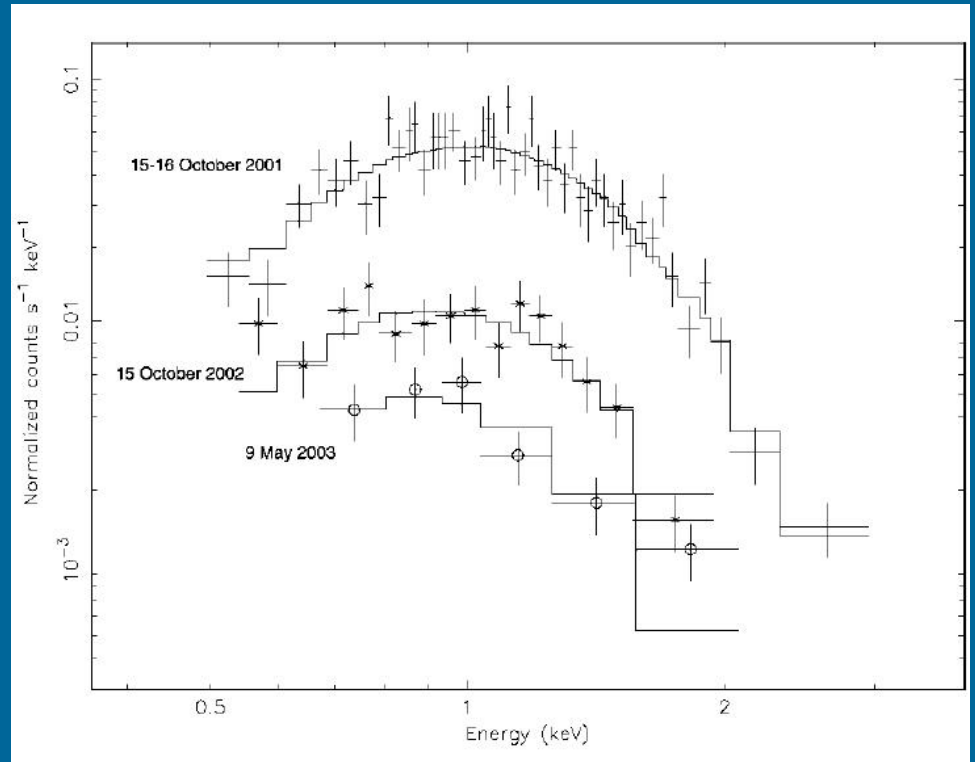
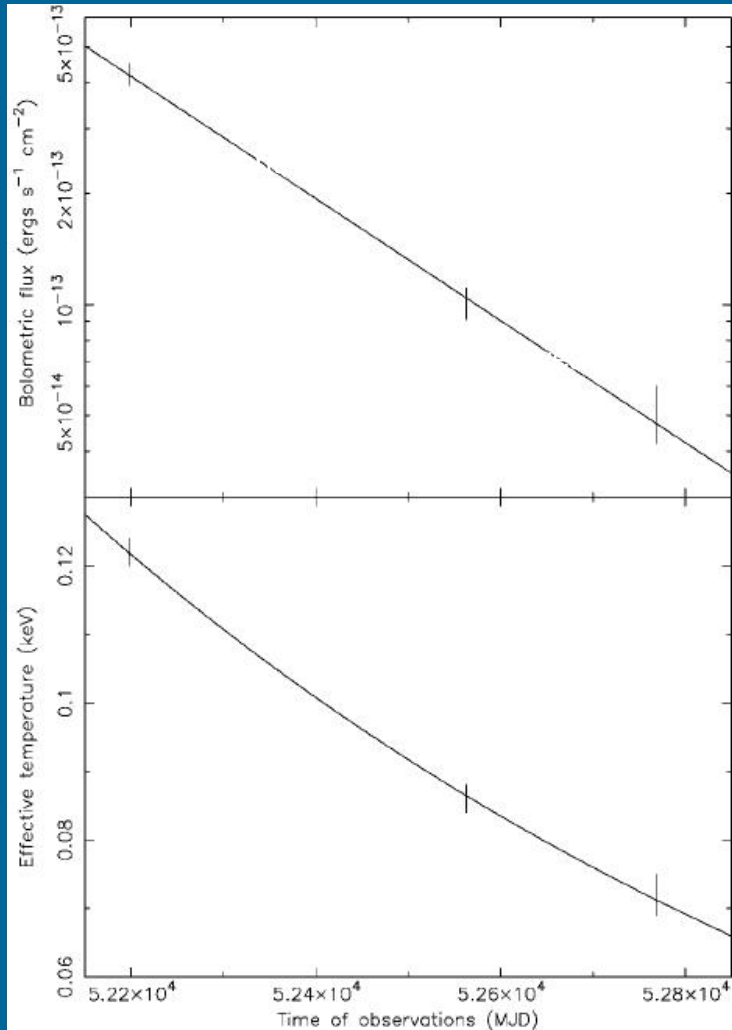
This deep crustal heating can maintain the temperature of the neutron star interior at a sufficiently high level to explain a persistent thermal X-ray radiation in quiescence (Brown *et al.*, 1998).”

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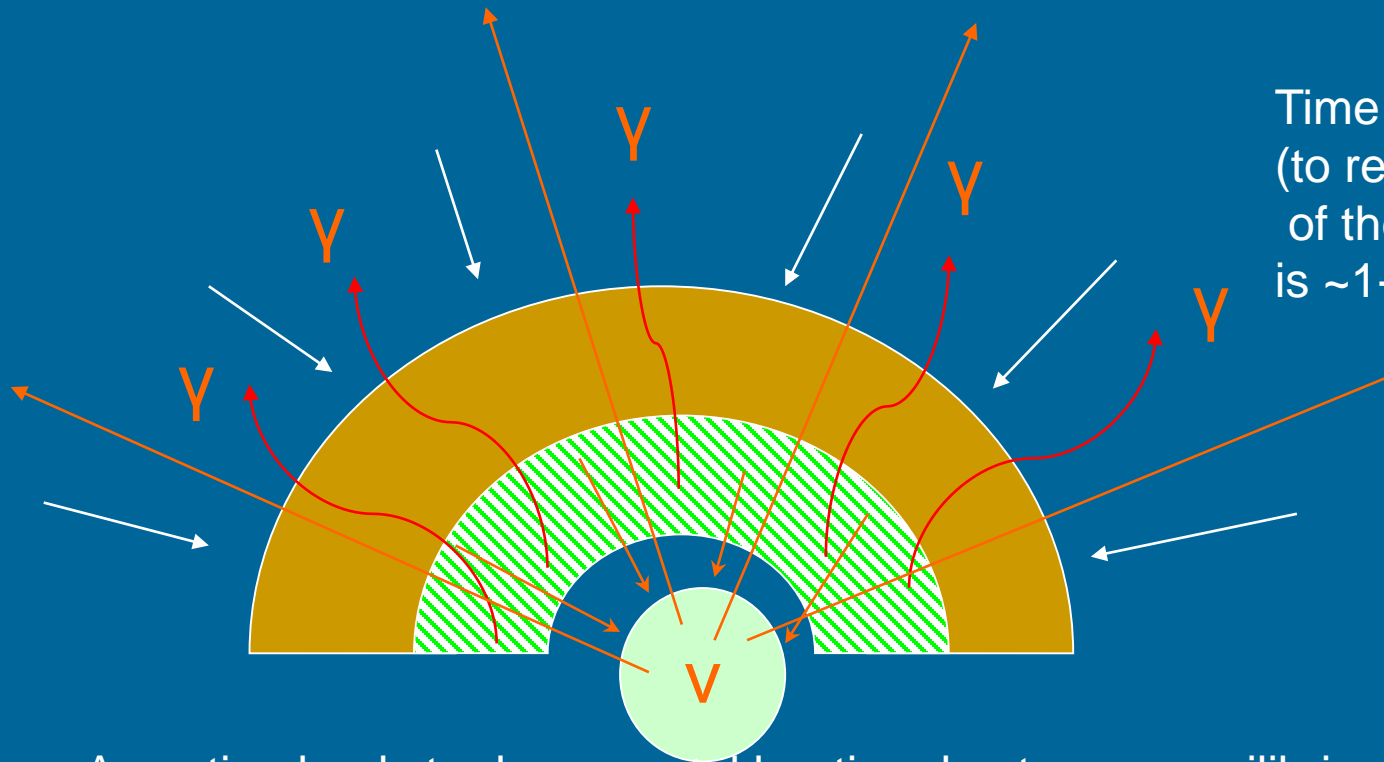
(quotation from the book by Haensel, Potekhin, Yakovlev)

# Cooling in soft X-ray transients

MXB 1659-29  
~2.5 years outburst



# Deep crustal heating and cooling



Time scale of cooling  
(to reach thermal equilibrium  
of the crust and the core)  
is  $\sim 1$ -100 years.

To reach the  
state “before”  
takes  $\sim 10^3$ - $10^4$  yrs

Accretion leads to deep crustal heating due to non-equilibrium nuclear reactions.  
After accretion is off:

- heat is transported inside and emitted by neutrinos
- heat is slowly transported out and emitted by photons

$$\rho \sim 10^{12} - 10^{13} \text{ g/cm}^3$$

See, for example, Haensel, Zdunik arxiv:0708.3996

New calculations appeared very recently 0811.1791 Gupta et al.



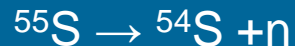
# Pycnonuclear reactions

Let us give an example from Haensel, Zdunik (1990)

We start with  $^{56}\text{Fe}$   
Density starts to increase



At  $^{56}\text{Ar}$ : neutron drip



Then from  $^{52}\text{S}$  we have a chain:



As  $Z$  becomes smaller  
the Coulomb barrier decreases.  
Separation between  
nuclei decreases, vibrations grow.



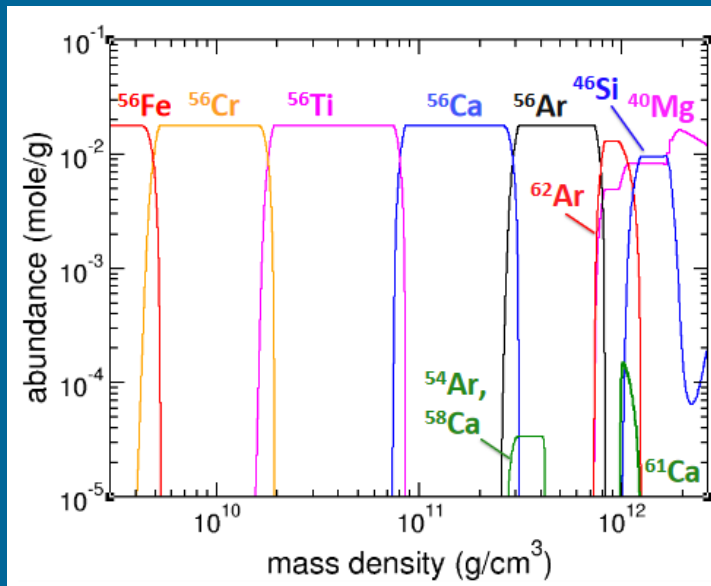
At  $Z=10$  (Ne) pycnonuclear reactions start.



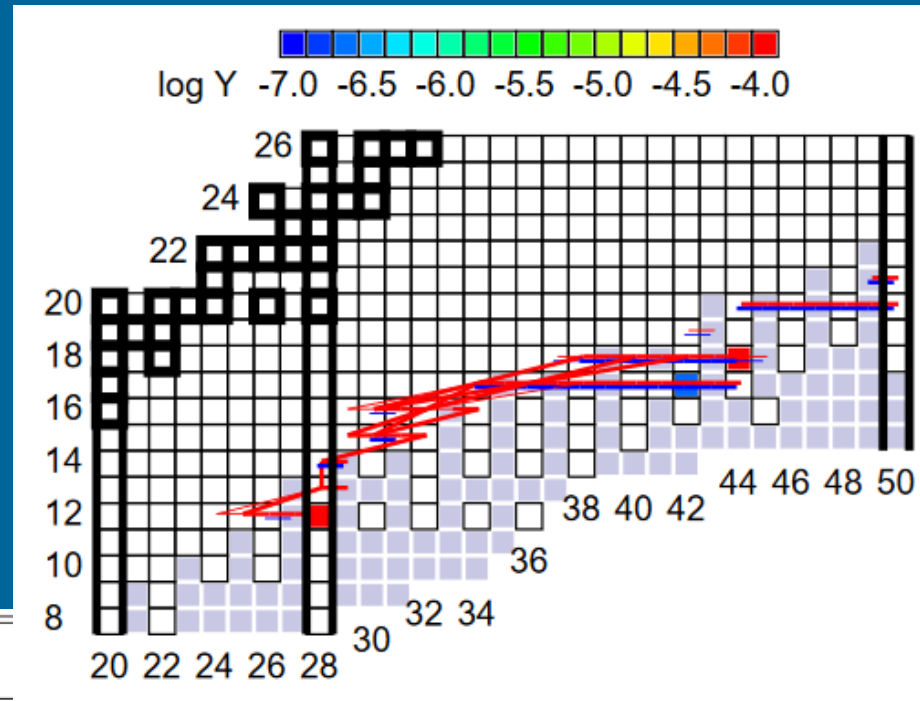
Then a heavy nuclei can react again:



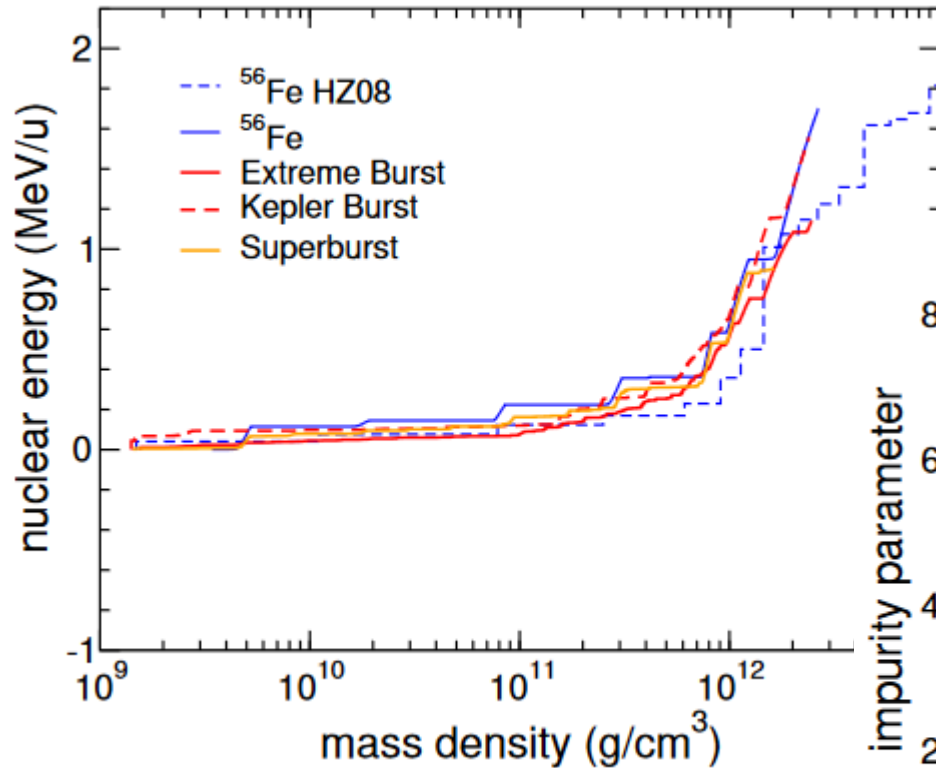
# Crust composition and reactions



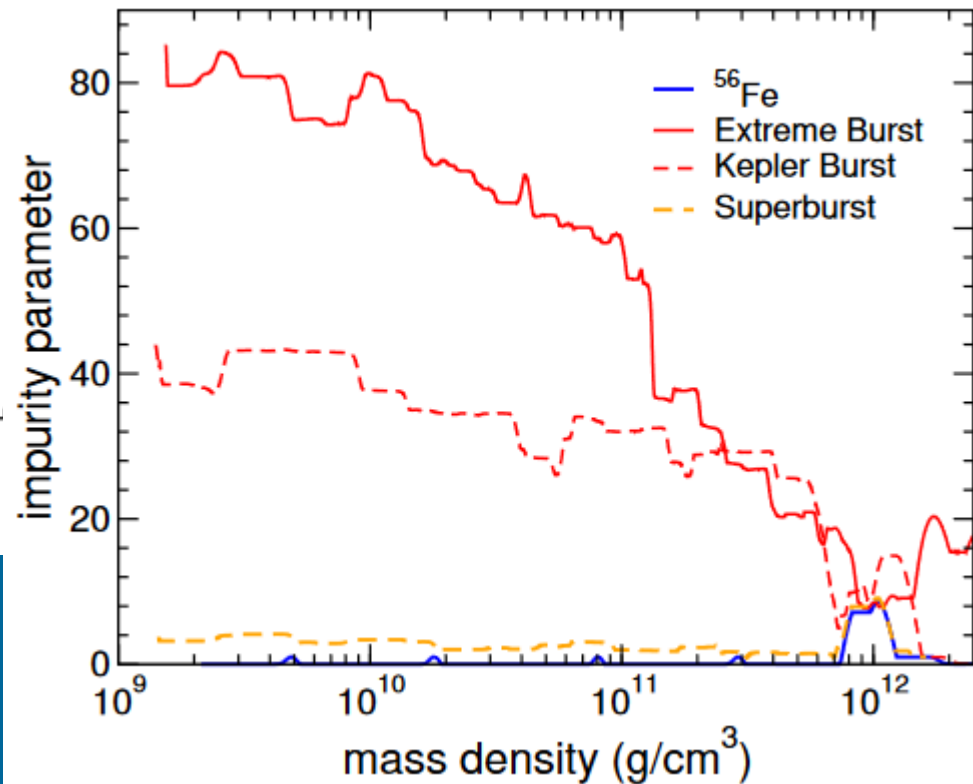
Transition	$P^a$	$\rho^b$	$\mu_e^c$	$X_n^d$
$^{56}\text{Fe} \rightarrow ^{56}\text{Cr}$	$3.4 \times 10^{27}$	$4.9 \times 10^9$	6.2	$< 10^{-25}$
$^{56}\text{Cr} \rightarrow ^{56}\text{Ti}$	$1.7 \times 10^{28}$	$1.8 \times 10^{10}$	9.6	$< 10^{-25}$
$^{56}\text{Ti} \rightarrow ^{56}\text{Ca}$	$1.1 \times 10^{29}$	$8.1 \times 10^{10}$	15.6	$< 10^{-25}$
$^{56}\text{Ca} \rightarrow ^{56}\text{Ar}, ^{54}\text{Ar}, ^{58}\text{Ca}$	$5.5 \times 10^{29}$	$2.9 \times 10^{11}$	23.3	$1.2 \times 10^{-18}$
$^{56}\text{Ar}, ^{54}\text{Ar}, ^{58}\text{Ca} \rightarrow ^{56}\text{Ar}$	$8.3 \times 10^{29}$	$4.2 \times 10^{11}$	25.9	$7.2 \times 10^{-20}$
$^{56}\text{Ar} \rightarrow ^{40}\text{Mg}, ^{62}\text{Ar}$	$1.8 \times 10^{30}$	$7.8 \times 10^{11}$	31.6	$5.4 \times 10^{-8}$
$^{40}\text{Mg}, ^{62}\text{Ar} \rightarrow ^{40}\text{Mg}, ^{48}\text{Si}$	$2.3 \times 10^{30}$	$1.1 \times 10^{12}$	33.5	0.13
$^{40}\text{Mg}, ^{48}\text{Si} \rightarrow ^{40}\text{Mg}$	$4.2 \times 10^{30}$	$2.8 \times 10^{12}$	37.1	0.54



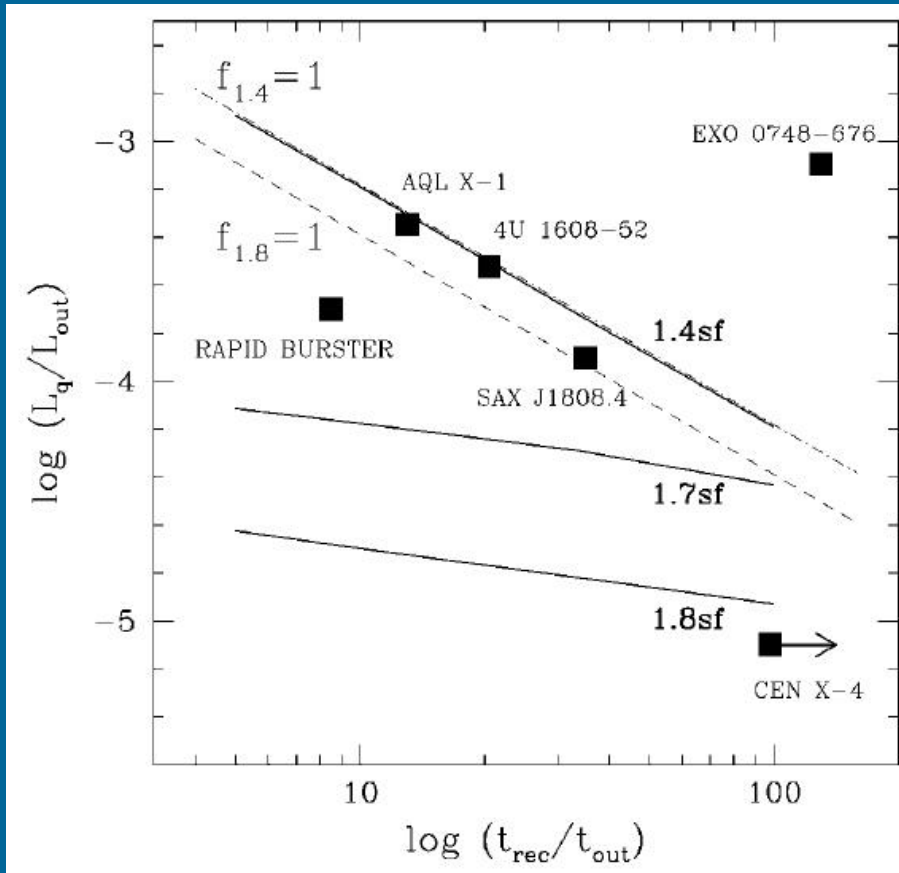
# Energy release vs. density and impurity



$$Q_{\text{imp}} = \sum_i Y_i (Z_i - \langle Z \rangle)^2 / \sum_i Y_i$$



# A simple model



$t_{rec}$  – time interval between outbursts

$t_{out}$  – duration of an outburst

$L_q$  – quiescent luminosity

$L_{out}$  – luminosity during an outburst

Dashed lines corresponds to the case when all energy is emitted from a surface by photons.

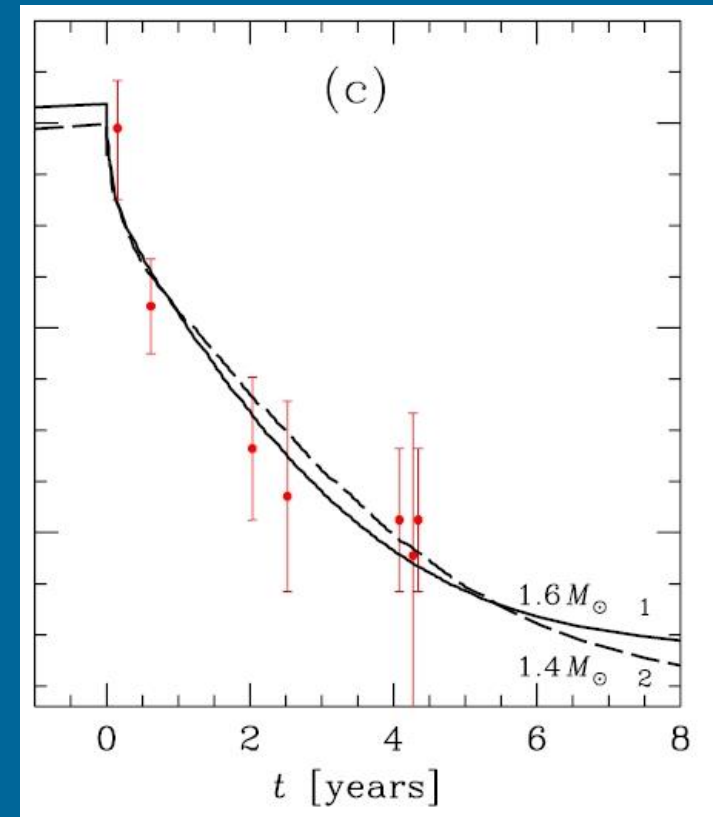
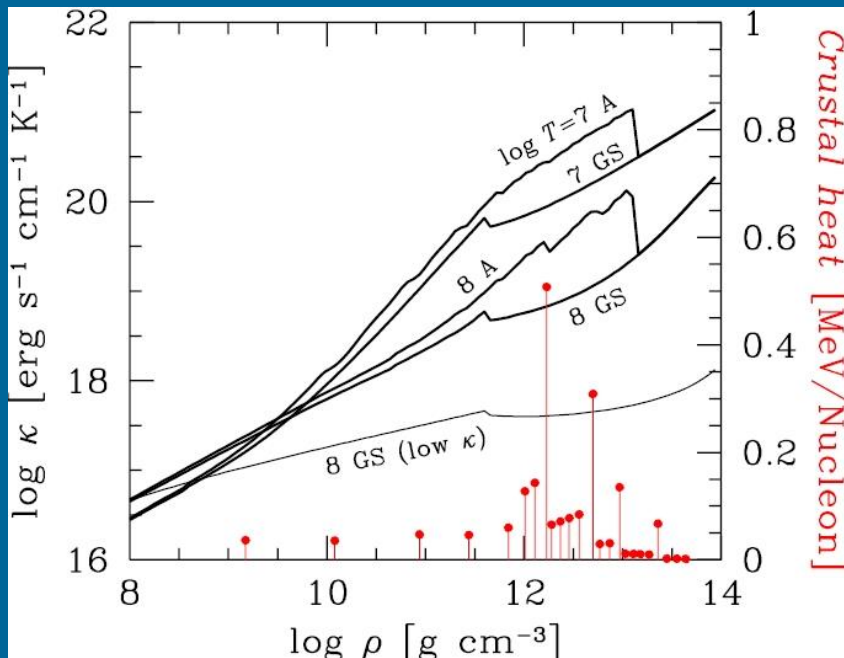
$$L_q \sim \frac{Q_{nuc}}{m_u} \langle \dot{M} \rangle \sim 6 \times 10^{32} \frac{\langle \dot{M} \rangle}{10^{-11} M_{\odot} \text{ yr}^{-1}} \text{ ergs s}^{-1}$$

# Deep crustal heating

~1.9 Mev per accreted nucleon  
Crust is not in thermal equilibrium with the core.  
After accretion is off the crust cools down and  
finally reach equilibrium with the core.

KS 1731-260

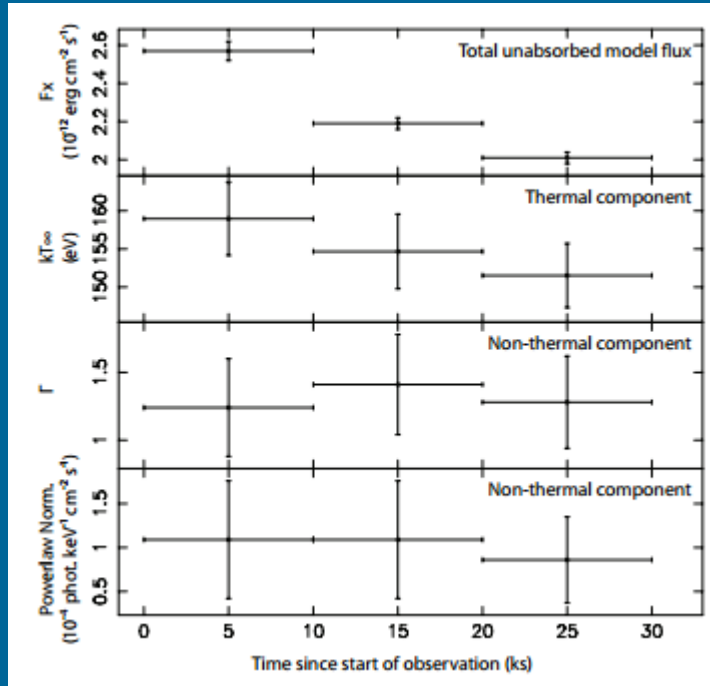
(see a recent model in 1202.3378)



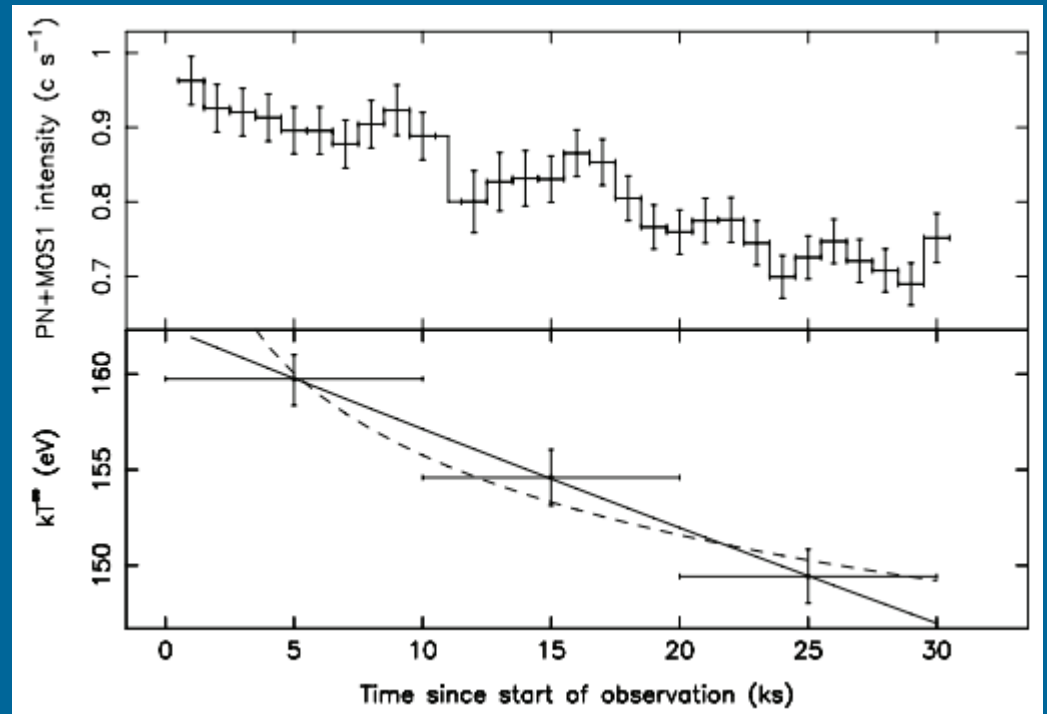
[Shternin et al. 2007]

See new results and discussion in 1702.08452

# Visible cooling of a NS in a binary

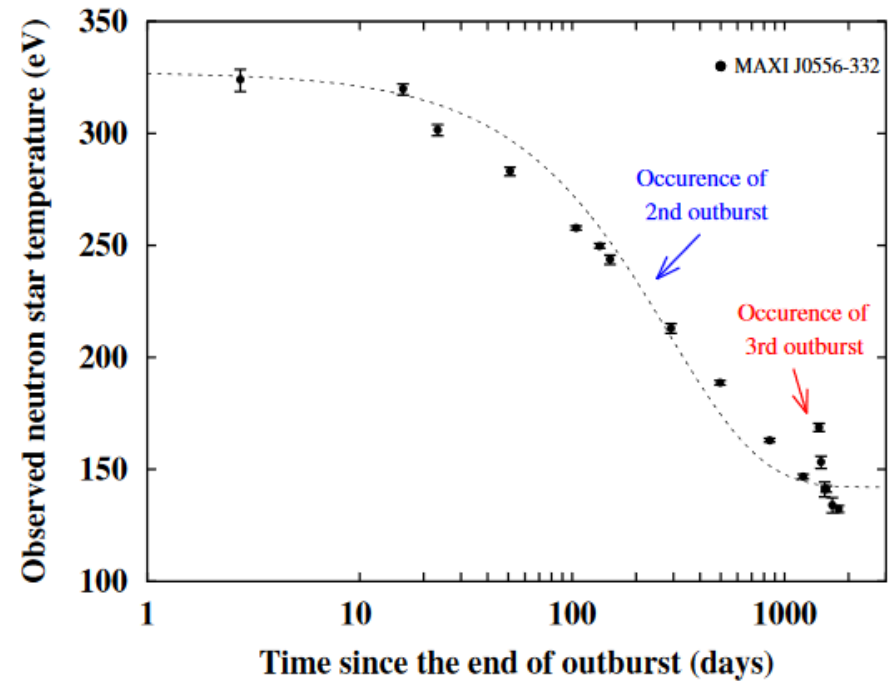
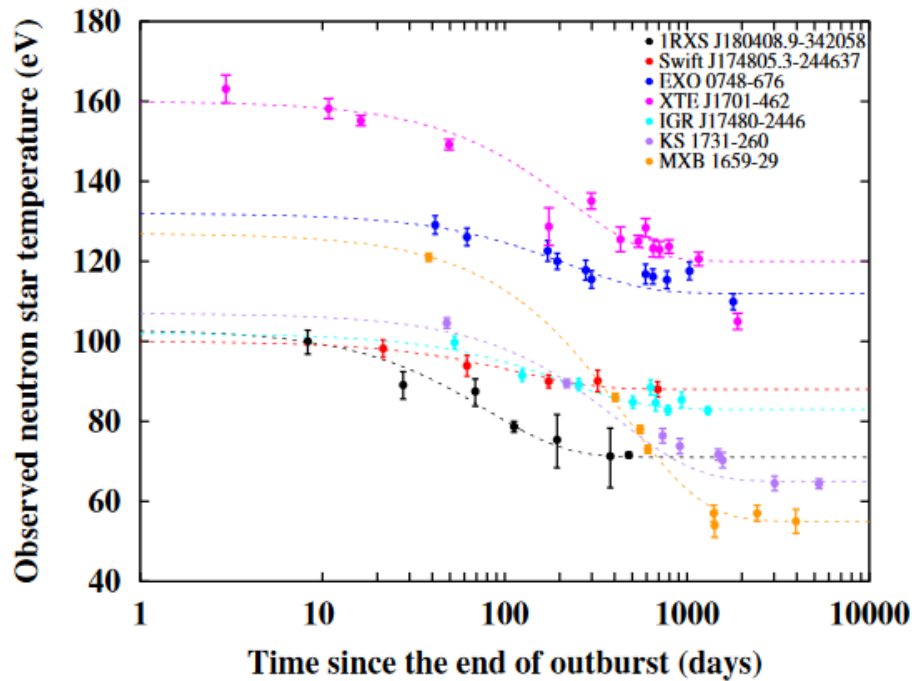


The authors interpret this as cooling of a layer located at a column density of  $y \simeq 5 \times 10^{12} \text{ g cm}^{-2}$  ( $\simeq 50 \text{ m}$  inside the neutron star), which is just below the ignition depth of superbursts.



XTE J1709-267

# Fitting cooling of known sources



Different systems allow to probe different regimes of cooling and different layers of the crust.

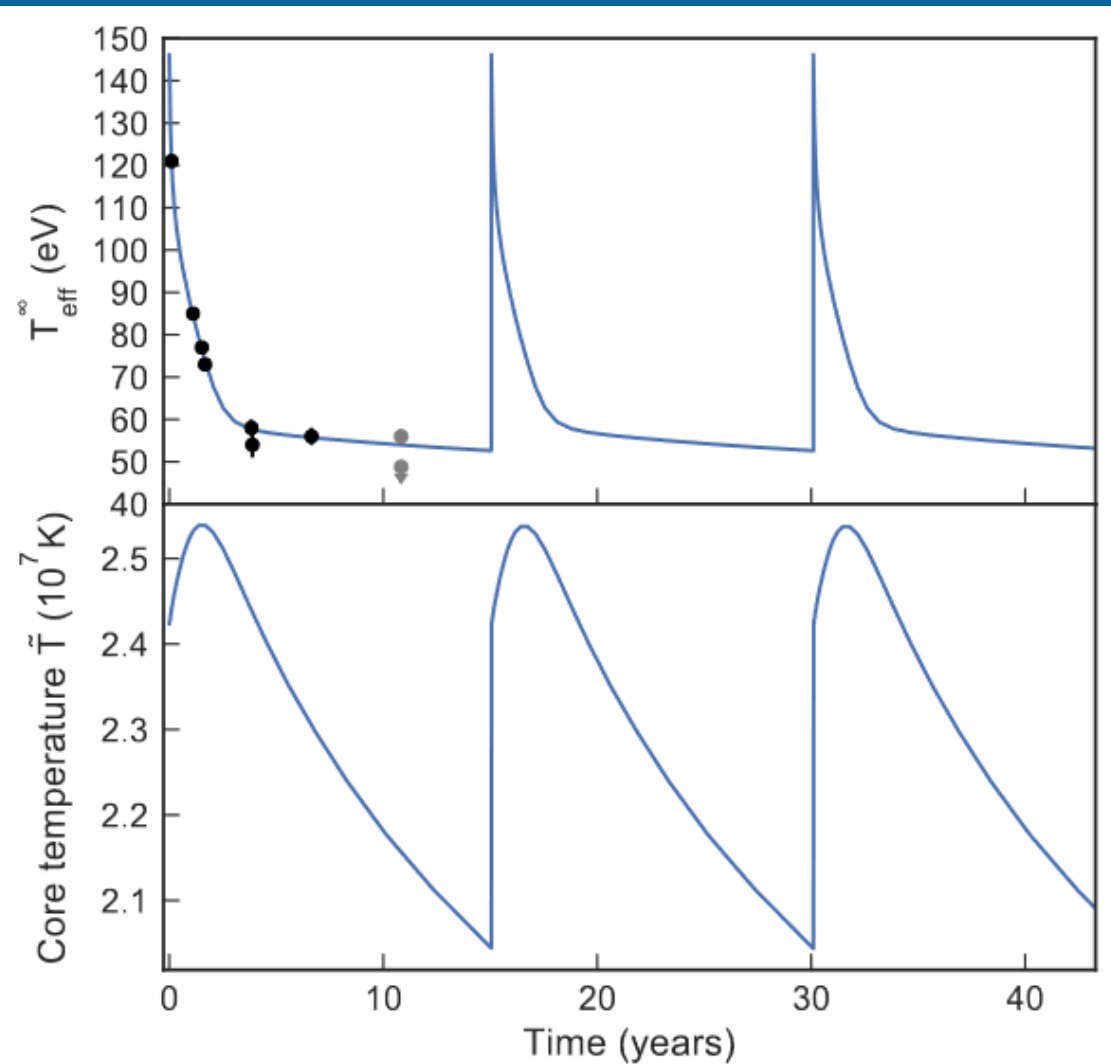
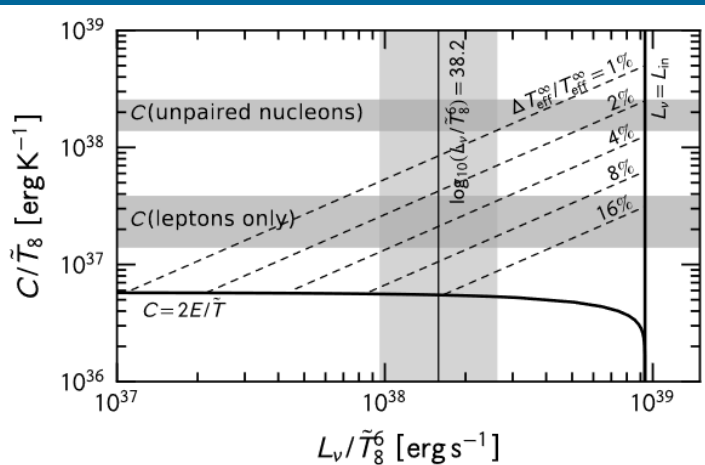
# Direct Urca in a cooling NS

MXB 1659-29

$$2.1 \times 10^{38} \text{ erg s}^{-1} \tilde{T}_8^6$$

$$C = 10^{37} \text{ erg K}^{-1} \tilde{T}_8$$

About 1% of the core volume available for direct URCA.



1801.00041



# Cooling and crustal properties

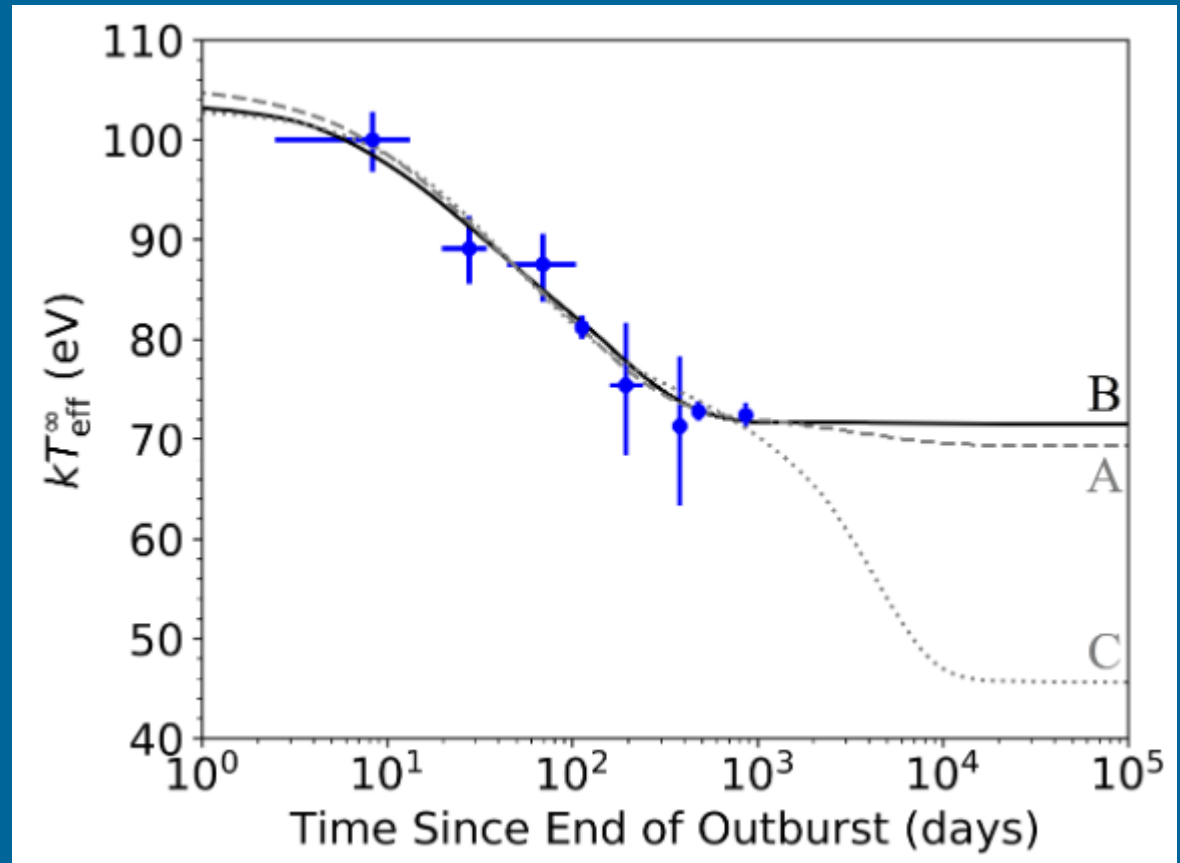
RXS J180408.9–342058  
LMXB

Rapid cooling  
down to thermal  
equilibrium between  
the core and the crust.

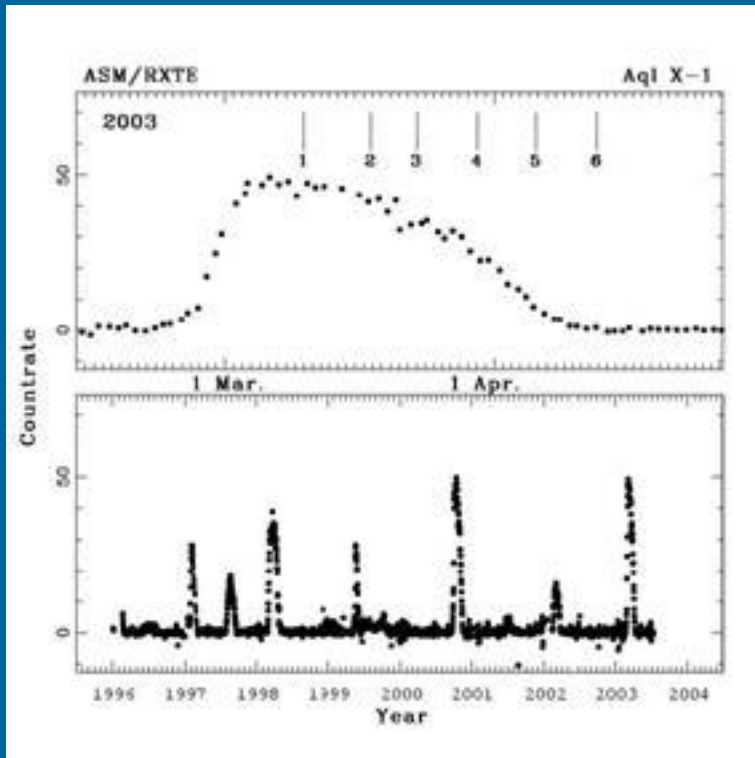
Deep crustal heating +  
shallow heat source.

The origin of the shallow  
heating is unknown.

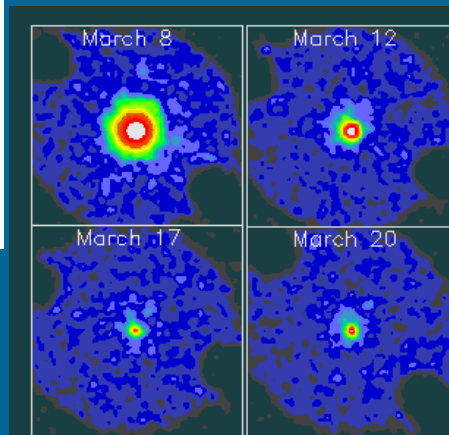
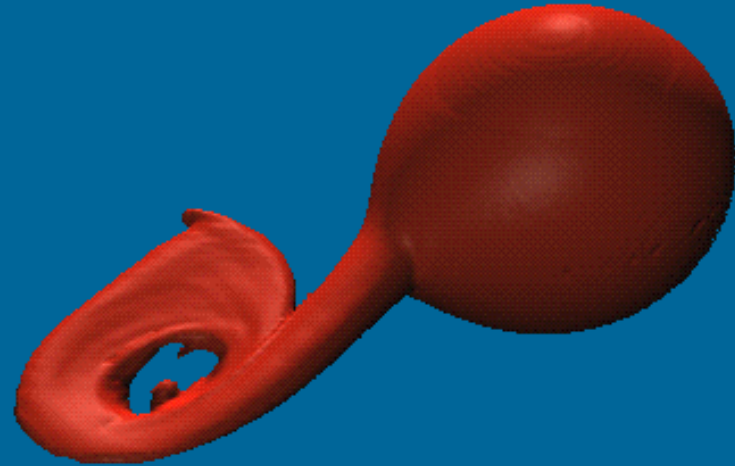
No DURCA.



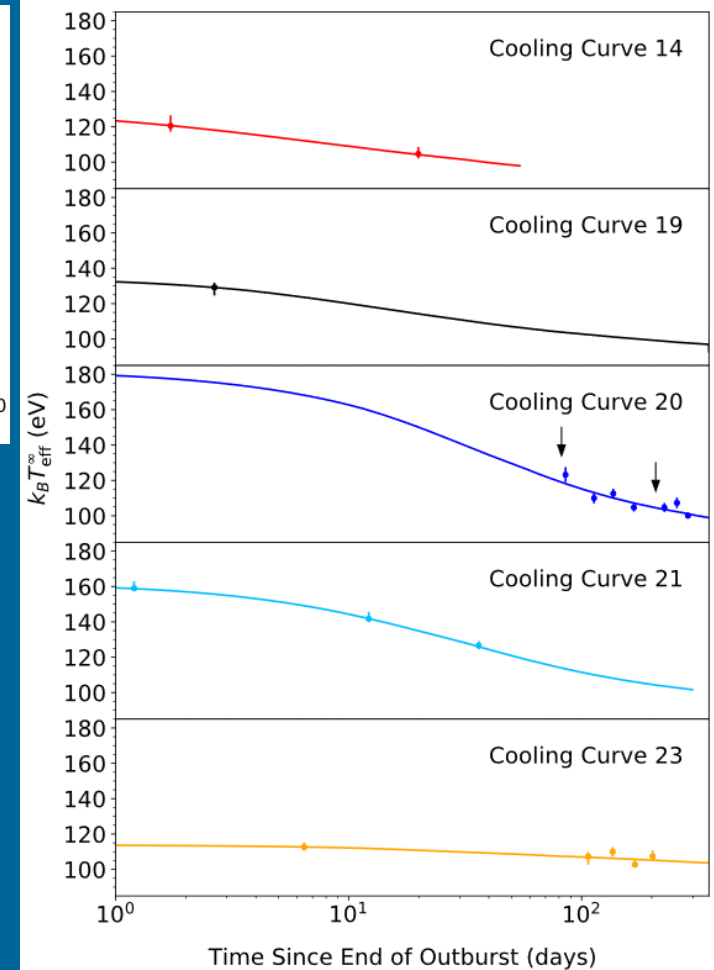
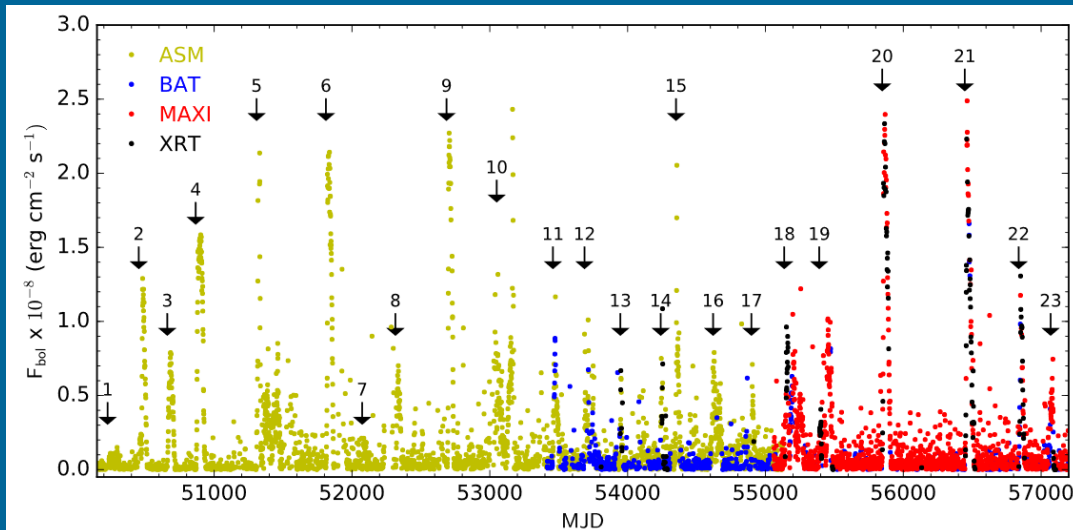
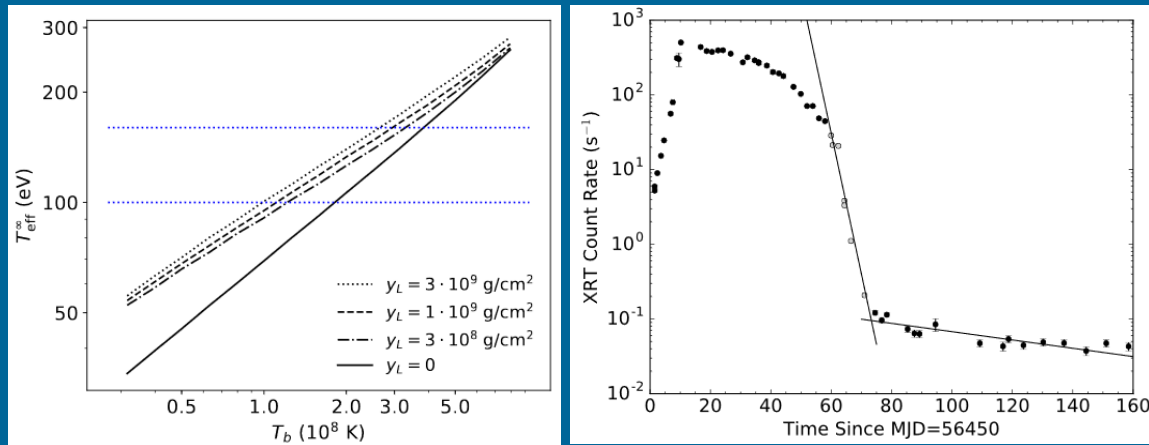
# Aql X-1 transient



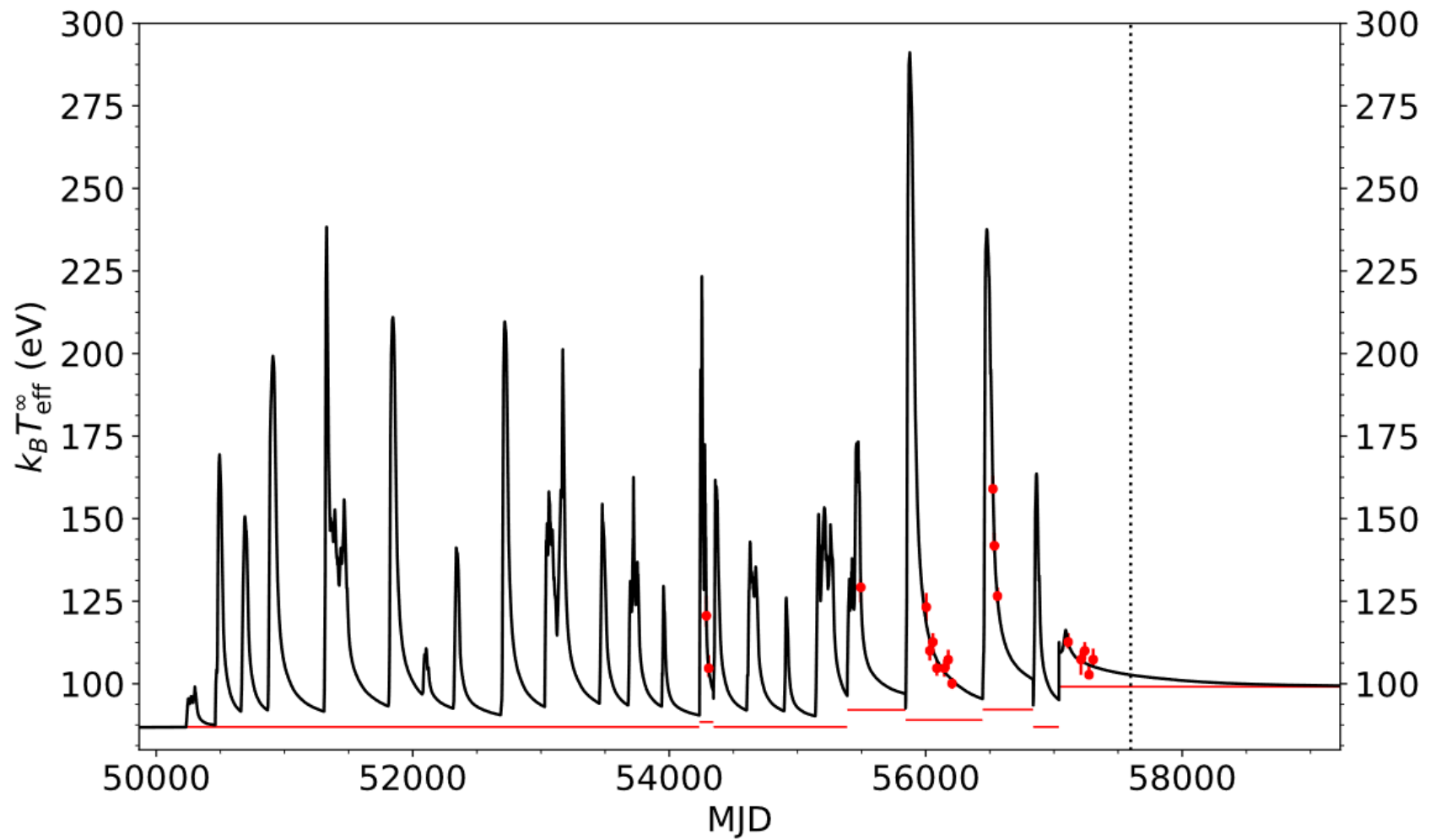
A NS with a K star.  
The NS is the hottest  
among SXTs.



# Aql X-1 modeling

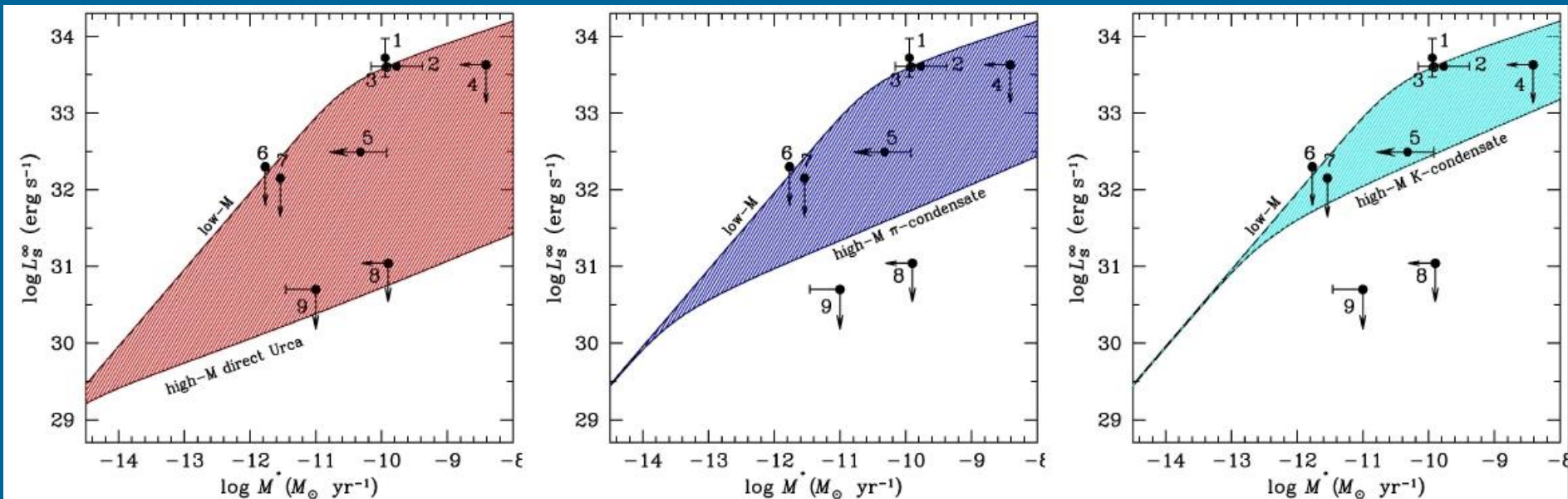


1802.06081



1802.06081

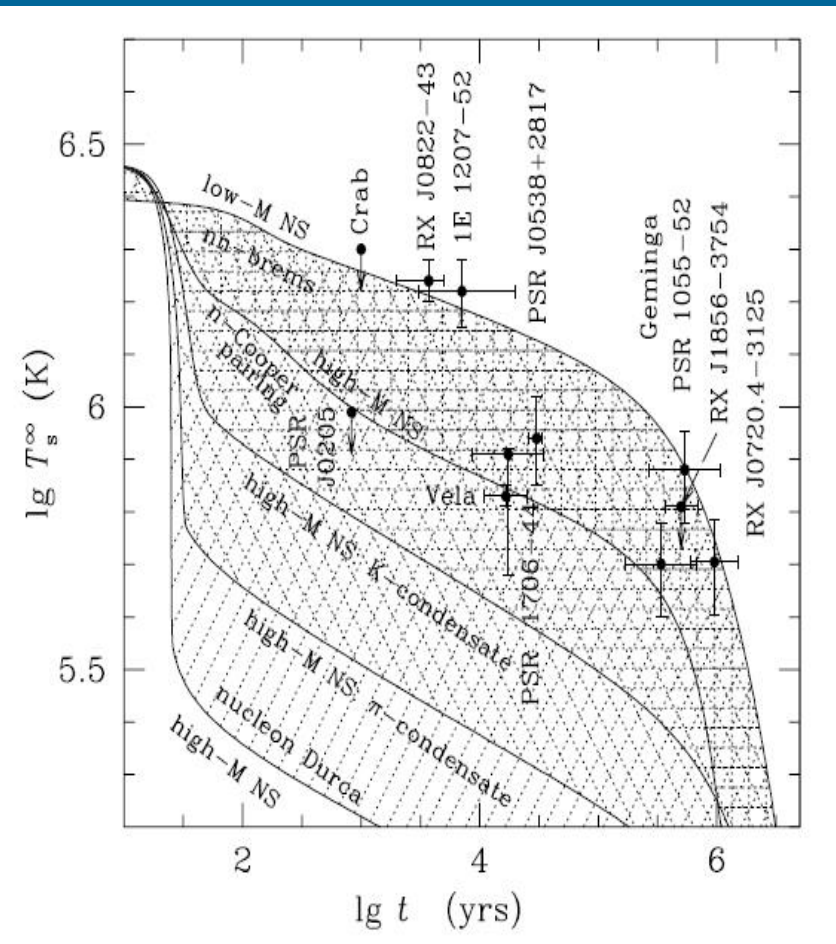
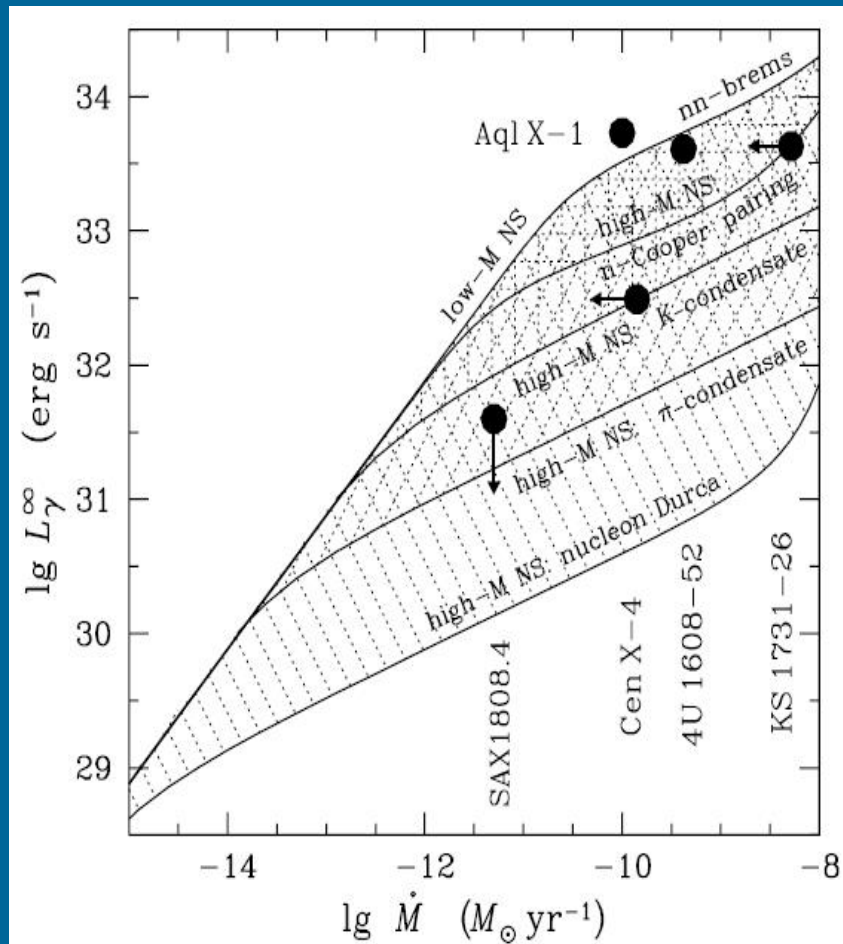
# Testing models with SXT



SXTs can be very important in confronting theoretical cooling models with data.

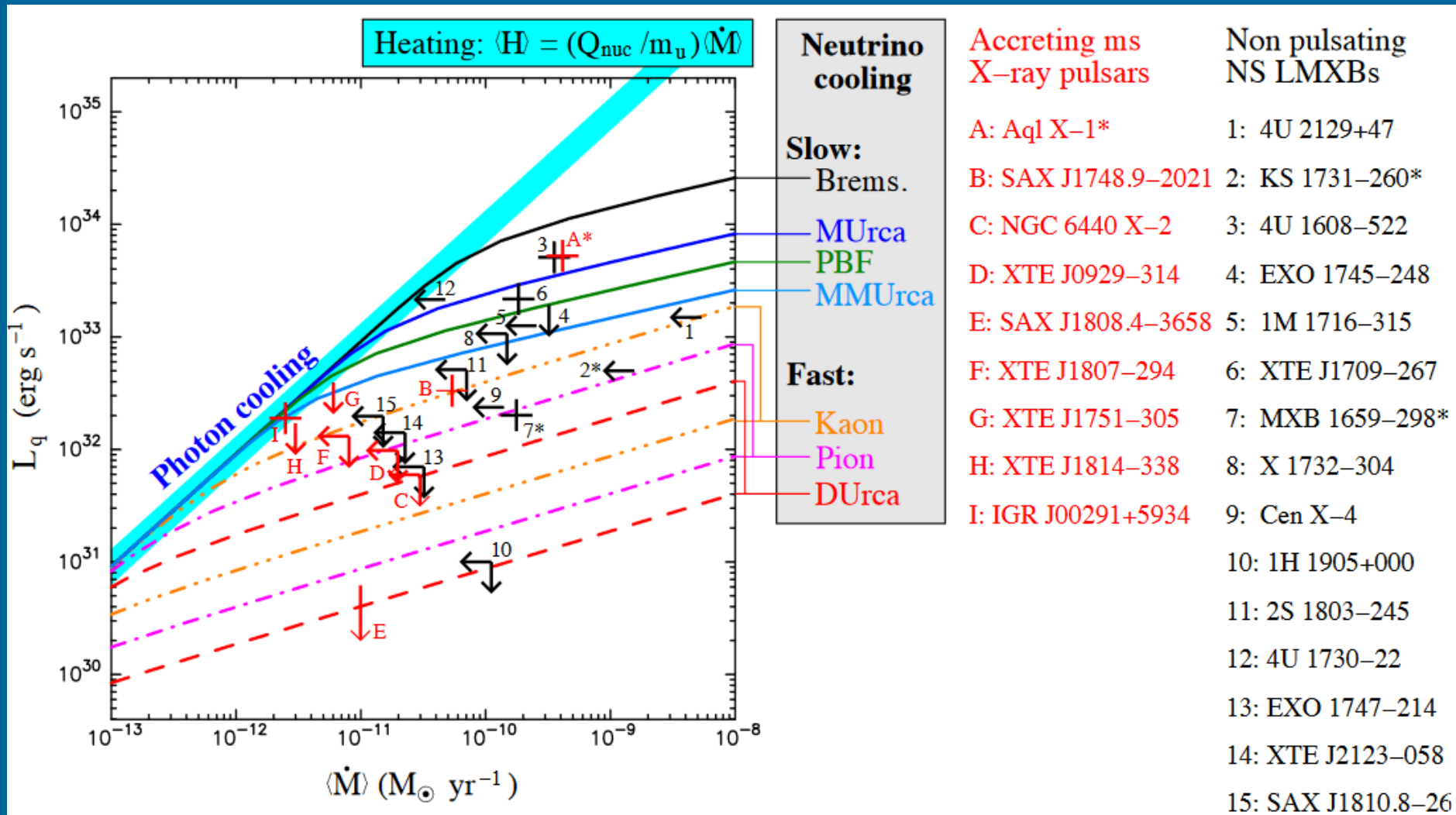
[from a presentation by Haensel, figures by Yakovlev and Levenfish]

# Theory vs. Observations: SXT and isolated cooling NSs





# Systems with deep crustal heating




# Conclusions

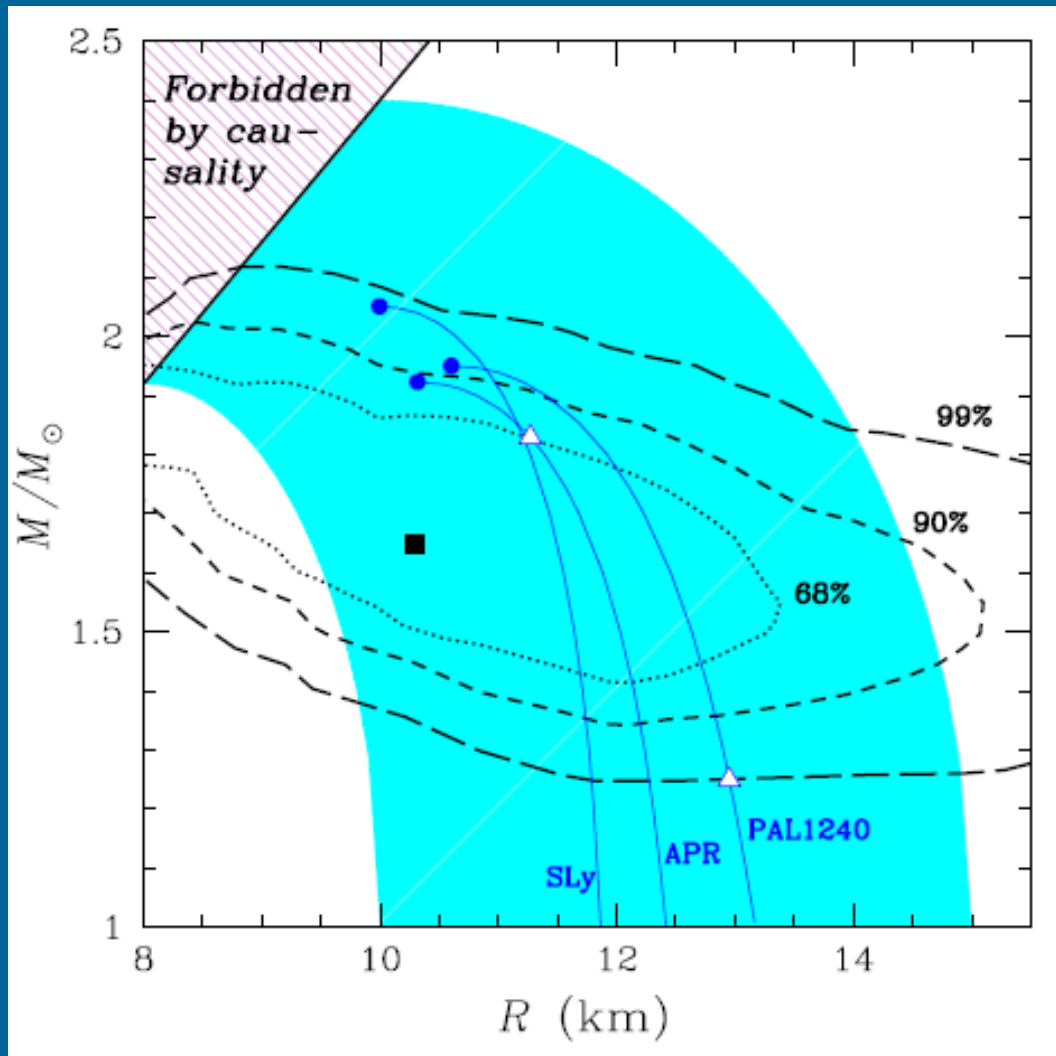
- NSs are born hot, and then cool down at first due to neutrino emission, and after – due to photon emission
- Observations of cooling provide important information about processes at high density at the NS interiors
- Two types of objects are studied:
  - isolated cooling NSs
  - NSs in soft X-ray transients

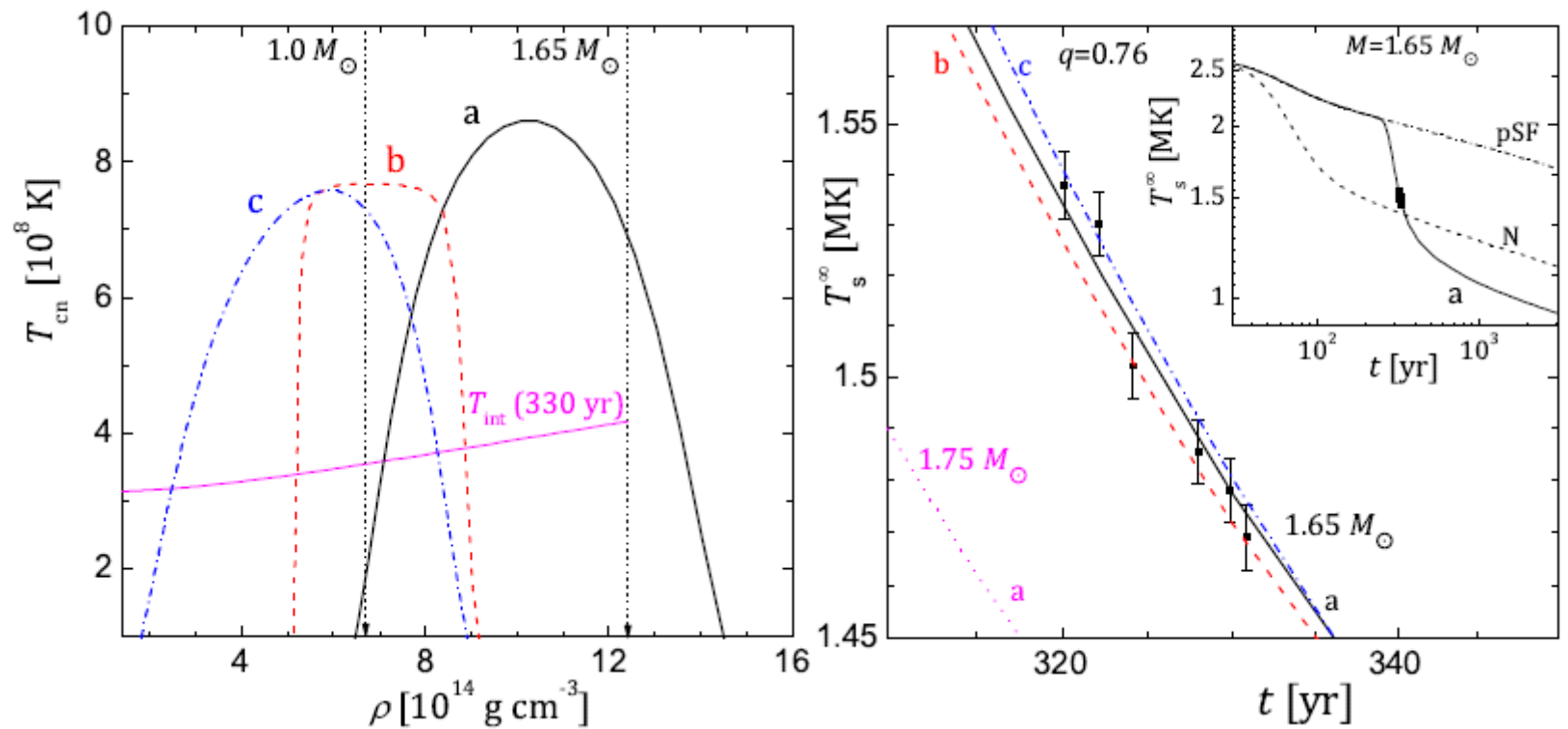


# Papers to read

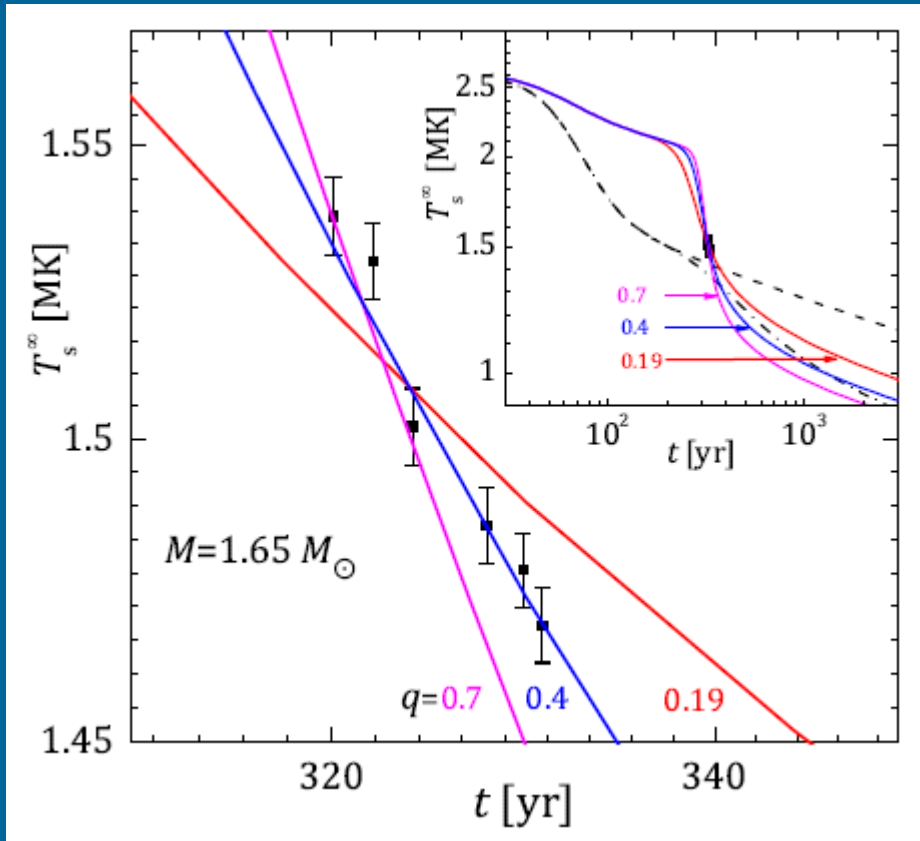
- Or astro-ph/0403657  
Or astro-ph/0508056  
Or astro-ph/0402143   
Or 1507.06186
- [arXiv:astro-ph/9906456](https://arxiv.org/abs/astro-ph/9906456) YΦH 1999
- 1709.07034 – about cooling of NSs in binaries

# M-R from spectral fit

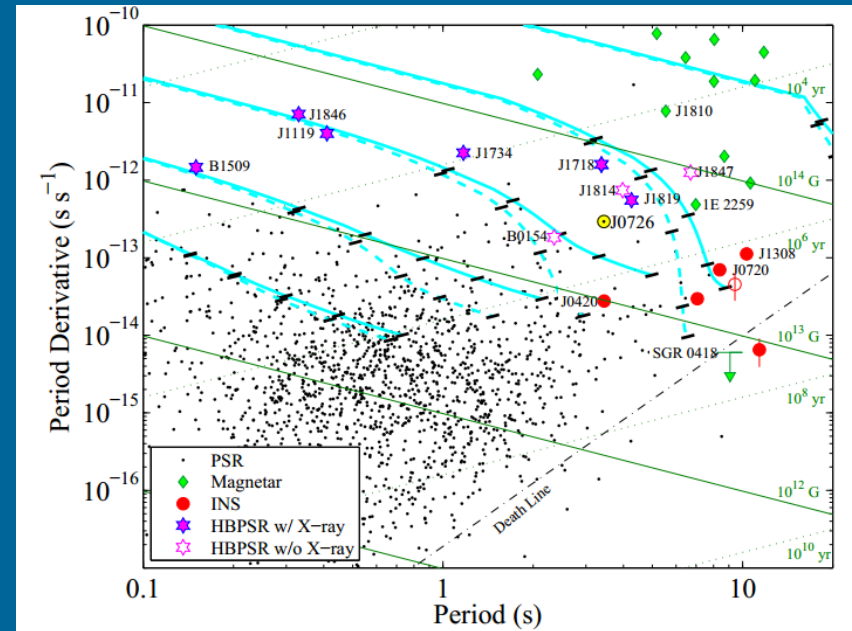
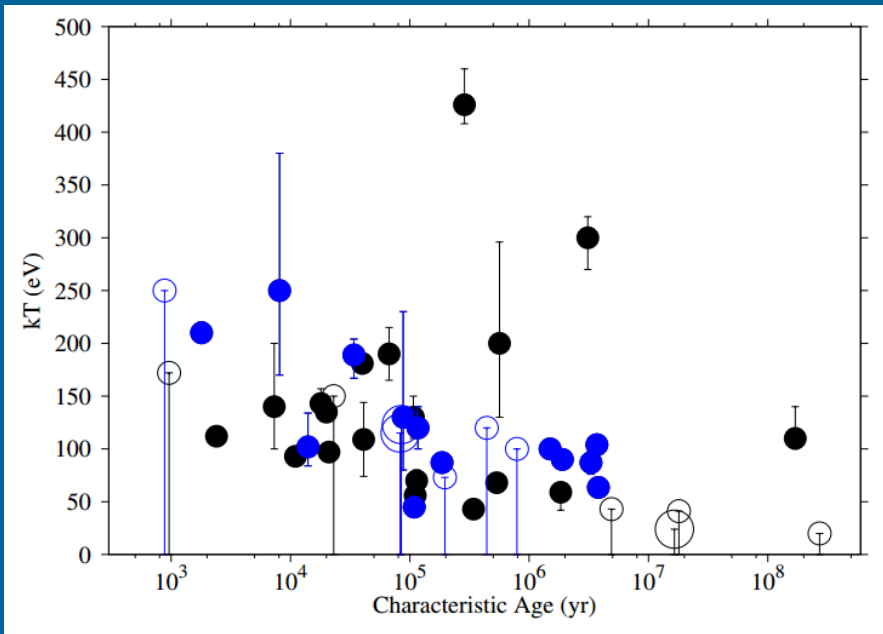




# Suppression in the axial-vector channel



# Cooling and grand unification for NSs



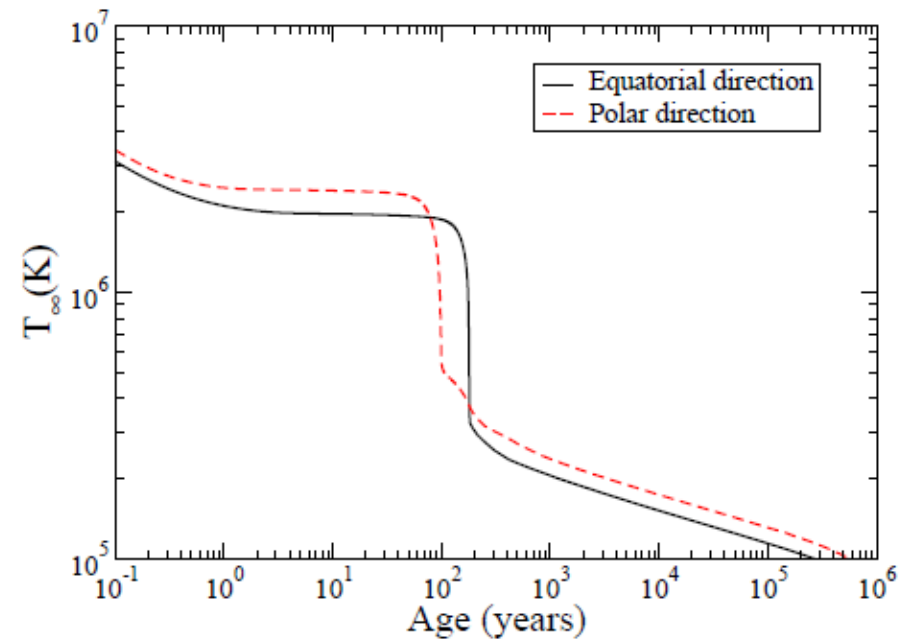
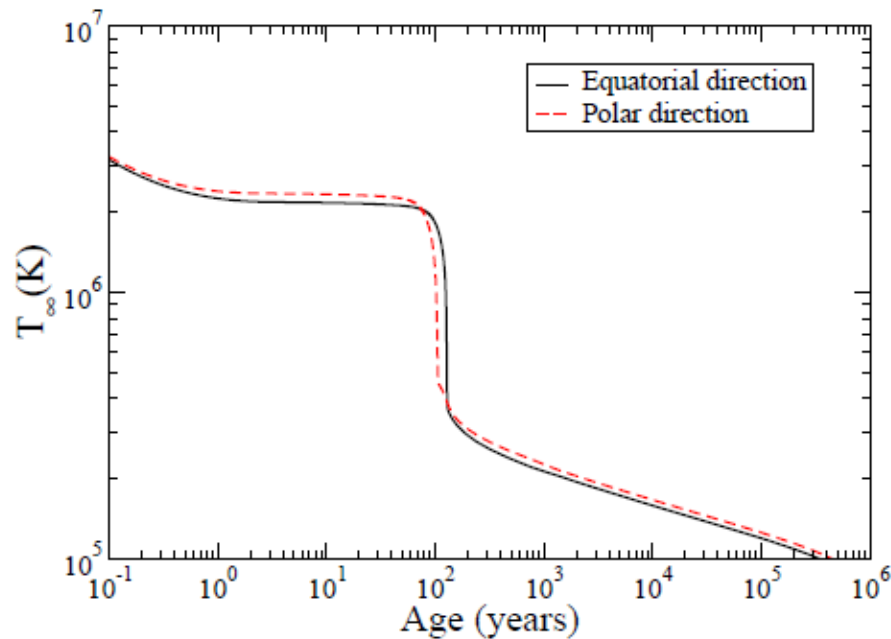
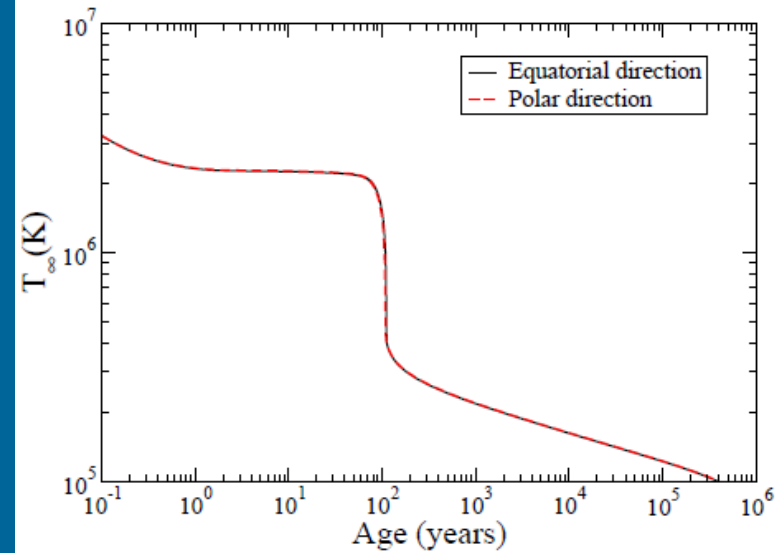
1301.2814

1111.2877

One study shows that highly magnetized NSs can be not hotter than NSs with standard magnetic fields.

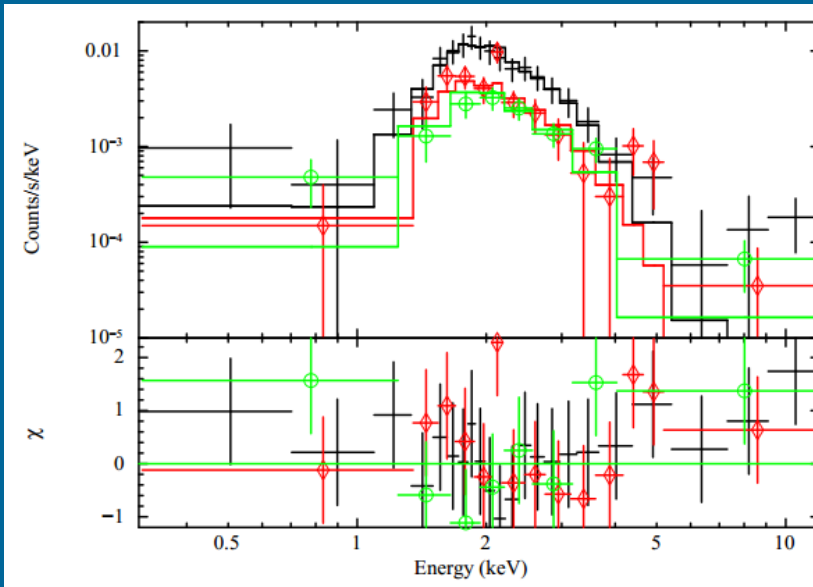
Another study demonstrates that some young PSRs with relatively large field are hot, similar to the M7.

# Влияние вращения

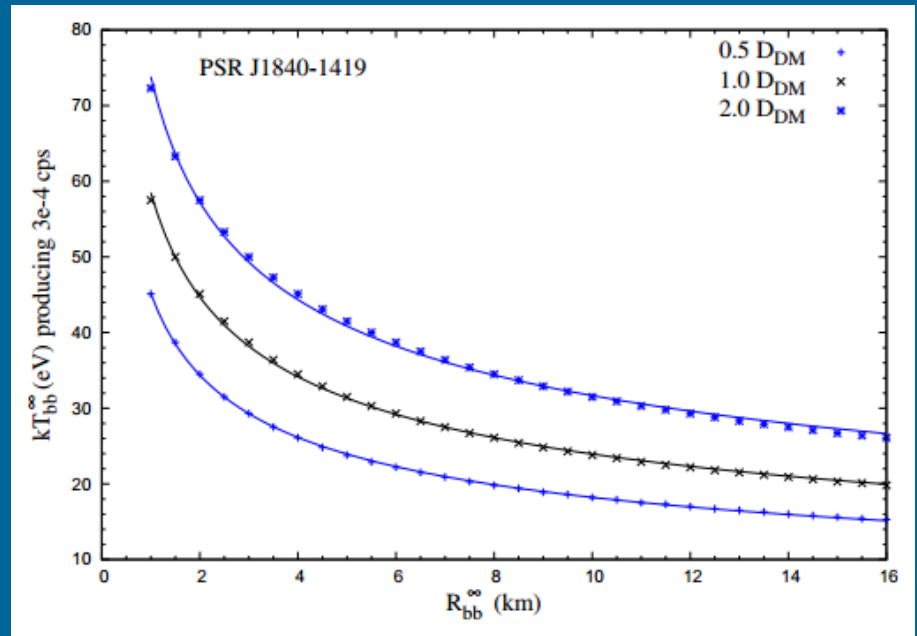


# Records

The hottest (in a binary, crustal heating)  
SAX J1750.8–2900.  $T \sim 150$  eV.  
1202.1531



The coldest. Isolated pulsar.  $T < 30$  eV  
PSR J18401419  
1301.2814



# Surface emission of neutron stars

---



# NS Radii

- A NS with homogeneous surface temperature and local blackbody emission

$$L = 4\pi R^2 \sigma T^4$$

$$F = \frac{L}{4\pi D^2} = (R/D)^2 \sigma T^4$$

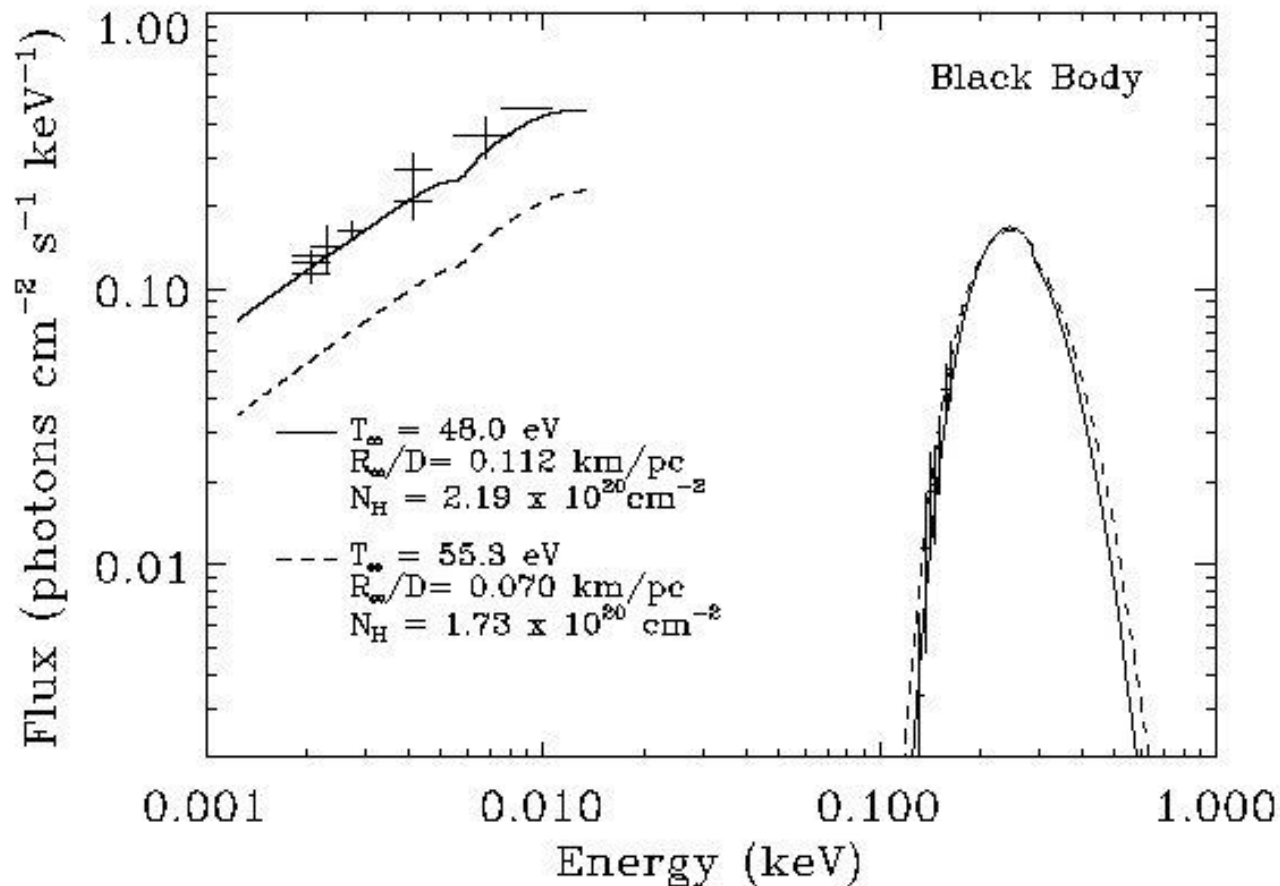
From dispersion  
measure

From X-ray  
spectroscopy

# NS Radii - II

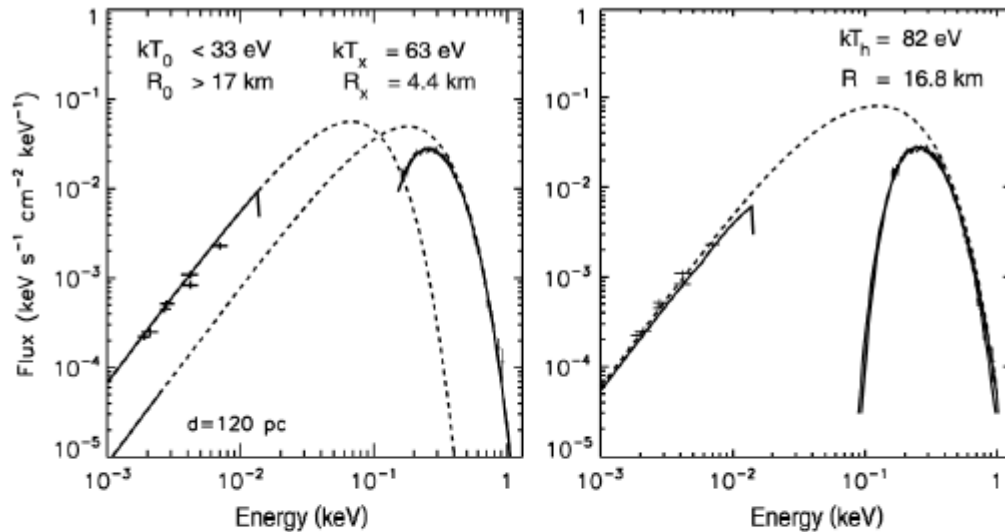
- Real life is a trifle more complicated...  
Atmospheres.
- Because of the strong B field
  - Photon propagation different
  - Surface temperature is not homogeneous
  - Local emission may be not exactly planckian
- Gravity effects are important

# Uncertainties in temperature

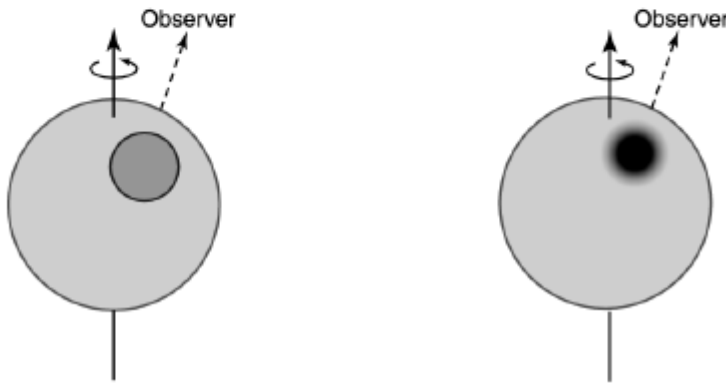


- Atmospheres (composition)
- Magnetic field
- Non-thermal contributions to the spectrum
- Distance
- Interstellar absorption
- Temperature distribution

# Non-uniform temperature distribution

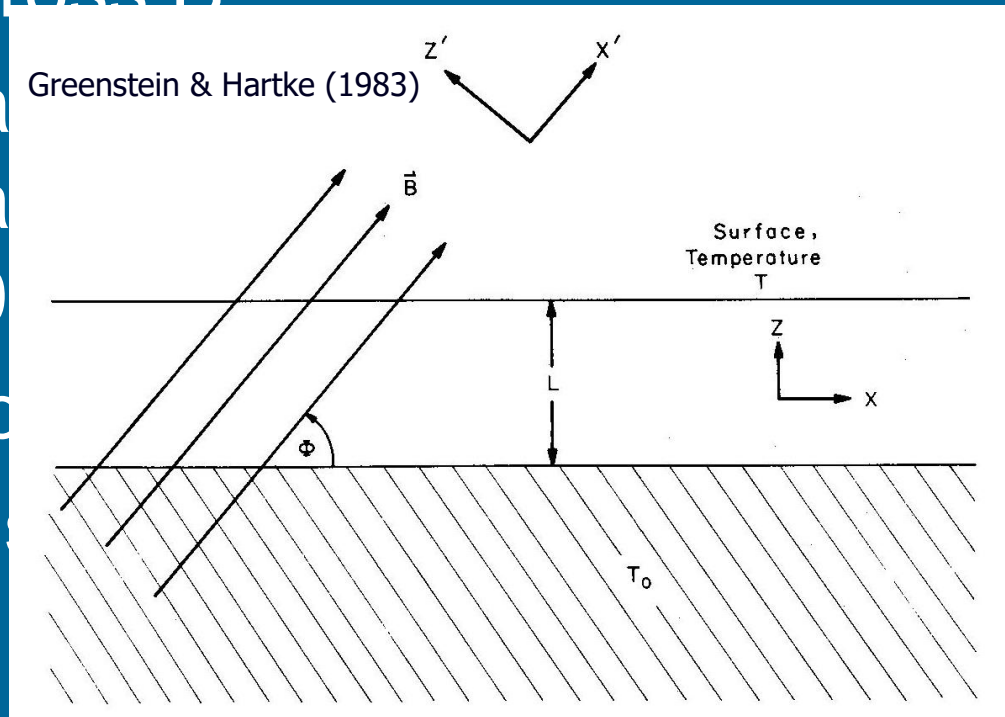


In the case of RX J1856 because of significant ( $\sim 6$ ) optical excess it was proposed that there is a spot, or there is a continuous temperature gradient.



# NS Thermal Maps

- Electrons move much more easily along  $B$  than across  $B$
- Thermal conductivity inside a metal is  $\rho \gg 10^{-8} \Omega \cdot \text{cm}$
- Envelope of the thermal map is  $B \sim \cos \theta$



$$T_s = \left[ \cos^2 \Theta + \left( K_{\text{perp}} / K_{\text{par}} \right) \sin^2 \Theta \right]^{1/4} T_{\text{pole}}$$

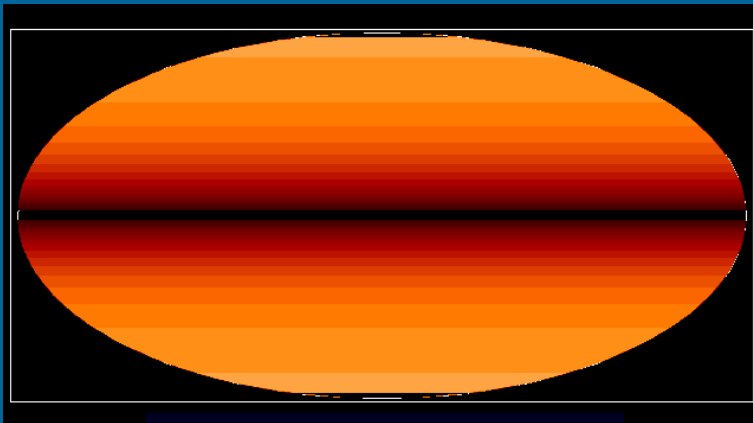
$$K_{\text{perp}} / K_{\text{par}} \ll 1$$

K - conductivity

$$T_s = |\cos \Theta|^{1/2} T_{\text{pole}}$$



Valid for strong fields:  $K_{\text{perp}} \ll K_{\text{par}}$



Core centered dipole



Core centered quadrupole

# Local Surface Emission

- Much like normal stars NSs are covered by an atmosphere
- Because of enormous surface gravity,  $g \approx 10^{14} \text{ cm/s}^2$ ,  $h_{\text{atm}} \approx 1\text{-}10 \text{ cm}$  ( $h_{\text{atm}} \sim kT/mg$ )
- Spectra depend on  $g$ , chemical composition and magnetic field
- Plane-parallel approximation (locally)

# Atmospheric composition

$A_1$  The lightest

---

$A_2$  Light

---

$A_3$  Heavy

---

$A_4$  The heaviest

---



As  $h \ll R$  we can consider only flat layers.

Due to strong gravity an atmosphere is expected to be separated: lighter elements on top.

Because of that even a small amount of light elements (hydrogen) results in its dominance in the properties of the atmosphere.

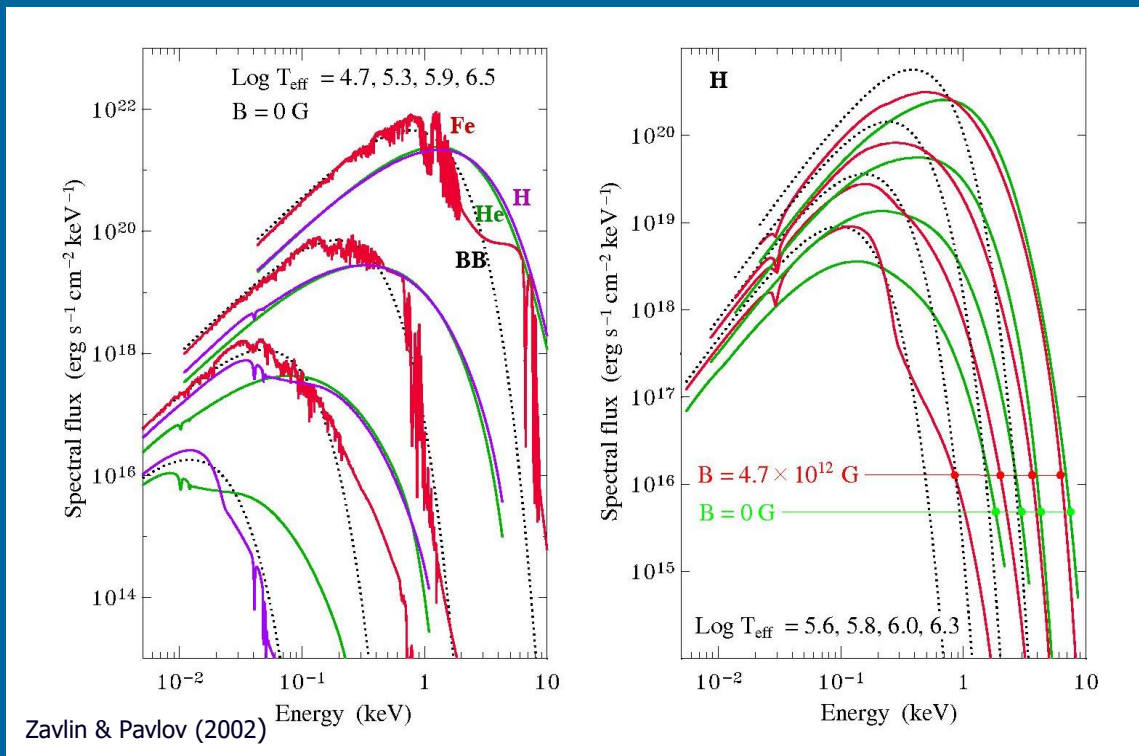
$10^{-20}$  solar mass of hydrogen is enough to form a hydrogen atmosphere.



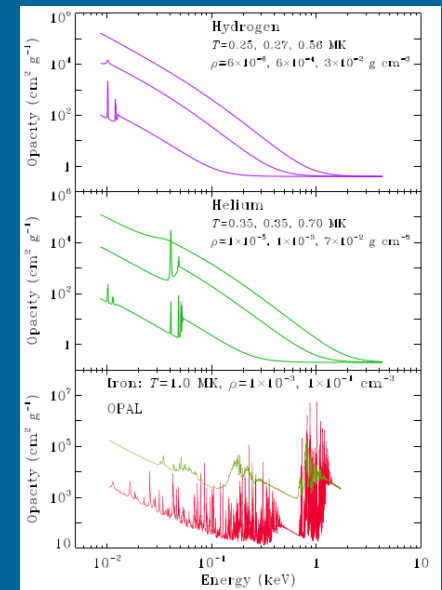
- Free-free absorption dominates

$$\kappa_\nu \propto \nu^{-3}, h\nu \gg kT$$

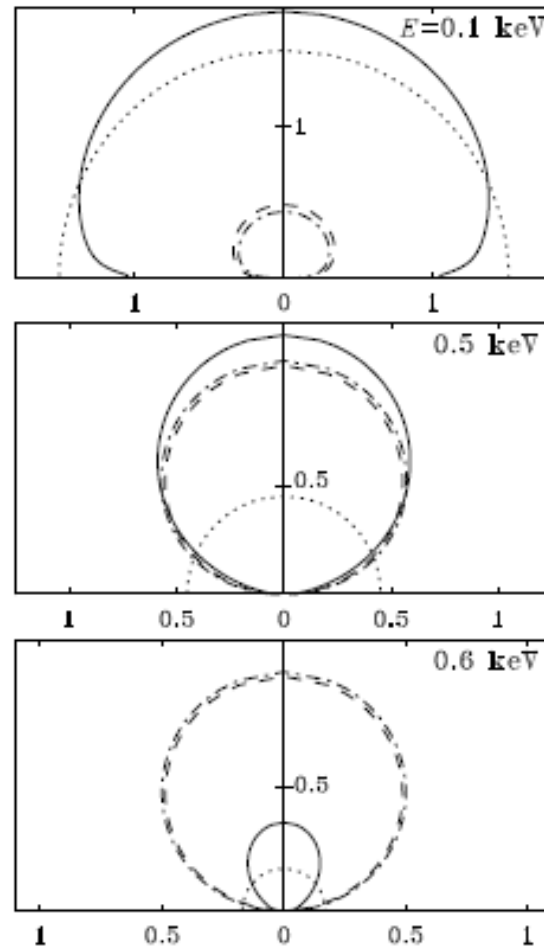
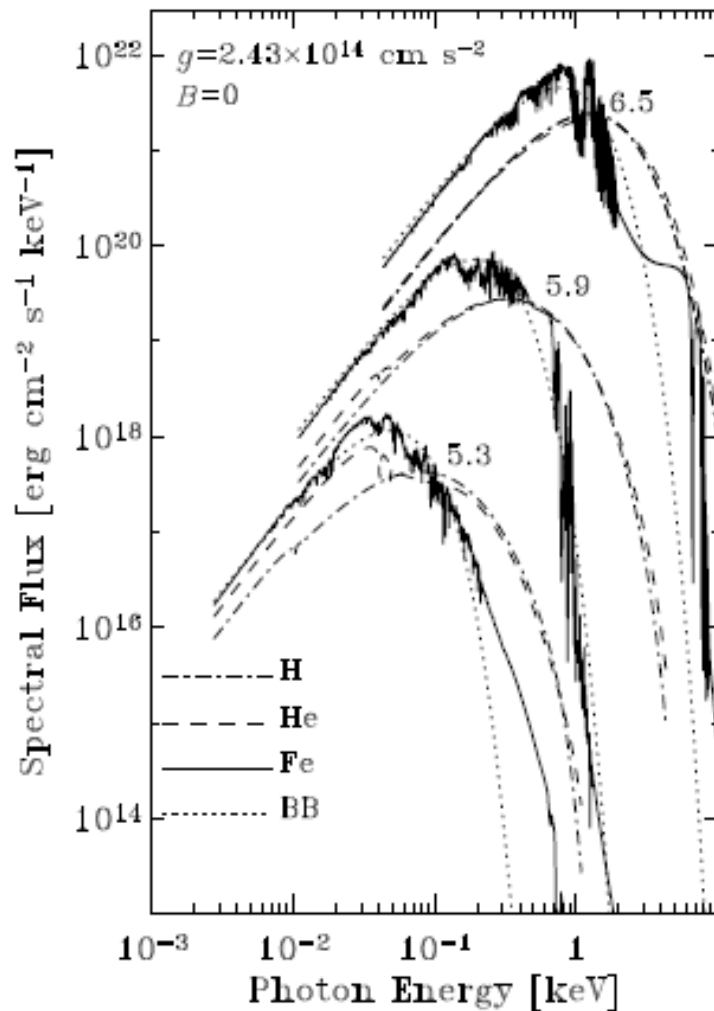
- High energy photons decouple deeper in the atmosphere where T is higher



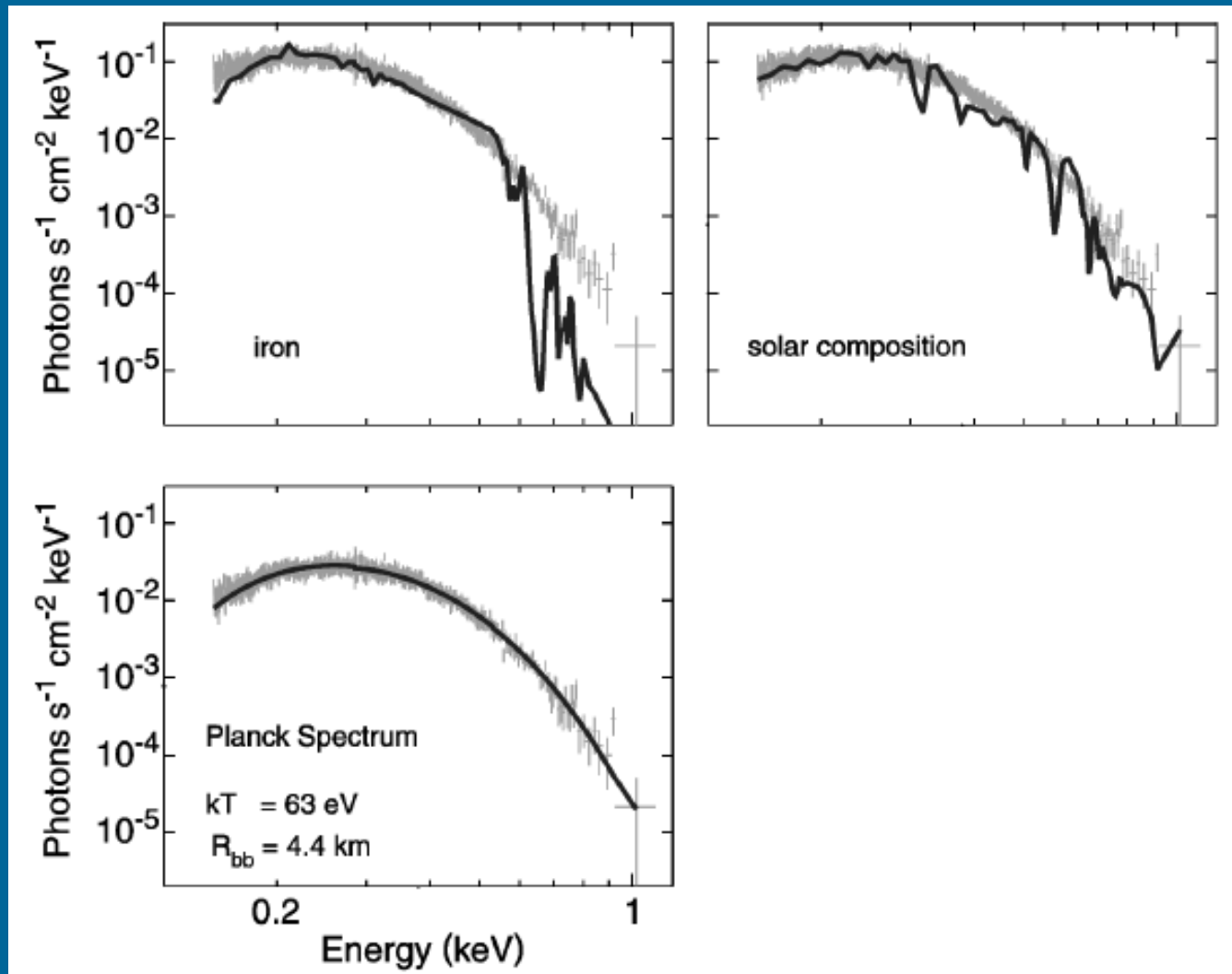
Rapid decrease of the light-element opacities with energy ( $\sim E^{-3}$ )



# Emission from different atmospheres



# Fitting the spectrum of RX J1856



# Different fits

PARAMETERS FROM MULTIWAVELENGTH FITS<sup>a</sup>

Model	$n_{\text{H}}$ ( $10^{20} \text{ cm}^{-2}$ )	$T_{\infty}$ (eV)	$R_{\infty}/D$ (km pc <sup>-1</sup> )	$T_{\infty}(R_{\infty}/D)^2$ [eV (km pc <sup>-1</sup> ) <sup>2</sup> ]	Luminosity <sup>b</sup> ( $10^{31} \text{ ergs s}^{-1}$ )	$P_{\text{ox}}^c$
BB .....	$2.2^{+0.3}_{-0.4}$	$48 \pm 2$	$0.11 \pm 0.01$	$0.60^{+0.05}_{-0.4}$	$1.55^{+0.23}_{-0.17}$	$3 \times 10^{-4}$
H .....	$1.0 \pm 0.1$	$26 \pm 1$	$0.27 \pm 0.01$	$1.94 \pm 0.01$	$0.6 \pm 0.01$	$< 10^{-14}$
Fe.....	$1.8 \pm 0.2$	$44 \pm 1$	$0.13 \pm 0.01$	$0.75 \pm 0.05$	$1.41^{+0.08}_{-0.06}$	$7 \times 10^{-7}$
Si-ash.....	$1.9^{+0.3}_{-0.2}$	$45^{+2}_{-1}$	$0.13 \pm 0.01$	$0.74^{+0.04}_{-0.05}$	$1.63^{+0.14}_{-0.21}$	0.53

<sup>a</sup> 3  $\sigma$  ranges, assuming  $z = 0.305$ . Weighting of the data is discussed in the text.

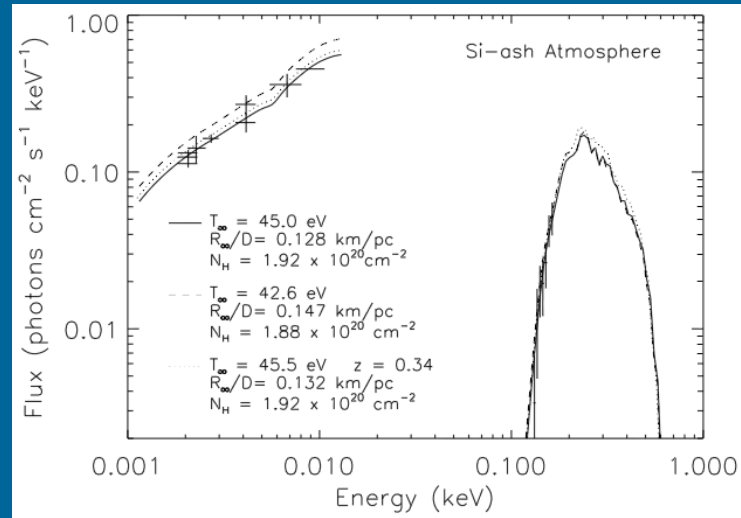
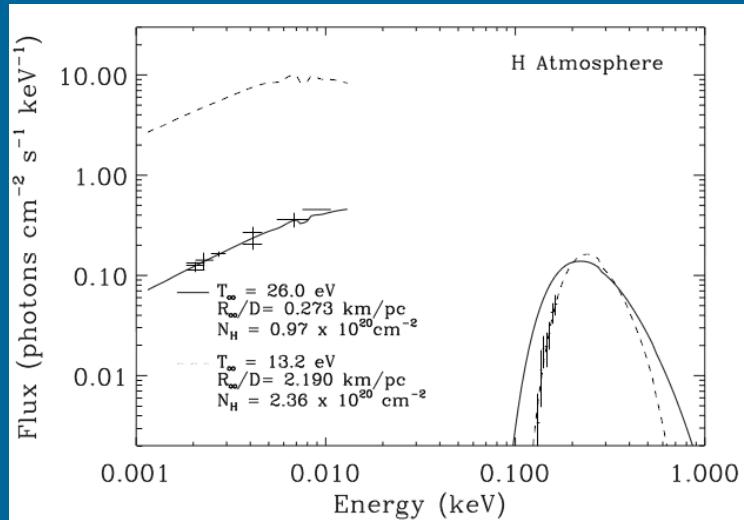
<sup>b</sup> Uncertainty does not include uncertainty in distance.

<sup>c</sup> The likelihood that the X-ray and optical parameters are the same.

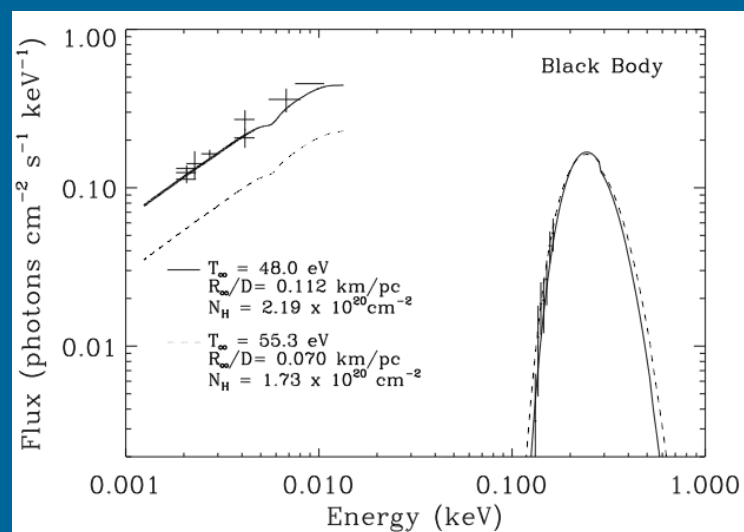
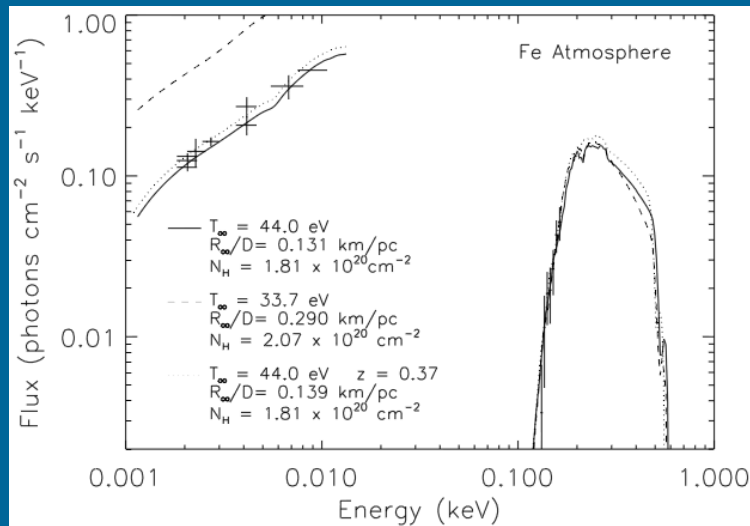
Fits of realistic spectra of cooling NSs give higher temperature (and so smaller emitting surfaces) for blackbody and heavy element atmospheres (Fe, Si).

$$T_{\text{BB}} \sim 2T_{\text{H}}$$

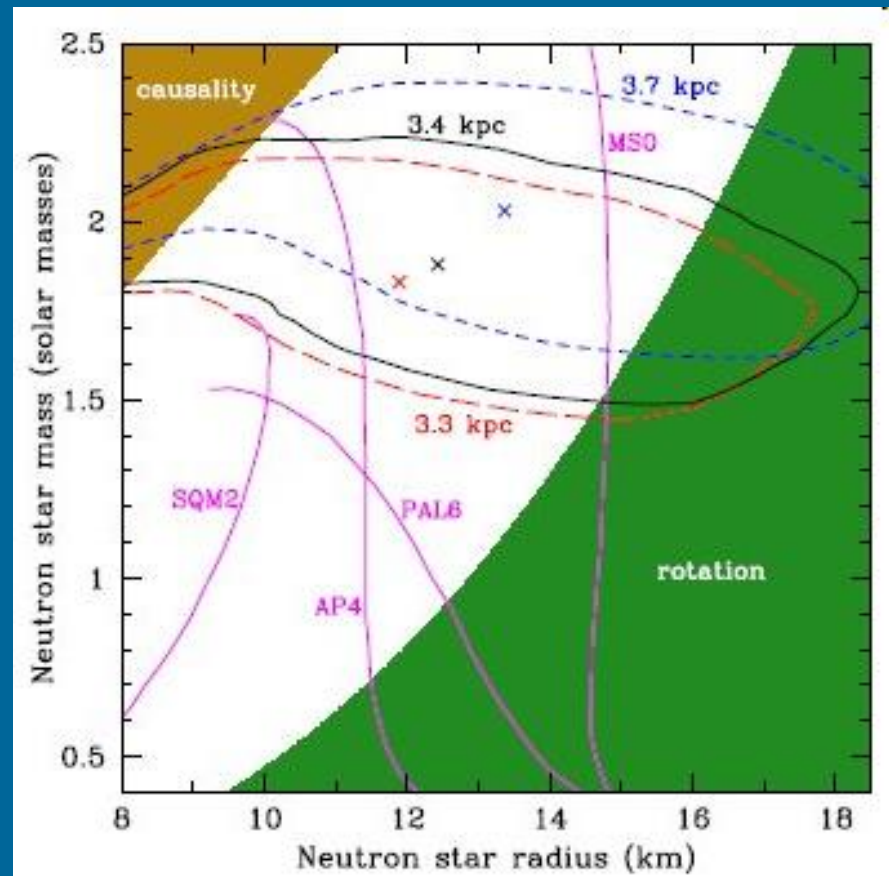
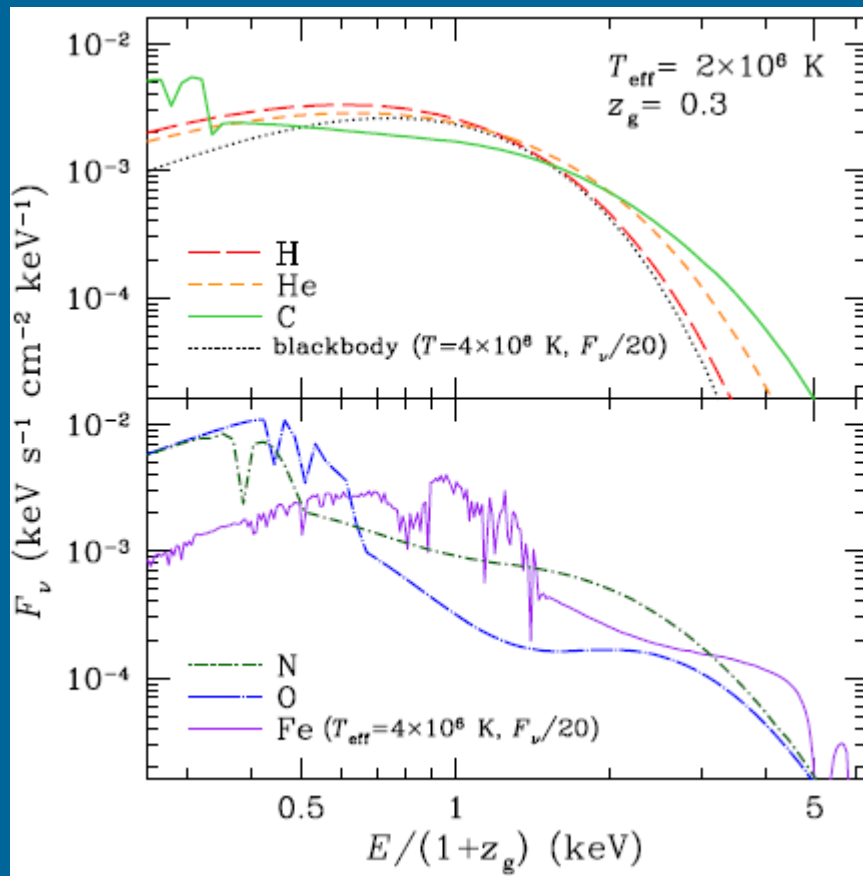
# Different fits



$$T_{\text{bb}} \sim T_{\text{Fe}} > T_{\text{H}}$$



# Cas A carbon atmosphere



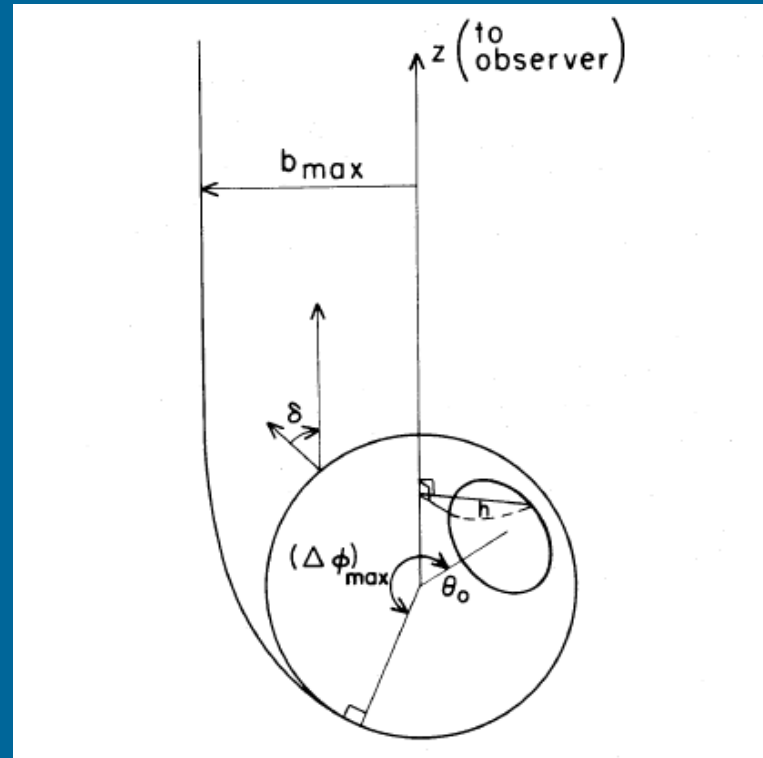
Low-field carbon atmosphere can fit the data.  
Before all fits provided a very small emitting area.

# Gravity Effects

- Redshift
- Ray bending

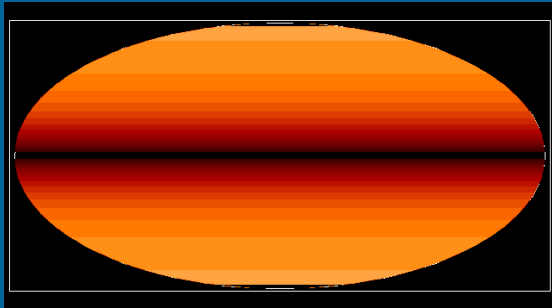
$$L_{\infty} = 4\pi R_{\infty}^2 \sigma T_{\infty}^4$$

$$4\pi\sigma T_{\infty}^4 \rightarrow \int_0^{2\pi} d\gamma \int_0^{2\pi} d\Phi \int_0^1 du^2 \int_{E_{\infty,1}}^{E_{\infty,2}} dE_{\infty} I(E, B, \cos \Theta, T_s, \gamma)$$



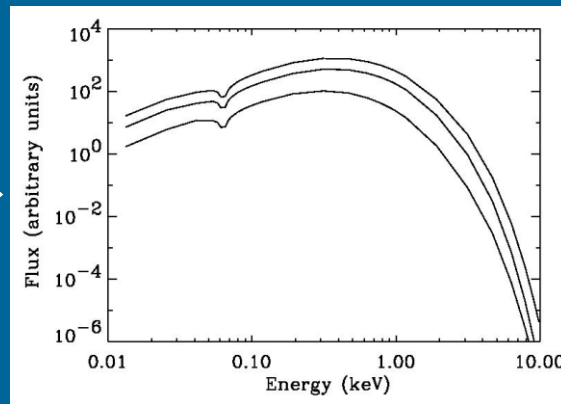
### STEP 1

Specify viewing geometry and B-field topology; compute the surface temperature distribution



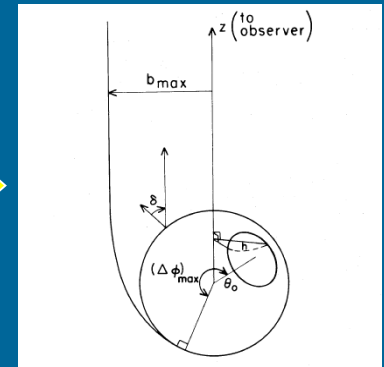
### STEP 2

Compute emission from every surface patch



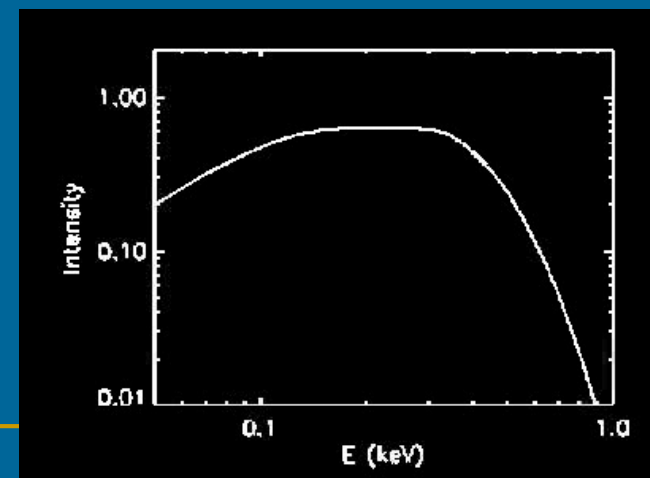
### STEP 3

GR ray-tracing to obtain the spectrum at infinity



### STEP 4

Predict lightcurve and phase-resolved spectrum  
Compare with observations





# The Seven X-ray dim Isolated NSs

- Soft thermal spectrum ( $kT \approx 50\text{-}100$  eV)
- No hard, non-thermal tail
- Radio-quiet, no association with SNRs
- Low column density ( $N_H \approx 10^{20} \text{ cm}^{-2}$ )
- X-ray pulsations in all (but one?) sources ( $P \approx 3\text{-}10$  s)
- Very faint optical counterparts
- Broad spectral features

# ICoNS: The Perfect Neutron Stars

ICoNS are key in neutron star astrophysics:  
these are the only sources for which we have  
a “clean view” of the star surface

- Information on the thermal and magnetic surface distributions
- Estimate of the star radius (and mass ?)
- Direct constraints on the EOS

# ICoNS: What Are They ?

- ICoNS *are* neutron stars
- Idea number 1: Powered by ISM accretion?  
 $\dot{M}_{\text{Bondi}} \sim n_{\text{ISM}}/v^3$  if  $v < 40$  km/s and  $D < 500$  pc  
(e.g. Treves et al 2000)
- Measured proper motions imply  $v > 100$  km/s
- Just cooling NSs

# Simple Thermal Emitters ?

Recent detailed observations of ICoNS allow direct testing of surface emission models

**“STANDARD MODEL”** thermal emission from the surface of a neutron star with a dipolar magnetic field and covered by an atmosphere

The optical excess

ICoNS lightcurves

The puzzle of RX J1856.5-3754

Spectral evolution of RX J0720.4-3125

Note a claim for an excess at harder (keV) X-rays: 1703.05995

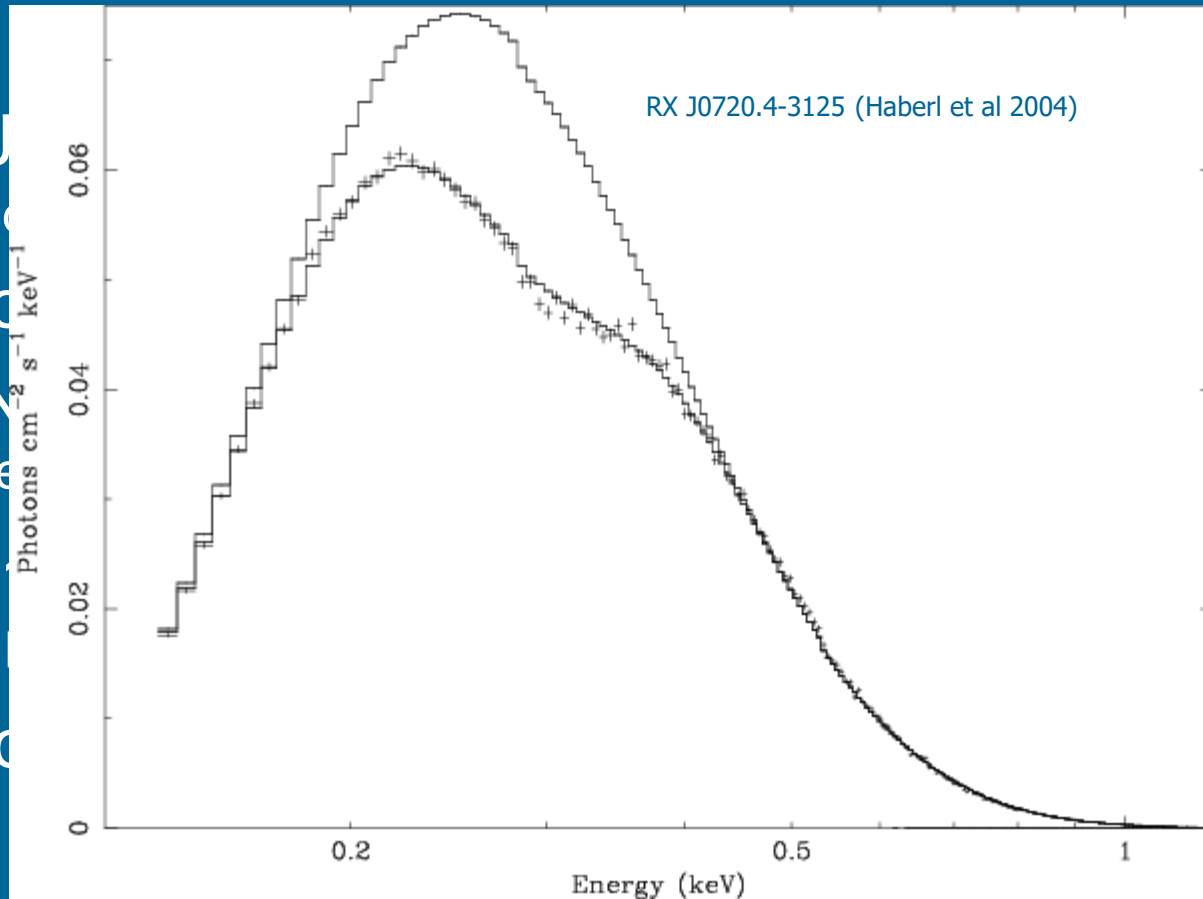
# The Magnificent Seven

Source	kT (eV)	P (s)	Amplitude/2	Optical
RX J1856.5-3754	60	7.06	1.5%	V = 25.6
RX J0720.4-3125 (*)	85	8.39	11%	B = 26.6
RX J0806.4-4123	96	11.37	6%	UV
RX J0420.0-5022	45	3.45	13%	B = 26.6
RX J1308.6+2127 (RBS 1223)	86	10.31	18%	$m_{50\text{CCD}} = 28.6$
RX J1605.3+3249 (RBS 1556)	96	3.39?	??	$m_{50\text{CCD}} = 26.8$
1RXS J214303.7+065419 (RBS 1774)	104	9.43	4%	B=27.4

(\*) variable source

# Featureless ? No Thanks !

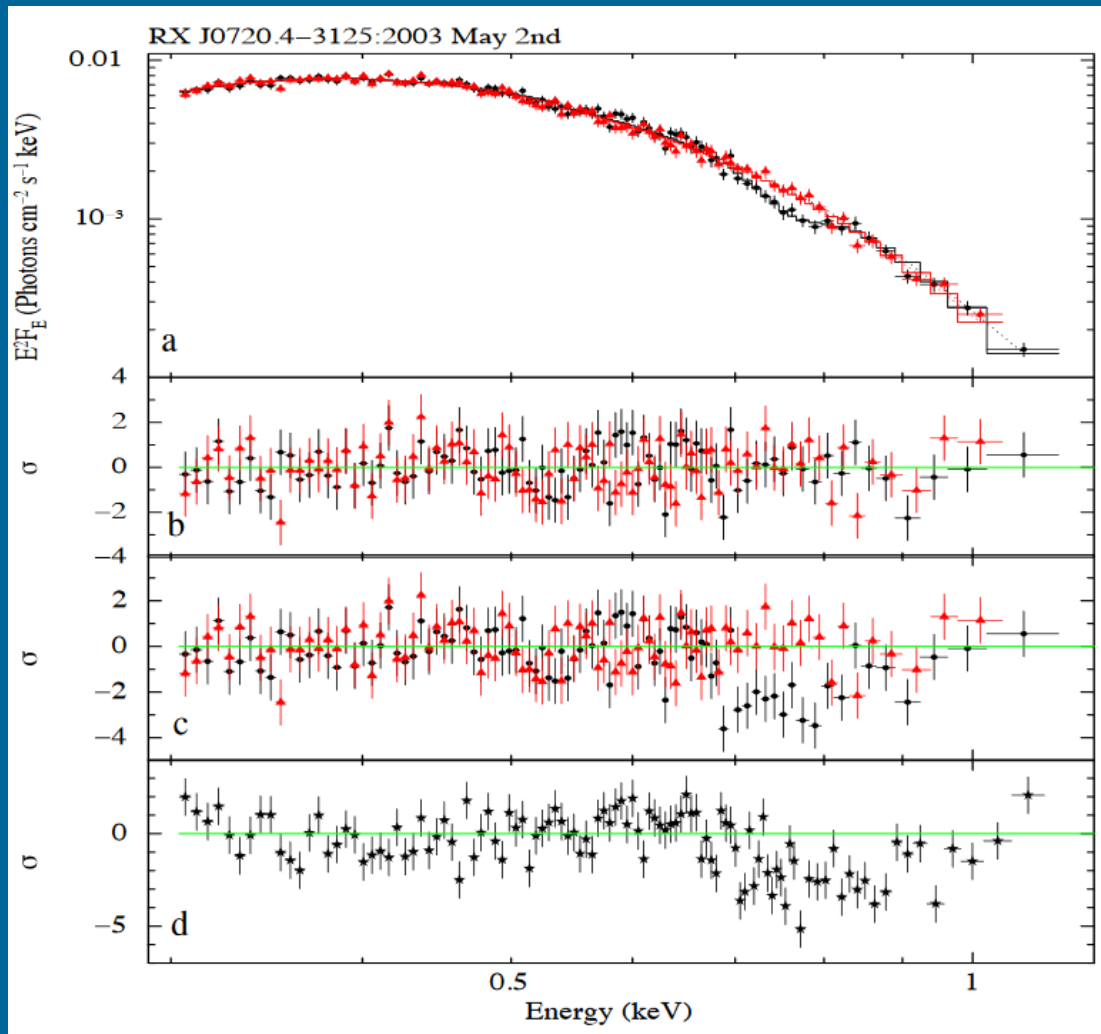
- RX J0720.4-3125 (Chandra)
- A broad ICoM-like Zane et al 2004
- $E_{\text{line}} = 2E_2$  in
- Proton ?



S  
her  
al 2004;  
th  $E_1 \sim$   
high B

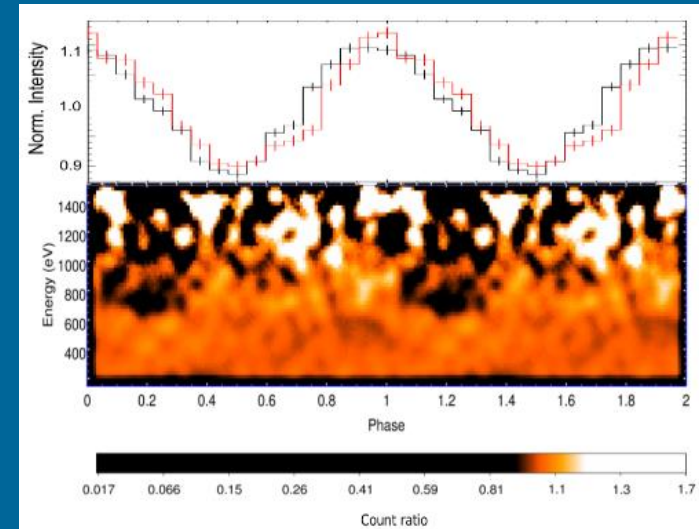
Source	Energy (eV)	EW (eV)	$B_{\text{line}}$ ( $B_{\text{sd}}$ ) ( $10^{13}$ G)	Notes
RX J1856.5-3754	no	no	?	-
RX J0720.4-3125	270	40	5 (2)	Variable line
RX J0806.4-4123	460	33	9	-
RX J0420.0-5022	330	43	7	-
RX J1308.6+2127	300	150	6 (3)	-
RX J1605.3+3249	450	36	9	-
1RXS J214303.7+065419	700	50	14	-

# Phase variable spectral feature



RX J0720.4-3125

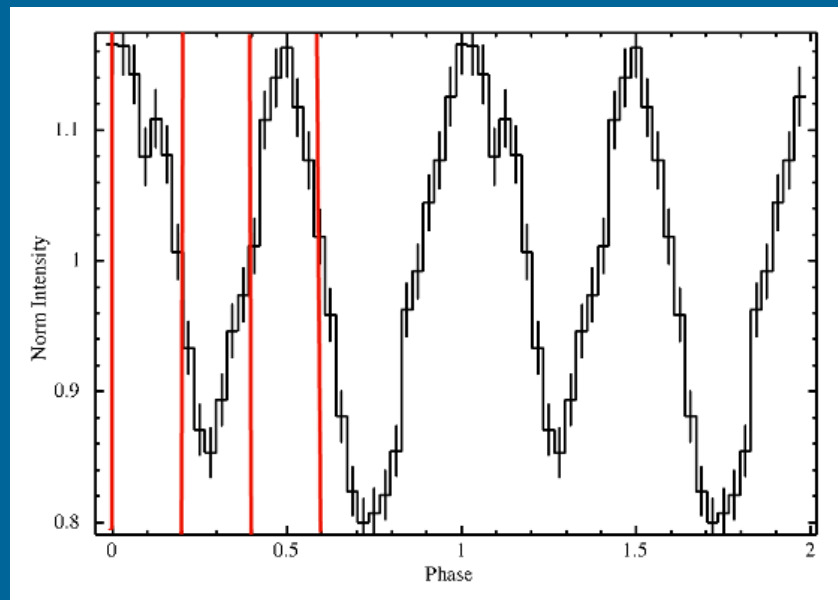
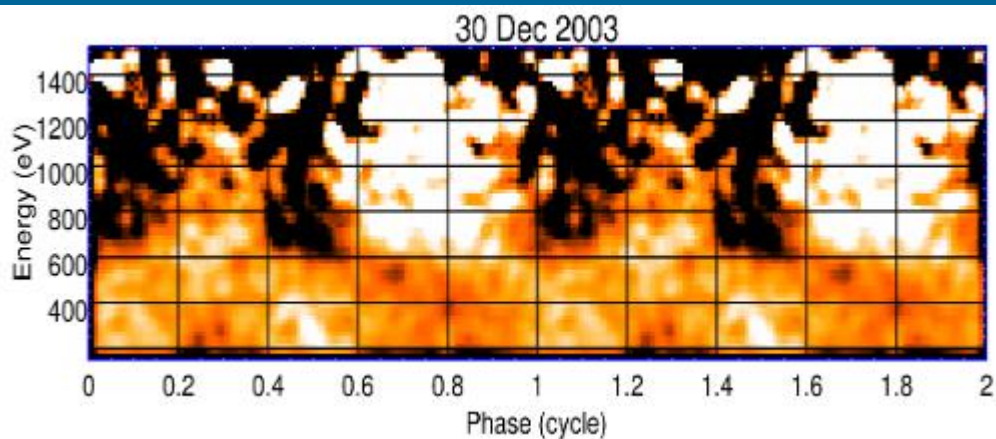
Black: phase 0.1-0.3  
red: phase 0.5-0.7





# More phase-dependent features in M7

RX J1308.6+2127

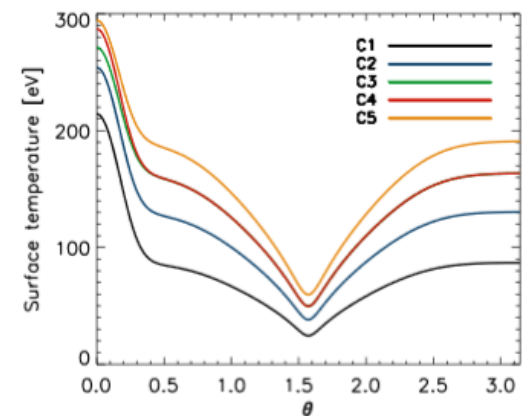
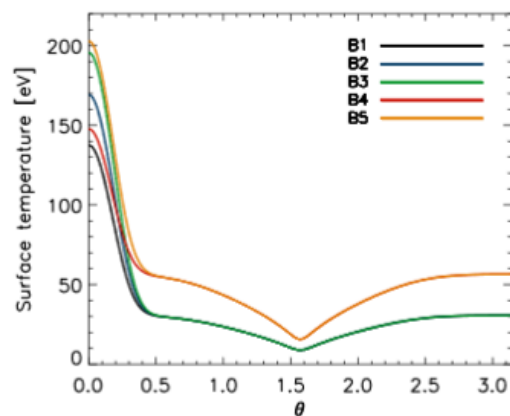
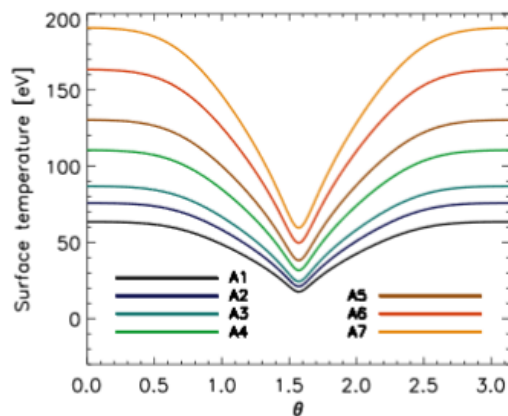


Parameter <sup>a</sup>	0–0.2	0.2–0.4	0.4–0.6	0.6–0.8	0.8–1
BB+GAUSS					
kT <sub>BB</sub> (eV)	77.7 <sup>+1.8</sup> <sub>-2.0</sub>	75.4 <sup>+2.2</sup> <sub>-2.5</sub>	84.9 <sup>+1.3</sup> <sub>-1.4</sub>	75.6 <sup>+2.1</sup> <sub>-2.7</sub>	84.9 <sup>+1.8</sup> <sub>-2.0</sub>
R <sub>BB</sub> (km)	4.3±0.5	5.6±1.0	2.6±0.1	5.8±1.1	3.4±0.3
Flux <sup>b</sup>	3.34 <sup>+0.04</sup> <sub>-0.09</sub>	3.67 <sup>+0.15</sup> <sub>-0.09</sub>	3.10 <sup>+0.05</sup> <sub>-0.07</sub>	3.68 <sup>+0.07</sup> <sub>-0.06</sub>	3.69±0.06
Unabs. Flux <sup>b</sup>	7.42±1.10	7.69 <sup>+1.70</sup> <sub>-1.01</sub>	6.63 <sup>+0.63</sup> <sub>-0.36</sub>	8.26 <sup>+1.35</sup> <sub>-1.39</sub>	7.77 <sup>+0.55</sup> <sub>-0.76</sub>
E <sub>1</sub> (eV)	173 <sup>+32</sup> <sub>-39</sub>	107 <sup>+44</sup> <sub>-54</sub>	256 <sup>+22</sup> <sub>-28</sub>	109 <sup>+41</sup> <sub>-59</sub>	198 <sup>+30</sup> <sub>-36</sub>
σ <sub>1</sub> (eV)	143 <sup>+13</sup> <sub>-12</sub>	169 <sup>+15</sup> <sub>-14</sub>	105 <sup>+11</sup> <sub>-11</sub>	168 <sup>+16</sup> <sub>-13</sub>	146 <sup>+14</sup> <sub>-12</sub>
Eq Width <sub>1</sub> (eV)	182 <sup>+2</sup> <sub>-8</sub>	204 <sup>+2</sup> <sub>-35</sub>	128 <sup>+10</sup> <sub>-14</sub>	203 <sup>+2</sup> <sub>-5</sub>	171 <sup>+11</sup> <sub>-29</sub>
NHP <sup>c</sup>	1.6×10 <sup>-1</sup>	1.5×10 <sup>-1</sup>	5.4×10 <sup>-2</sup>	1.3×10 <sup>-1</sup>	2.8×10 <sup>-3</sup>
χ <sub>ν</sub> <sup>2</sup>	1.12	1.12	1.20	1.13	1.35
dof	141	149	139	147	150

1703.05336

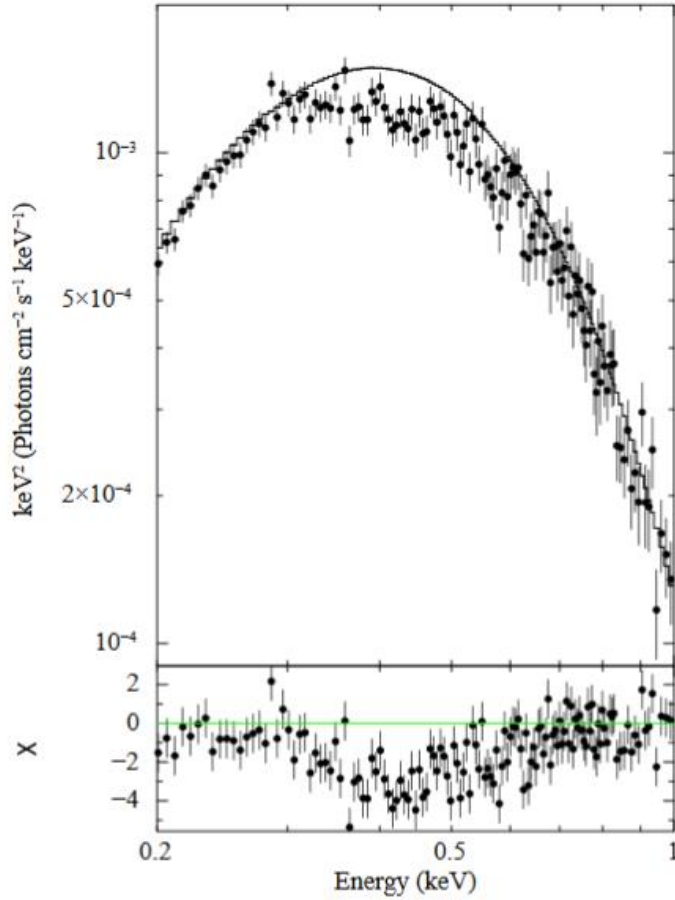
# Non-uniform temperature distribution

Source	Class	$B_{dip}$ [ $10^{12}$ G]	$N_H$ [ $10^{20}$ cm $^{-2}$ ]	$kT_{bb}$ [eV]	$E_0$ [eV]	$ E_w $ [eV]	PF %	Refs.
RX J0720.4-3125	XINS	49	1.0	84-94	311*	0-70	11	[1]
RX J0806.4-4123	XINS	51	0.9	95	486*	30	6	[2]
RX J1308.6+2127	XINS	68	3.7	93	390*	150	18	[3]
RX J1605.3+3249	XINS	148 <sup>†</sup>	0	99	400*	70	5 <sup>†</sup>	[4]
RX J2143.0+0654	XINS	40	2.3	104	750	50	4	[5]
2XMM J1046-5943 <sup>‡</sup>	?	?	26	135	1350*	90	<4	[6]
1E 1207.4-5209	CCO	0.2	13	155,290	740,1390	60,100	4-14**	[7]
PSR J1740+1000	RPP	37	9.7	94	550-650	50-230	30	[8]
PSR J1819-1458	RPP	100	124	112	1120*	400	34	[9]
XTE J1810-197	MAG	410	73	300	1150	35	17-47**	[10]

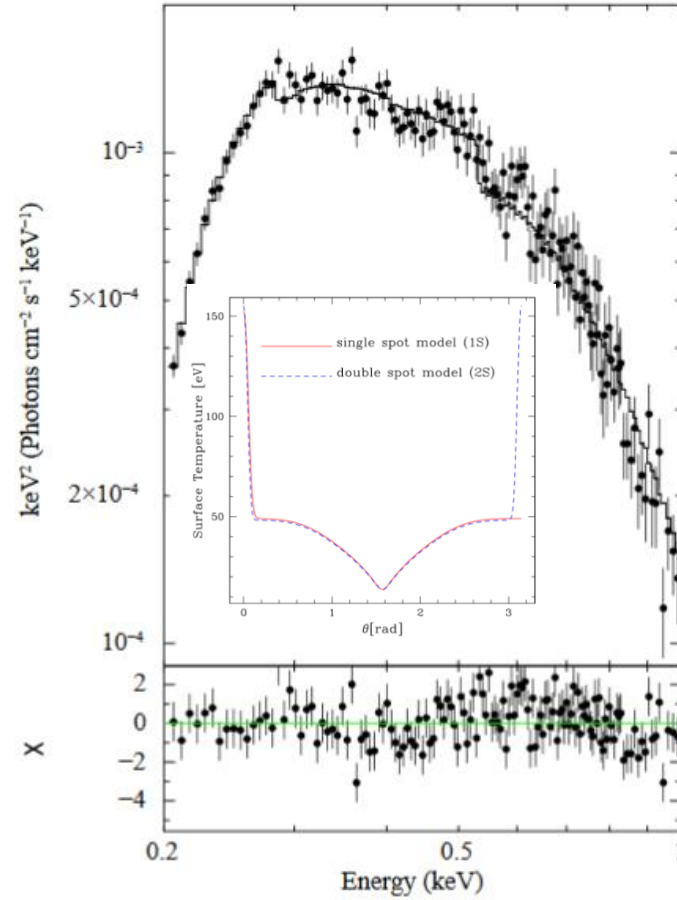


# RX J0806.4-412

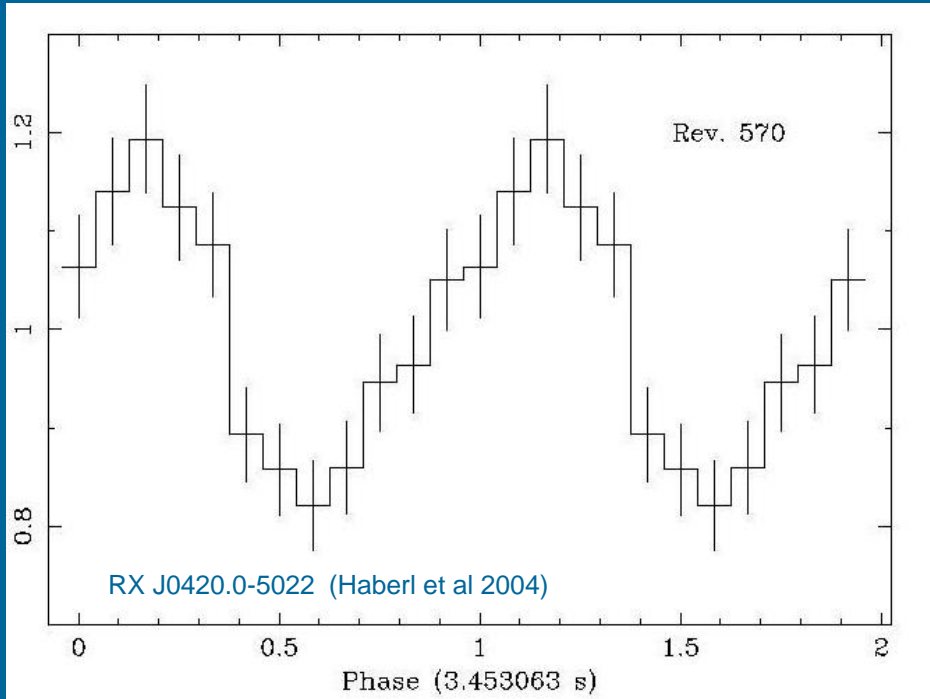
BB+line



Non-uniform distribution



# Pulsating ICoNS - I



- Quite large pulsed fractions
- Skewed lightcurves
- Harder spectrum at pulse minimum
- Phase-dependent absorption features

# Pulsating ICoNS - II

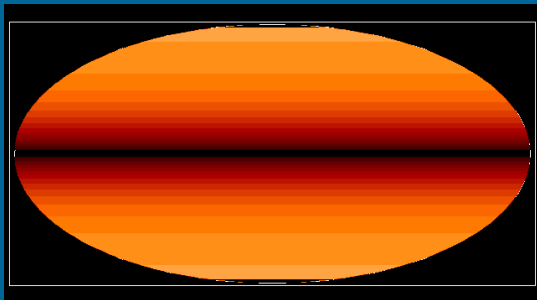
Core-centred  
dipole field

+

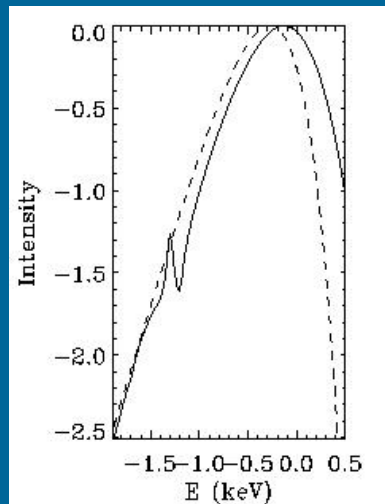
Atmosphere  
emission

=

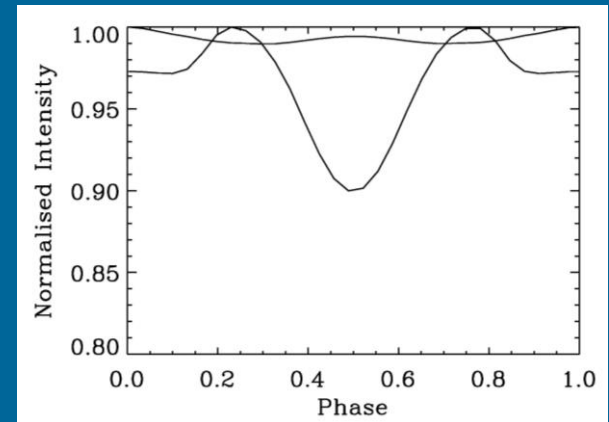
Too small  
pulsed fractions  
Symmetrical  
pulse profiles  
(Zane & Turolla 2006)



+

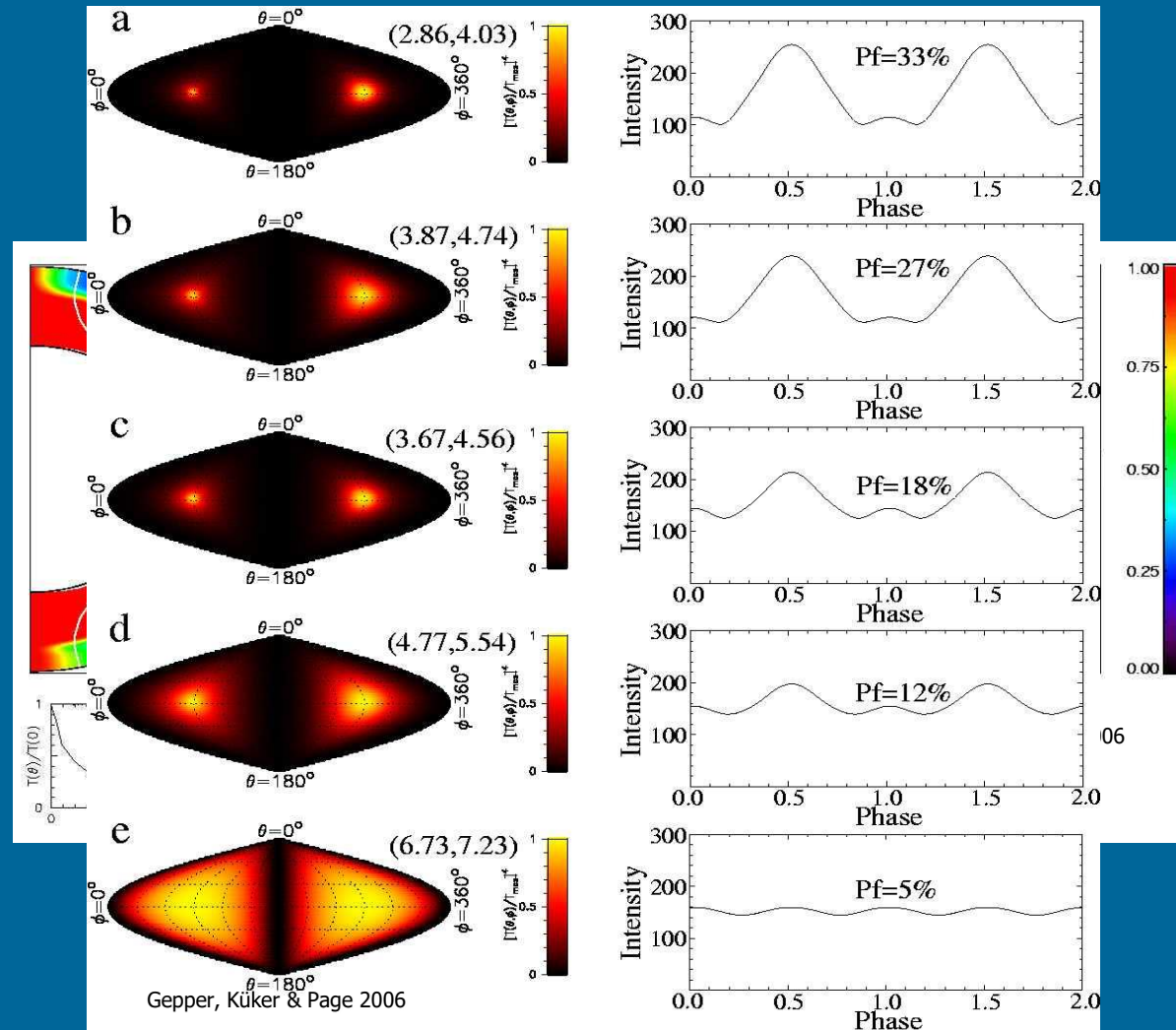


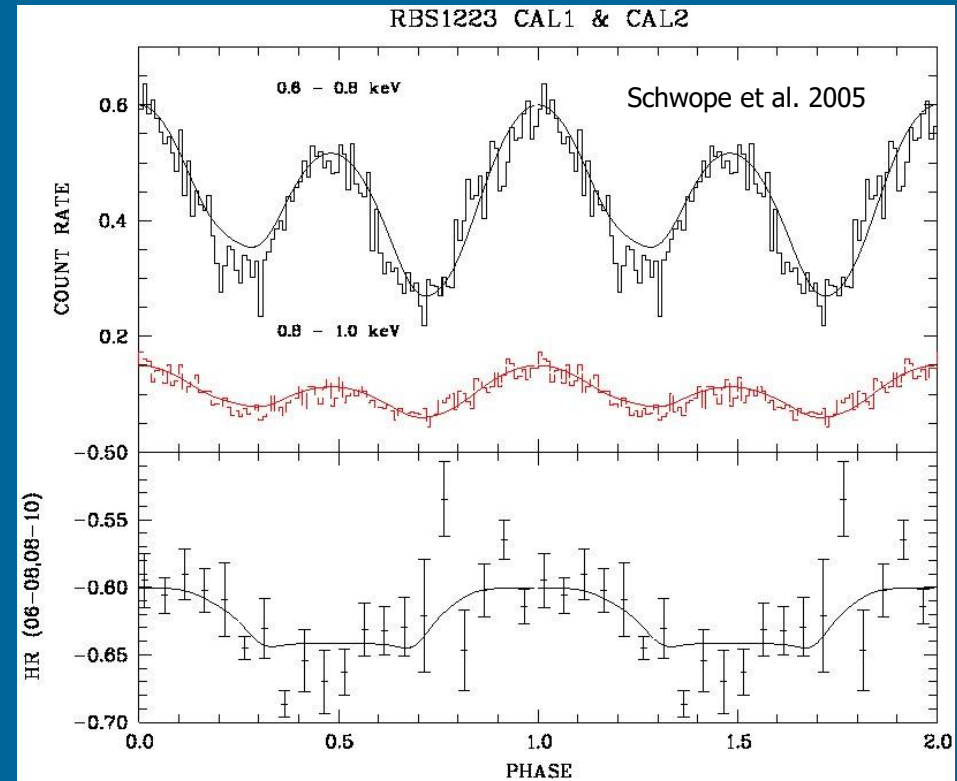
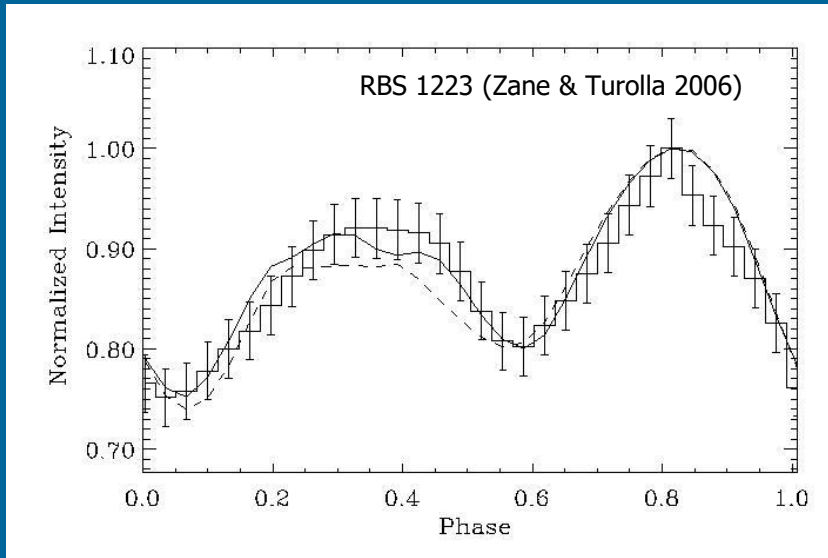
=



# Crustal Magnetic Fields

- Star centred dipole + poloidal/toroidal field in the envelope (Geppert, Küker & Page 2005; 2006)
- Purely poloidal crustal fields produce a steeper meridional temperature gradient
- Addition of a toroidal component introduces a N-S asymmetry





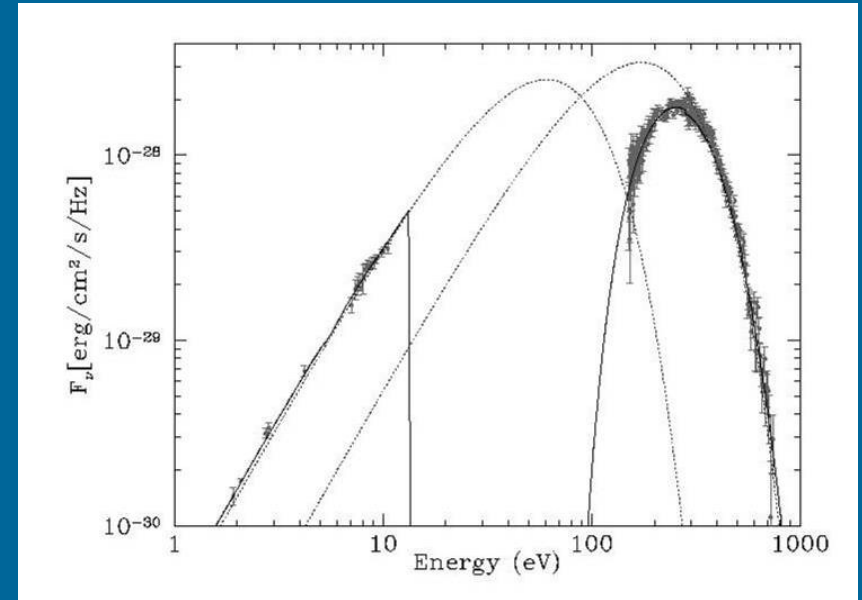
Indications for non-antipodal caps (Schwope et al 2005)

Need for a non-axisymmetric treatment of heat transport



# RX J1856.5-3754 - I

Blackbody featureless spectrum in the 0.1-2 keV band (Chandra 500 ks DDT, Drake et al 2002); possible broadband deviations in the XMM 60 ks observation (Burwitz et al 2003)



RX J1856 multiwavelength SED (Braje & Romani 2002)

Thermal emission from NSs is not expected to be a featureless BB ! H, He spectra are featureless but only blackbody-like (harder). Heavy elements spectra are closer to BB but with a variety of features



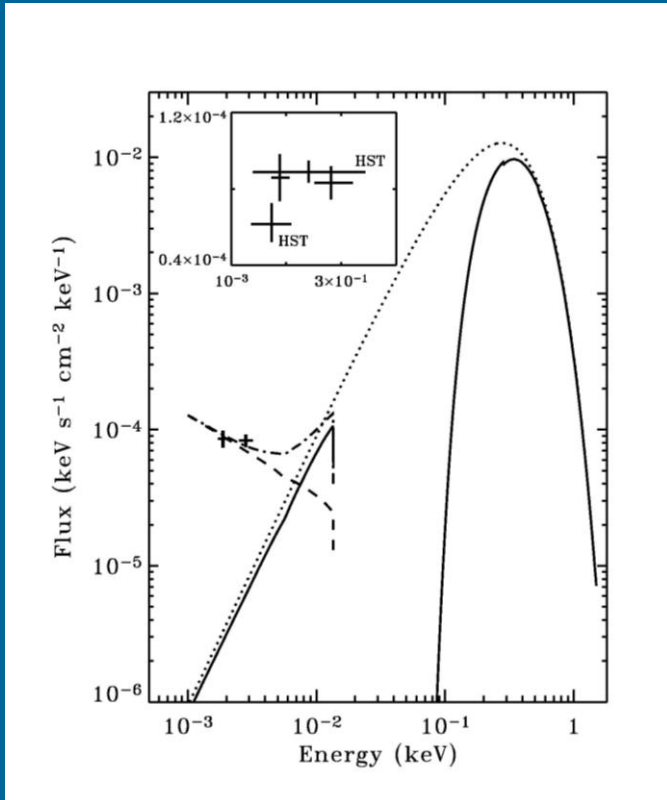
# RX J1856.5-3754 - II

What spectrum ?  
The optical excess ?

- A quark star (Drake et al 2002; Xu 2002; 2003)
- A NS with hotter caps and cooler equatorial region (Pons et al 2002; Braje & Romani 2002; Trümper et al 2005)
- A bare NS (Burwitz et al 2003; Turolla, Zane & Drake 2004; Van Adelsberg et al 2005; Perez-Azorin, Miralles & Pons 2005)

A perfect BB ?

# The Optical Excess

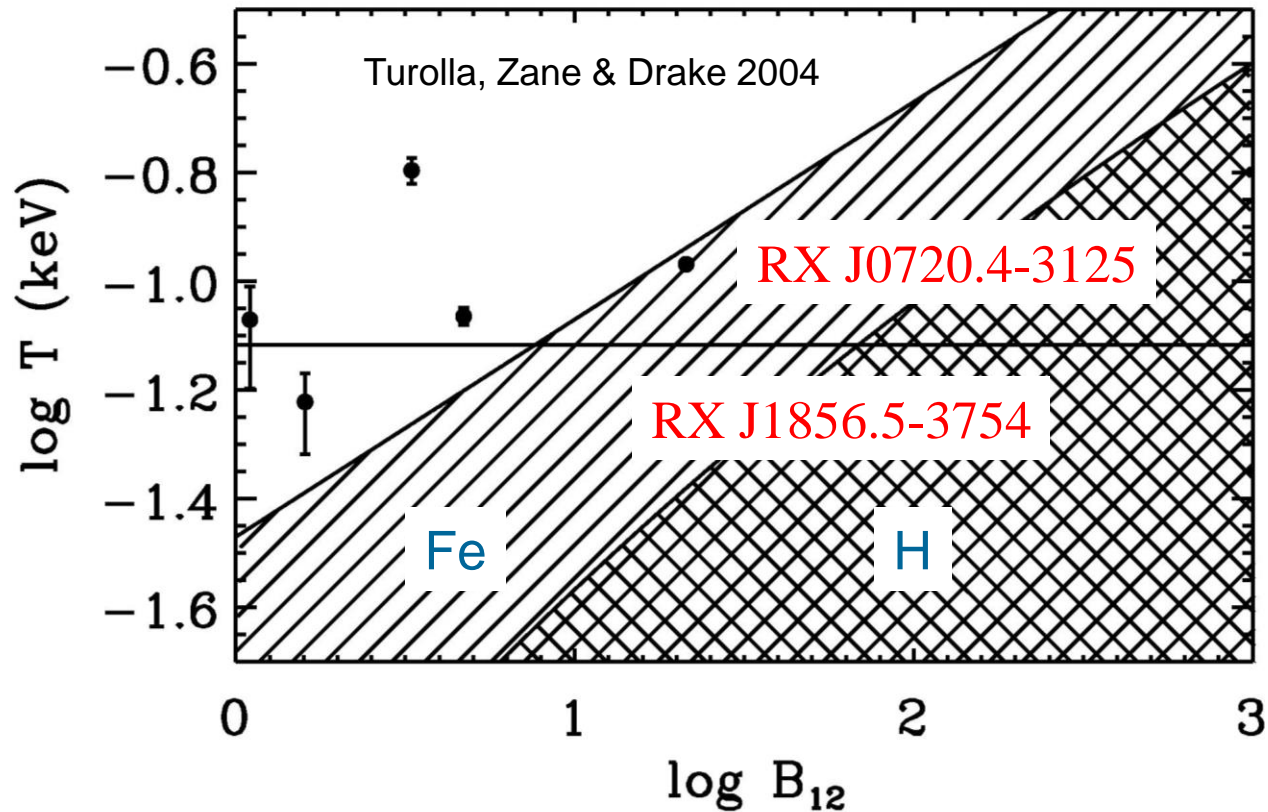
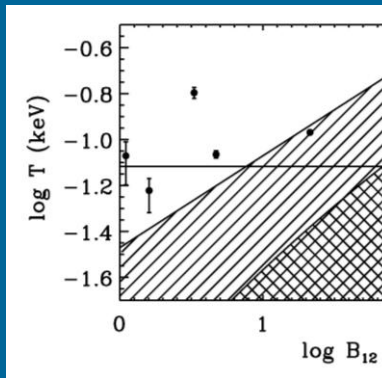


RX J1605 multiwavelength SED (Motch et al 2005)

- In the most of the sources with a confirmed optical counterpart  $F_{\text{opt}} \approx 5-10 \times B_{\nu}(T_{\text{BB},X})$
- $F_{\text{opt}} \approx \nu^2$  ?
- Deviations from a Rayleigh-Jeans continuum in RX J0720 (Kaplan et al 2003) and RX J1605 (Motch et al 2005). A non-thermal power law ?

# Bare Neutron Stars

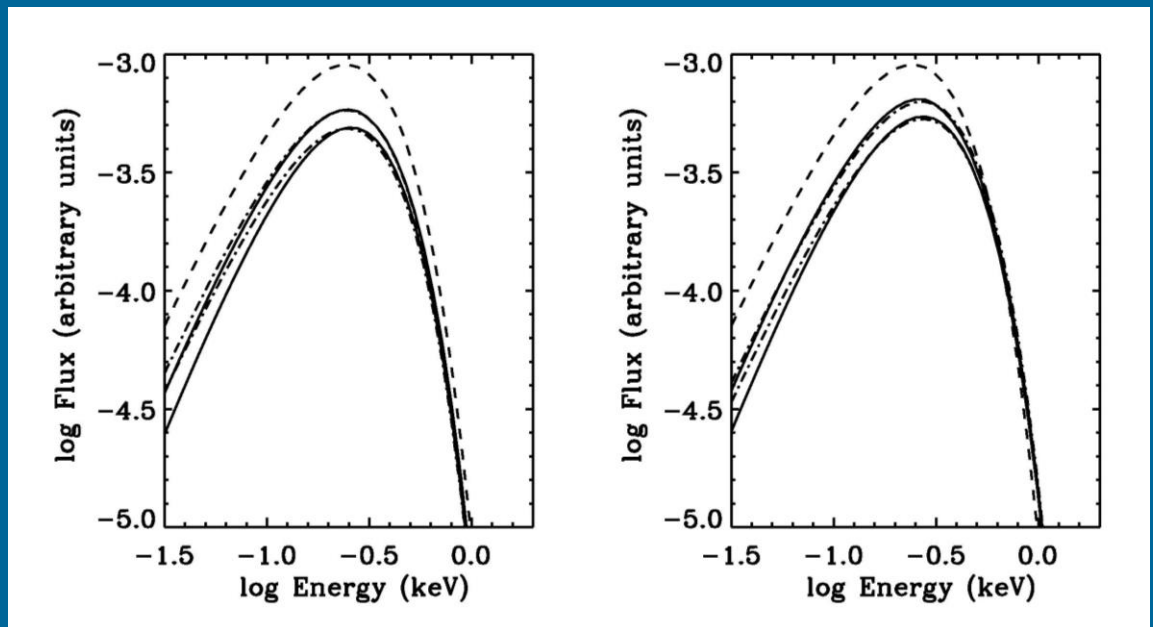
- At  $B \gg B_0 \sim 2.35 \times 10^9$  G atoms



# Spectra from Bare NSs - I

The cold electron gas approximation. Reduced emissivity expected below  $\omega_p$  (Lenzen & Trümper 1978; Brinkmann 1980)

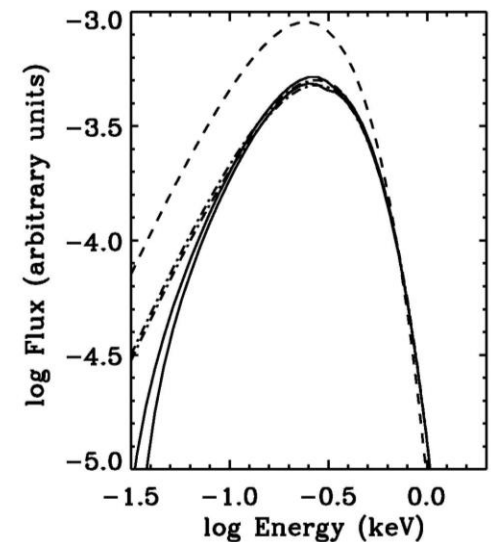
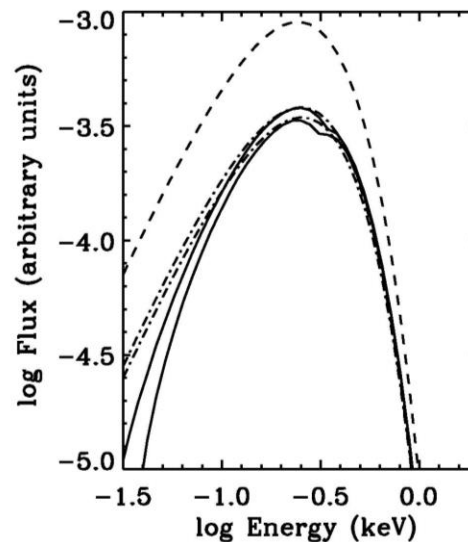
Spectra are very close to BB in shape in the 0.1 - 2 keV range, but depressed wrt the BB at  $T_{\text{eff}}$ . Reduction factor  $\sim 2 - 3$ .



# Spectra from Bare NS - II

Proper account for damping of free electrons by lattice interactions (e-phonon scattering; Yakovlev & Urpin 1980; Potekhin 1999)

Spectra deviate more from BB. Fit in the 0.1 – 2 keV band still acceptable. Features may be present. Reduction factors higher.



# Is RX J1856.5-3754 Bare ?

- Fit of X-ray data in the 0.15-2 keV band acceptable
- Radiation radius problem eased
- Optical excess may be produced by reprocessing of surface radiation in a very rarefied atmosphere (Motch, Zavlin & Haberl 2003; Zane, Turolla & Drake 2004; Ho et al. 2006)
- Details of spectral shape (features, low-energy behaviour) still uncertain

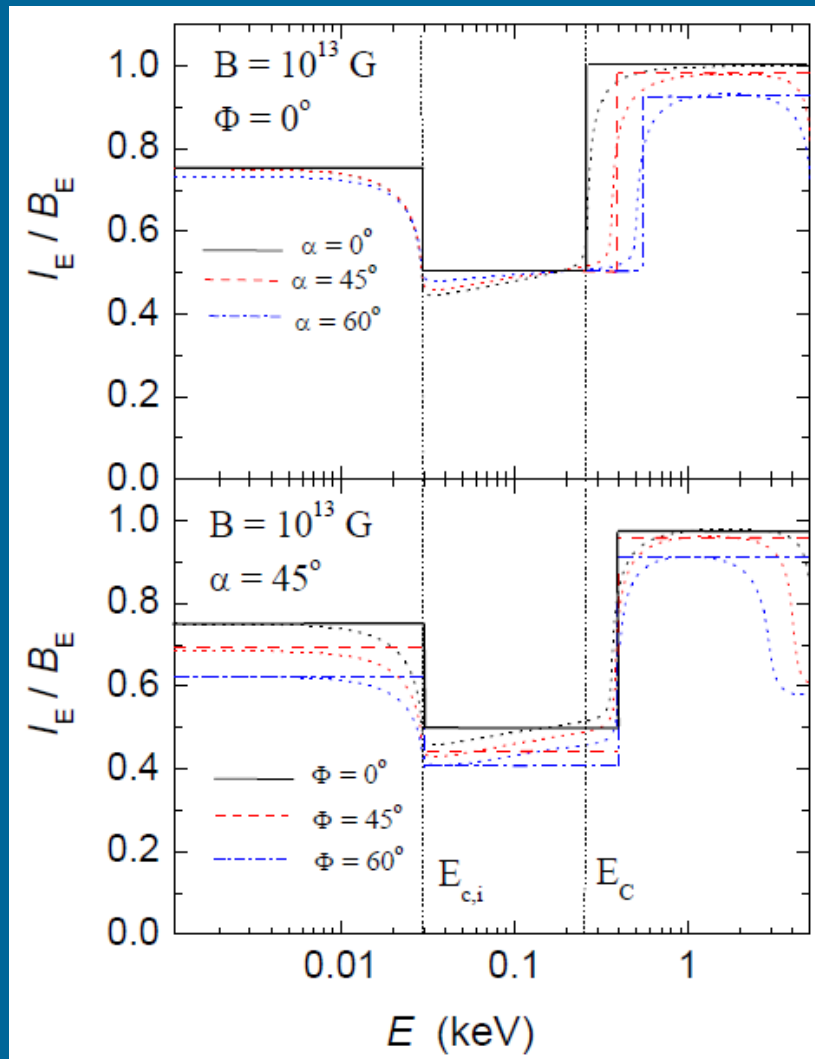
$$R_{\infty} = 4.25 f_E^{-1/2} \left( \frac{D}{100 \text{ pc}} \right) \left( \frac{T_{BB}}{60 \text{ keV}} \right)^{-2} \text{ km}$$

Does the atmosphere keep the star surface temperature ?



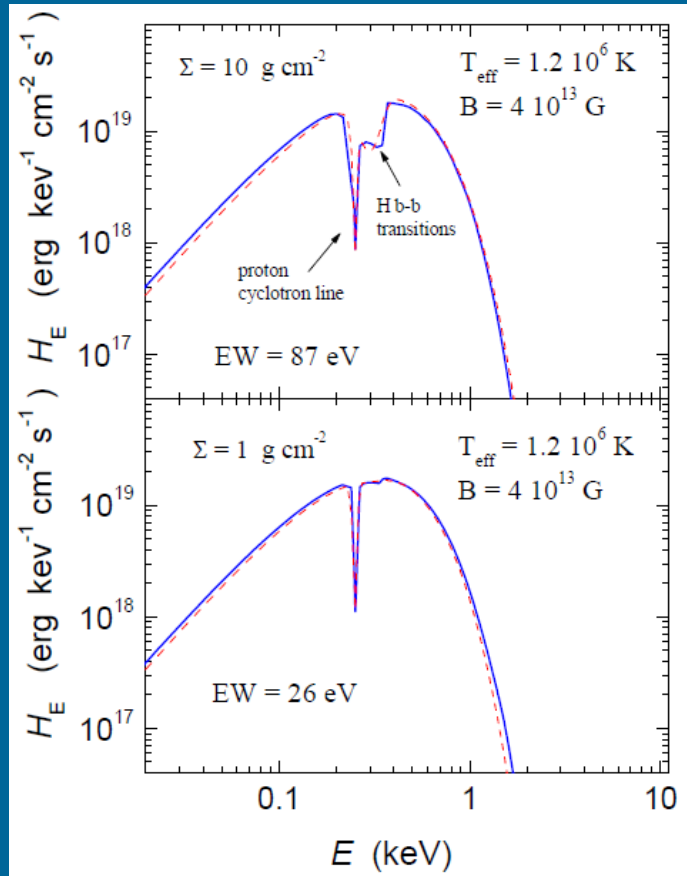
What is the ion contribution to the dielectric tensor ?  
(Van Adelsberg et al. 2005; Perez-Azorin, Miralles & Pons 2005)

# Condensed iron surface emissivity

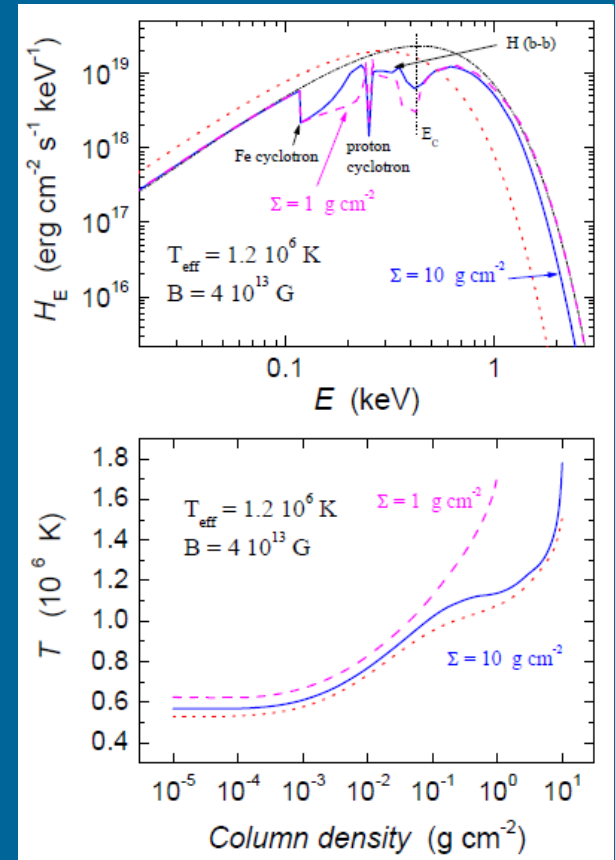


Free ions approximation.

# Thin hydrogen magnetized atmosphere above blackbody and iron condensed surface



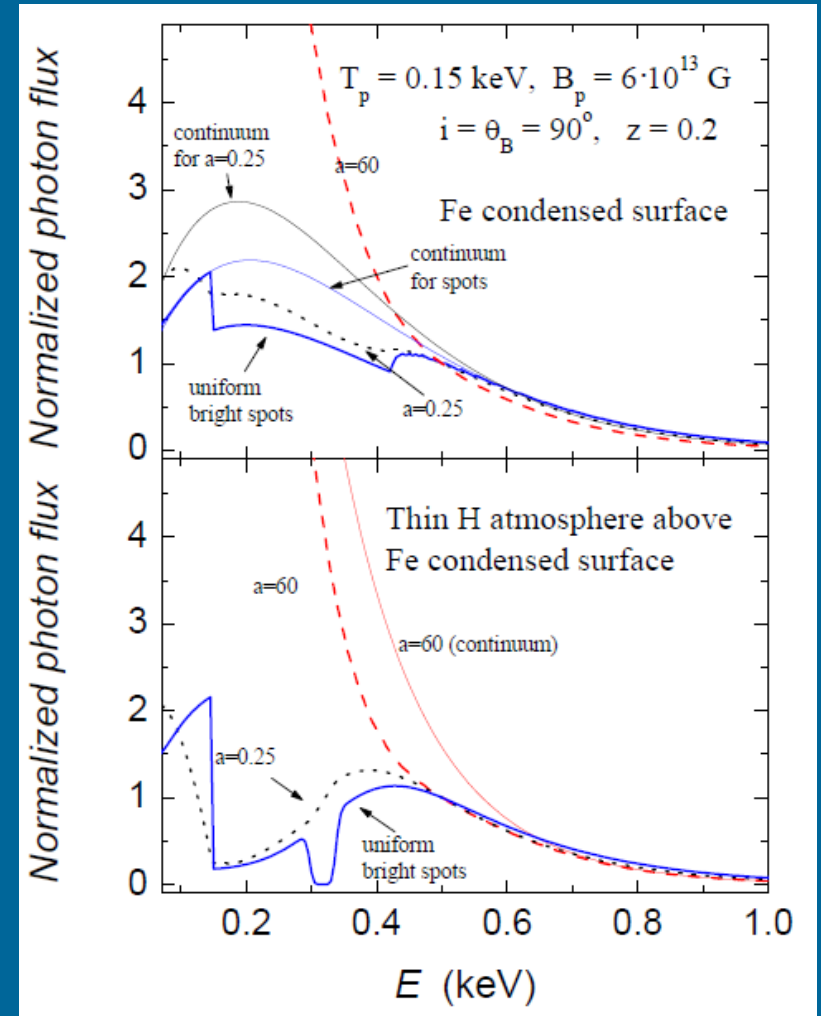
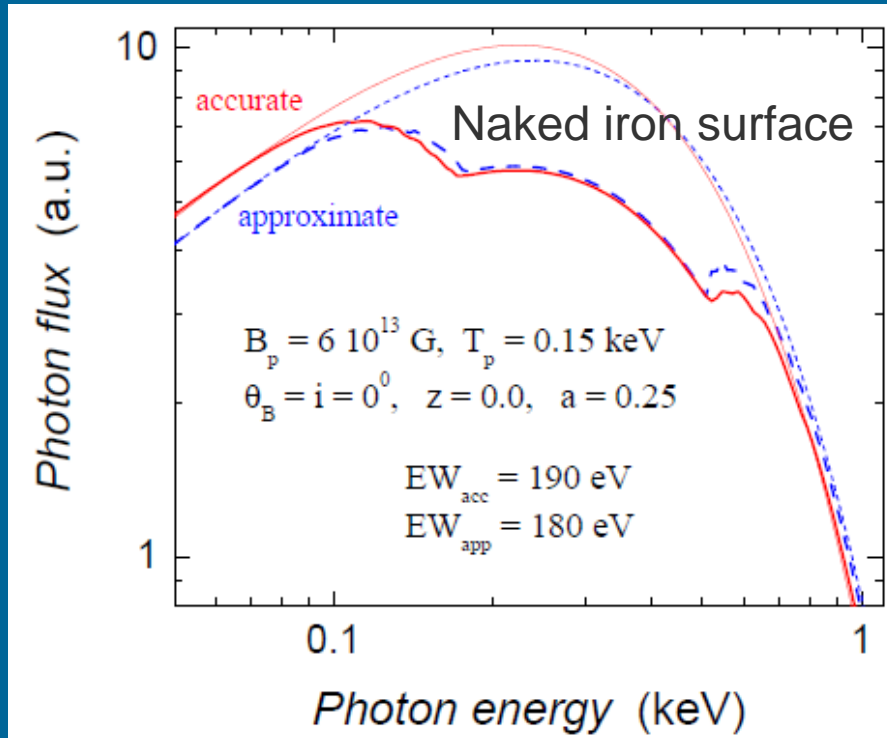
Below atmosphere was a blackbody spectrum



Below – iron condensed surface

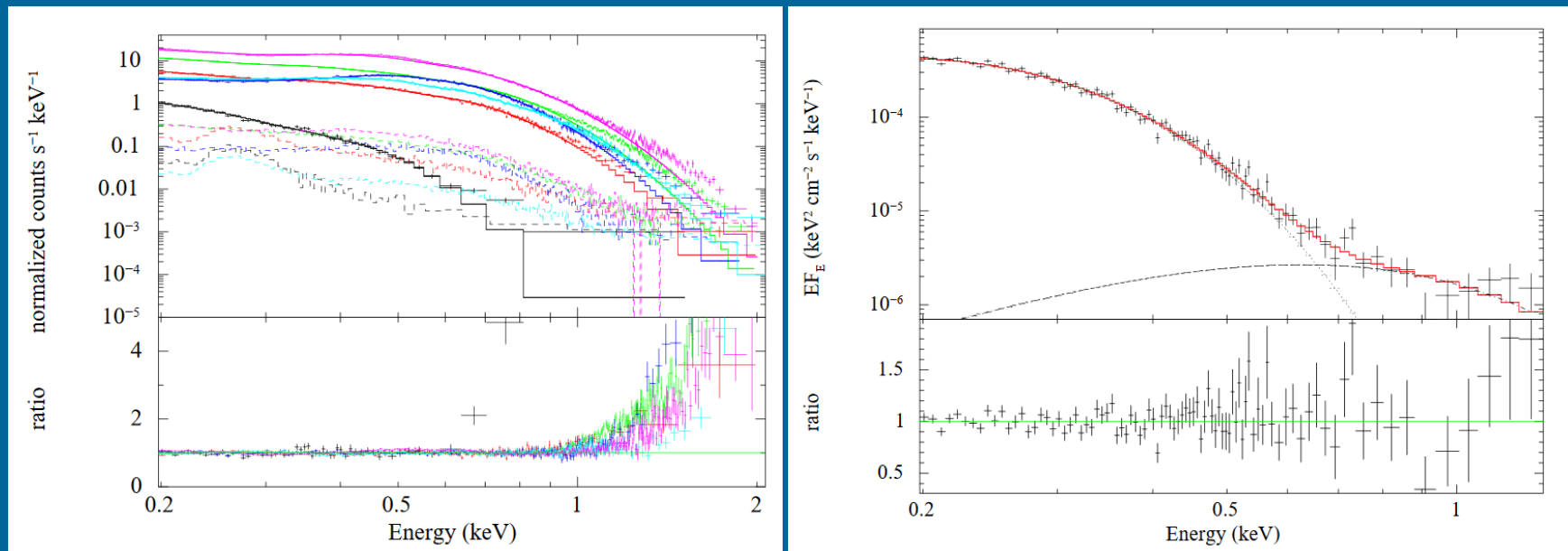


# Let us make it realistic

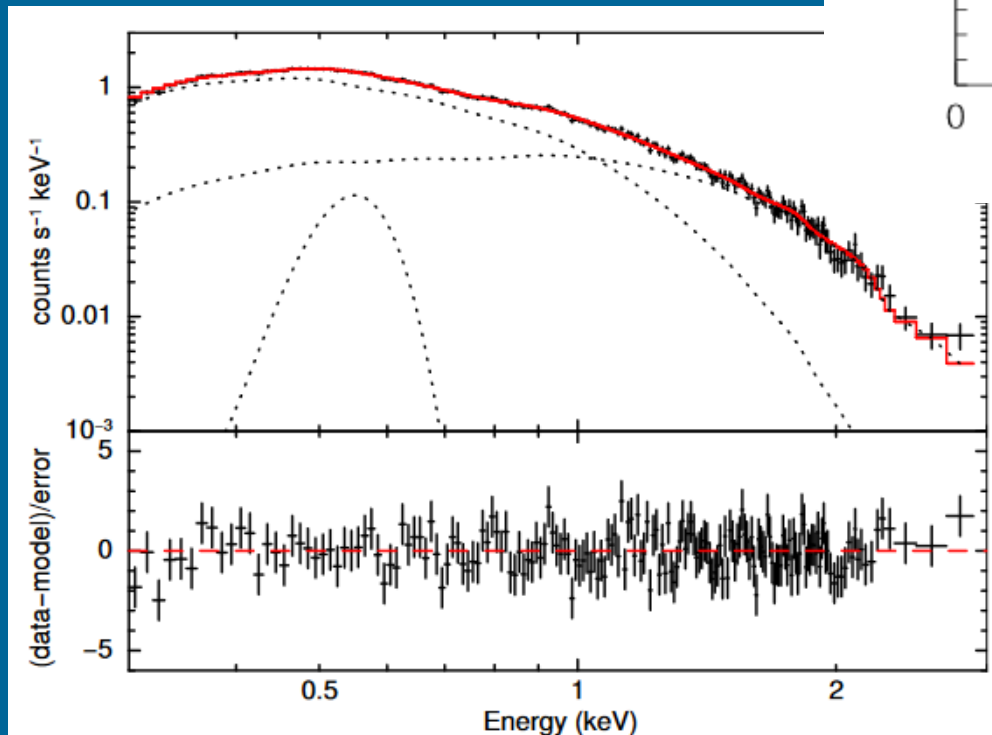
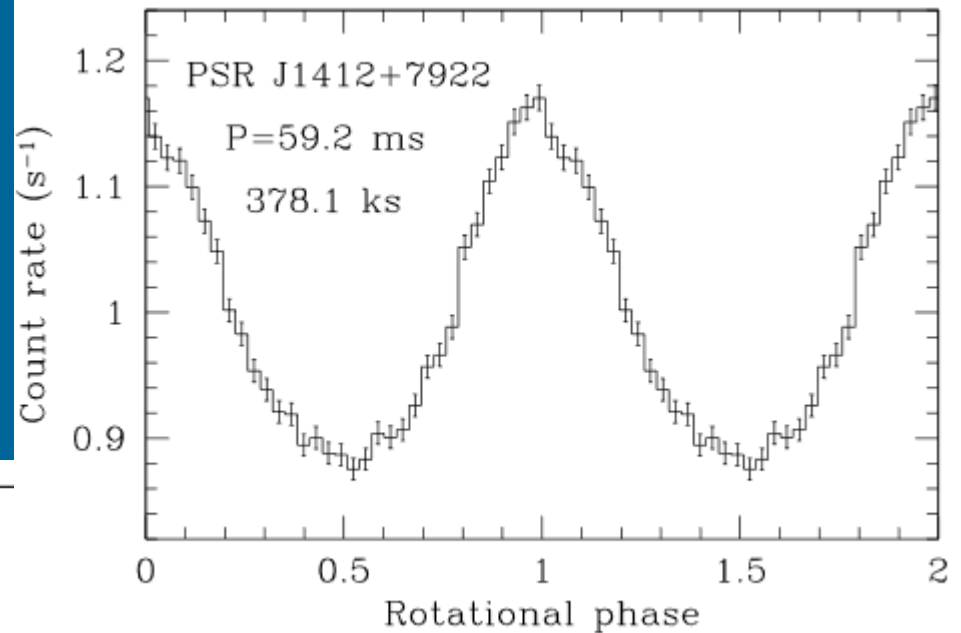


# Excess at $>1$ keV?

Analysis of spectra of M7 demonstrated a strange excess at energies  $> 1$  keV. This is somehow similar to what magnetars demonstrate.

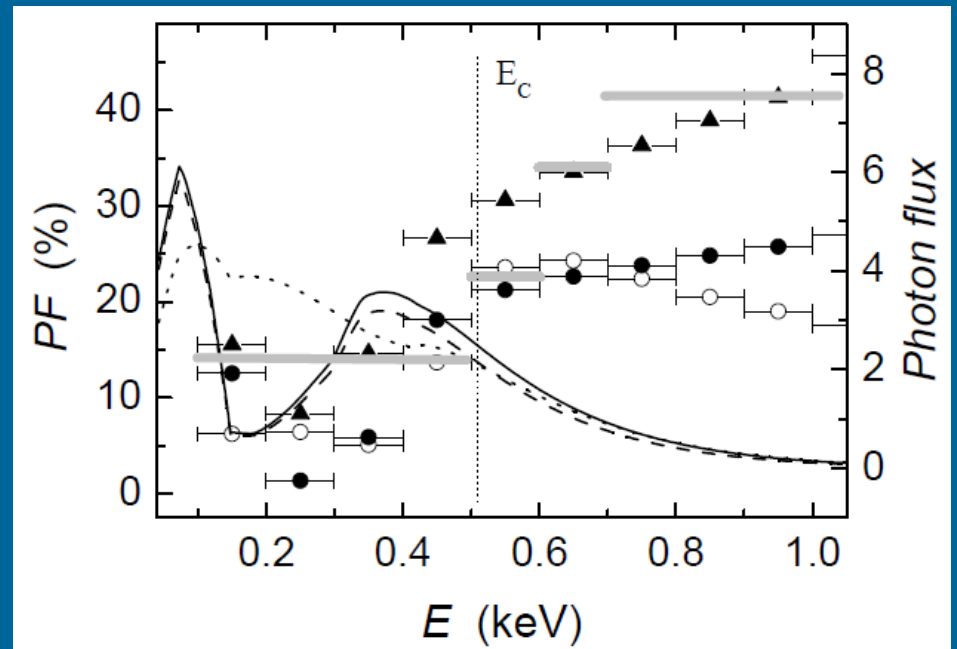
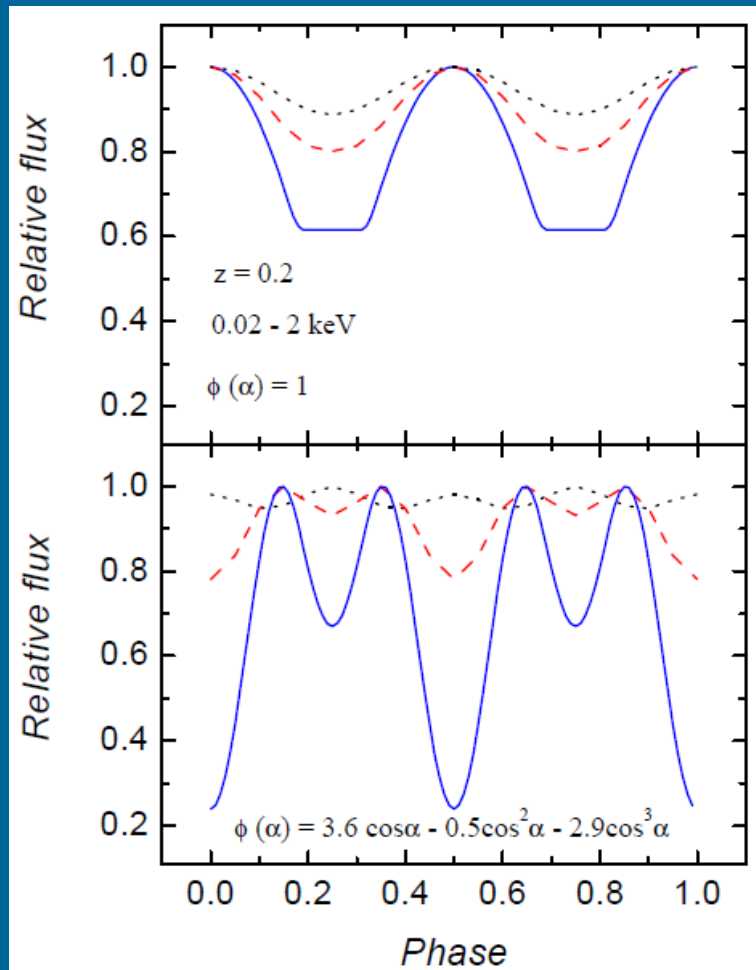


# Calvera spectrum



$kT_1$ (keV)	$0.154 \pm 0.004$
$R_1$ (km) <sup>b</sup>	$2.21^{+0.08}_{-0.07}$
$kT_2$ (keV)	$0.319^{+0.013}_{-0.012}$
$R_2$ (km) <sup>b</sup>	$0.37 \pm 0.04$

# Light curves and pulsed fraction



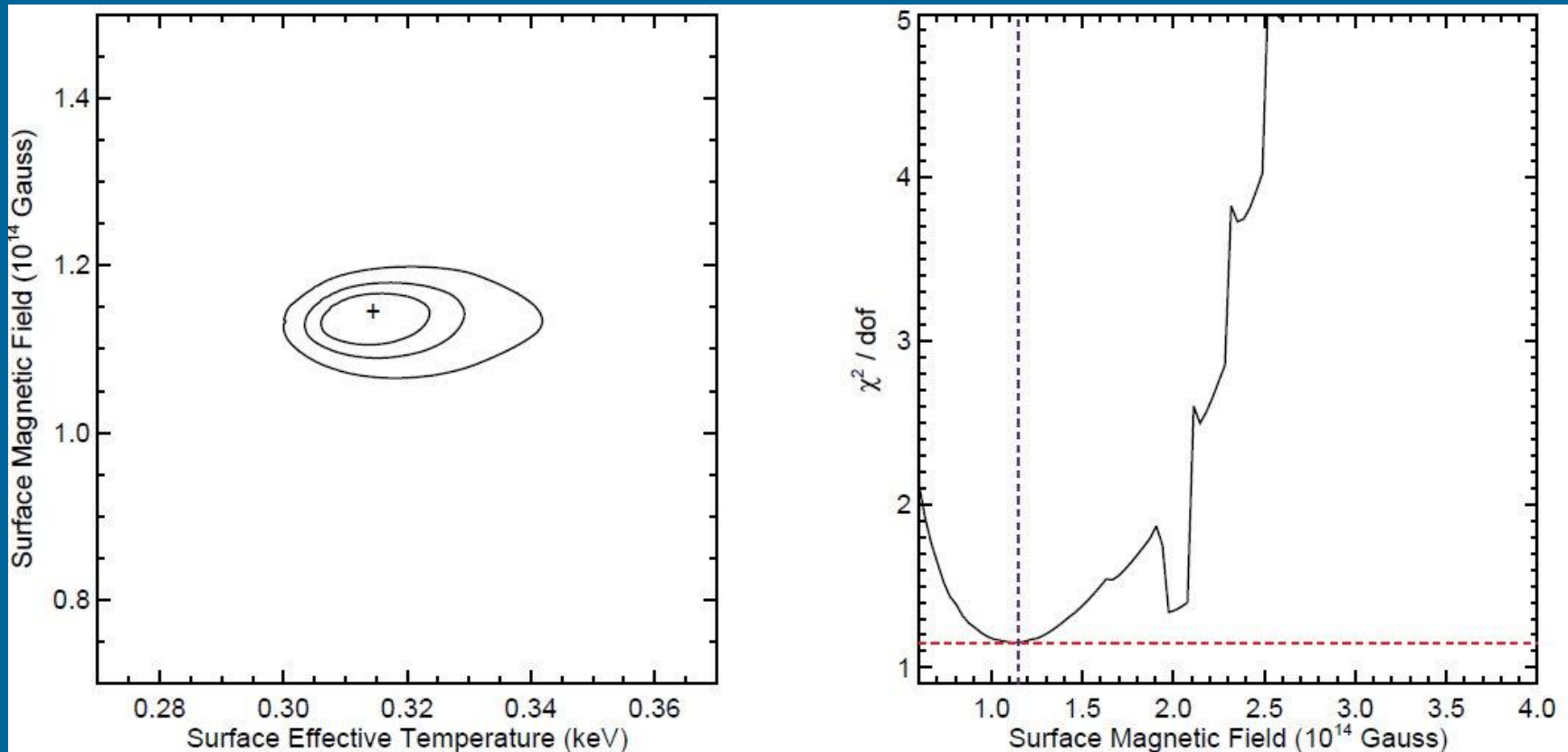
1010.0125

1006.3292

# Low-field magnetar SGR 0418+5729

Fitting parameters of the magnetized atmosphere it is possible to show, that the low-field solution is not acceptable.

This can be due to non-dipolar field components.



# Conclusions

- Emission from cooling NSs is more complicated than a simple blackbody
- Light bending (gravity)
- Atmospheres
- Magnetic field distribution - effects on properties of atmospheres and emission
- Magnetic field (including toroidal) in the crust – non-uniform temp.distr.
- Condensate
- Rotation at  $\sim$ msec periods can smear spectral lines

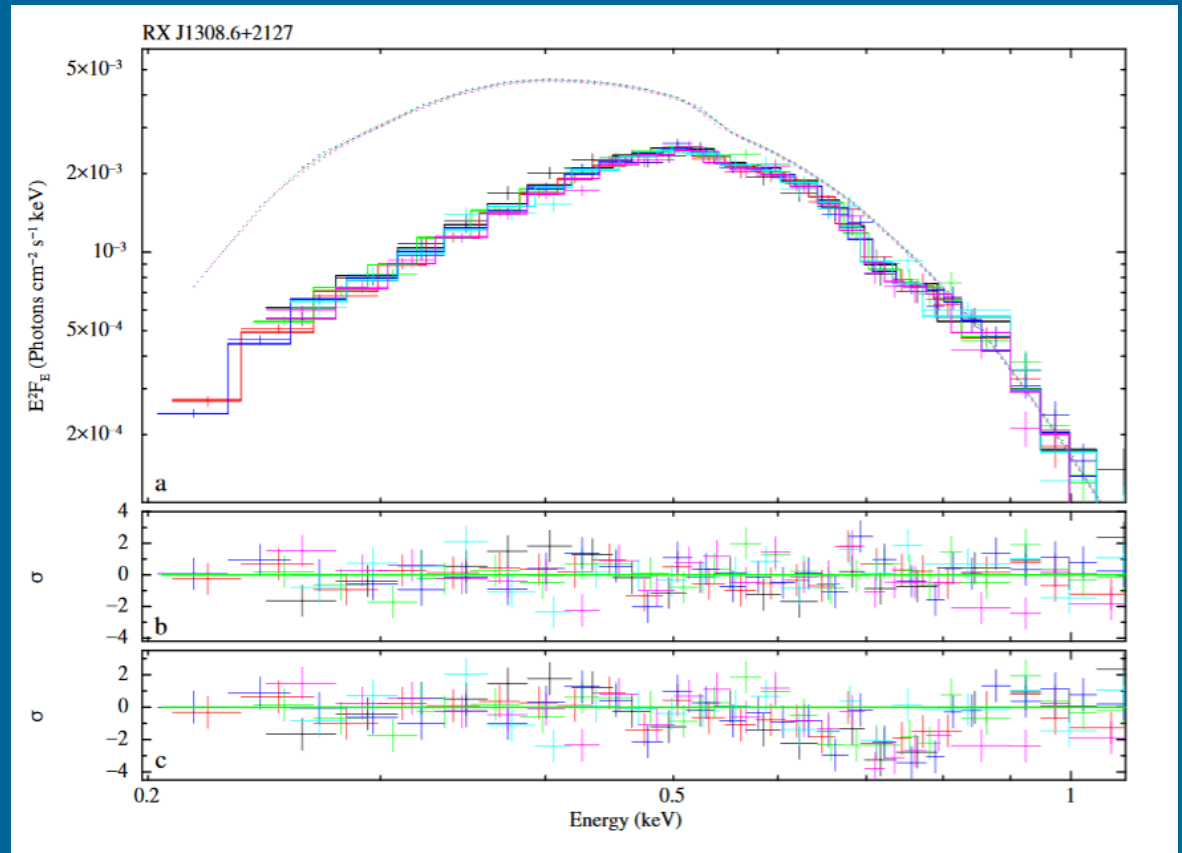
# Papers to read

- [astro-ph/0702426](#) ←
- [arXiv: 0801.1143](#)  
or [astro-ph/0609066](#) } Reviews on the M7
- [astro-ph/0206025](#) ←
- [arXiv: 0905.3276](#) } Recent calculations of spectra from magnetized atmos.
- [arXiv: 1006.3292](#) }
- [arXiv: 1210.0916](#) – review
- [arXiv: 1409.7666](#) - review

# Phase-resolved spectra and features

RX J1308.6+2127

A feature at the energy of  $\sim 740$  eV  
and an equivalent width of  $\sim 15$  eV



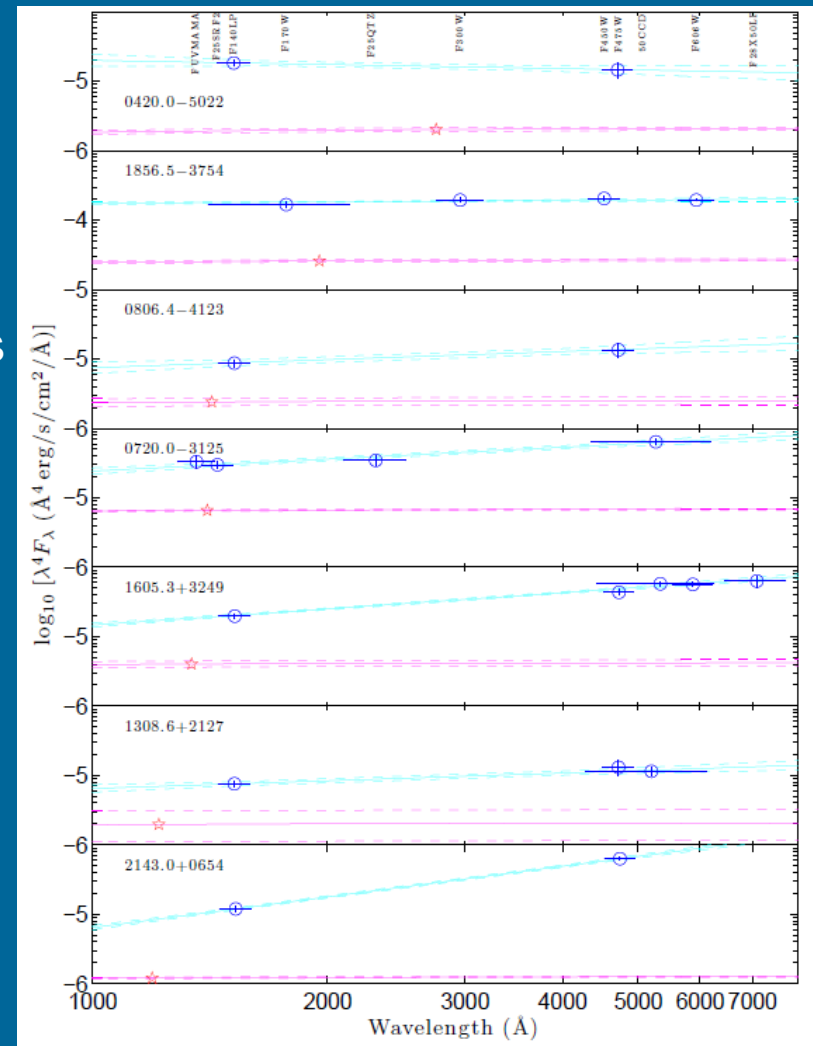


# All in optics and UV

All seven objects have confirmed optical and ultraviolet counterparts.

The Rayleigh-Jeans tail would be flat.  
The best-fit power-laws with  $\pm 1\sigma$  uncertainties are shown by the cyan lines.  
The extrapolations of the X-ray blackbodies with  $\pm 1\sigma$  uncertainties are shown by the magenta lines.

$kT \rightarrow$



New data: Kaplan et al. 1105.4178

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Does the atmosphere keep the star surface temperature ?

